TRAINING-WEEK IN ACCELERATOR PHYSICS

Elias Métral

- **Programme of the week**

	Morning **(lectures: 2 × 45 min)**	**Afternoon** **(problem solving, individual work)**
MO 27/05/13	Introduction and luminosity	Exercises on luminosity
TU 28/05/13	Transverse beam dynamics	Exercises on transverse beam dynamics
WE 29/05/13	Longitudinal beam dynamics	Exercises on longitudinal beam dynamics
TH 30/05/13	Collective effects (space charge, impedances and related instabilities, beam-beam and e-cloud)	Tutorial on MAD-X code (for transverse beam dynamics) + Exercises on collective effects
FR 31/05/13	Feedback and hand-out of last problem (to be solved after the course)	Reserve time

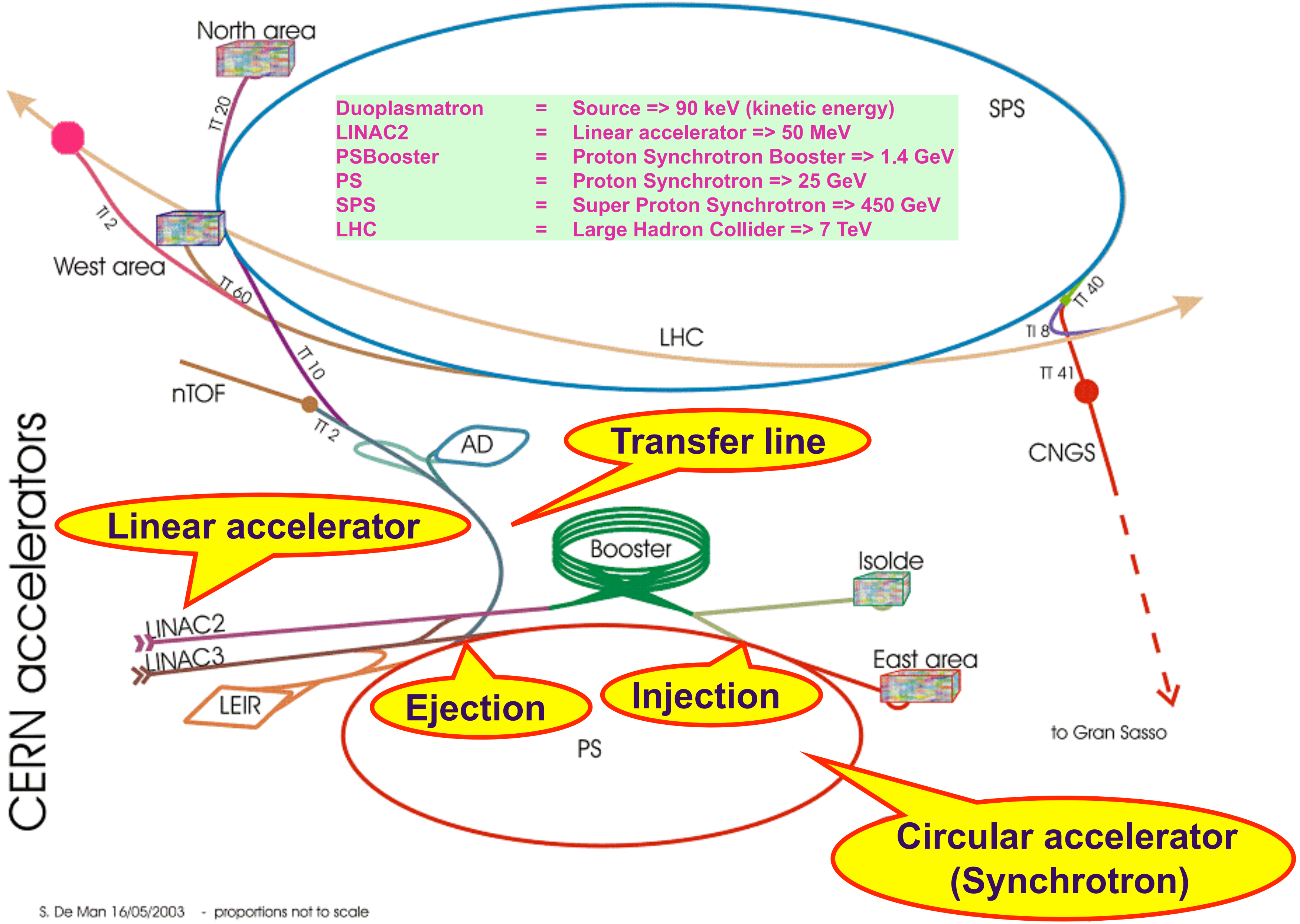
- **Introduction**

Elias.Metral@cern.ch
Tel.: 00 41 76 487 48 09
http://emetral.web.cern.ch/emetral/

CONCEPTS AND PREREQUISITES

- **BEAM DYNAMICS** describes the motion of a charged particle beam in an accelerator
- **LOW-INTENSITY PARTICLE BEAMS** can be modeled by using single-particle dynamics, in which particles are tracked through the external electromagnetic fields (from the guiding and focusing magnets in the transverse planes, RF cavities in the longitudinal plane, etc.) => Classical mechanics (linear and nonlinear), electrodynamics, physical or engineering mathematics and special relativity
- **HIGH-INTENSITY (and or HIGH-DENSITY) PARTICLE BEAMS** require a more complicated description which involves interactions between the beam particles and between the beam particles and their environment (and/or other particles) => Plasma physics. High-intensity (and or high-density) effects are very important because they usually pose an upper limitation to the number of particles that can be injected into an accelerator

