LUMINOSITY

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- Colliders vs. fixed-target experiments
- General definition of luminosity
- Simplest formula for Head-On (HO) collisions
- Some complications
  - Crossing angle
  - Transverse beam offset
  - Hourglass effect
- Integrated luminosity and maximization
- Pile-up, luminosity leveling and luminous region
- Summary: How to reach high luminosity?

COLLIDERS VS. FIXED-TARGET EXPERIMENTS (1/2)

- Using the relativistic equations given in Introduction, it can be seen that

$$\sqrt{s} = E_{CM} = \sqrt{m_{01}^2 \, c^4 + m_{02}^2 \, c^4 + 2 \left(E_1 \, E_2 - \vec{p}_1 . \vec{p}_2 \, c^2 \right)}$$

  - For a fixed target ($\vec{p}_2 = 0$) and if we neglect the masses (i.e. if we are at sufficiently high energy)

$$\sqrt{s} = \sqrt{2 \, E_1 \, m_{02} \, c^2}$$

  - For a collider ($\vec{p}_2 = - \, \vec{p}_1$) and if we neglect the masses (i.e. if we are at sufficiently high energy)

$$\sqrt{s} = 2 \, E_1$$

COLLIDERS VS. FIXED-TARGET EXPERIMENTS (2/2)

- **Numerical applications (for the available energy in the CM, i.e. to create new particles)**
  - LHC (p^+p^+, 7 TeV / beam)
    - Collider mode => 14 TeV
    - Fixed-target mode => ~ 115 GeV (i.e. ~ 122 times less)
  - LEP (e^+e^-, 105 GeV / beam)
    - Collider mode => 210 GeV
    - Fixed-target mode => ~ 0.3 GeV (i.e. 626 times less)

GENERAL DEFINITION OF LUMINOSITY (1/7)

- **By definition, the luminosity *L* is the time-averaged integral over the interaction volume Ω of the number of reactions per unit time and volume**

$$L = \frac{1}{\sigma_r \, T_b} \int_0^{T_b} \int_\Omega \frac{d^2 N}{dt \, d\Omega} \, dt \, d\Omega$$

Total cross section of the reaction [m^2]

Inverse of the bunch collision frequency [s]

$$T_b^{-1} = f_b = f_{rev} \, M$$

Interaction volume where the 2 beams collide [m^3]: dΩ = ds dx dy

GENERAL DEFINITION OF LUMINOSITY (2/7)

- **The number of reactions per unit time and unit volumes satisfies the following relation associated with the Lorentz transformation of the variables => Luminosity density *S***

$$S = \frac{1}{\sigma_r}\,\frac{d^2 N}{d t \, d \Omega} = N_1 \, N_2 \, \rho_1 (x , y , s , t) \, \rho_2 (x , y , s , t) \, M_{KLF}$$

Numbers of particles / bunch (1 and 2)

Density of bunch 1

Density of bunch 2

Møller Kinematic Luminosity Factor

$$\int d\Omega \, \rho_1 = 1 \qquad \int d\Omega \, \rho_2 = 1$$

$$M_{KLF} = \sqrt{ (\vec{v}_1 - \vec{v}_2)^2 - \frac{(\vec{v}_1 \times \vec{v}_2)^2}{c^2} }$$

Correction factor that makes *S* a relativistic invariant

GENERAL DEFINITION OF LUMINOSITY (3/7)

- **Møller Kinematic Luminosity Factor is linked to the relative velocity between the 2 beams v_{21} (see Introduction)**

$$M_{KLF} = v_{21} \left(1 - \frac{\vec{v}_1 \, . \vec{v}_2}{c^2} \right)$$

$\vec{v}_1$ **= Velocity in the Laboratory frame of all particles of bunch 1**

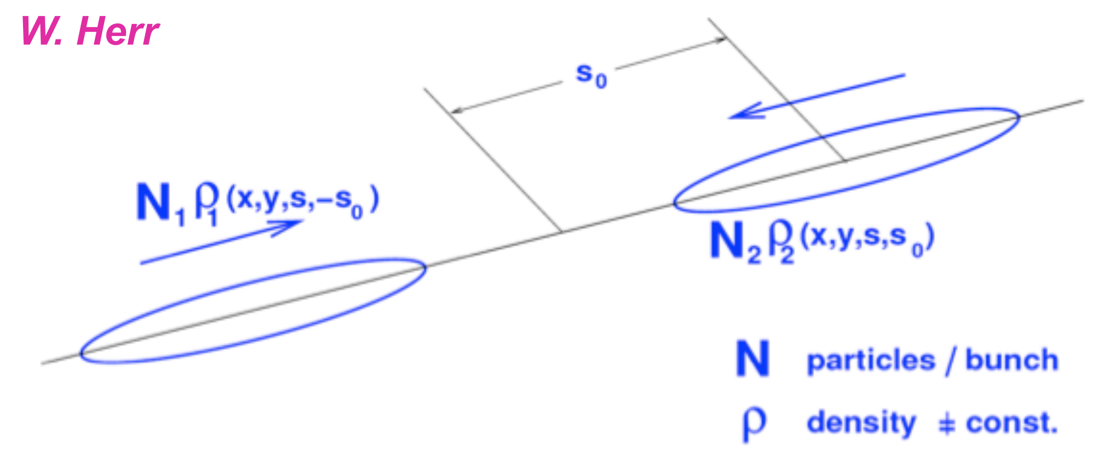
$\vec{v}_2$ **= Velocity in the Laboratory frame of all particles of bunch 2**

=>

$$L = M \, N_1 \, N_2 \, f_{rev} \, M_{KLF} \int_0^{T_b} \int_\Omega \rho_1 (x , y , s , t) \, \rho_2 (x , y , s , t) \, d t \, d \Omega$$

GENERAL DEFINITION OF LUMINOSITY (4/7)

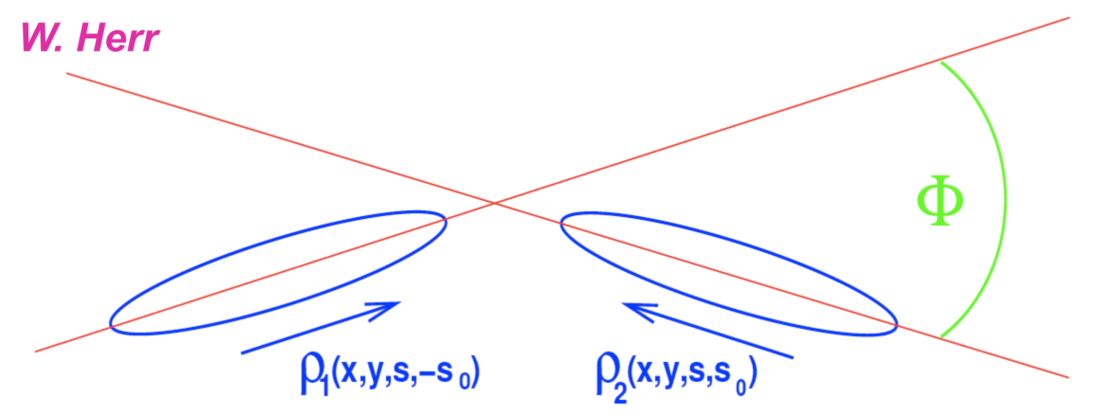
- Collision without crossing angle



$$s_0 = c\ t$$

Time variable

- Collision with crossing angle (general case)



