PBL SCENARIO ON ACCELERATORS: SUMMARY

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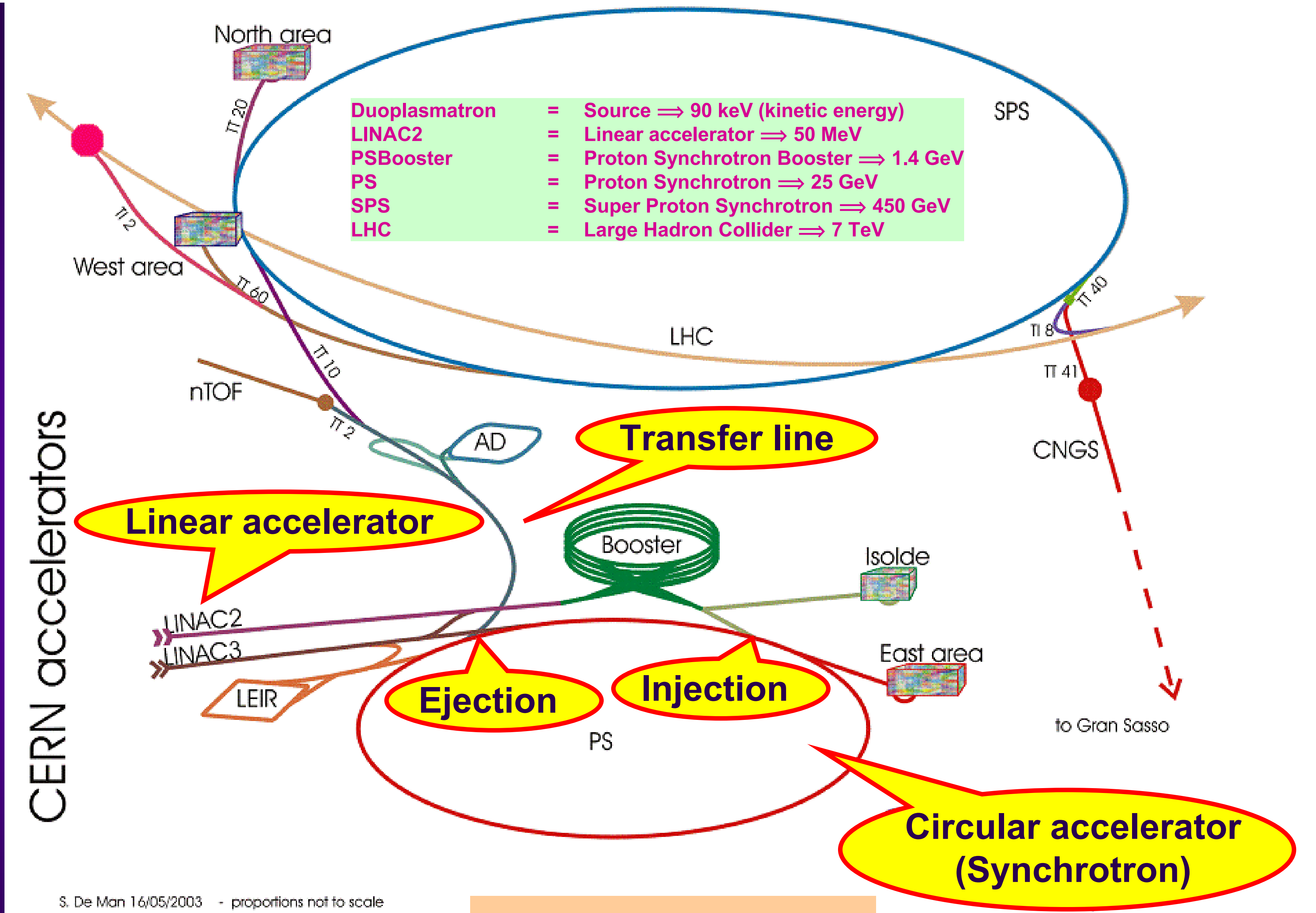
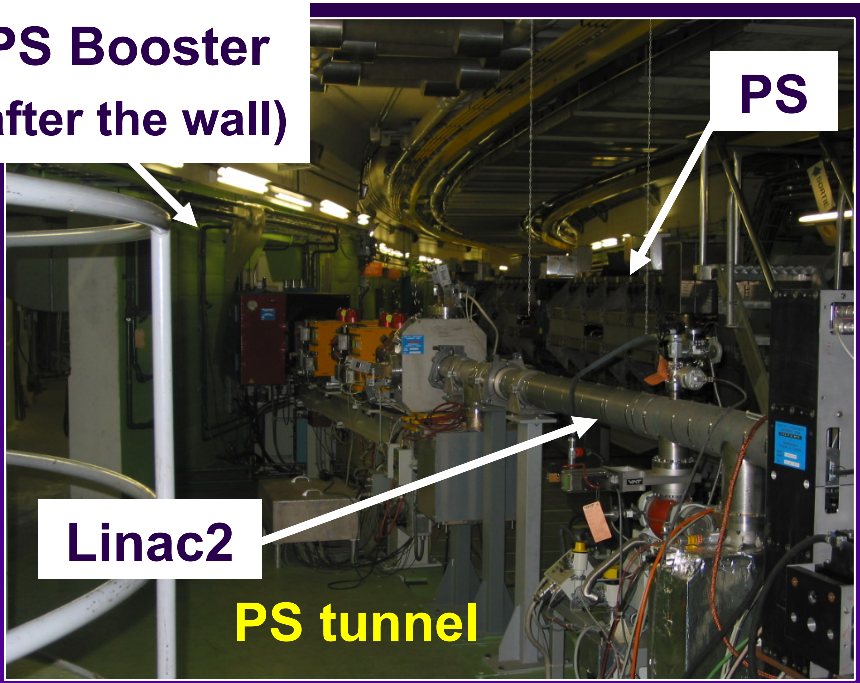
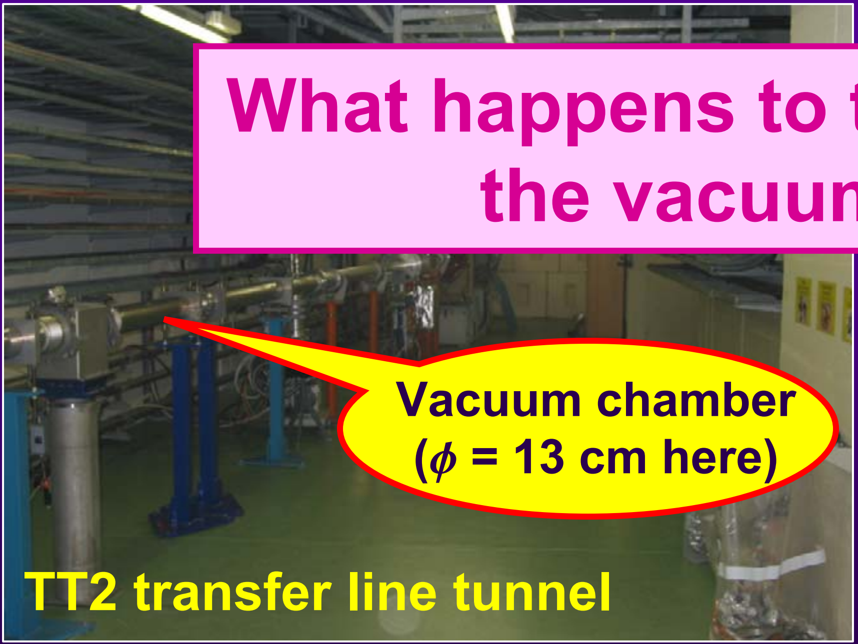
- **CERN accelerators and CERN Control Centre**
- **Machine luminosity**
- **Transverse beam dynamics + space charge**
- **Longitudinal beam dynamics**
- **Solution of the transverse problem**
- **Solution of the longitudinal problem**
- **Synchrotron radiation**

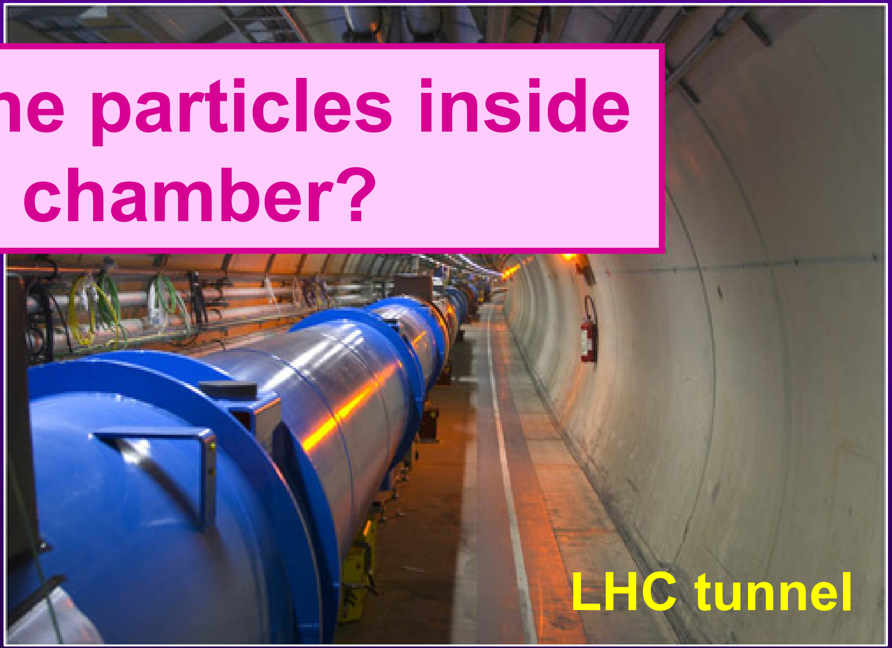

North area
SPS
Duoplasmatron = Source ⟹ 90 keV (kinetic energy)
LINAC2 = Linear accelerator ⟹ 50 MeV
PSBooster = Proton Synchrotron Booster ⟹ 1.4 GeV
PS = Proton Synchrotron ⟹ 25 GeV
SPS = Super Proton Synchrotron ⟹ 450 GeV
LHC = Large Hadron Collider ⟹ 7 TeV
TT 20
TI 2
West area
TT 60
LHC
TT 40
TI 8
TT 41
TT 10
nTOF
TT 2
AD
Transfer line
CNGS
CERN accelerators
Linear accelerator
Booster
Isolde
LINAC2
LINAC3
LEIR
Ejection
Injection
East area
PS
to Gran Sasso
Circular accelerator (Synchrotron)
S. De Man 16/05/2003 - proportions not to scale

LHC beam



What happens to the particles inside the vacuum chamber?





CERN Control Centre (CCC)

MACHINE LUMINOSITY (1/3)

Book p. 162 + Ref. [5]

$$L = \frac{N_{events/second}}{\sigma_{event}}$$

Number of events per second generated in the collisions

Cross-section for the event under study

[cm^{-2} s^{-1}]

The Luminosity depends only on the beam parameters ⇒ It is independent of the physical reaction

MACHINE LUMINOSITY (2/3)

⇒ **For a Gaussian (round) beam distribution**

$$L = \frac{N_b^2 \, M \, f_{rev} \, \gamma_r}{4 \, \pi \, \varepsilon_n \, \beta^*} \, F$$

- Number of particles per bunch
- Number of bunches per beam
- Revolution frequency
- Relativistic velocity factor
- Normalized transverse beam emittance
- *β*-function at the collision point
- Geometric reduction factor due to the crossing angle at the IP

- **PEAK LUMINOSITY for ATLAS&CMS in the LHC =** $L_{peak} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

MACHINE LUMINOSITY (3/3)