Example of the LHC p beam in the injector chain



Duoplasmatron = Source => 90 keV (kinetic energy)
LINAC2 = Linear accelerator => 50 MeV
PSBooster = Proton Synchrotron Booster => 1.4 GeV
PS = Proton Synchrotron => 25 GeV
SPS = Super Proton Synchrotron => 450 GeV
LHC = Large Hadron Collider => 7 TeV

Transfer line

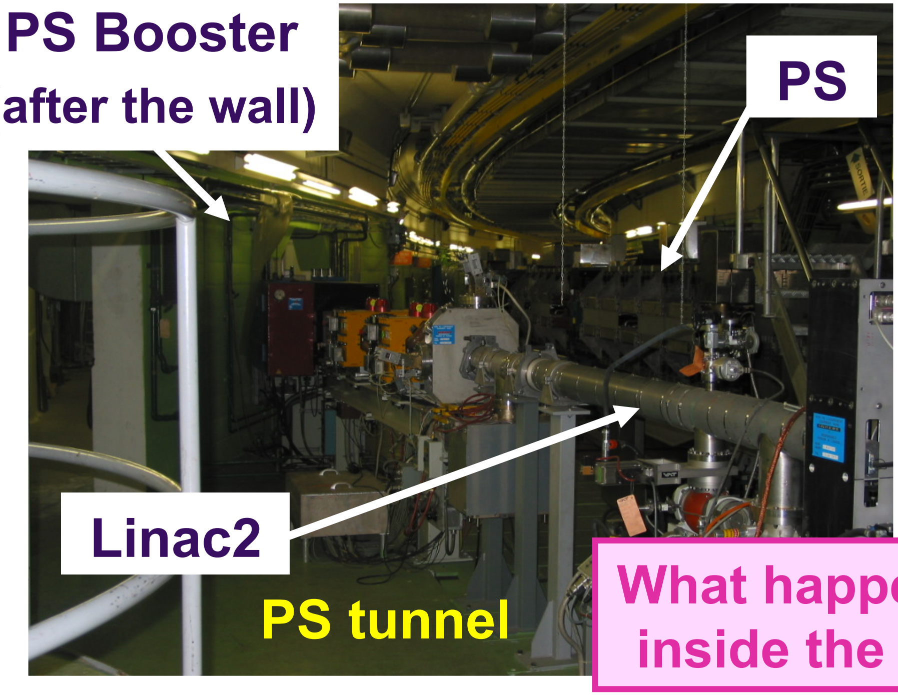
Linear accelerator

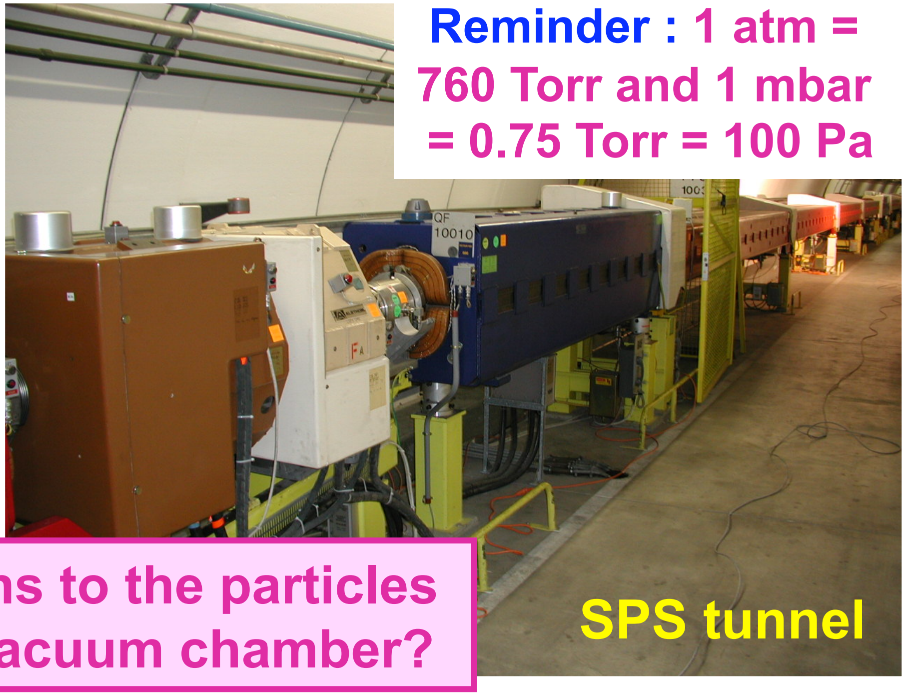
Ejection

Injection

Circular accelerator (Synchrotron)