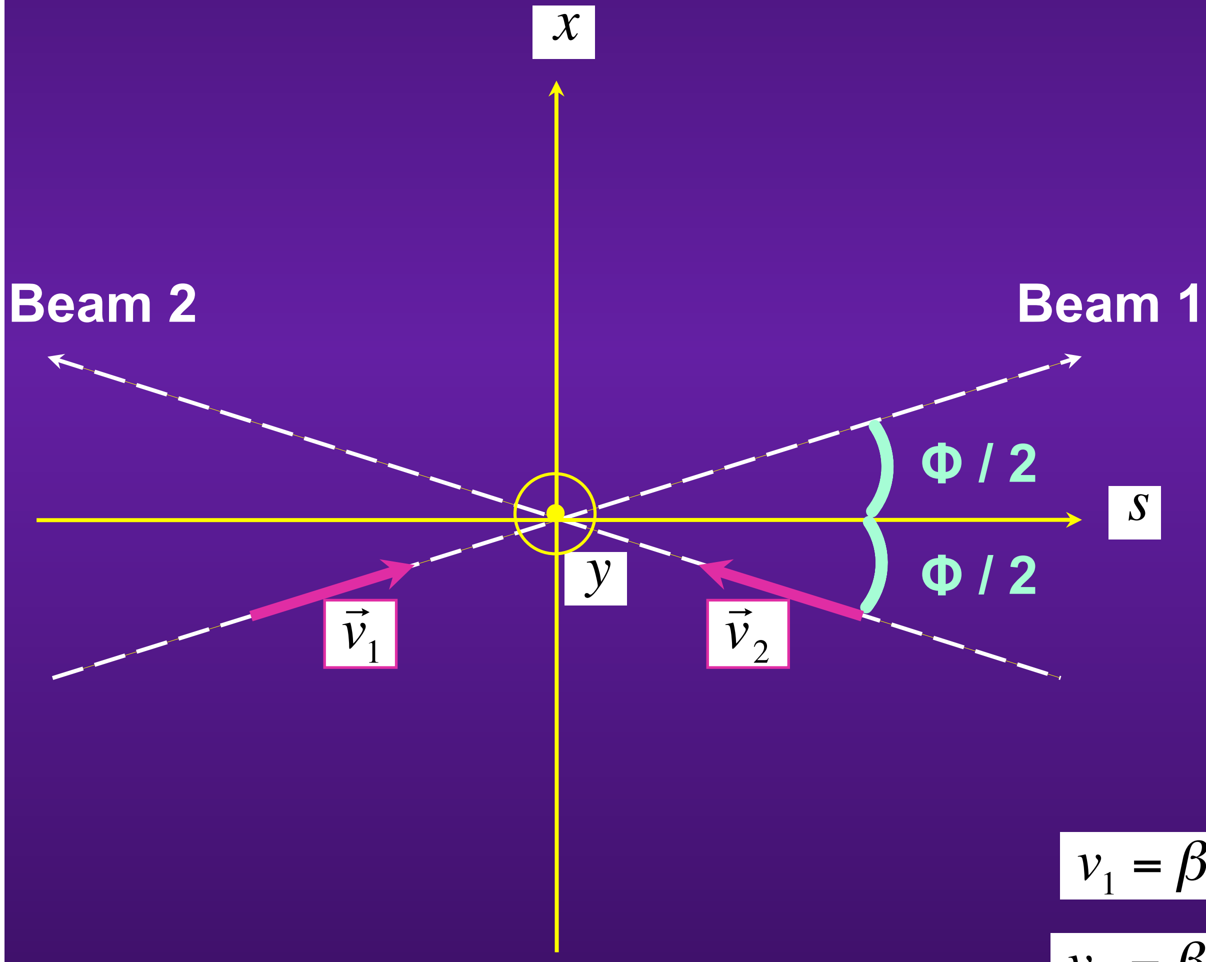
=>

$$L = M\ N_1\ N_2\ f_{rev}\ \frac{M_{KLF}}{c} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\,\rho_2(x,y,s,s_0)\,dx\,dy\,ds\,ds_0$$

GENERAL DEFINITION OF LUMINOSITY (5/7)

- **Møller Kinematic Luminosity Factor (general case with crossing angle)**

$$M_{KLF} = \sqrt{\left(\vec{v}_1 - \vec{v}_2\right)^2 - \frac{\left(\vec{v}_1 \times \vec{v}_2\right)^2}{c^2}}$$



$$\vec{v}_1 = \left| \begin{array}{c} v_1 \sin\frac{\Phi}{2} \\ 0 \\ v_1 \cos\frac{\Phi}{2} \end{array} \right.$$

$$\vec{v}_2 = \left| \begin{array}{c} v_2 \sin\frac{\Phi}{2} \\ 0 \\ -v_2 \cos\frac{\Phi}{2} \end{array} \right.$$

$$v_1 = \beta_1 \, c$$

$$v_2 = \beta_2 \, c$$

GENERAL DEFINITION OF LUMINOSITY (6/7)

=> $$\vec{v}_1 - \vec{v}_2 = \left| \begin{matrix} (v_1 - v_2) \sin \frac{\Phi}{2} \\ 0 \\ (v_1 + v_2) \cos \frac{\Phi}{2} \end{matrix} \right.$$ **and** $$\vec{v}_1 \times \vec{v}_2 = \left| \begin{matrix} 0 \\ 2 \, v_1 \, v_2 \cos \frac{\Phi}{2} \sin \frac{\Phi}{2} \\ 0 \end{matrix} \right.$$

=> $$M_{KLF} = \sqrt{v_1^2 + v_2^2 + 2 \, v_1 \, v_2 \cos \Phi - \frac{v_1^2 \, v_2^2}{c^2} \sin^2 \Phi}$$

$v_1 = \beta_1 \, c$

$v_2 = \beta_2 \, c$

=> $$\boxed{\frac{M_{KLF}}{c} = \sqrt{\beta_1^2 + \beta_2^2 + 2 \, \beta_1 \, \beta_2 \cos \Phi - \beta_1^2 \, \beta_2^2 \sin^2 \Phi}}$$

GENERAL DEFINITION OF LUMINOSITY (7/7)

- If $\beta_1 = \beta_2 = \beta$ => $$\frac{M_{KLF}}{c} = 2\,\beta\,\cos\frac{\Phi}{2}\,\sqrt{1-\beta^2\,\sin^2\frac{\Phi}{2}}$$
- If $\beta_1 = \beta_2 = \beta = 1$ => $$\frac{M_{KLF}}{c} = 2\,\cos^2\frac{\Phi}{2}$$
- If $\Phi = 0$ => $$\frac{M_{KLF}}{c} = 2$$

SIMPLEST FORMULA FOR HEAD-ON COLLISIONS (1/4)

- **Luminosity in the absence of crossing angle (and transverse beam offset and hourglass effect)**

$$L = M \, N_1 \, N_2 \, f_{rev} \, 2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho_1 (x , y , s , - s_0) \, \rho_2 (x , y , s , s_0) \, d x \, d y \, d s \, d s_0$$

  - **Assuming that the densities are uncorrelated in all planes**

$$\rho_1 (x , y , s , - s_0) = \rho_{1x} (x) \rho_{1y} (y) \rho_{1s} (s - s_0)$$

$$\rho_2 (x , y , s , s_0) = \rho_{2x} (x) \rho_{2y} (y) \rho_{2s} (s + s_0)$$

  - **Assuming Gaussian distributions in all dimensions**

$$\rho_{1x} (x) = \frac{1}{\sigma_{1x} \sqrt{2 \pi}} \, e^{-\frac{x^2}{2 \sigma_{1x}^2}} \quad , \cdots$$

SIMPLEST FORMULA FOR HEAD-ON COLLISIONS (2/4)

=>

$$L = \frac{2\, M\, N_1\, N_2\, f_{rev}}{(2\,\pi)^3\, \sigma_{1x}\, \sigma_{2x}\, \sigma_{1y}\, \sigma_{2y}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{e^{-\frac{x^2}{2\sigma_{1x}^2}}\, e^{-\frac{y^2}{2\sigma_{1y}^2}}\, e^{-\frac{(s-s_0)^2}{2\sigma_{1s}^2}}\, e^{-\frac{x^2}{2\sigma_{2x}^2}}\, e^{-\frac{y^2}{2\sigma_{2y}^2}}\, e^{-\frac{(s+s_0)^2}{2\sigma_{2s}^2}}}{\sigma_{1s}\, \sigma_{2s}}\, d\,x\, d\,y\, d\,s\, d\,s_0$$

- **Assuming** $\sigma_{1s} = \sigma_{2s} = \sigma_s$ => $e^{-\frac{(s-s_0)^2}{2\sigma_{1s}^2}}\, e^{-\frac{(s+s_0)^2}{2\sigma_{2s}^2}} = e^{-\frac{s^2}{\sigma_s^2}}\, e^{-\frac{s_0^2}{\sigma_s^2}}$

and $\iint \frac{e^{-\frac{s^2}{\sigma_s^2}}\, e^{-\frac{s_0^2}{\sigma_s^2}}}{\sigma_s^2}\, d\,s\, d\,s_0 = \pi$ **(see Useful relations in Introduction)**