- **INTEGRATED LUMINOSITY** $$L_{\text{int}} = \int_0^T L(t)\, dt$$

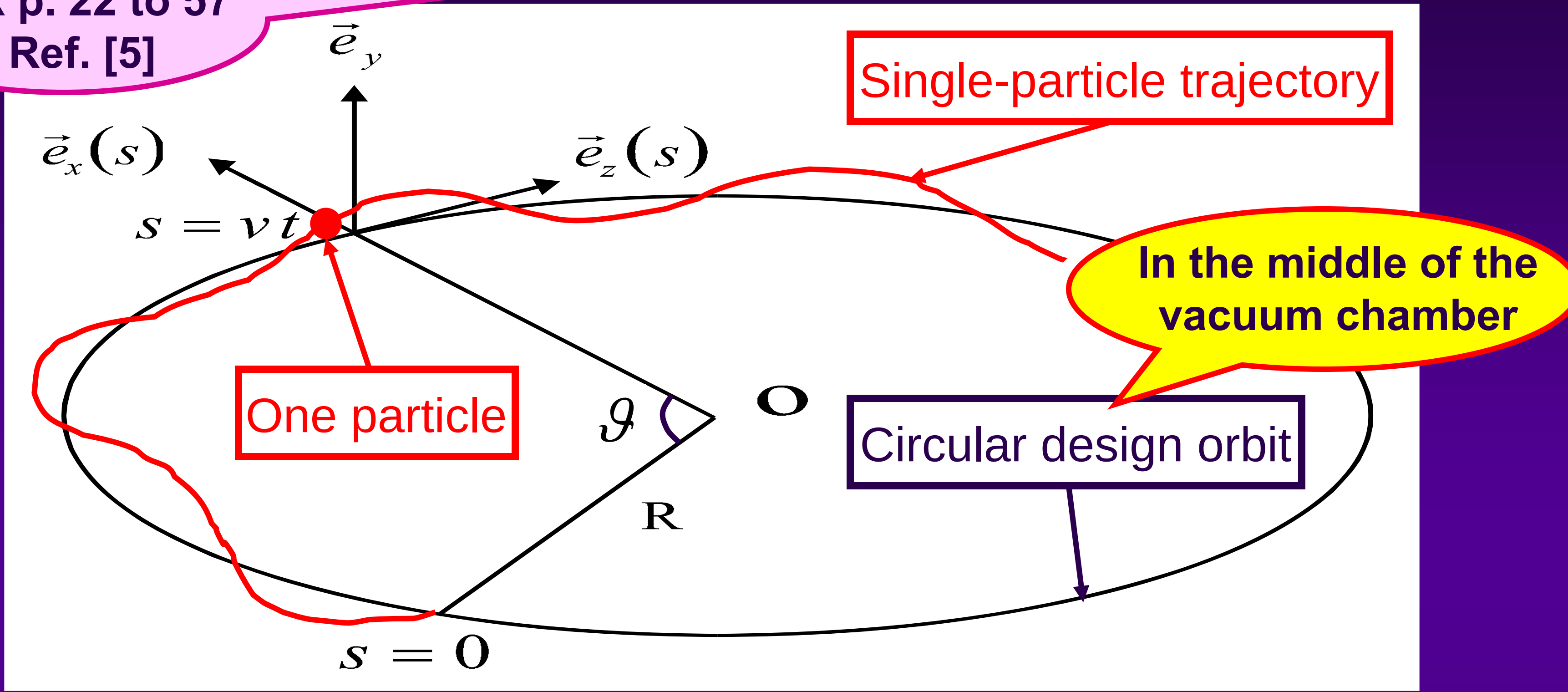
  $\Rightarrow$ **The real figure of merit =** $L_{\text{int}}\ \sigma_{event} = \text{number of events}$

- **LHC integrated Luminosity expected per year: [80-120] fb^{-1}**

Reminder: 1 barn = 10^{-24} cm^2 and femto = 10^{-15}

TRANSVERSE BEAM DYNAMICS (1/16)

Book p. 22 to 57 + Ref. [5]



- **The motion of a charged particle (proton) in a beam transport channel or a circular accelerator is governed by the LORENTZ FORCE**

$$\vec{F} = e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- **The motion of particle beams under the influence of the Lorentz force is called BEAM OPTICS**

TRANSVERSE BEAM DYNAMICS (2/16)

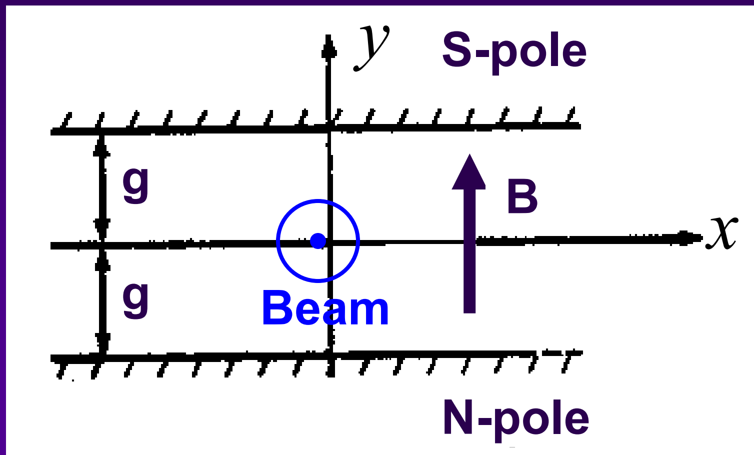
- The Lorentz force is applied as a
  - BENDING FORCE (using DIPOLES) to guide the particles along a predefined ideal path, the DESIGN ORBIT, on which – ideally – all particles should move
  - FOCUSING FORCE (using QUADRUPOLES) to confine the particles in the vicinity of the ideal path, from which most particles will unavoidably deviate

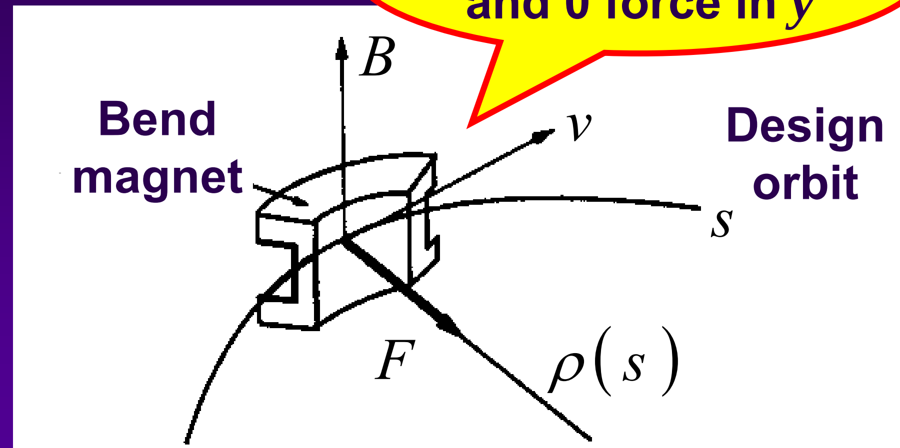
- LATTICE = Arrangement of magnets along the design orbit

- The ACCELERATOR DESIGN is made considering the beam as a collection of non-interacting single particles

DIPOLE = Bending magnet

Constant force in x and 0 force in y





⇒ **A particle, with a constant energy, describes a circle in equilibrium between the centripetal magnetic force and the centrifugal force**

- **BEAM RIGIDITY**

$$B\,\rho\,[\mathrm{T\,m}] = 3.3356\;p_0\,[\mathrm{GeV/c}]$$

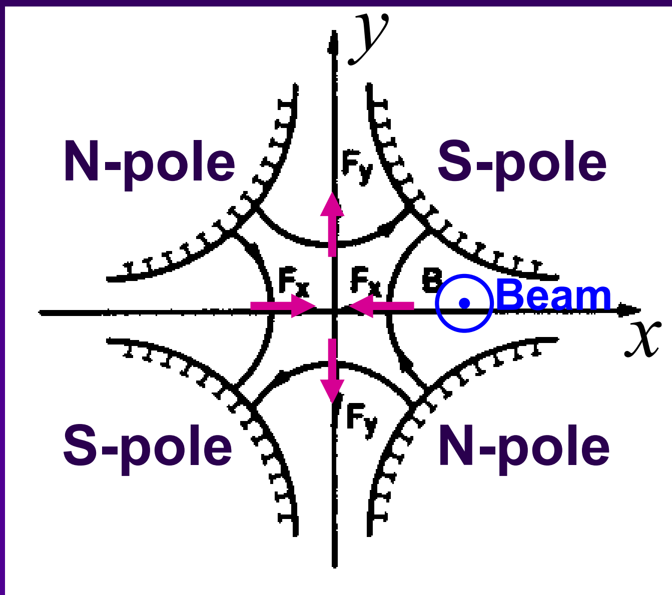
Magnetic field

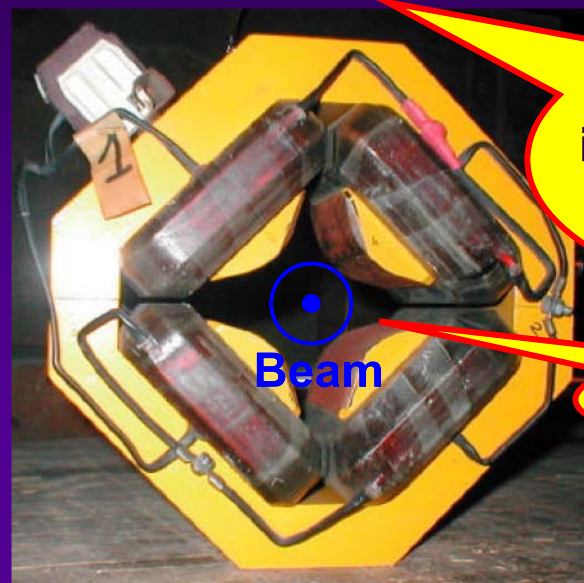
Curvature radius of the dipoles

Beam momentum

TRANSVERSE BEAM DYNAMICS (4/16)

QUADRUPOLE = Focusing magnet





In x (and Defocusing in y) ⇒ F-type. Permuting the N- and S- poles gives a D-type

Linear force in x&y

$$\Rightarrow \quad x''(s) + K\,x(s) = 0$$: **Equation of a harmonic oscillator**

- **From this equation, one can already anticipate the elliptical shape of the particle trajectory in the phase space (x, x') by integration**

$$x'^2(s) + K\,x^2(s) = \text{Constant}$$

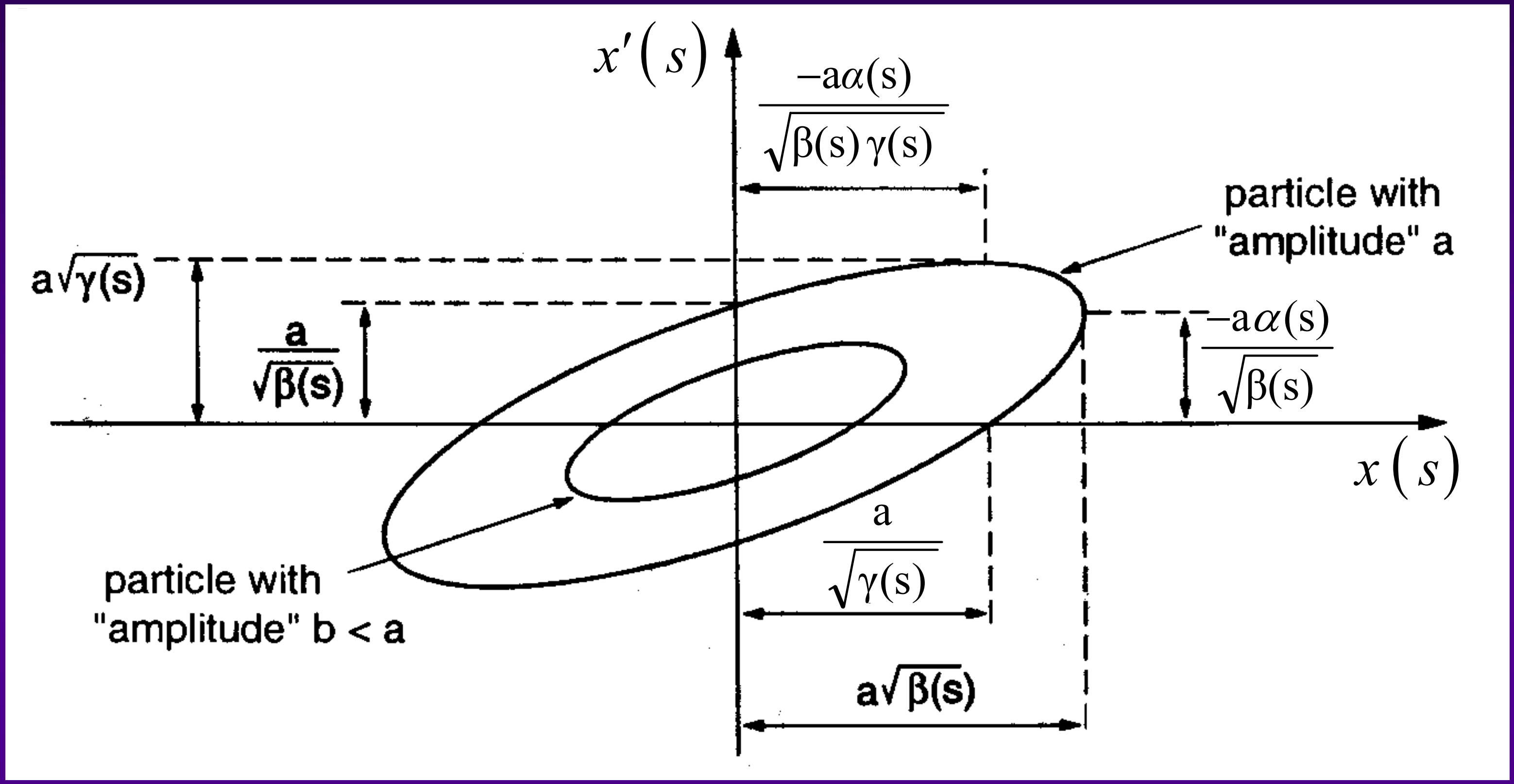
TRANSVERSE BEAM DYNAMICS (5/16)