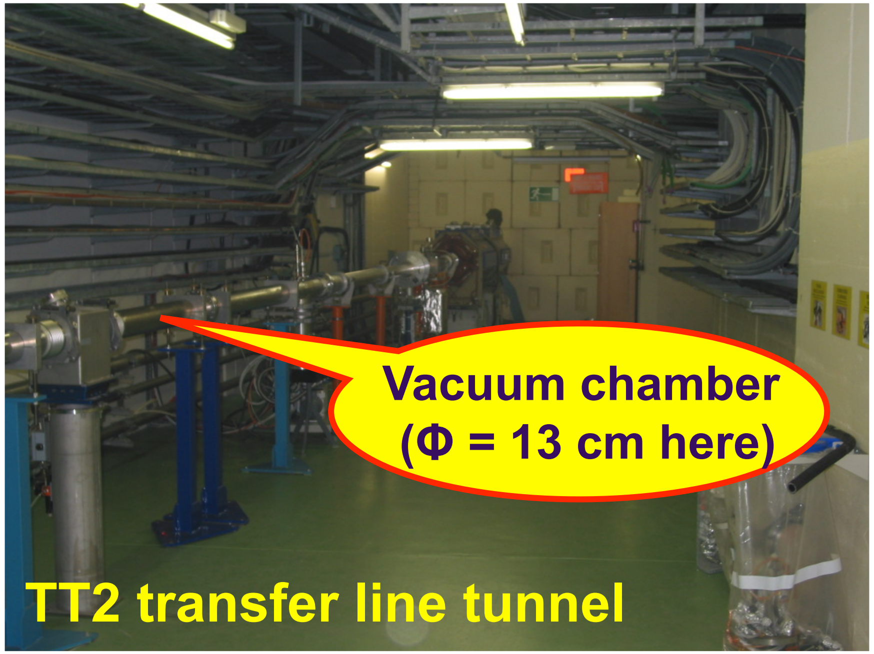
S. De Man 16/05/2003 - proportions not to scale

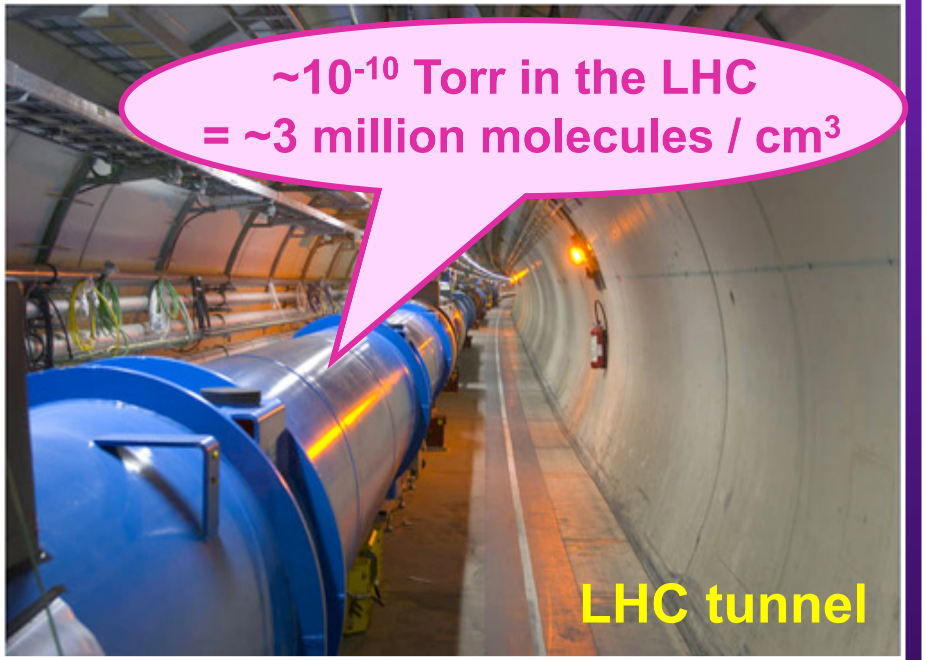




Reminder : 1 atm = 760 Torr and 1 mbar = 0.75 Torr = 100 Pa

What happens to the particles inside the vacuum chamber?