=>

$$L = \frac{M\, N_1\, N_2\, f_{rev}}{4\,\pi^2\, \sigma_{1x}\, \sigma_{2x}\, \sigma_{1y}\, \sigma_{2y}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{x^2}{2\sigma_{1x}^2}}\, e^{-\frac{y^2}{2\sigma_{1y}^2}}\, e^{-\frac{x^2}{2\sigma_{2x}^2}}\, e^{-\frac{y^2}{2\sigma_{2y}^2}}\, d\,x\, d\,y$$

SIMPLEST FORMULA FOR HEAD-ON COLLISIONS (3/4)

- **Assuming** $\sigma_{1x} = \sigma_{2x} = \sigma_x$, $\sigma_{1y} = \sigma_{2y} = \sigma_y$, as $\iint \frac{e^{-\frac{x^2}{\sigma_x^2}} \, e^{-\frac{y^2}{\sigma_y^2}}}{\sigma_x \, \sigma_y} \, dx \, dy = \pi$,

one finally obtains

$$L = \frac{M \, N_1 \, N_2 \, f_{rev}}{4 \, \pi \, \sigma_x \, \sigma_y}$$

Let's call it *L*₀

- If $\sigma_{1x} \neq \sigma_{2x}$, $\sigma_{1y} \neq \sigma_{2y}$ =>

$$L = \frac{M \, N_1 \, N_2 \, f_{rev}}{2 \, \pi \, \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \, \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}}$$

SIMPLEST FORMULA FOR HEAD-ON COLLISIONS (4/4)

- **Assuming**

$$\sigma_{1x} = \sigma_{2x} = \sigma_x$$

$$\sigma_{1y} = \sigma_{2y} = \sigma_y$$

$$\sigma_x = \sigma_y = \sigma$$

$$N_1 = N_2 = N_b$$

=>

$$\boxed{L_0 = \frac{M\, N_b^2\, f_{rev}\, \beta\, \gamma}{4\, \pi\, \beta^*\, \varepsilon_n}}$$

using

$$\varepsilon_n = \beta\, \gamma\, \varepsilon = \beta\, \gamma\, \frac{\sigma^2}{\beta^*}$$

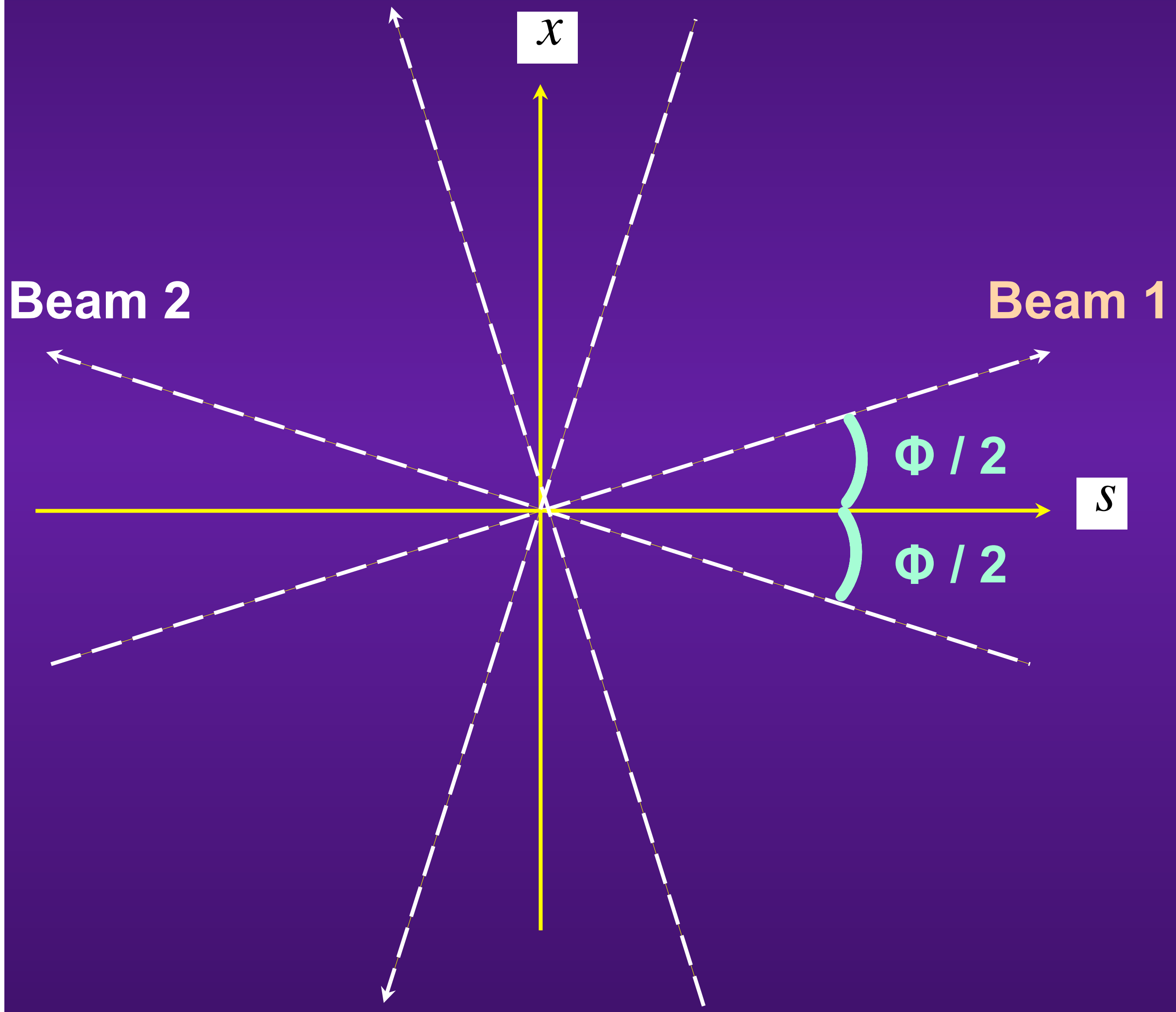
Normalized transverse beam emittance

***β*-function at the collision point**

- **Numerical application for LHC => $L_0 = 1.2 \times 10^{34}$ cm^{-2}s^{-1}**

SOME COMPLICATIONS: CROSSING ANGLE (1/5)

- Luminosity in the presence of crossing angle (in the x-s plane)



- Rotation of Φ /2 for beam 1
- Rotation of - Φ /2 for beam 2

SOME COMPLICATIONS: CROSSING ANGLE (2/5)

=> The following relations are thus obtained (see also Useful relations in Introduction)

$$\begin{pmatrix} x_1 \\ s_1 \end{pmatrix} = \begin{bmatrix} \cos\frac{\Phi}{2} & -\sin\frac{\Phi}{2} \\ \sin\frac{\Phi}{2} & \cos\frac{\Phi}{2} \end{bmatrix} \begin{pmatrix} x \\ s \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ s_2 \end{pmatrix} = \begin{bmatrix} \cos\frac{\Phi}{2} & \sin\frac{\Phi}{2} \\ -\sin\frac{\Phi}{2} & \cos\frac{\Phi}{2} \end{bmatrix} \begin{pmatrix} x \\ s \end{pmatrix}$$

$$L_{CA} = 2\cos^2\frac{\Phi}{2}\, M\, N_1\, N_2\, f_{rev}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho_{1x}(x_1)\rho_{1y}(y_1)\rho_{1s}(s_1 - s_0)\,\rho_{2x}(x_2)\rho_{2y}(y_2)\rho_{2s}(s_2 + s_0)\,dx\,dy\,ds\,ds_0$$

SOME COMPLICATIONS: CROSSING ANGLE (3/5)

- Assuming $\sigma_{1x} = \sigma_{2x} = \sigma_x$, $\sigma_{1y} = \sigma_{2y} = \sigma_y$, $\sigma_{1s} = \sigma_{2s} = \sigma_s$, $y_1 = y_2 = y$, and using the relation given in the