- **Along the accelerator K is not constant and depends on s (and is periodic) $\Rightarrow$ The equation of motion is then called HILL'S EQUATION**

- **The solution of the Hill's equation is a pseudo-harmonic oscillation with varying amplitude and frequency called BETATRON OSCILLATION**

- **An invariant, i.e. a constant of motion, (called COURANT-SNYDER INVARIANT) can be found from the solution of the Hill's equation**

  $\Rightarrow$ Equation of an ellipse (motion for one particle) in the phase space plane (x, x'), with area $\pi\, a^2$

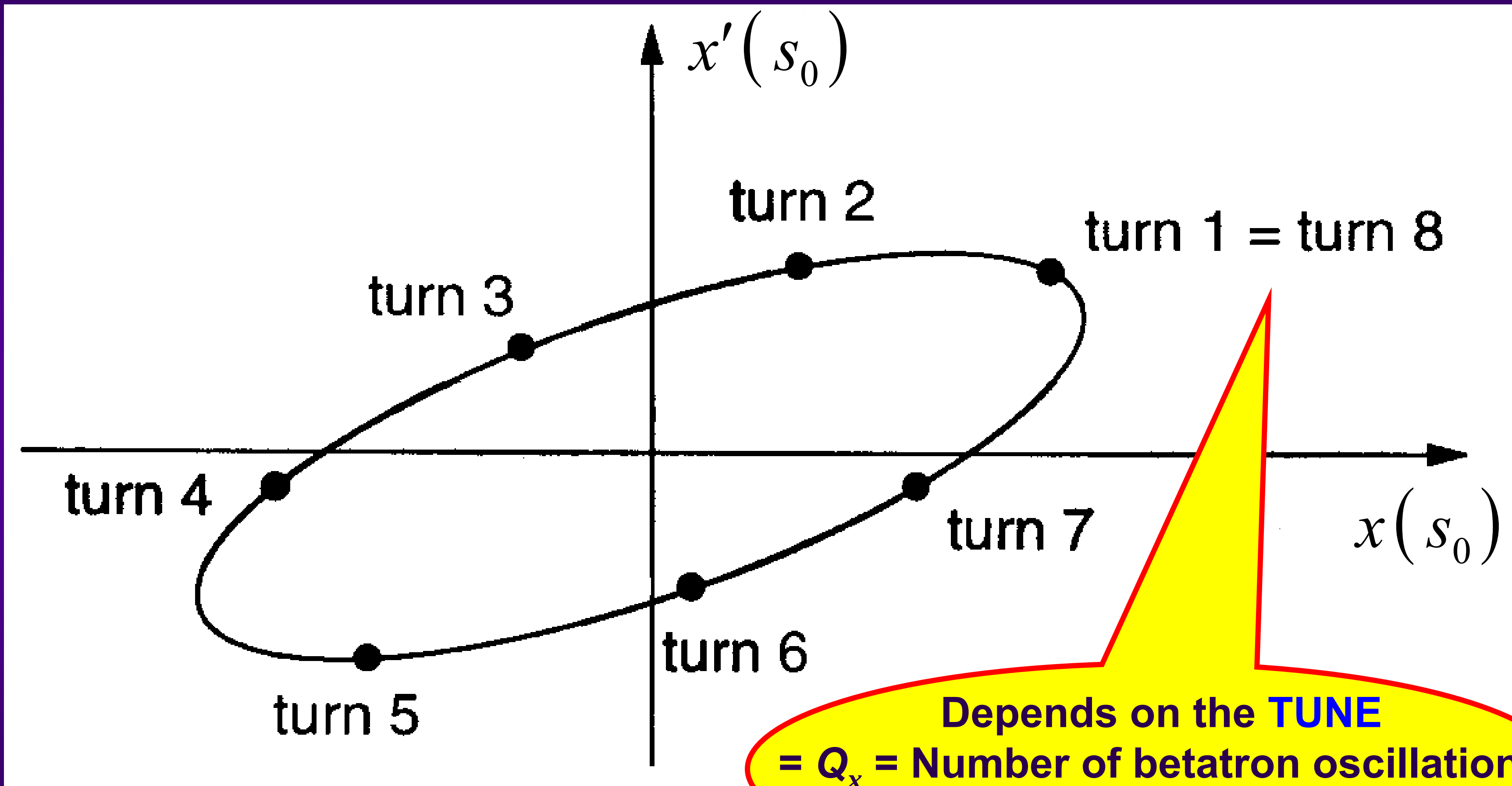


$\alpha(s)$ $\beta(s)$ $\gamma(s)$ **are called TWISS PARAMETERS**

- **The shape and orientation of the phase plane ellipse evolve along the machine, but not its area**

TRANSVERSE BEAM DYNAMICS (7/16)

Stroboscopic representation or POINCARÉ MAPPING



Depends on the TUNE
= Q_x = Number of betatron oscillations per machine revolution

TRANSVERSE BEAM DYNAMICS (8/16)

- **MATRIX FORMALISM: The previous (linear) equipments of the accelerator (extending from s_0 to s) can be described by a matrix, $M\left(s\,/\,s_0\right)$, called TRANSFER MATRIX, which relates (x, x') at s_0 and (x, x') at s**

$$\begin{bmatrix} x\left(s\right) \\ x'\left(s\right) \end{bmatrix} = M\left(s\,/\,s_0\right) \begin{bmatrix} x\left(s_0\right) \\ x'\left(s_0\right) \end{bmatrix}$$

- **The transfer matrix over one revolution period is then the product of the individual matrices composing the machine**

- **The transfer matrix over one period is called the TWISS MATRIX**

- **Once the Twiss matrix has been derived the Twiss parameters can be obtained at any point along the machine**

Book p. 62 to 67
+ Ref. [5]

- **In practice, particle beams have a finite dispersion of momenta about the ideal momentum p_0. A particle with momentum $p \neq p_0$ will perform betatron oscillations around A DIFFERENT CLOSED ORBIT from that of the reference particle**

⇒ Displacement of

$$x_\Delta(s) = D_x(s)\frac{p - p_0}{p_0} = D_x(s)\frac{\Delta p}{p_0}$$

$D_x(s)$ **is called the DISPERSION FUNCTION**

TRANSVERSE BEAM DYNAMICS (10/16)

- **BEAM EMITTANCE = Measure of the spread in phase space of the points representing beam particles ⇒ 3 definitions**

  1) **In terms of the phase plane "amplitude" a_q enclosing q % of the particles**

$$\iint_{\substack{\text{ellipse of} \\ \text{"amplitude" } a_q}} dx\, dx' = \pi\, \varepsilon_x^{(q\%)}$$

[mm mrad] or [µm]

  2) **In terms of the 2nd moments of the particle distribution**

$$\varepsilon_x^{(\text{stat})} \equiv \sqrt{<x^2><x'^2> - <x\,x'>^2}$$

Determinant of the covariance matrix

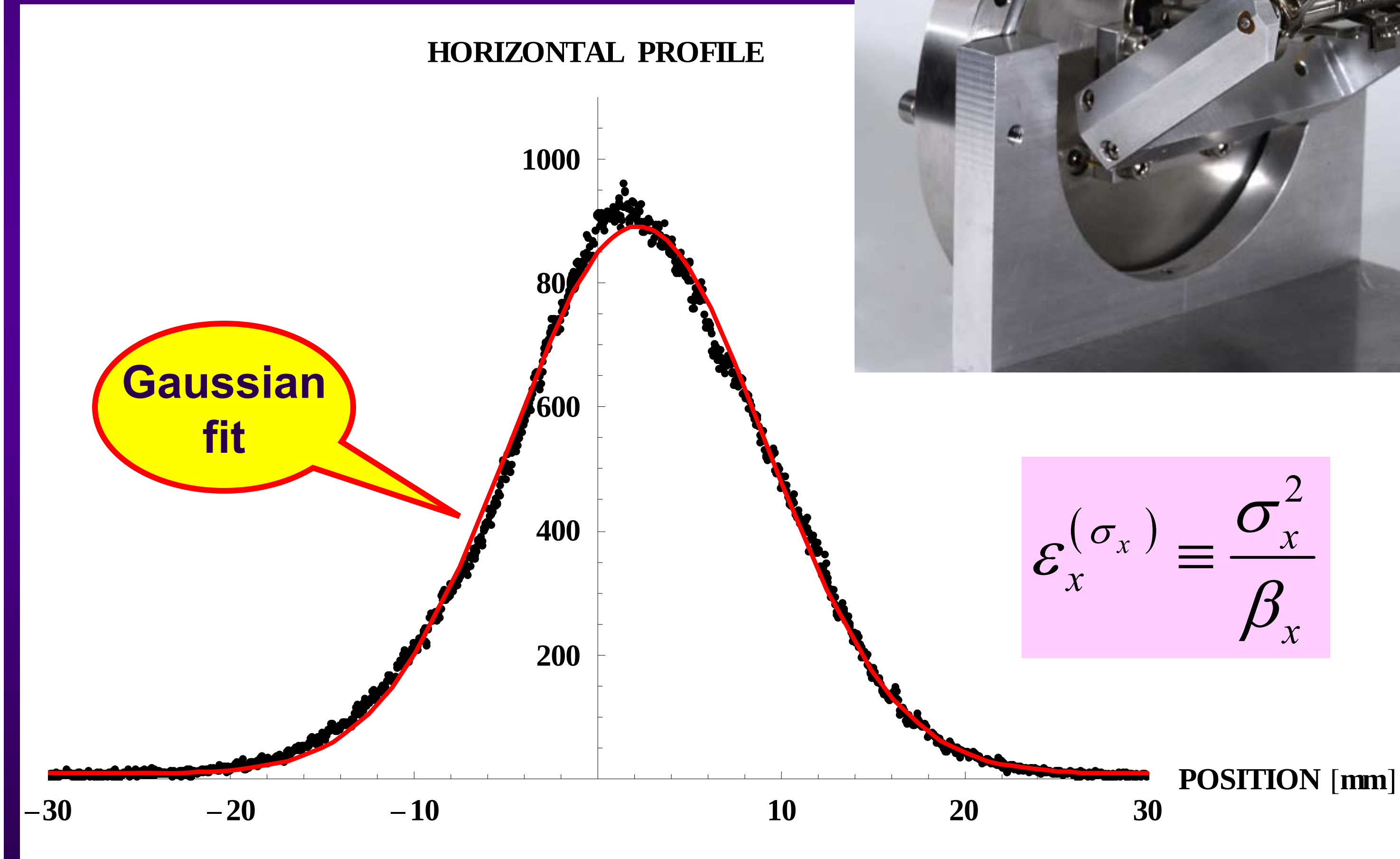
  3) **In terms of σ_x the standard deviation of the particle distribution in real space (= projection onto the x-axis)**