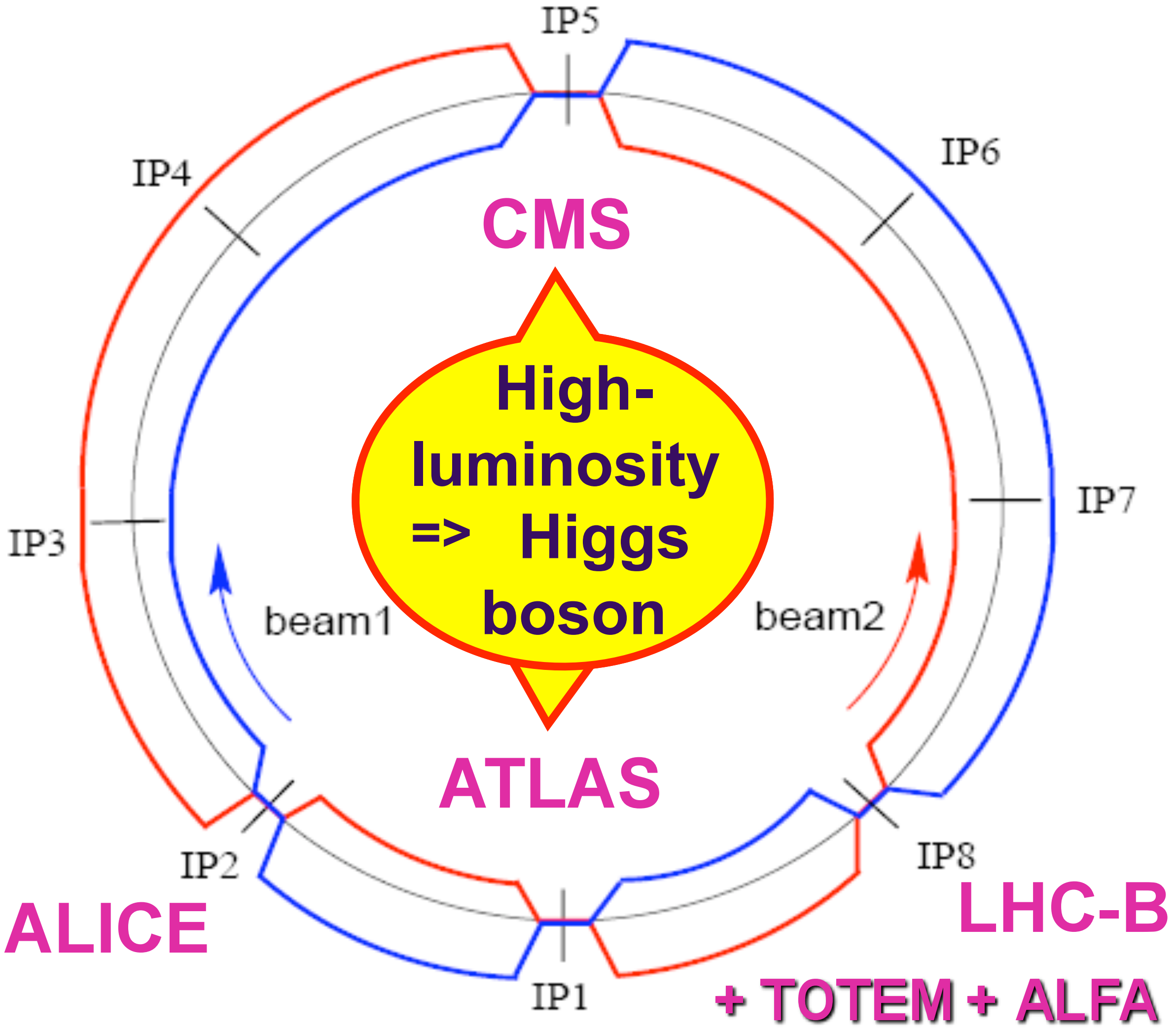


~10^{-10} Torr in the LHC = ~3 million molecules / cm^3

LAYOUT OF THE LHC

Courtesy W. Herr

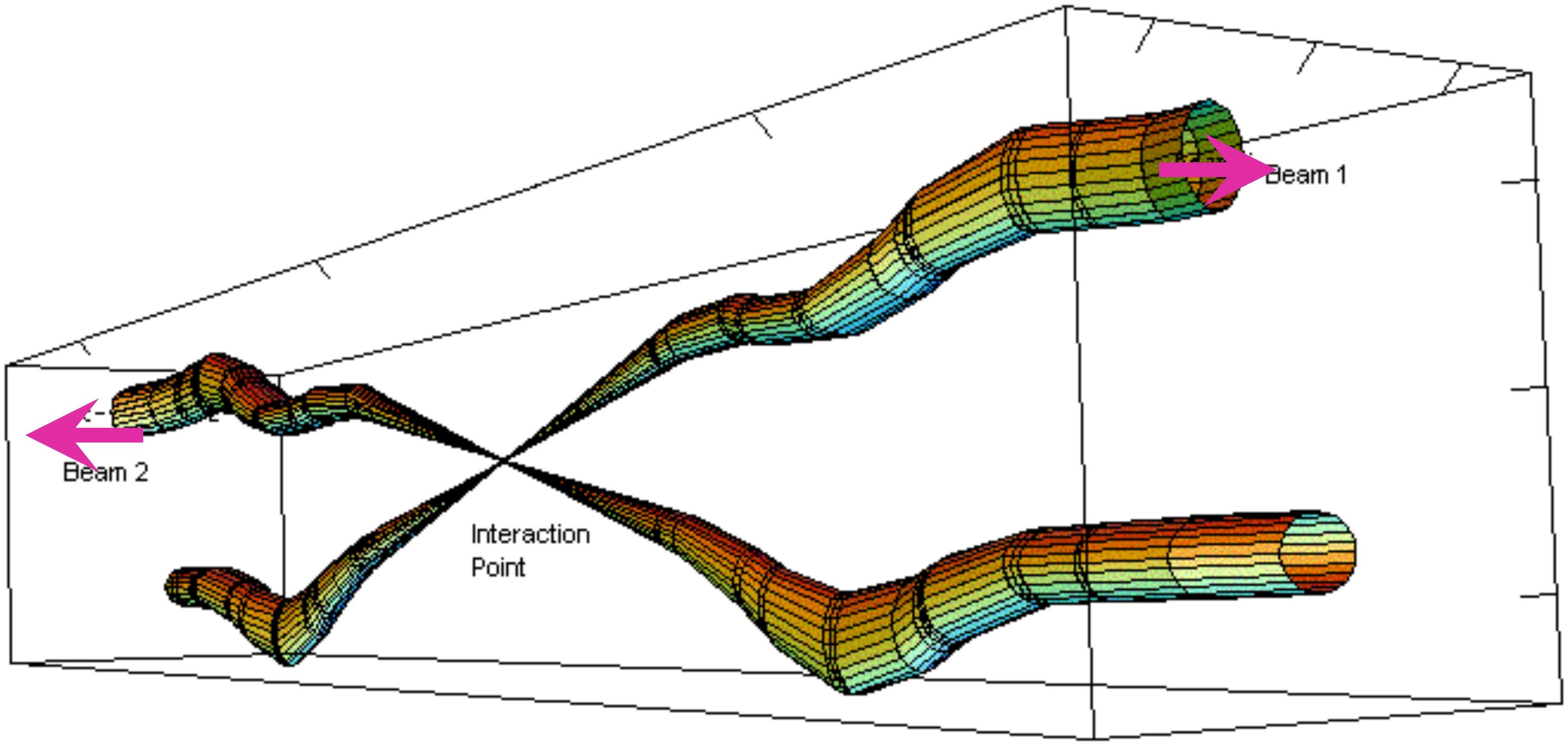
IP = Interaction Point



High-luminosity => Higgs boson

+ TOTEM + ALFA

COLLISION in IP1 (ATLAS)



Relative beam sizes around IP1 (Atlas) in collision

⇒ Vertical crossing angle in IP1 (ATLAS) and horizontal one in IP5 (CMS)

FIGURE OF MERIT for a synchrotron / collider: Brightness / luminosity

- **(2D) BEAM BRIGHTNESS** $B = \dfrac{I}{\pi^2 \; \varepsilon_x \; \varepsilon_y}$

Beam current

Transverse emittances

- **MACHINE LUMINOSITY** $L = \dfrac{N_{events/\sec ond}}{\sigma_r}$

Number of events per second generated in the collisions

Cross-section of the reaction

[cm^{-2} s^{-1}]

- **The Luminosity depends only on the beam parameters**
  ⇒ It is independent of the physical reaction
- **Reliable procedures to compute and measure**

⇒ For a Gaussian (round) beam distribution

$$L = \frac{N_b^2 \; M \; f_0 \; \gamma}{4 \; \pi \; \varepsilon_n \; \beta^*} \; F_{ca}$$

Number of particles per bunch

Number of bunches per beam

Revolution frequency

Relativistic mass factor

Normalized rms transverse beam emittance

β-function at the collision point

Geometric reduction factor due to the crossing angle at the IP

- **PEAK LUMINOSITY for ATLAS&CMS in the LHC =** $L_{peak} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

Number of particles per bunch	N_b	1.15×10^{11}
Number of bunches per beam	M	2808
Revolution frequency	f_0	11245 Hz
Relativistic velocity factor	γ	7461 (=> E = 7 TeV)
β-function at the collision point	β^*	55 cm
Normalised rms transverse beam emittance	ε_n	3.75×10^{-4} cm
Geometric reduction factor	F_{ca}	0.84

$$F_{ca} = 1 / \sqrt{1 + \left(\frac{\theta_c \, \sigma_s}{2 \, \sigma^*} \right)^2}$$

Full crossing angle at the IP	θ_c	285 µrad
Rms bunch length	σ_s	7.55 cm
Transverse rms beam size at the IP	σ^*	16.7 µm

- **INTEGRATED LUMINOSITY** $L_{\text{int}} = \int_0^T L(t)\, dt$

⇒ The real figure of merit = $L_{\text{int}}\, \sigma_r = \text{number of events}$