Useful relations in Introduction $\int_{-\infty}^{+\infty} e^{-\left(a\,t^2 + b\,t + c\right)}\,dt = \sqrt{\frac{\pi}{a}}\;e^{\frac{b^2}{4\,a} - c}$,

yields $\boxed{L_{CA} = L_0\;F_{CA}}$, with

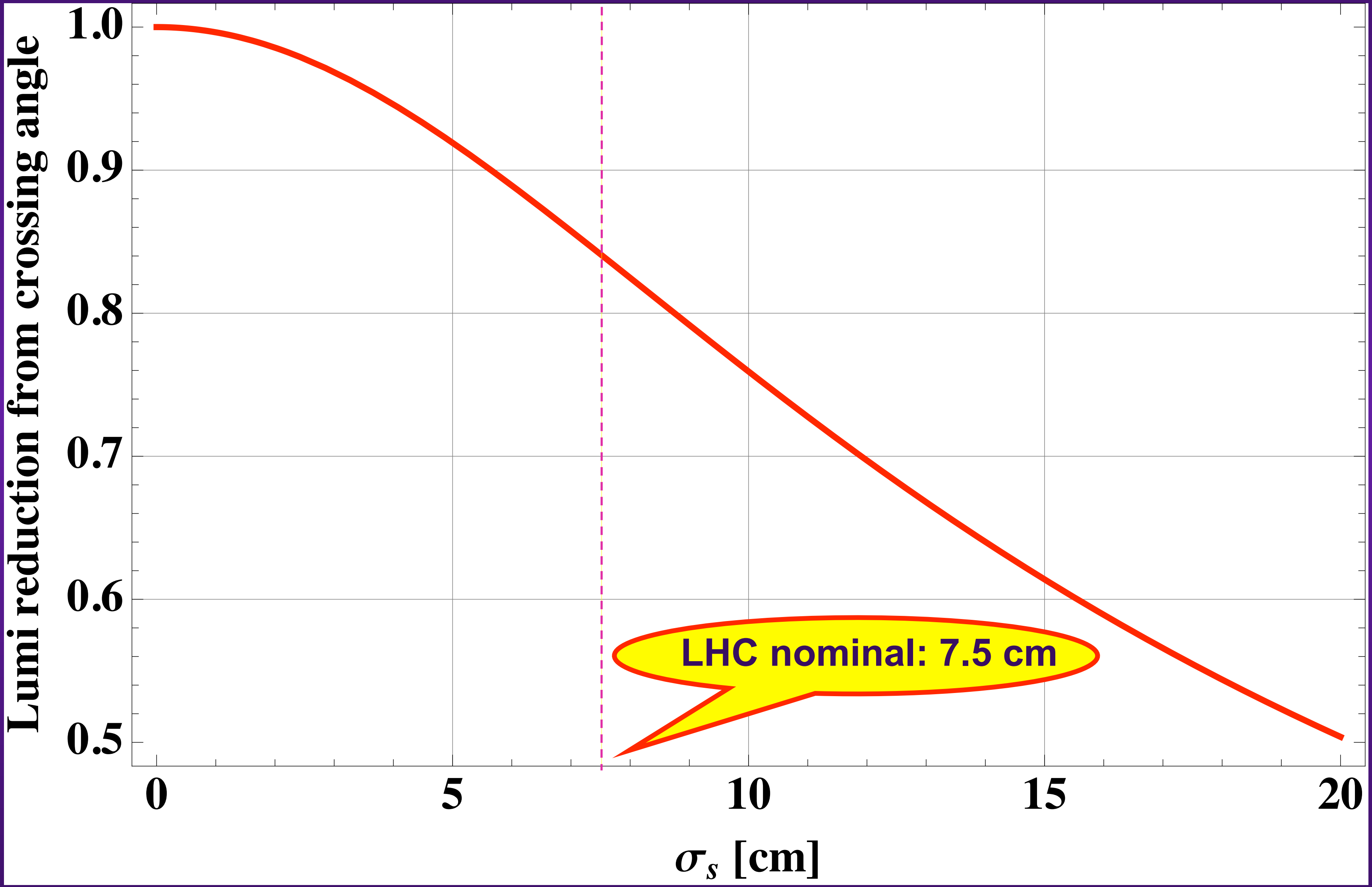
$$\boxed{F_{CA} = \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x}\tan\frac{\Phi}{2}\right)^2}}}$$

tan Φ / 2 ~ Φ / 2

- Numerical application for LHC

=> $F_{CA} = 0.84$

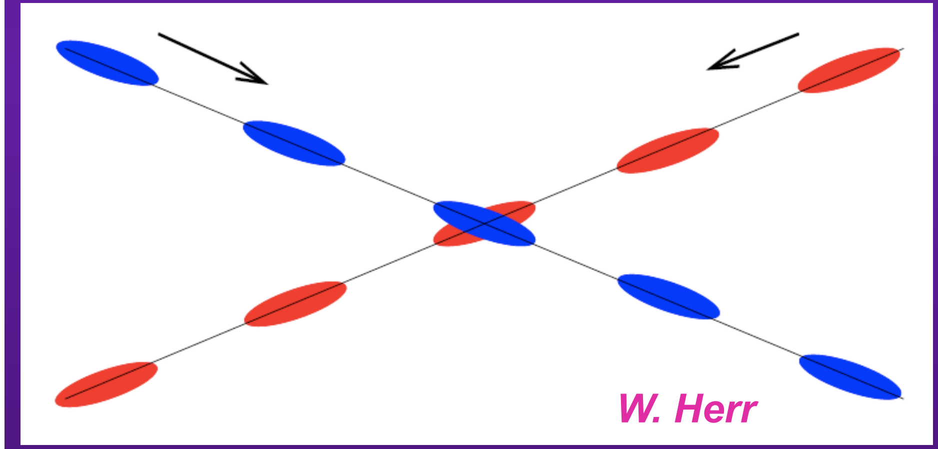
SOME COMPLICATIONS: CROSSING ANGLE (4/5)



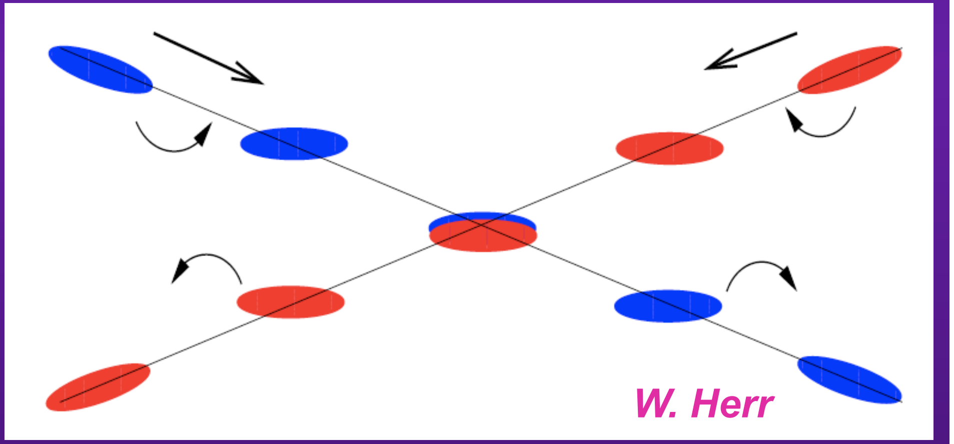
SOME COMPLICATIONS: CROSSING ANGLE (5/5)

- **Possibility to compensate for this geometric loss factor => Use "Crab Cavities"**