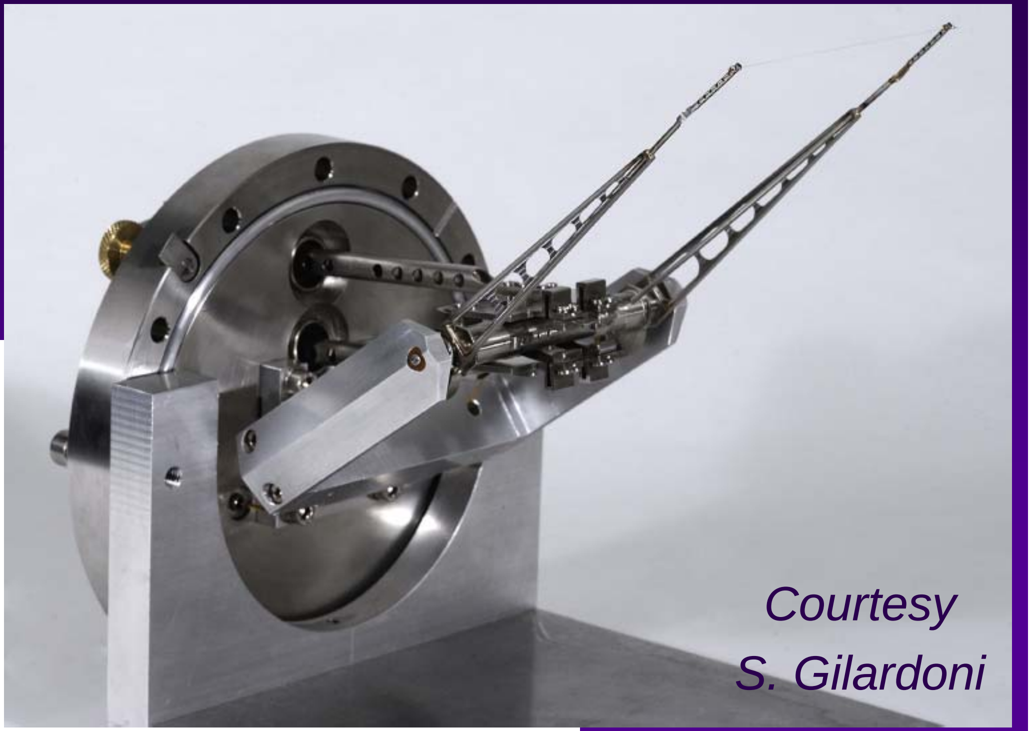
$$\varepsilon_x^{(\sigma_x)} \equiv \frac{\sigma_x^2}{\beta_x}$$

TRANSVERSE BEAM DYNAMICS (11/16)

◆ **FAST WIRE SCANNER**

⇒ **Measures the transverse beam profiles by detecting the particles scattered from a thin wire swept rapidly through the beam**

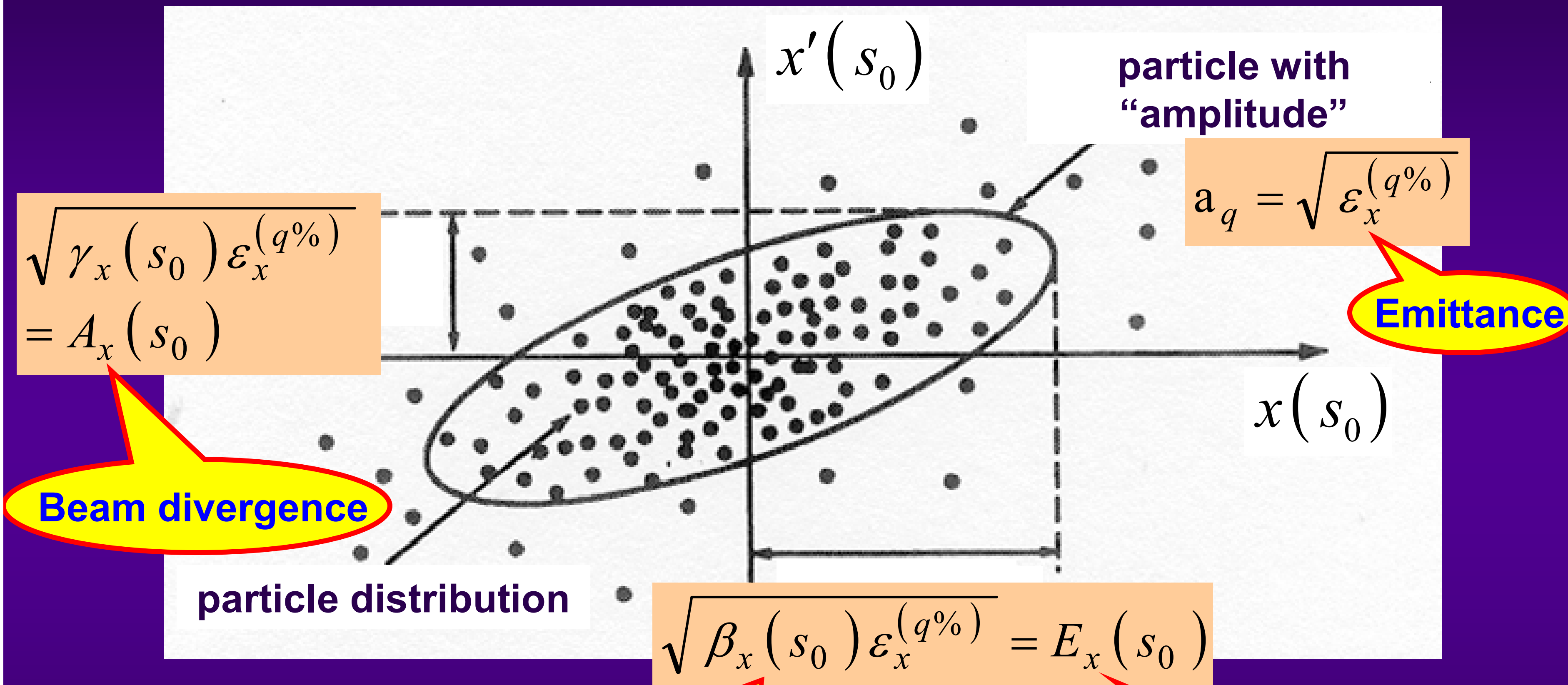




Courtesy S. Gilardoni

$$\varepsilon_x^{(\sigma_x)} \equiv \frac{\sigma_x^2}{\beta_x}$$

TRANSVERSE BEAM DYNAMICS (12/16)



The *β*-function reflects the size of the beam and depends only on the lattice

TRANSVERSE BEAM DYNAMICS (13/16)

- **MACHINE mechanical (i.e. from the vacuum chamber) ACCEPTANCE or APERTURE = Maximum beam emittance**

- **NORMALIZED BEAM EMITTANCE**

Relativistic factors

$$\varepsilon_{x,norm}^{(\sigma_x)} = \beta_r \, \gamma_r \, \varepsilon_x^{(\sigma_x)}$$

$\Rightarrow$ **The normalized emittance is conserved during acceleration (in the absence of collective effects…)**

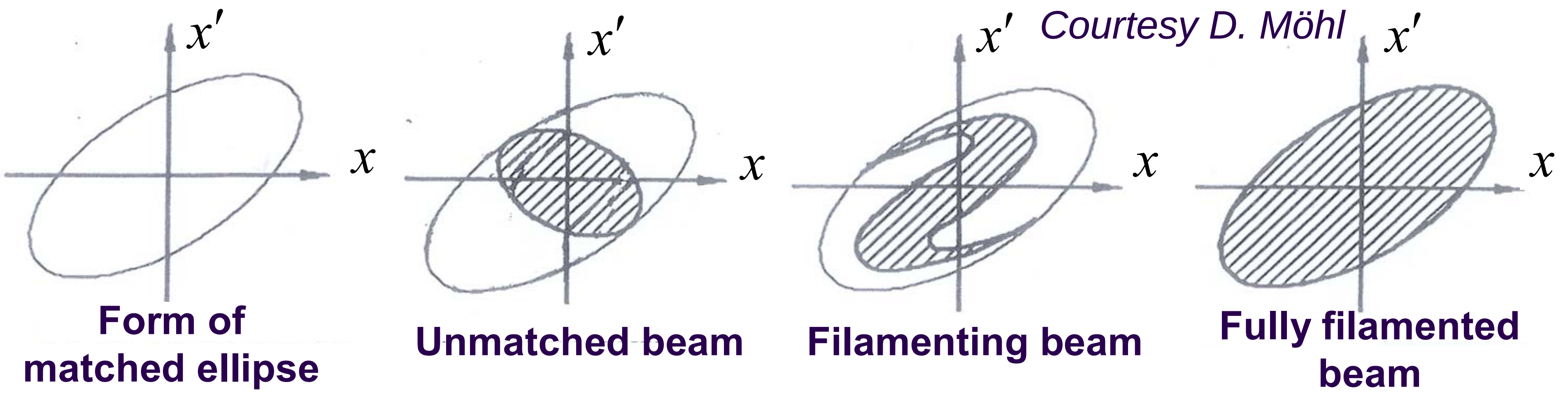
- **ADIABATIC DAMPING: As $\beta_r \, \gamma_r$ increases proportionally to the particle momentum *p*, the (physical) emittance decreases as 1 / *p***

- **However, many phenomena may affect (increase) the emittance**

- **An important challenge in accelerator technology is to preserve beam emittance and even to reduce it (by COOLING)**

TRANSVERSE BEAM DYNAMICS (14/16)

- **BETATRON MATCHING = The phase space ellipses at the injection (ejection) point of the circular machine, and the exit (entrance) of the beam transport line, should be homothetic. To do this, the Twiss parameters are modified using quadrupoles. If the ellipses are not homothetic, there will be a dilution (i.e. a BLOW-UP) of the emittance**



- **DISPERSION MATCHING = D_x and D'_x should be the same at the injection (ejection) point of the circular machine, and the exit (entrance) of the beam transport line. If there are different, there will be also a BLOW-UP, but due to a missteering (because the beam is not injected on the right orbit)**

- **In the presence of extra (NONLINEAR) FORCES, the Hill's equation takes the general form**

$$x''(s) + K_x(s)\,x(s) = P_x(x, y, s)$$

Any perturbation

- **Perturbation terms in the equation of motion may lead to UNSTABLE motion, called RESONANCES, when the perturbating field acts in synchronism with the particle oscillations**

TRANSVERSE BEAM DYNAMICS (16/16)

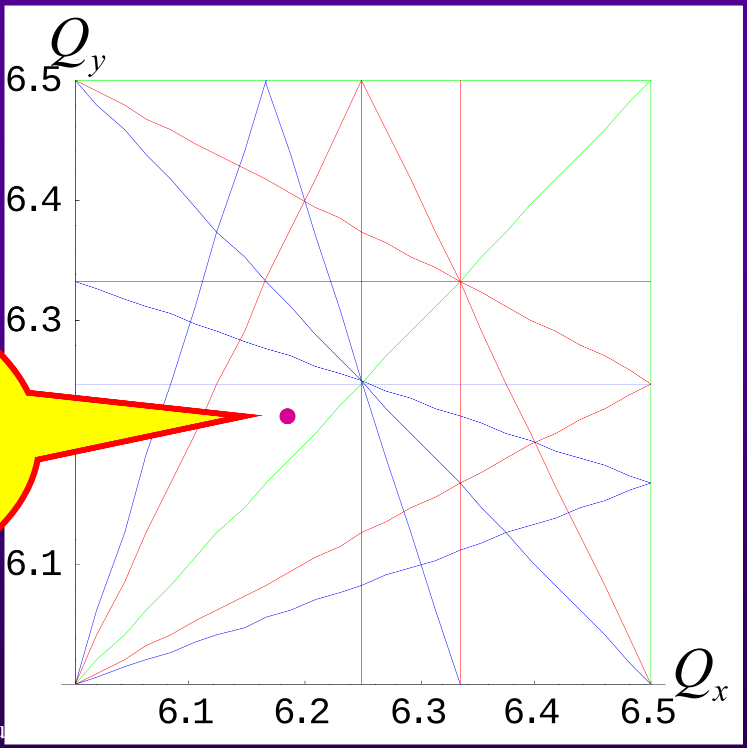
Book p. 88-89 + Ref. [5]

- **General RESONANCE CONDITIONS** $M\,Q_x + N\,Q_y = P$

  where *M*, *N* and *P* are integers, *P* being non-negative, |M| + |N| is the order of the resonance and *P* is the order of the perturbation harmonic

- **Plotting the resonance lines for different values of *M*, *N*, and *P* in the (Q_x, Q_y) plane yields the so-called RESONANCE or TUNE DIAGRAM**

This dot in the tune diagram is called the WORKING POINT (case of the PS, here)