- LHC integrated Luminosity expected per year (~10^7 s): [80-120] fb^{-1}

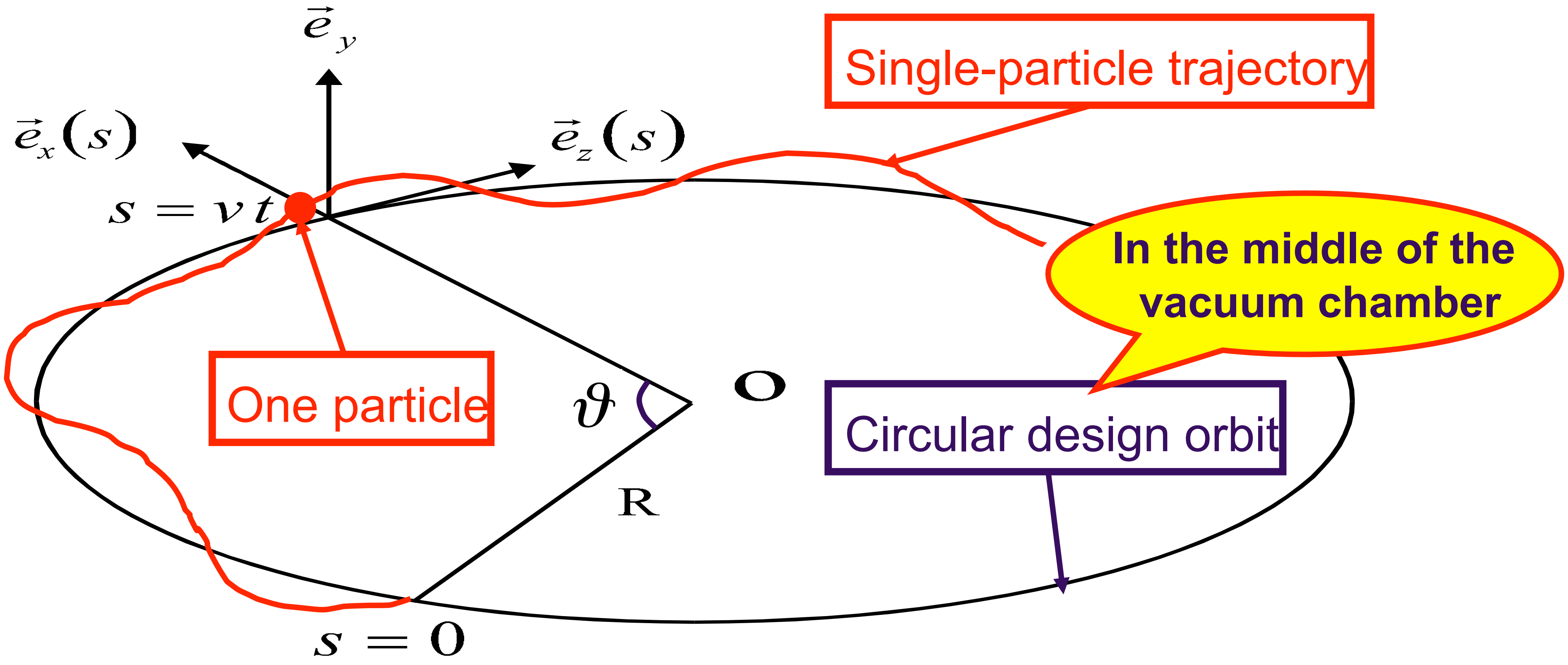
Reminder: 1 barn = 10^{-24} cm^2
and femto = 10^{-15}

ESTIMATIONS MADE BEFORE THE LHC STARTED

- The total proton-proton cross section at 7 TeV is ~ 110 mbarns:
  - Inelastic ➔ σ_{in} = 60 mbarns
  - Single diffractive ➔ σ_{sd} = 12 mbarns
  - Elastic ➔ σ_{el} = 40 mbarns
- The cross section from elastic scattering of the protons and diffractive events will not be seen by the detectors as it is only the inelastic scatterings that give rise to particles at sufficient high angles with respect to the beam axis
- Inelastic event rate at nominal luminosity = $10^{34} \times 60 \times 10^{-3} \times 10^{-24}$ = 600 millions / second per high-luminosity experiment

- The bunch spacing in the LHC is 25 ns → Crossing rate of 40 MHz
- However, there are bigger gaps (for the kickers) → Average crossing rate = number of bunches × revolution frequency = 2808 × 11245 = 31.6 MHz
- (600 millions inelastic events / second) / (31.6 × 10^6) = 19 inelastic events per crossing
- Total inelastic events per year (~10^7 s) = 600 millions × 10^7 = 6 × 10^{15} ~ 10^{16}
- The LHC experimental challenge is to find rare events at levels of 1 in 10^{13} or more → ~ 1000 Higgs events in each of the ATLAS and CMS experiments expected per year