WITHOUT CRAB CAVITIES



WITHOUT CRAB CAVITIES

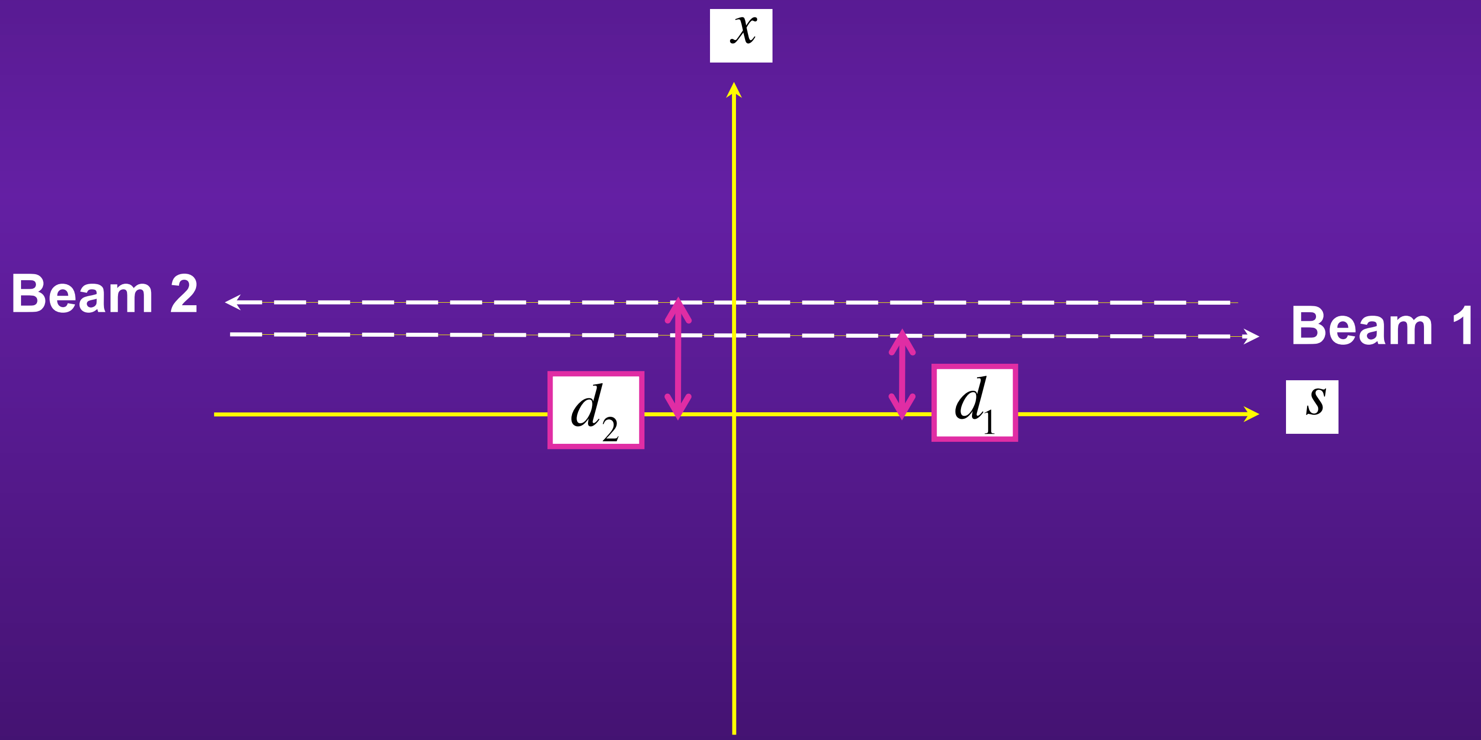


=> Already used in leptons machine (KEK-B, Japan) and planned to be used for the upgrade of the LHC

SOME COMPLICATIONS: TRANSVERSE OFFEST (1/3)

- Luminosity in the presence of a transverse offset (but no crossing angle)

$$x_1 = x + d_1 \qquad x_2 = x + d_2$$



SOME COMPLICATIONS: TRANSVERSE OFFEST (2/3)

$$L_{TO} = 2\, M\, N_1\, N_2\, f_{rev} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho_{1x}(x_1)\,\rho_{1y}(y)\,\rho_{1s}(s-s_0)\,\rho_{2x}(x_2)\,\rho_{2y}(y)\,\rho_{2s}(s+s_0)\,dx\,dy\,ds\,ds_0$$

- Assuming $\sigma_{1x} = \sigma_{2x} = \sigma_x$, $\sigma_{1y} = \sigma_{2y} = \sigma_y$, $\sigma_{1s} = \sigma_{2s} = \sigma_s$, $y_1 = y_2 = y$, and using the relation given in the

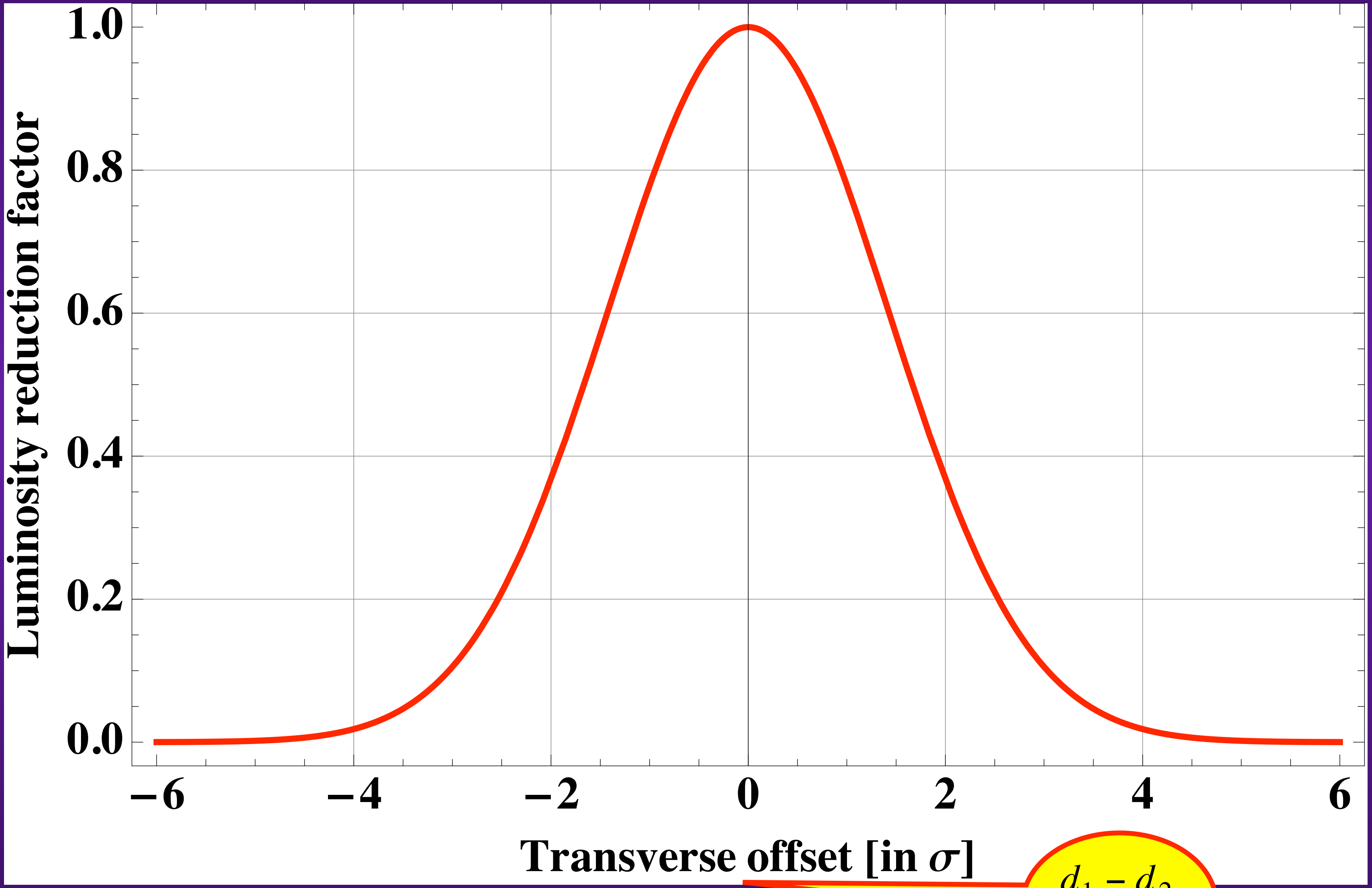
  Useful relations in Introduction, yields

$$L_{TO} = L_0\, F_{TO}$$

with

$$F_{TO} = e^{-\left(\frac{d_1 - d_2}{2\sigma_x}\right)^2}$$

SOME COMPLICATIONS: TRANSVERSE OFFEST (3/3)