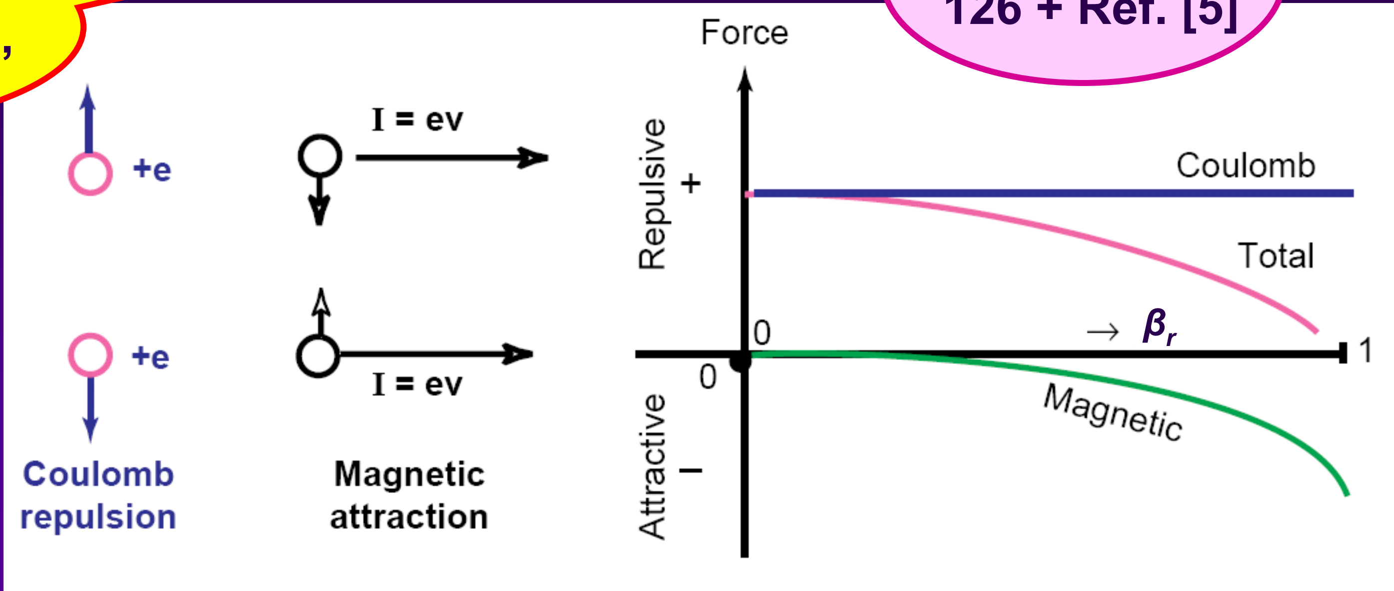
- **Each line has a finite width, proportional to the strength of the imperfection which drives it**
- **The dot is in fact not a dot because all the particles do not have exactly the same tune ⟹ There is a tune spread**

SPACE CHARGE (1/4)

Very important effect for LINAC2, PSB and PS!

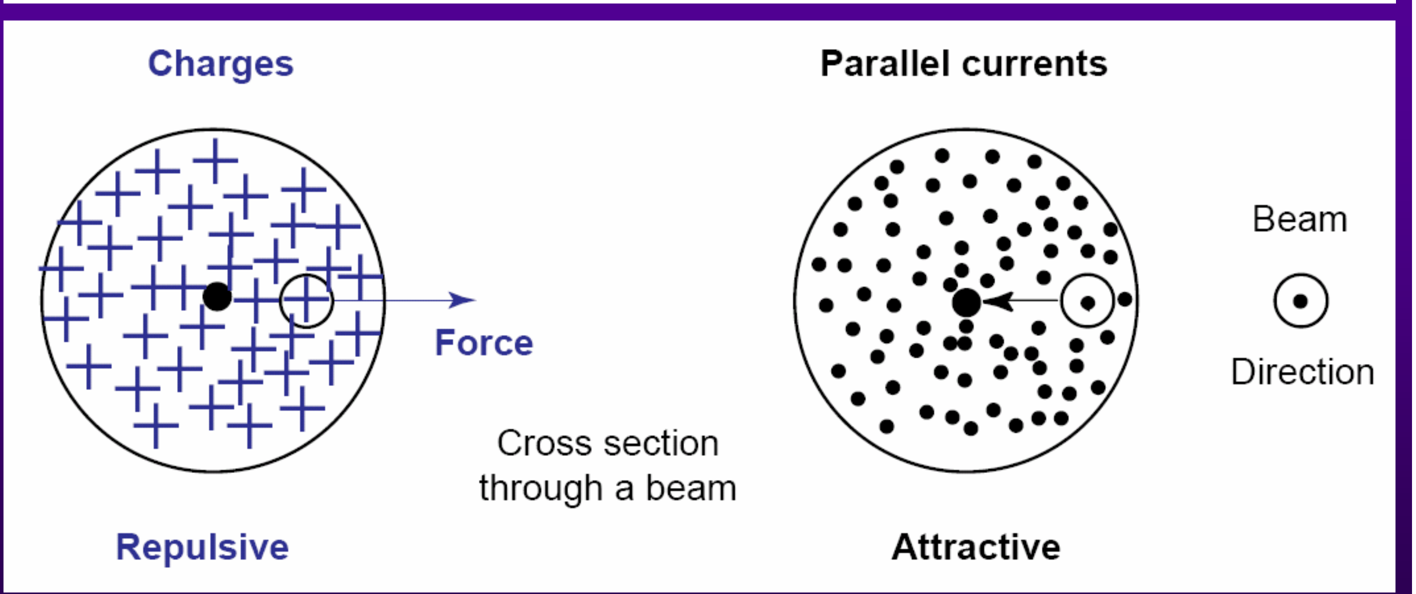
Book p. 124 to 126 + Ref. [5]

2 particles at rest or travelling

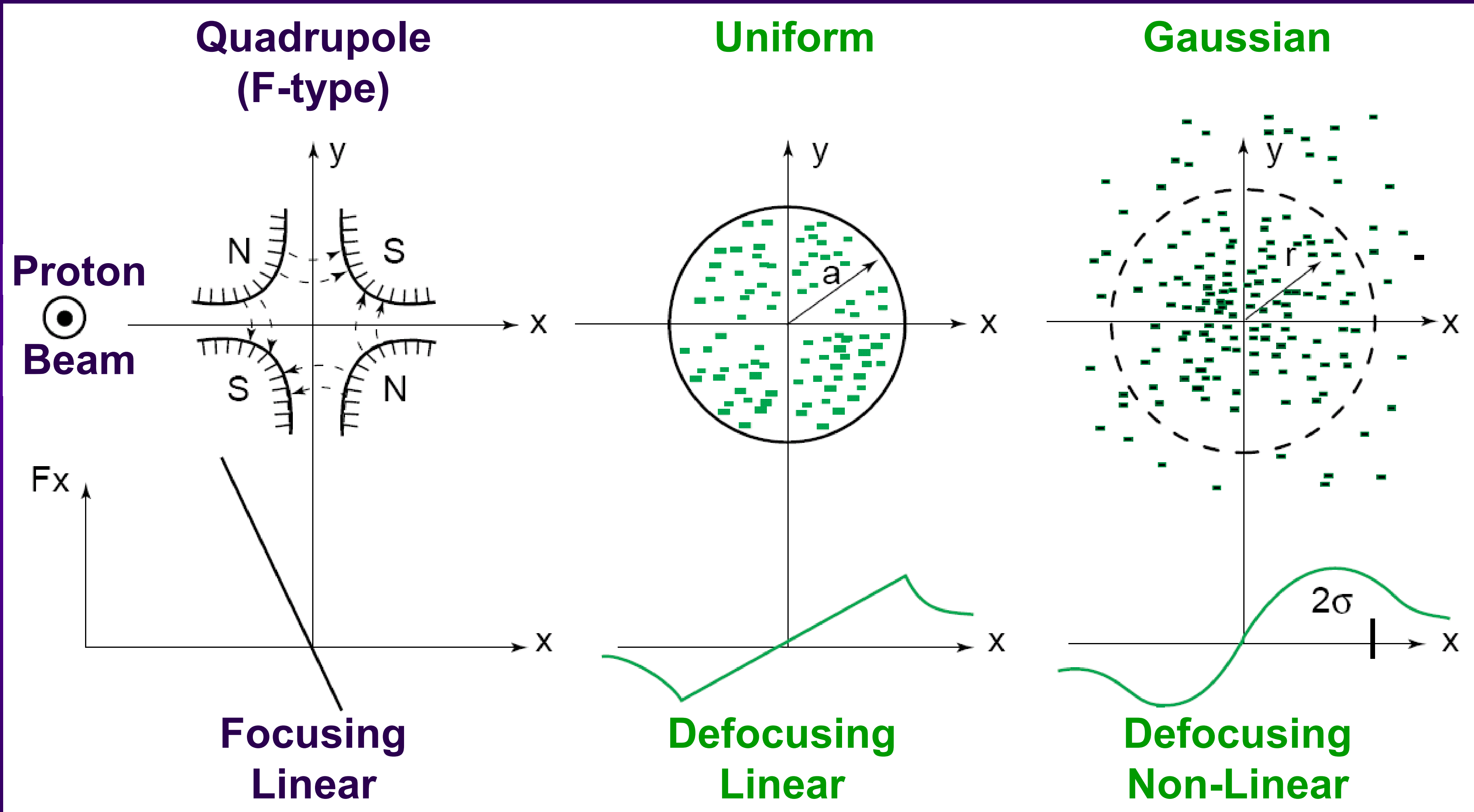


Courtesy K. Schindl

Many charged particles travelling in an unbunched beam with circular cross-section



SPACE CHARGE (2/4)

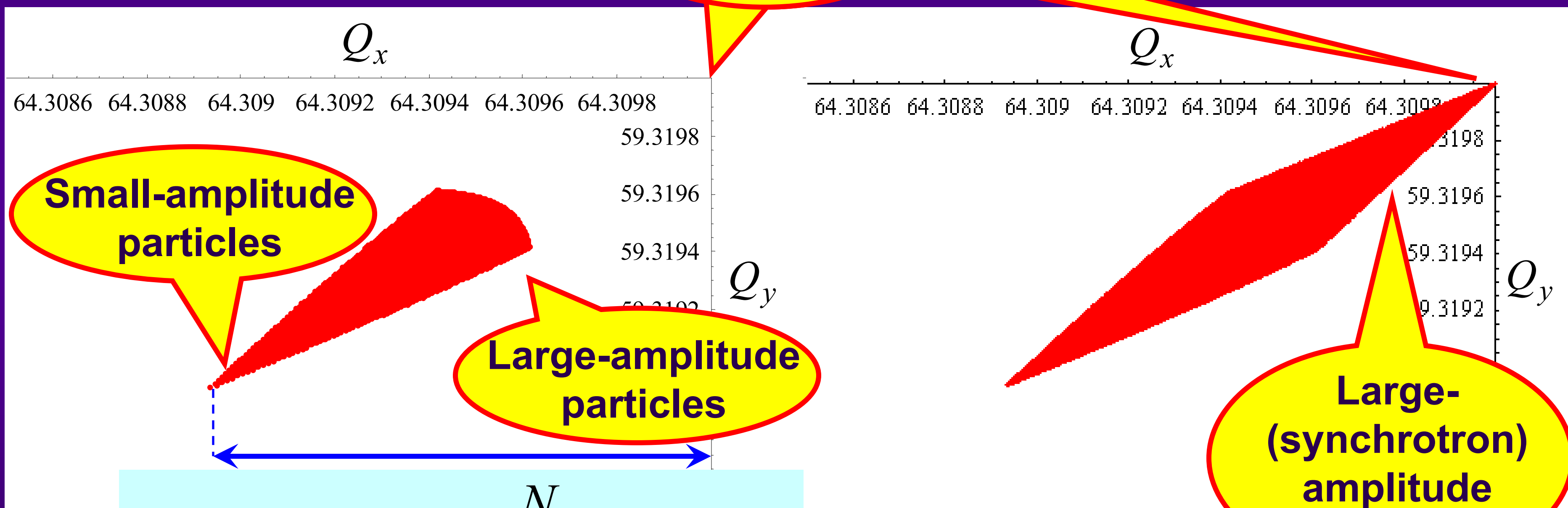


Courtesy K. Schindl

SPACE CHARGE (3/4)