ACCELERATOR MODEL



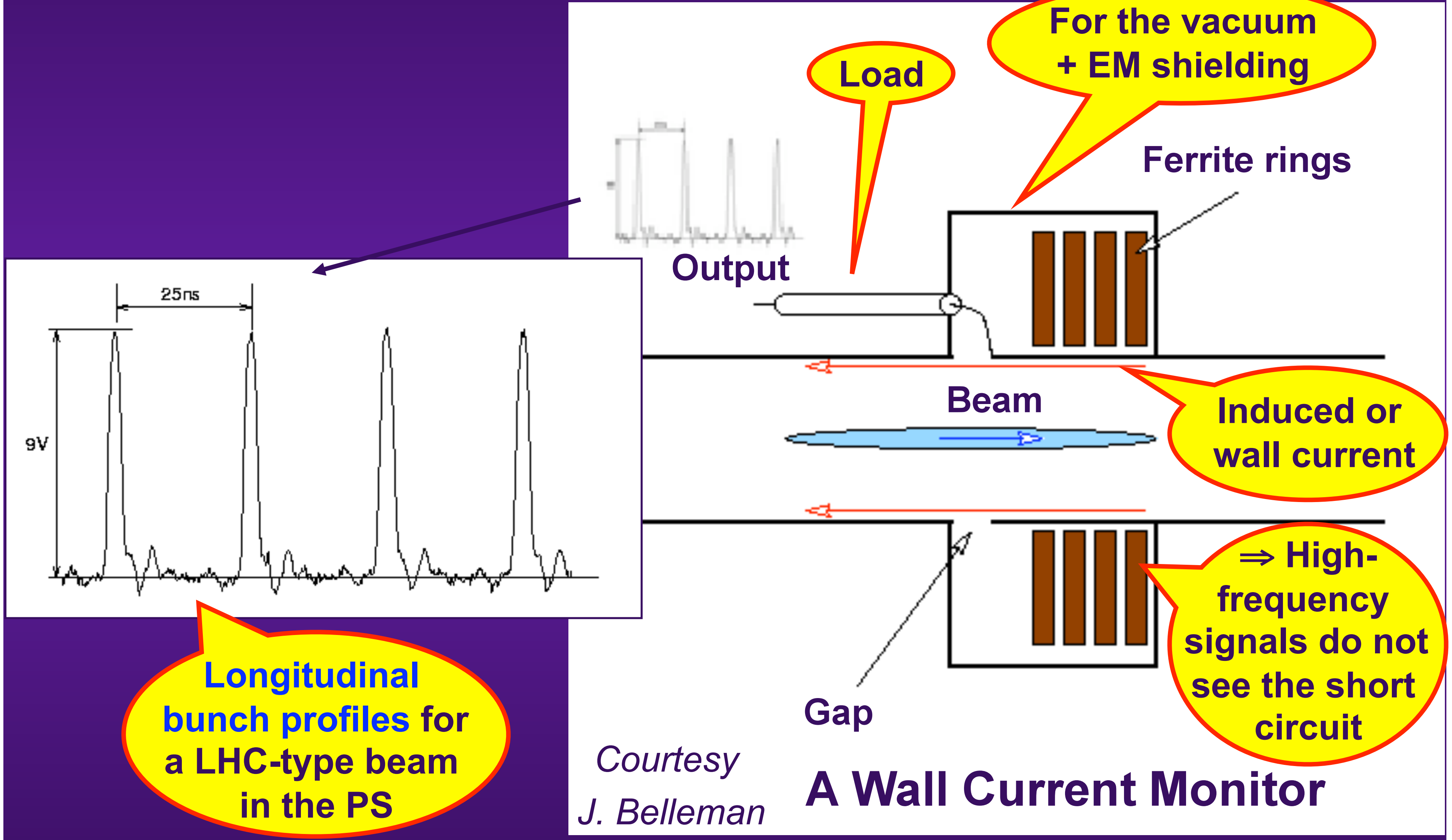
Single-particle trajectory

In the middle of the vacuum chamber

One particle

Circular design orbit

WALL CURRENT MONITOR = Device used to measure the instantaneous value of the beam current