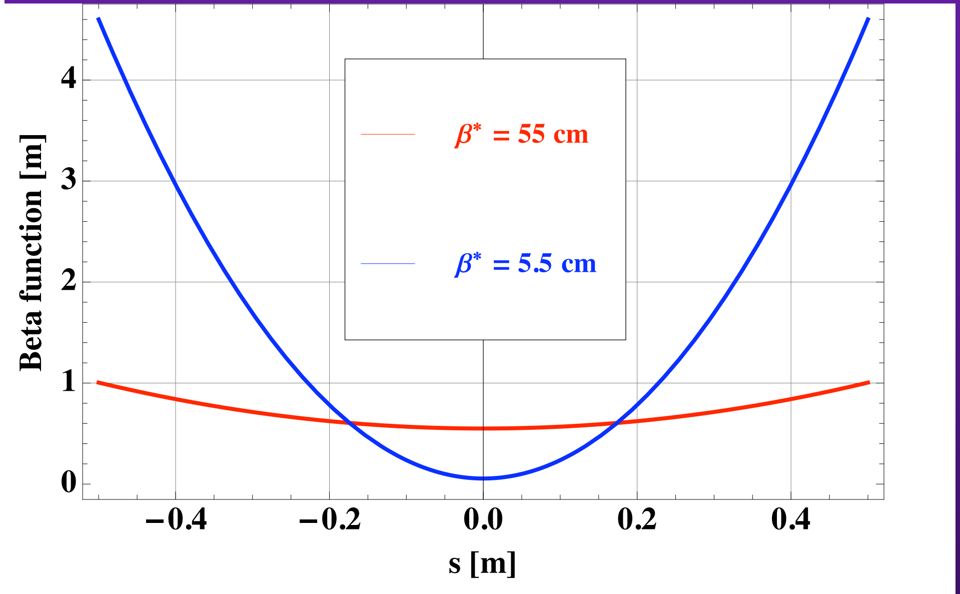


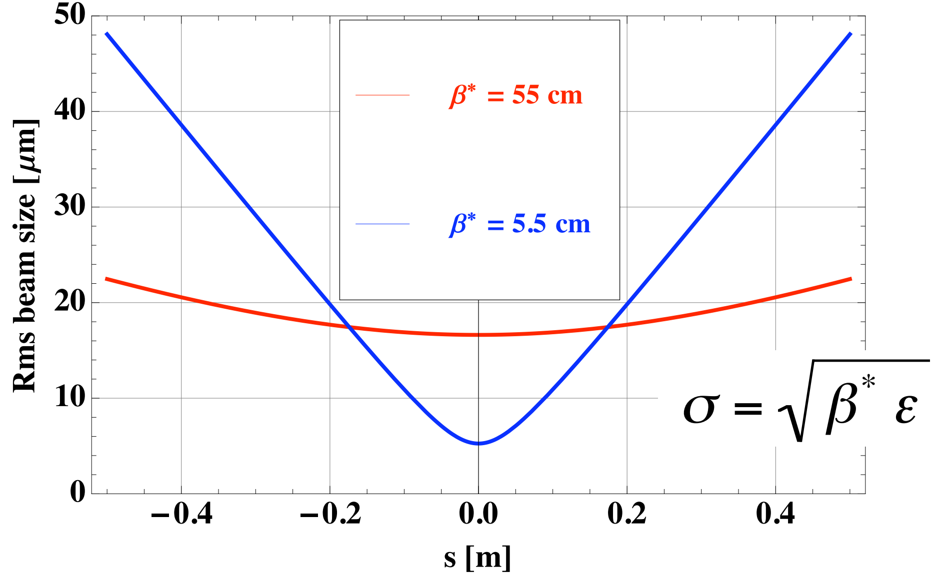
$$\frac{d_1 - d_2}{\sigma_x}$$

SOME COMPLICATIONS: HOURGLASS EFFECT (1/3)

- **Luminosity in the presence of the Hourglass effect (and crossing angle in the x-s plane but no transverse offset)**
  - Close to the IP, the beta function is given by (see course on Transverse Beam Dynamics)

$$\beta(s) = \beta^* \left[1 + \left(\frac{s}{\beta^*} \right)^2 \right]$$





$$\sigma = \sqrt{\beta^* \varepsilon}$$

SOME COMPLICATIONS: HOURGLASS EFFECT (2/3)

- **Following the same approach as before, yields**

$$L_{CA\,\&\,HG} = \frac{\cos\frac{\Phi}{2}\,N_1\,N_2\,M\,f_{rev}}{4\,\pi^{3/2}\,\sigma_s\,\sigma_x^*\,\sigma_y^*}\int_{-\infty}^{+\infty} d\,s\,\frac{e^{-s^2\left\{\frac{\sin^2\frac{\Phi}{2}}{\sigma_x^{*2}\left[1+\left(\frac{s}{\beta^*}\right)^2\right]}+\frac{\cos^2\frac{\Phi}{2}}{\sigma_s^2}\right\}}}{\left[1+\left(\frac{s}{\beta^*}\right)^2\right]}$$