⇒ INCOHERENT (single-particle) tunes

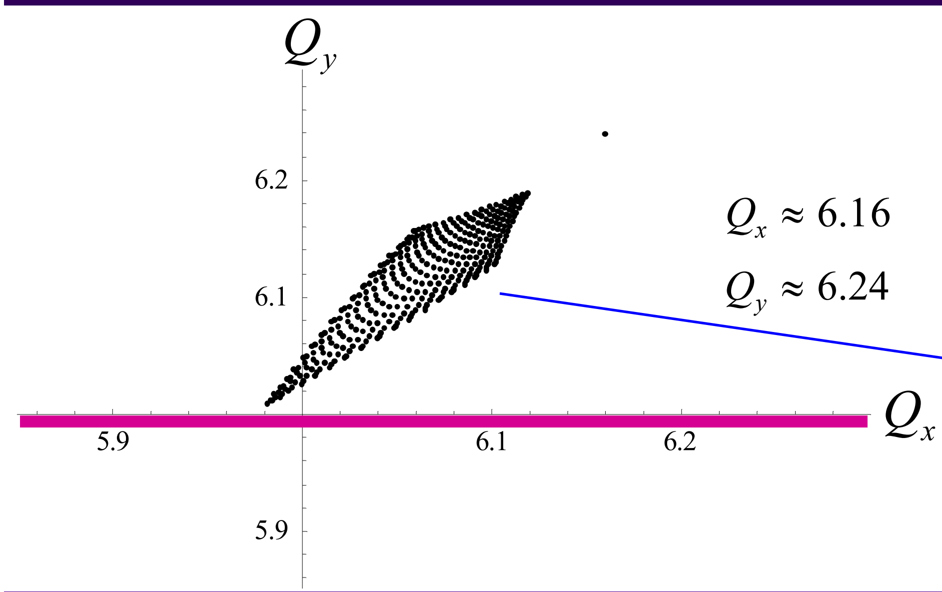
- 2D tune footprint
- 3D tune footprint

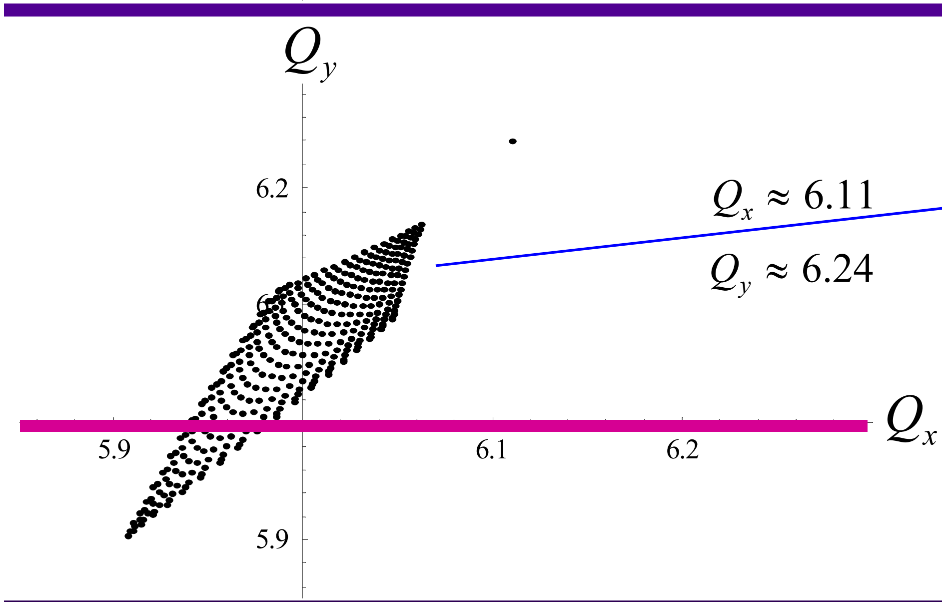


$$\Delta_0 \propto -\frac{N_b}{\beta_r\,\gamma_r^2\,\varepsilon_{rms}^{norm}}$$

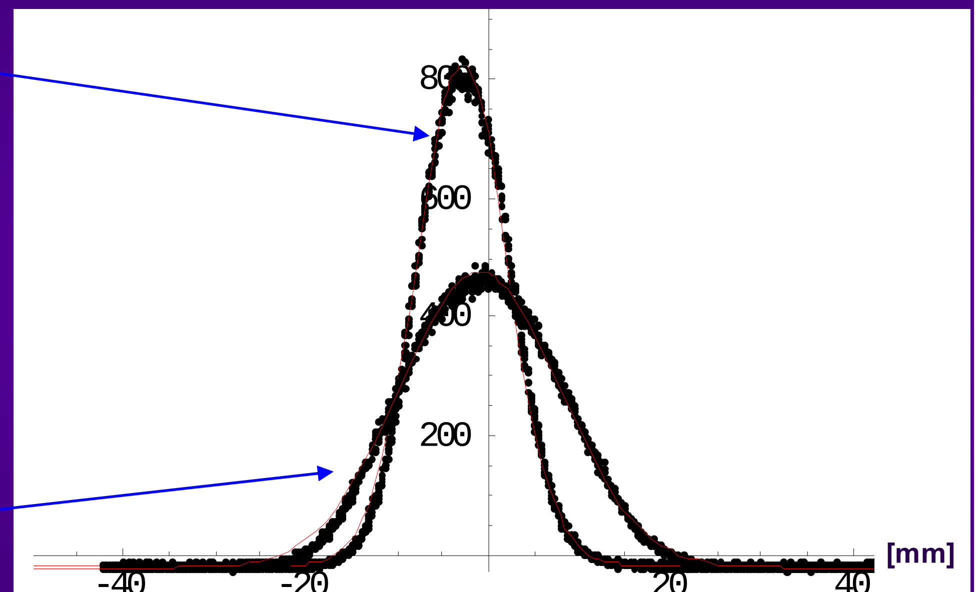
$= \text{Linear space - charge tune shift}$

SPACE CHARGE (4/4)





Horizontal bunch profile + Gaussian fit

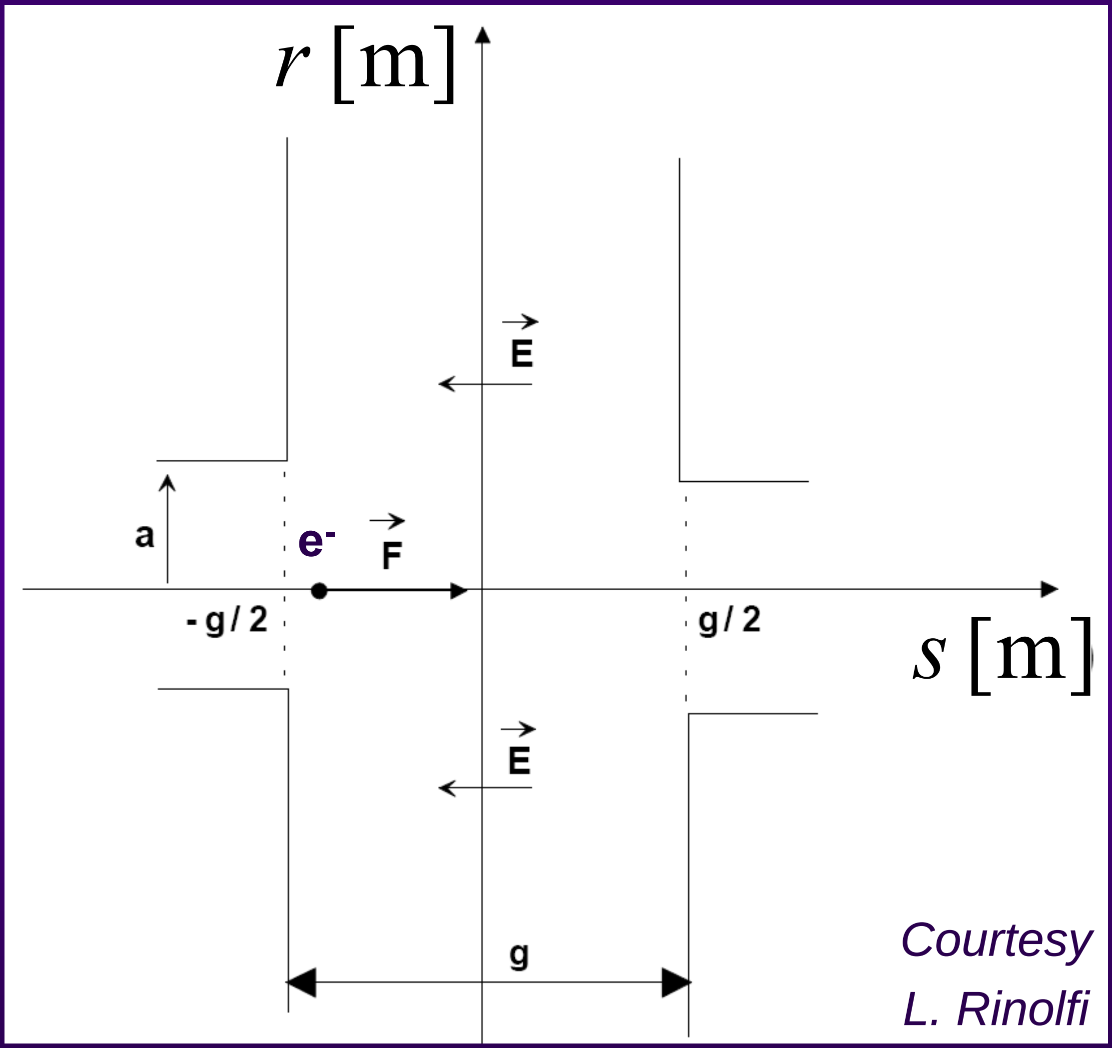


$$\varepsilon_x^{(\sigma_x)} \equiv \frac{\sigma_x^2}{\beta_x}$$

$\Rightarrow$ **Emittance blow-up**

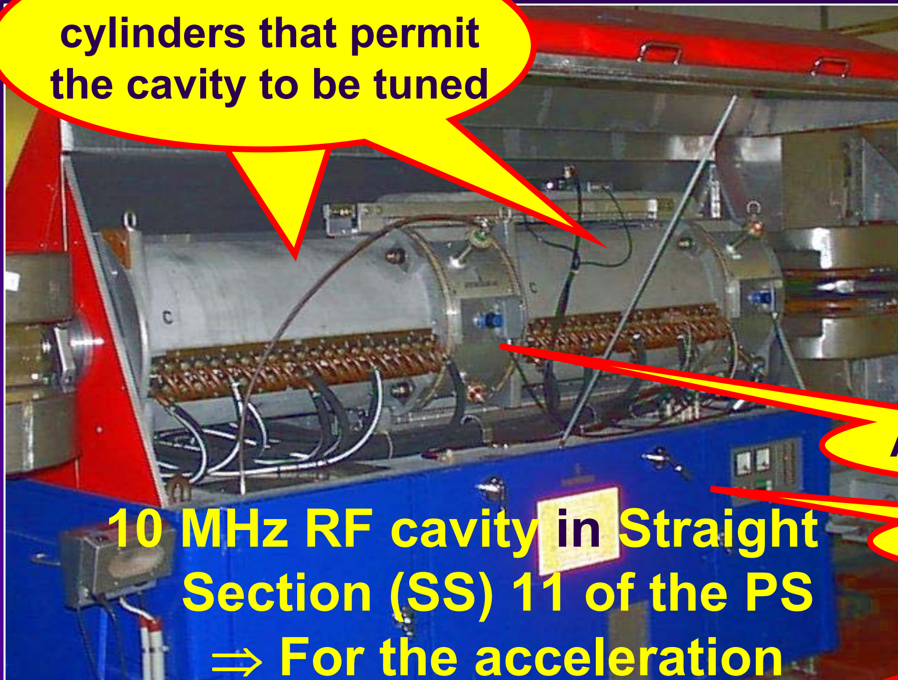
LONGITUDINAL BEAM DYNAMICS (1/8)

- **The electric field is used to accelerate or decelerate the particles, and is produced by one or more RF (Radio-Frequency) CAVITIES**

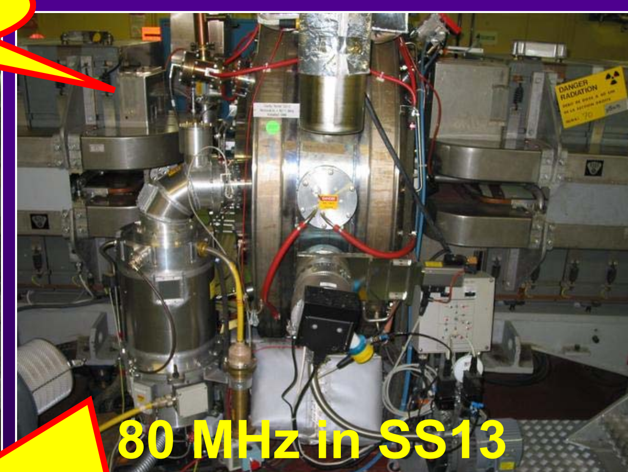


Book p. 58 to 73 + Ref. [5]

LONGITUDINAL BEAM DYNAMICS (2/8)



RF gymnastics



World Radio Geneva: 88.4 MHz

LONGITUDINAL BEAM DYNAMICS (3/8)

- **TRANSITION ENERGY: The increase of energy has 2 contradictory effects**

  - **An increase of the particle's velocity**
  - **An increase of the length of the particle's trajectory**

