

GENERAL FORMULA TO DEDUCE THE SPACE CHARGE TUNE SPREAD FROM A QUADROPOLAR PICK-UP MEASUREMENT

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Abstract

- ◆ Wish to measure the incoherent direct space charge tune spread in the CERN LHC injectors with quadrupolar pick-ups
- ◆ CERN LHC injectors, as other machines, are running close to the coupling resonance where the usual formula should not work as it neglects the coupling induced by space charge between transverse planes
- ◆ A new formula, which should be valid also close to the coupling resonance, is presented and some examples are shown => Differences up to a factor ~ 1.5-2 can be observed
- ◆ Next step should be to check it with simulations and beam-based measurements

◆ Coupled equations to be solved

(starting from the 2D transverse envelope equations, with small perturbations on top of equilibrium beam sizes)

$$\frac{d^2 \Delta a}{d\phi^2} + Q_a^2 \Delta a = K R^2 \Delta b$$

$$\frac{d^2 \Delta b}{d\phi^2} + Q_b^2 \Delta b = K R^2 \Delta a$$

$$Q_a = 2Q_{x0} + \Delta Q_a = 2Q_{x0} - \frac{K_{sc} R^2 (2a_0 + 3b_0)}{2Q_{x0} a_0 (a_0 + b_0)^2}$$

$$Q_b = 2Q_{y0} + \Delta Q_b = 2Q_{y0} - \frac{K_{sc} R^2 (2b_0 + 3a_0)}{2Q_{y0} b_0 (a_0 + b_0)^2}$$

$$\Delta Q_{x, \text{spread}}^{SC} = -\Delta Q_{x, \text{linear shift}}^{SC} = \frac{K_{sc} R^2}{Q_{x0} a_0 (a_0 + b_0)}$$

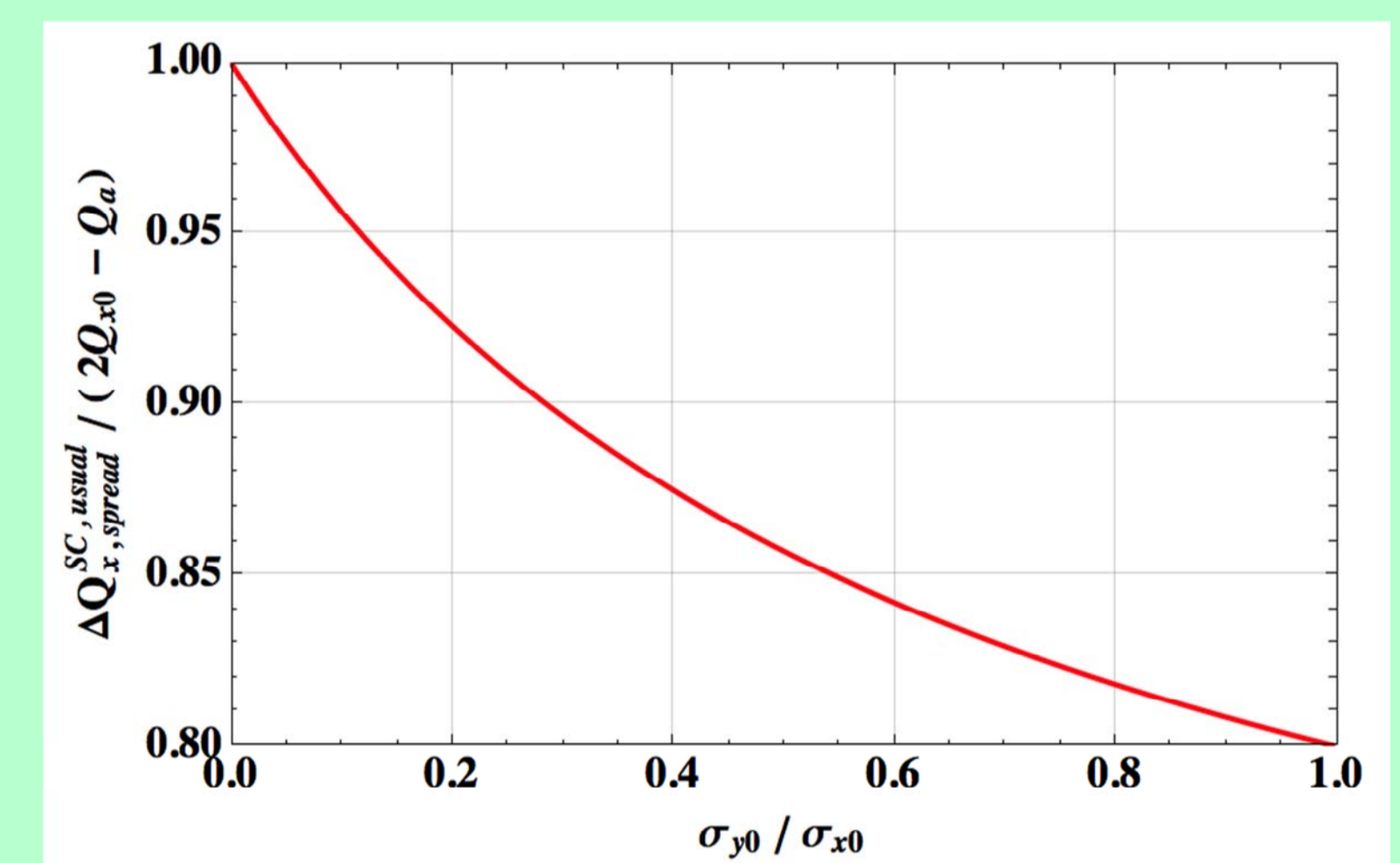
$$K = \frac{2K_{sc}}{(a_0 + b_0)^2}$$

◆ Far from the coupling resonance induced by space charge

$$\Delta a = \Delta a_0 e^{jQ_a \phi}$$

$$\Delta b = \Delta b_0 e^{jQ_b \phi}$$

$$\Delta Q_{x, \text{spread}}^{SC} = \frac{2Q_{x0} - Q_a}{\frac{1}{2} \left(3 - \frac{1}{1 + \frac{\sigma_{y0}}{\sigma_{x0}}} \right)}$$



◆ Close to the coupling resonance induced by space charge

$$\Delta Q_{x, \text{spread}}^{SC, \text{new}}(x, y, q) = \frac{1}{6 + 9x + 6x^2} \left[q(3 + 7x + 7x^2 + 3x^3) + 4xy + 10x^2y + 6x^3y \mp (1+x) \sqrt{q^2(9 - 2x^2 + 9x^4) + 4qx(-6 - x + 6x^2 + 9x^3)y + 4x^2(2 + 3x)^2 y^2} \right]$$

$$x = \frac{\sigma_{y0}}{\sigma_{x0}}$$

$$y = Q_{y0} - Q_{x0}$$

$$q = 2Q_{x0} - Q_u$$

◆ Some examples