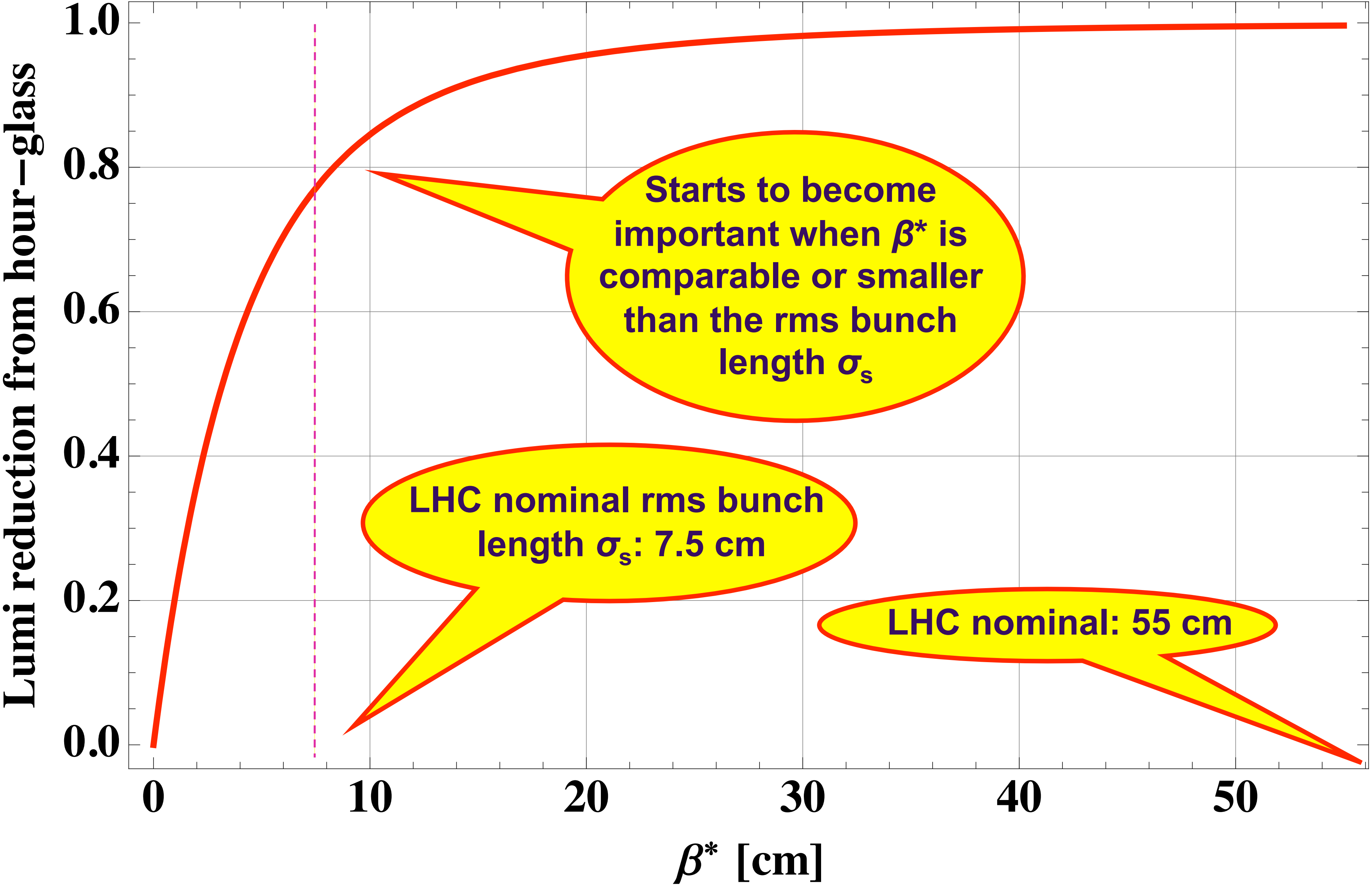
=>

$$\boxed{L_{CA\,\&\,HG} = L_{CA}\;F_{HG}}$$

with

$$\boxed{F_{HG} = \frac{\sqrt{\frac{\sin^2\frac{\Phi}{2}}{\sigma_x^{*2}}+\frac{\cos^2\frac{\Phi}{2}}{\sigma_s^2}}}{\sqrt{\pi}}\int_{-\infty}^{+\infty} d\,s\,\frac{e^{-s^2\left\{\frac{\sin^2\frac{\Phi}{2}}{\sigma_x^{*2}\left[1+\left(\frac{s}{\beta^*}\right)^2\right]}+\frac{\cos^2\frac{\Phi}{2}}{\sigma_s^2}\right\}}}{1+\left(\frac{s}{\beta^*}\right)^2}}$$

SOME COMPLICATIONS: HOURGLASS EFFECT (3/3)



Starts to become important when β^* is comparable or smaller than the rms bunch length σ_s

LHC nominal rms bunch length σ_s: 7.5 cm

LHC nominal: 55 cm

INTEGRATED LUMINOSITY AND MAXIMIZATION (1/4)

- Integrated luminosity $L_{\text{int}} = \int_0^T L(t)\,dt$

- Real figure of merit $L_{\text{int}}\,\sigma_r = \text{number of events}$

- Let's assume some luminosity lifetime behaviour => Exponential decay (due to intensity decay, emittance growth, etc.)

$$L(t) = L_{peak}\, e^{-\frac{t}{\tau_l}}$$

Luminosity lifetime

- What is the best run time t_r?

INTEGRATED LUMINOSITY AND MAXIMIZATION (2/4)

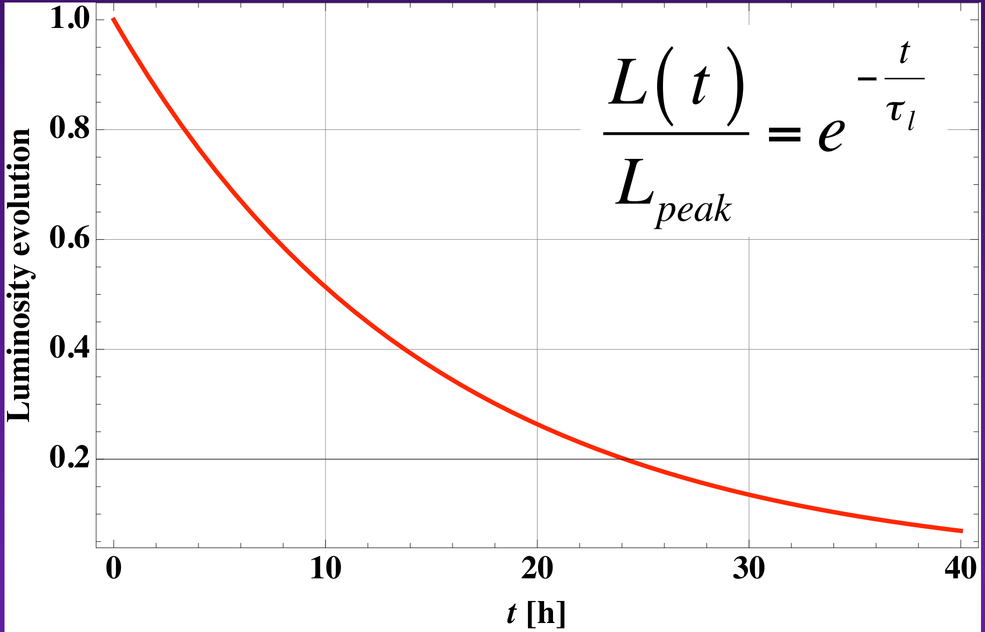
- Let's call t_p the preparation time (time needed to put the beams in collision after the end of the previous physics fill) => Optimization of t_r and t_p gives the maximum luminosity

$$\langle L \rangle = \frac{1}{t_r + t_p} \int_0^{t_r + t_p} L(t)\, dt$$

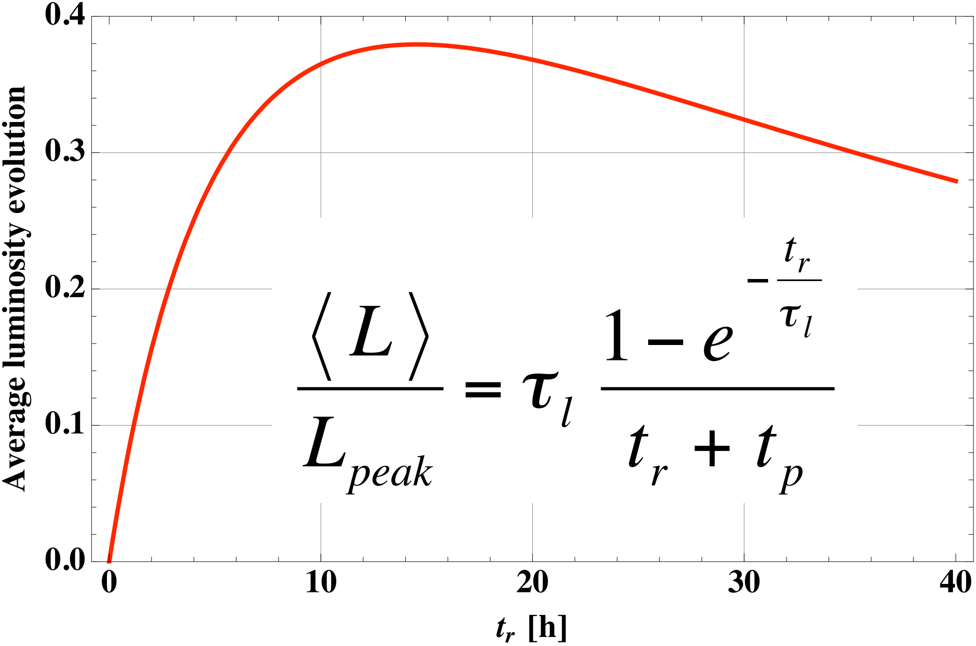
=>

$$\langle L \rangle = L_{peak}\, \tau_l \, \frac{1 - e^{-\frac{t_r}{\tau_l}}}{t_r + t_p}$$

INTEGRATED LUMINOSITY AND MAXIMIZATION (3/4)



$$\frac{L(t)}{L_{peak}} = e^{-\frac{t}{\tau_l}}$$



$$\frac{\langle L \rangle}{L_{peak}} = \tau_l \, \frac{1 - e^{-\frac{t_r}{\tau_l}}}{t_r + t_p}$$

$\tau_l = 15 \text{ h}$

$\tau_p = 10 \text{ h}$

INTEGRATED LUMINOSITY AND MAXIMIZATION (4/4)

- **The average luminosity is maximum when**

$$t_r \approx \tau_l \ln\left(1 + \sqrt{2\,\frac{t_p}{\tau_l}} + \frac{t_p}{\tau_l}\right)$$

Gives ~ 15.5 h…

PILE-UP, LEVELING AND LUMINOUS REGION (1/3)