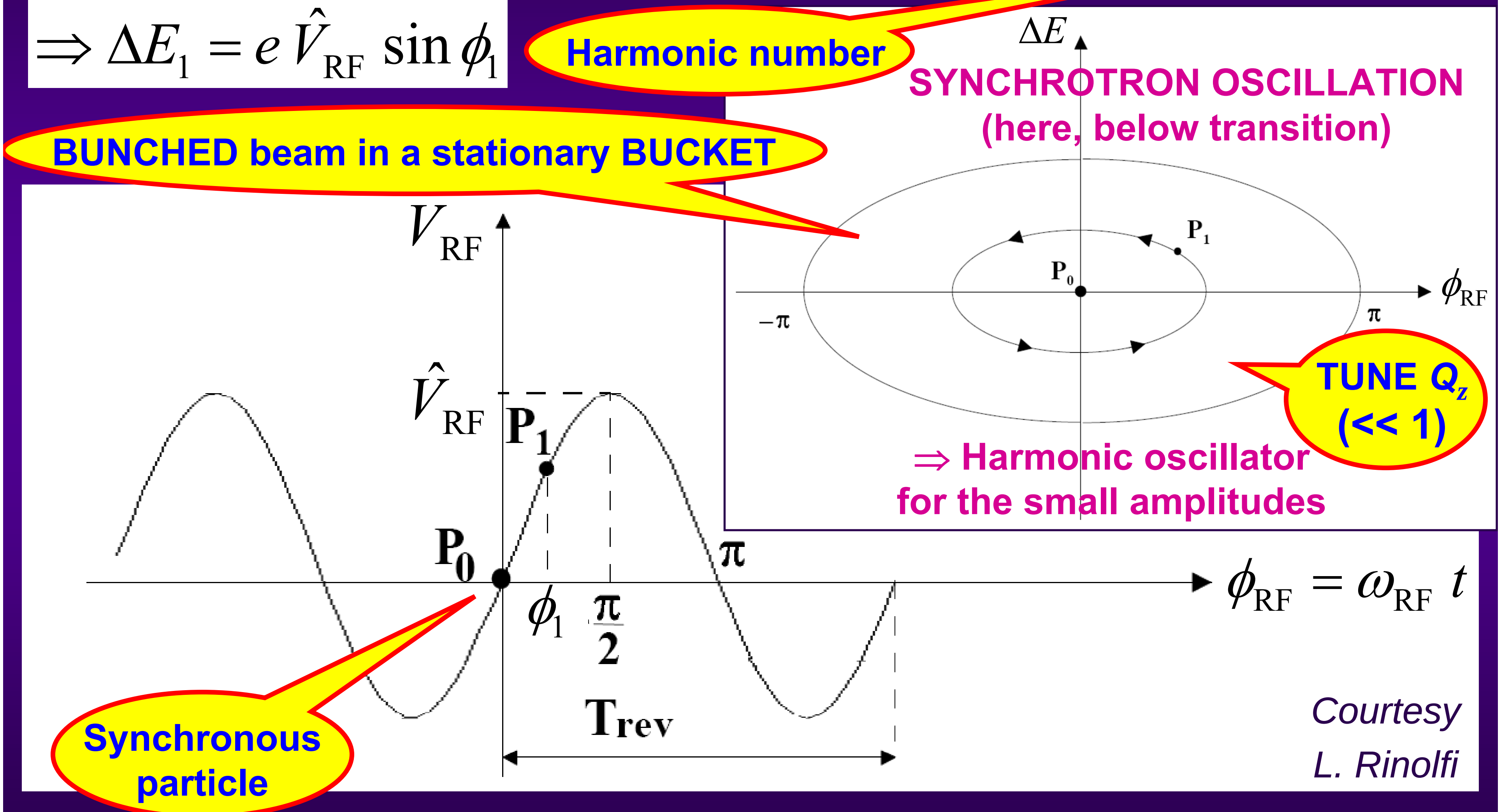
According to the variations of these 2 parameters, the revolution frequency evolves differently

  - **Below transition energy: The velocity increases faster than the length ⇒ The revolution frequency increases**
  - **Above transition energy: It is the opposite case ⇒ The revolution frequency decreases**
  - **At transition energy: The variation of the velocity is compensated by the variation of the trajectory ⇒ A variation of energy does not modify the frequency**

LONGITUDINAL BEAM DYNAMICS (4/8)

- **Sinusoidal voltage applied** $V_{RF} = \hat{V}_{RF} \sin \phi_{RF}(t)$ $\omega_{RF} = h \, \omega_{rev}$

$$\Rightarrow \Delta E_1 = e \, \hat{V}_{RF} \sin \phi_1$$

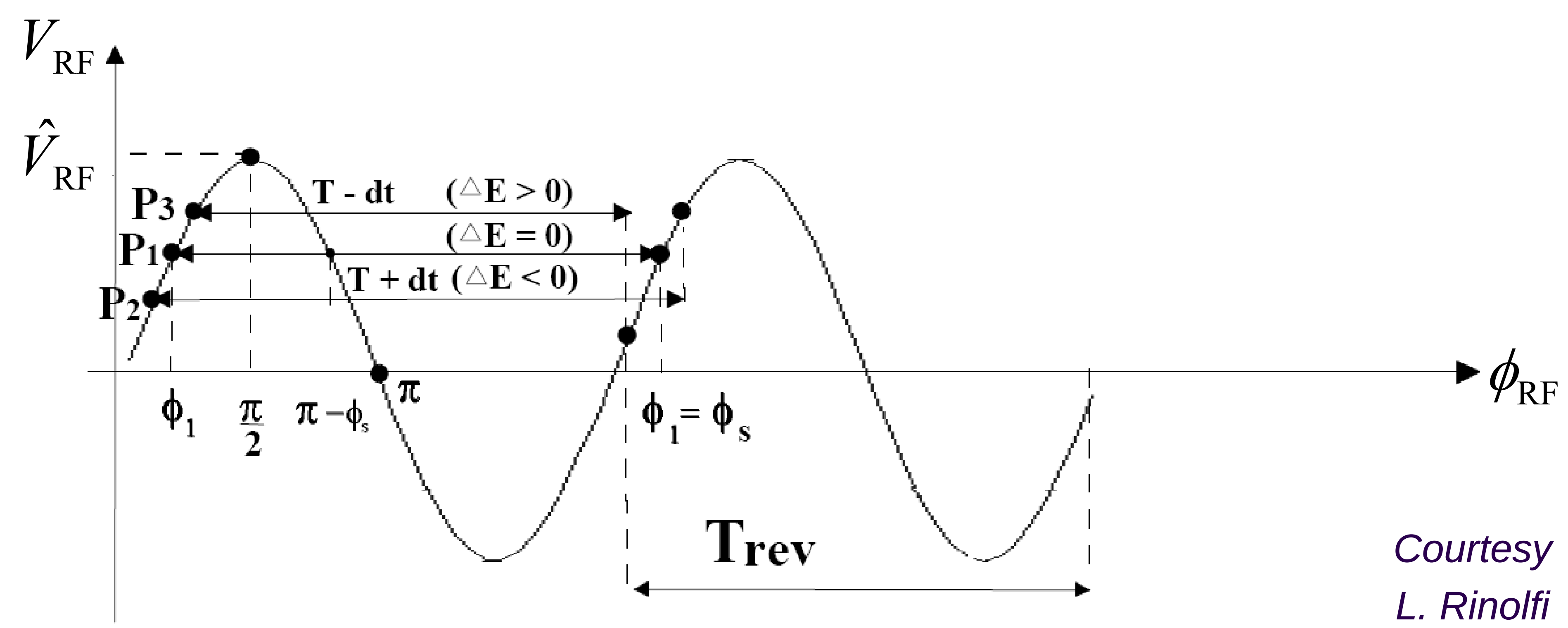


LONGITUDINAL BEAM DYNAMICS (5/8)

- **Synchrotron oscillation during acceleration (below transition)** $\phi_1 = \phi_s \neq 0$

Synchronous phase

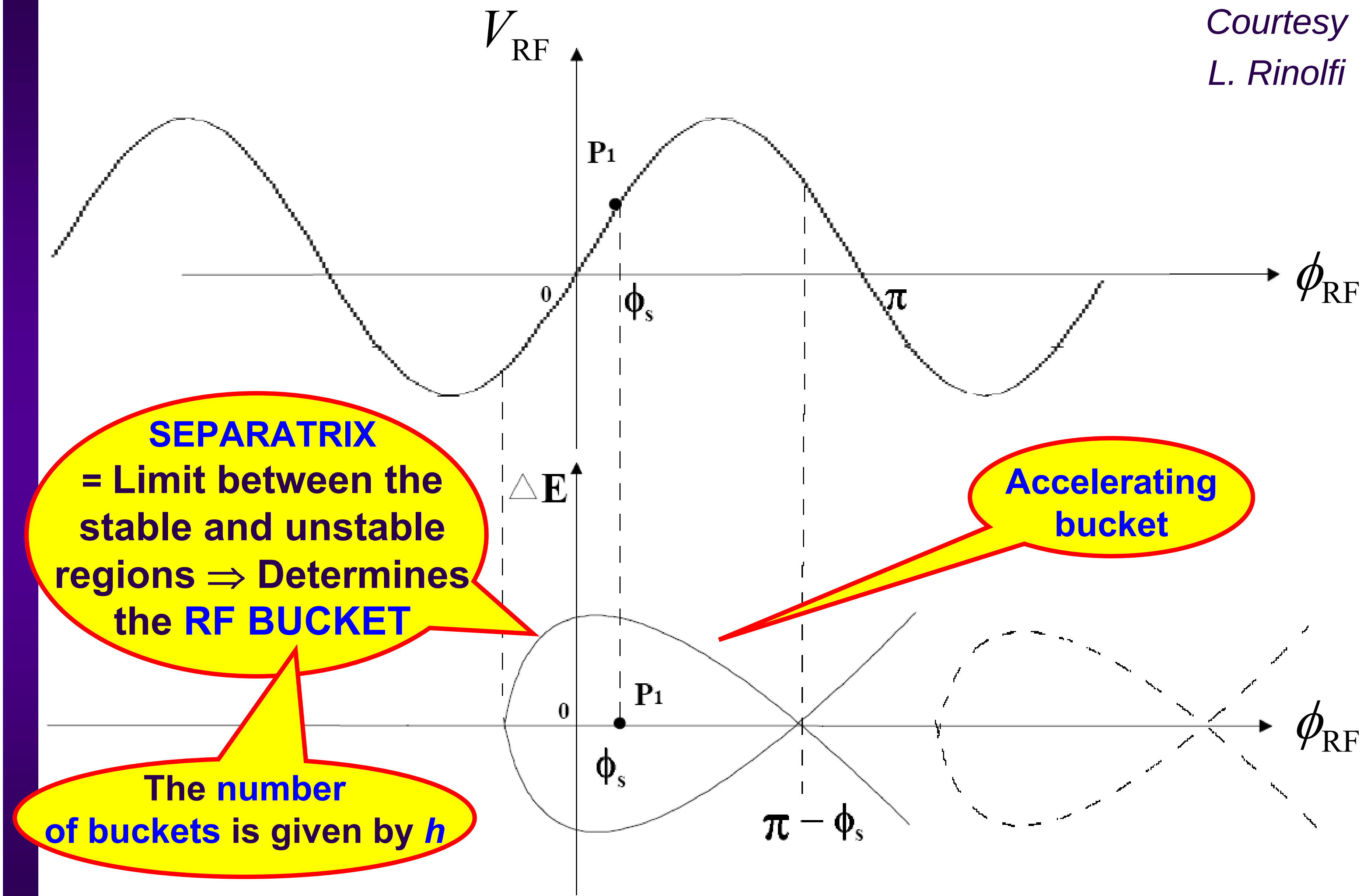
Above transition, the stable phase is $\pi - \phi_s$



Courtesy L. Rinolfi

LONGITUDINAL BEAM DYNAMICS (6/8)

Courtesy
L. Rinolfi



LONGITUDINAL BEAM DYNAMICS (7/8)

$\Rightarrow$ $$\tau''(t) + \omega_s^2 \, \tau(t) = 0$$: **Equation of a harmonic oscillator**

τ = time interval between the passage of the synchronous particle and the particle under consideration

$$\omega_s = \sqrt{\frac{|\eta \cos \phi_s| \hat{V}_{\mathrm{RF}} \, h}{2 \pi \beta_r^2 (E/e)}} \, \omega_{rev}$$

= momentum compaction factor α_p

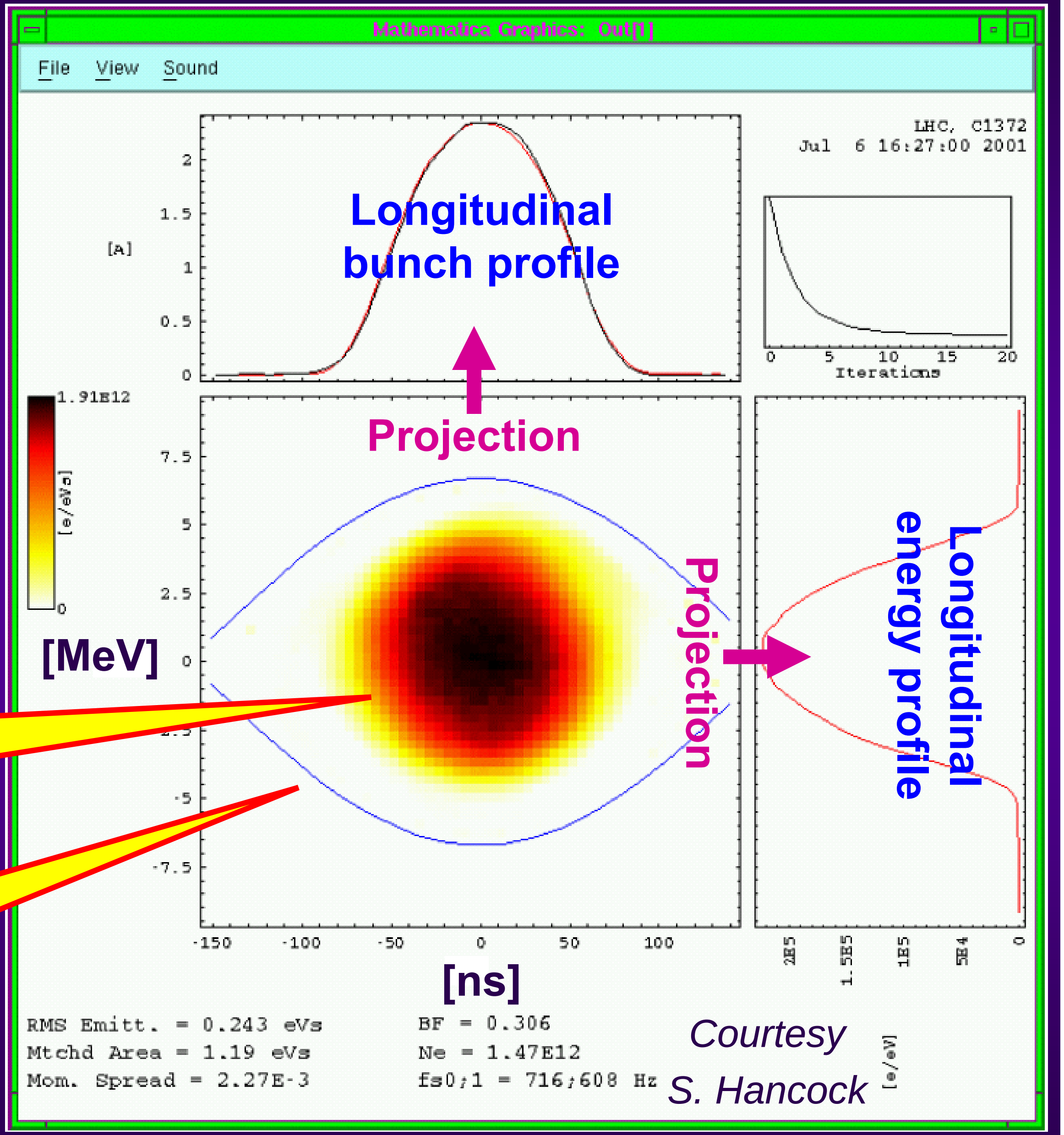
$$\eta = \gamma_{tr}^{-2} - \gamma^{-2} = (\Delta T / T_0) / (\Delta p / p_0)$$

Slip factor (sometimes defined with a negative sign…)

$\Rightarrow$ $$Q_z = \frac{\omega_s}{\omega_{rev}}$$: **Synchrotron tune**

Number of synchrotron oscillations per machine revolution

LONGITUDINAL BEAM DYNAMICS (8/8)



Surface = Longitudinal EMITTANCE of the bunch = ε_L [eV.s]

Surface = Longitudinal ACCEPTANCE of the bucket

SOLUTION OF THE TRANSVERSE PROBLEM (1/2)

- **Doubling the peak luminosity $\Longrightarrow$ Increasing the intensity (per bunch) by $\sqrt{2}$, i.e. by ~ 40%**

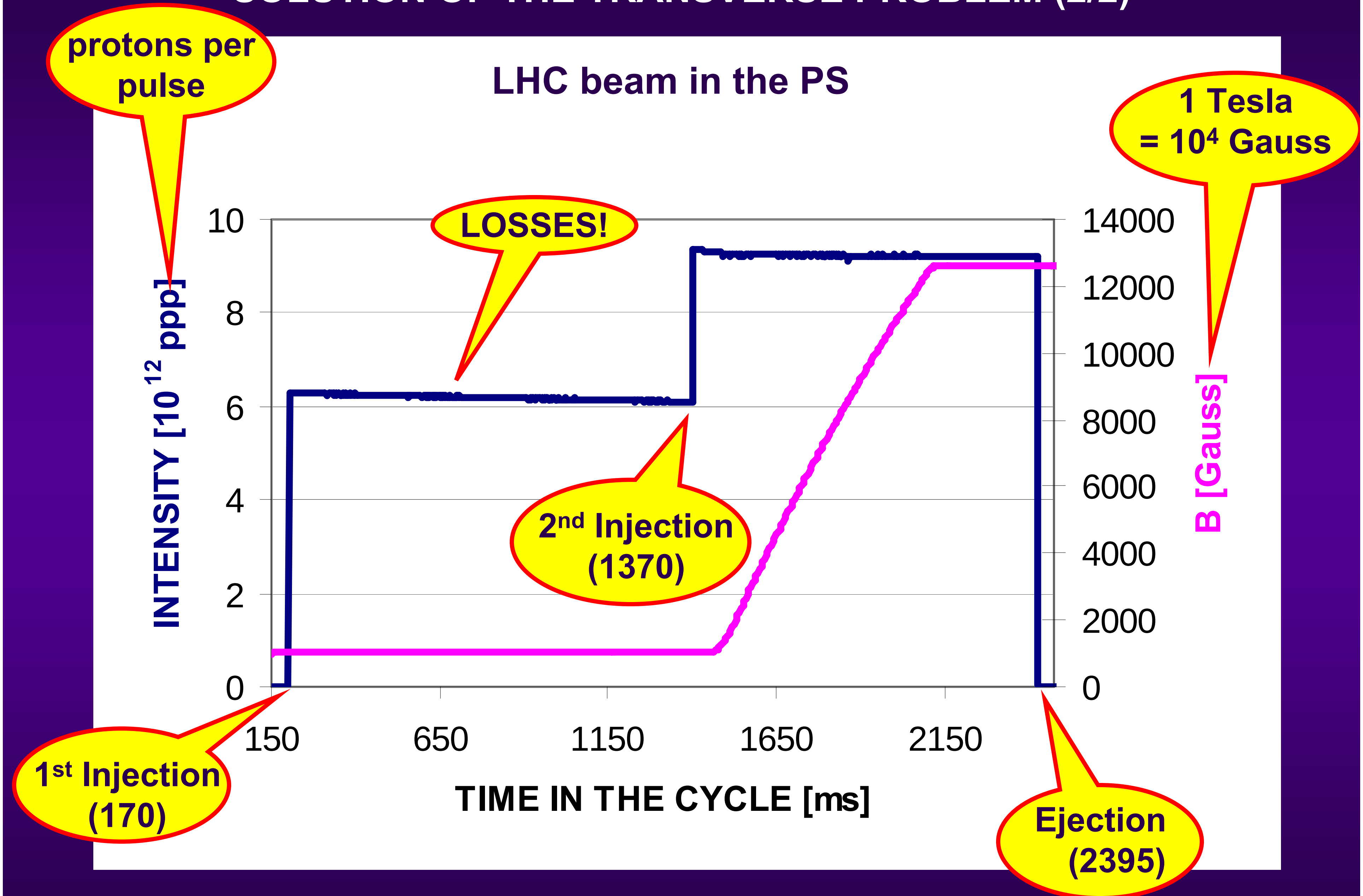
- **$\Longrightarrow$ The space charge tune spread is increased by 40% (assuming the same transverse emittance)**

- **Losses are observed and reduced by tuning the working point $\Longrightarrow$ The space charge tune spread can be placed in a better position in the tune diagram where it overlaps less dangerous resonances…**

- **The losses did not disappear completely $\Longrightarrow$ See next (real) picture. One has either to reduce the density N_b / ϵ_n or compensate the resonances if one wants to suppress these losses**

- **The transverse emittances still have to be checked $\Longrightarrow$ Because, as seen before, some resonances can lead to emittance blow-up…and emittance blow-up leads to less luminosity…**

SOLUTION OF THE TRANSVERSE PROBLEM (2/2)



SOLUTION OF THE LONGITUDINAL PROBLEM

1) Are the 1020 Gauss OK?

- This is the very important formula of the beam rigidity which has to be used here. It is given by

$$B[\mathrm{T}]\,\rho[\mathrm{m}] = 3.3356\; p[\mathrm{GeV/c}]\,.$$

The numerical application yields B = 1020 Gauss. The value given is good and therefore this is the RF voltage which is not the good one. Why? If the longitudinal emittance of the beam sent to the SPS is too large and the RF voltage at PS injection is not the good one, it means that the blow-up of the longitudinal emittance is due to a longitudinal mismatch between the PSB and the PS…

2) We know that the 60 kV is not the good value. What is the good one?

- In a stationary bucket (as it is the case in the PS at injection), the synchronous phase below transition energy is $\Phi_s = 0$. Therefore, the bucket (half) height $\Delta E_{\max}$ is given by the formula with F = 2. Concerning the bucket (half) length $\Delta t_{\max}$, it is also given by the formula with $\hat{\phi} = \pi$ in a stationary bucket.

- The longitudinal matching condition between the PSB and PS is given by

$$\left(\frac{\Delta E_{\max}}{\Delta t_{\max}}\right)_{PS} = \left(\frac{\Delta E_{\max}}{\Delta t_{\max}}\right)_{PSB}.$$

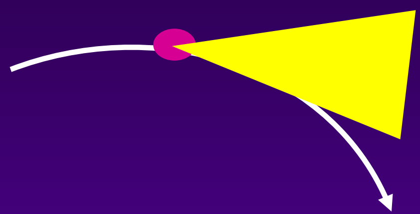
As the beam energy is the same at extraction of the PSB and injection in the PS, this leads to

$$\hat{V}_{RF}^{PS} = \hat{V}_{RF}^{PSB} \times \left|\frac{\eta_{PS}}{\eta_{PSB}}\right| \times \frac{h_{PSB}}{h_{PS}} \times \left(\frac{R_{PS}}{R_{PSB}}\right)^2.$$

The numerical application gives $\hat{V}_{RF}^{PS} = 25\ \mathrm{kV}$. The RF voltage should therefore be 25 kV and not 60 kV. With 60 kV the beam coming from the PSB is not matched longitudinally. It will start to oscillate in the RF bucket finding after some time a new matching condition but with a larger longitudinal emittance, as observed by the SPS operator…

SYNCHROTRON RADIATION (1/2)

Book p. 111 to 122



- Power radiated by a particle (due to bending)

$$P_{\perp} = \frac{q^2 \, c \, \beta^4 \, E^4}{6 \, \pi \, \varepsilon_0 \, \rho^2 \, E_0^4}$$

Particle total energy

Curvature radius of the dipoles

Particle rest energy

- Energy radiated in one ring revolution

$$U_0 = \frac{q^2 \, \beta^3 \, E^4}{3 \, \varepsilon_0 \, E_0^4 \, \rho}$$

- Average (over the ring circumference) power radiation

$$P_{av} = \frac{U_0}{T_0}$$

Revolution period

SYNCHROTRON RADIATION (2/2)

	LEP	LHC
ρ [m]	3096.175	2803.95
p_0 [GeV/c]	104	7000
U_0	3.3 GeV	6.7 keV

The RF system had therefore to compensate for an energy lost of **~3%** of the total beam energy per turn!

The total average (over the ring circumference) power radiation (per beam) is **3.9 kW** (2808 bunches of 1.15 10^{11} protons)