- Pile-Up (PU) = Number of events / crossing for a given luminosity

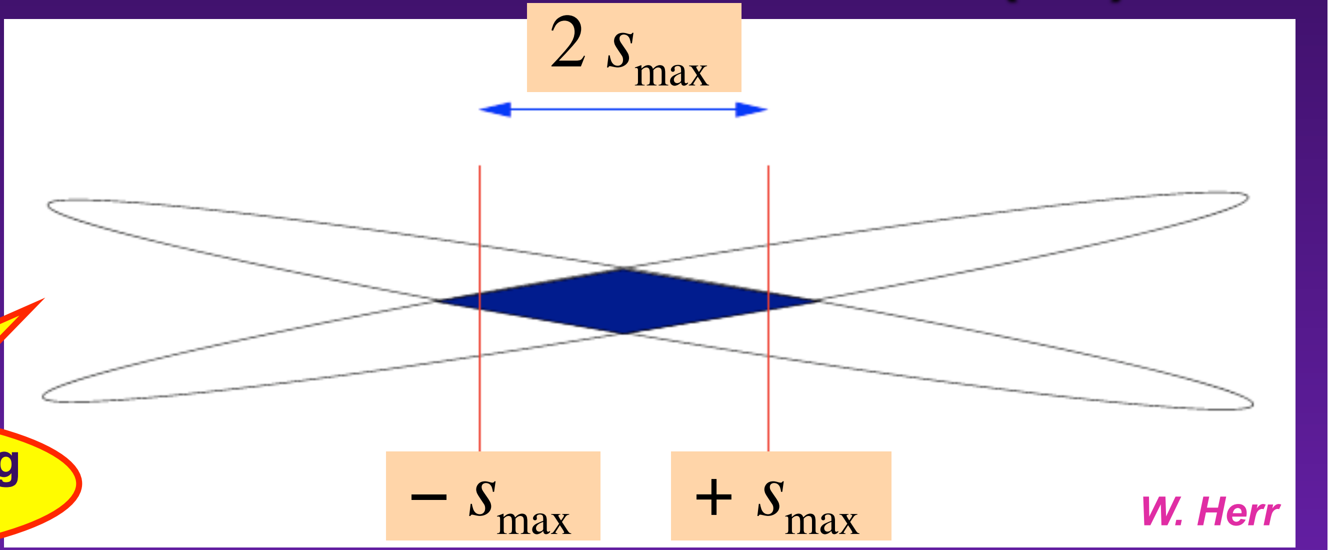
$$\text{PU} = \frac{L\,\sigma_r}{M\ f_{rev}}$$

  - This is a limit coming from the experiments' detectors => Better to have larger number of bunches (for the same beam intensity)

- In case the pile-up is too big, luminosity leveling techniques could be used to remain at the limit => Playing with the different parameters which can reduce the luminosity (transverse beam offset, β^*, etc.)

PILE-UP, LEVELING AND LUMINOUS REGION (2/3)

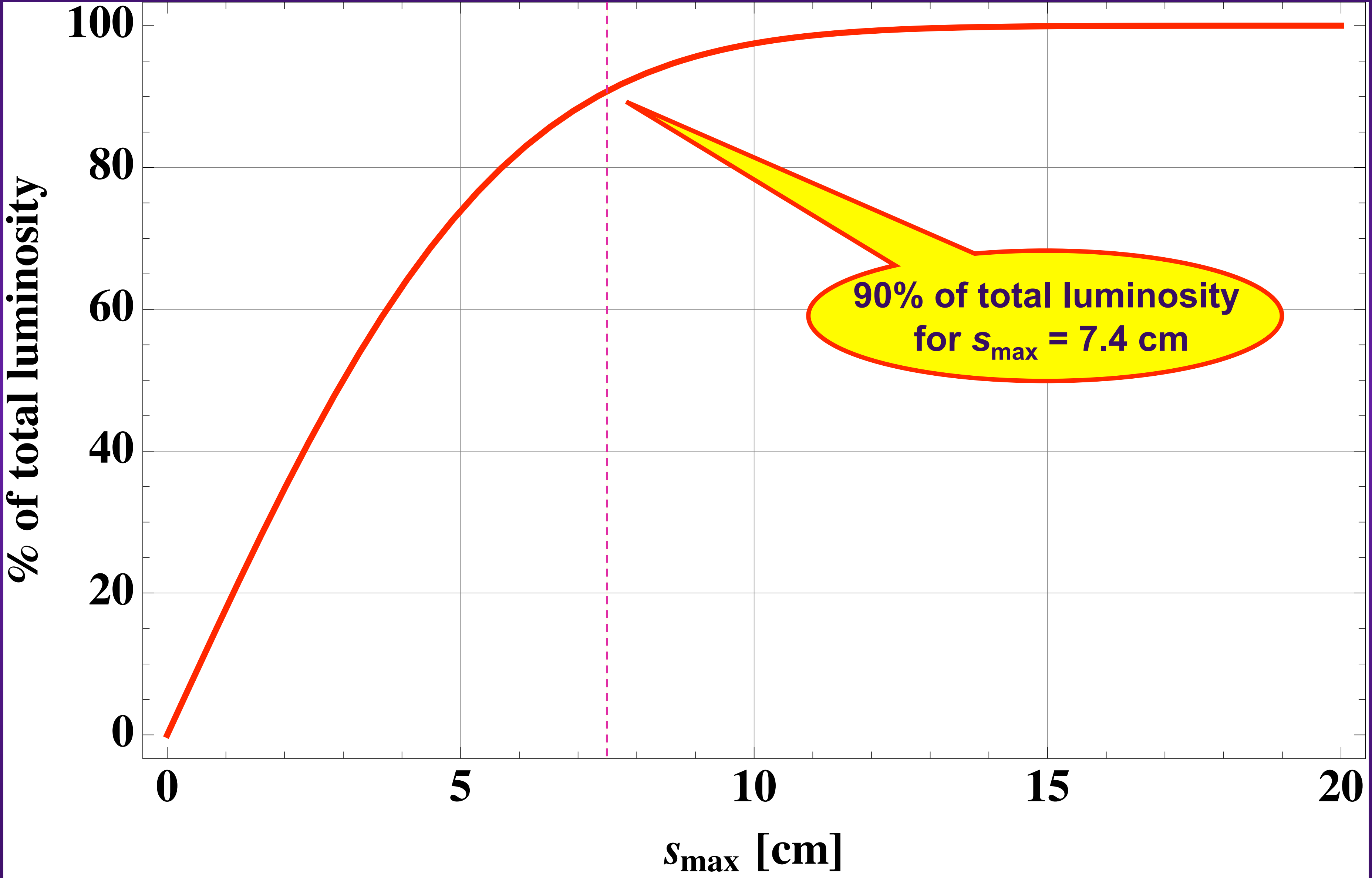
- **Luminous region**



In the case of crossing angle only

$$\frac{L_{CA}\left(s_{\max}\right)}{L_{CA}\left(+\infty\right)} = \int_{-s_{\max}}^{+s_{\max}} d\,s\,\frac{e^{-s^{2}\left\{\frac{\sin^{2}\frac{\Phi}{2}}{\sigma_x^{*2}\left[1+\left(\frac{s}{\beta^{*}}\right)^{2}\right]}+\frac{\cos^{2}\frac{\Phi}{2}}{\sigma_s^{2}}\right\}}}{\left[1+\left(\frac{s}{\beta^{*}}\right)^{2}\right]} \Bigg/ \int_{-\infty}^{+\infty} d\,s\,\frac{e^{-s^{2}\left\{\frac{\sin^{2}\frac{\Phi}{2}}{\sigma_x^{*2}\left[1+\left(\frac{s}{\beta^{*}}\right)^{2}\right]}+\frac{\cos^{2}\frac{\Phi}{2}}{\sigma_s^{2}}\right\}}}{\left[1+\left(\frac{s}{\beta^{*}}\right)^{2}\right]}$$

PILE-UP, LEVELING AND LUMINOUS REGION (3/3)



90% of total luminosity for s_{max} = 7.4 cm

SUMMARY: HOW TO REACH HIGH LUMINOSITY?

- High beam intensities
  - High bunch intensity => More efficient (for the same beam intensity) but pile-up issue for the experiments' detectors
  - High number of bunches => Less efficient but better for the pile-up
- Small transverse beam sizes (small transverse emittance and beta function at the IP)
- High energy
- Small crossing angle
- Small transverse offset
- Short bunches