Load

For the vacuum + EM shielding

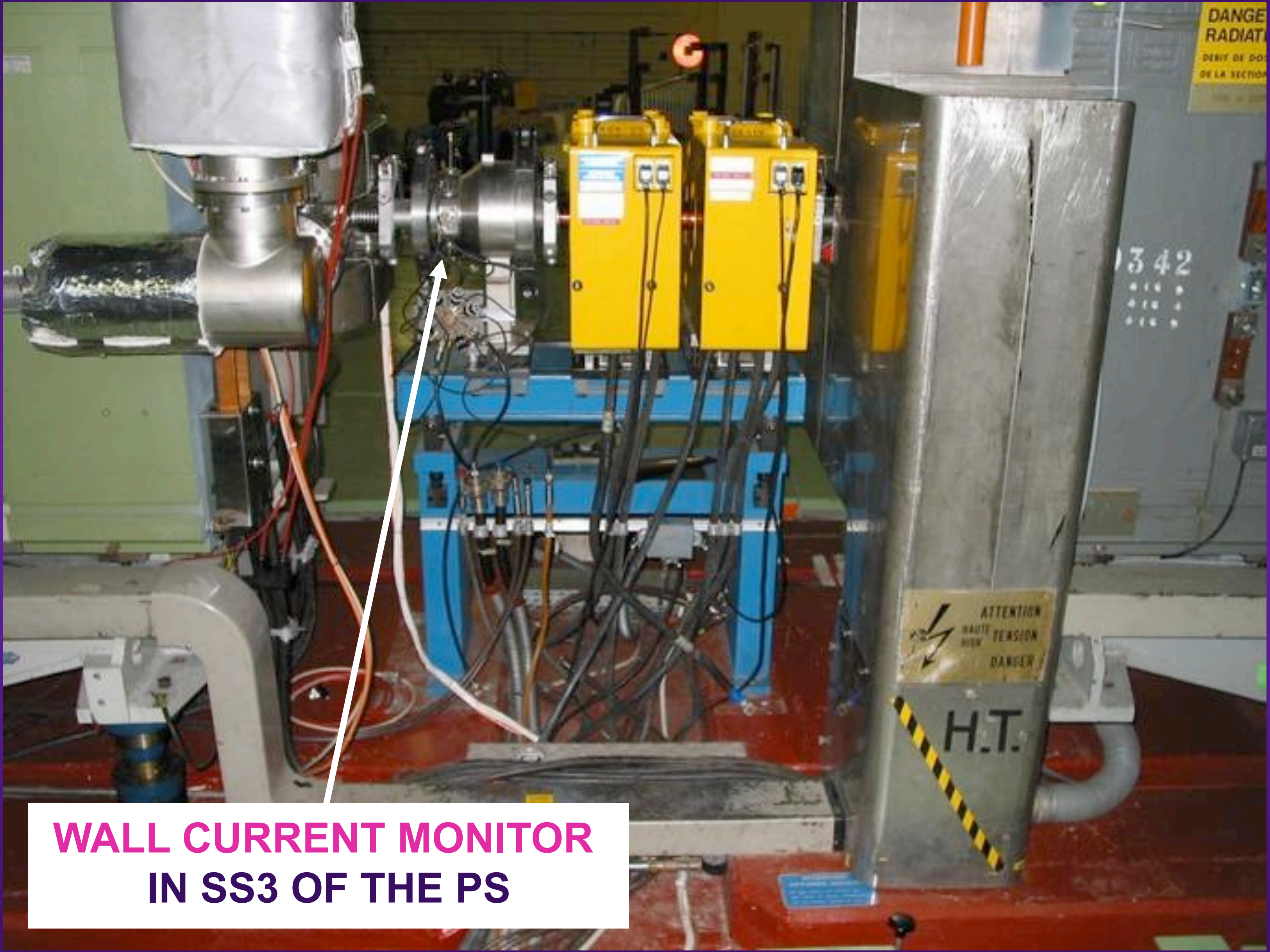
Induced or wall current

⇒ High-frequency signals do not see the short circuit

Longitudinal bunch profiles for a LHC-type beam in the PS

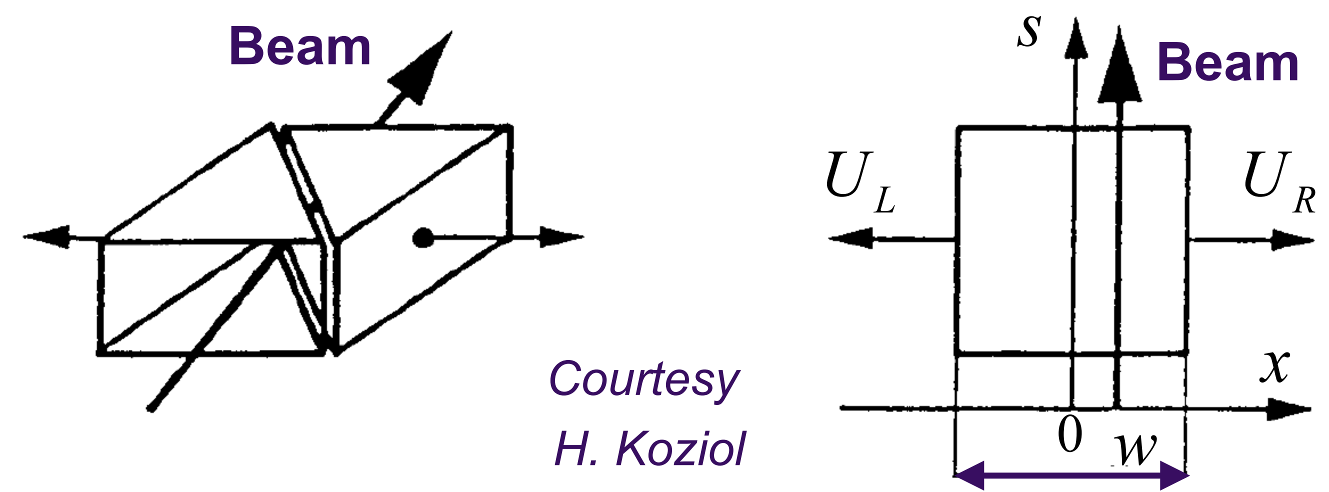
Courtesy J. Belleman

A Wall Current Monitor



WALL CURRENT MONITOR
IN SS3 OF THE PS

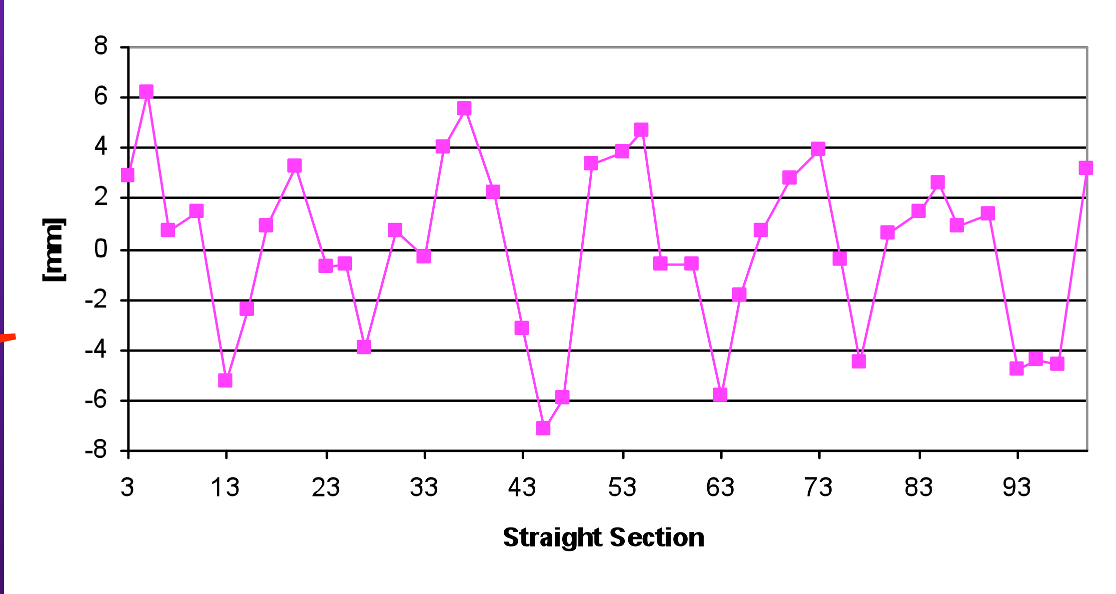
(Transverse) beam POSITION PICK-UP MONITOR



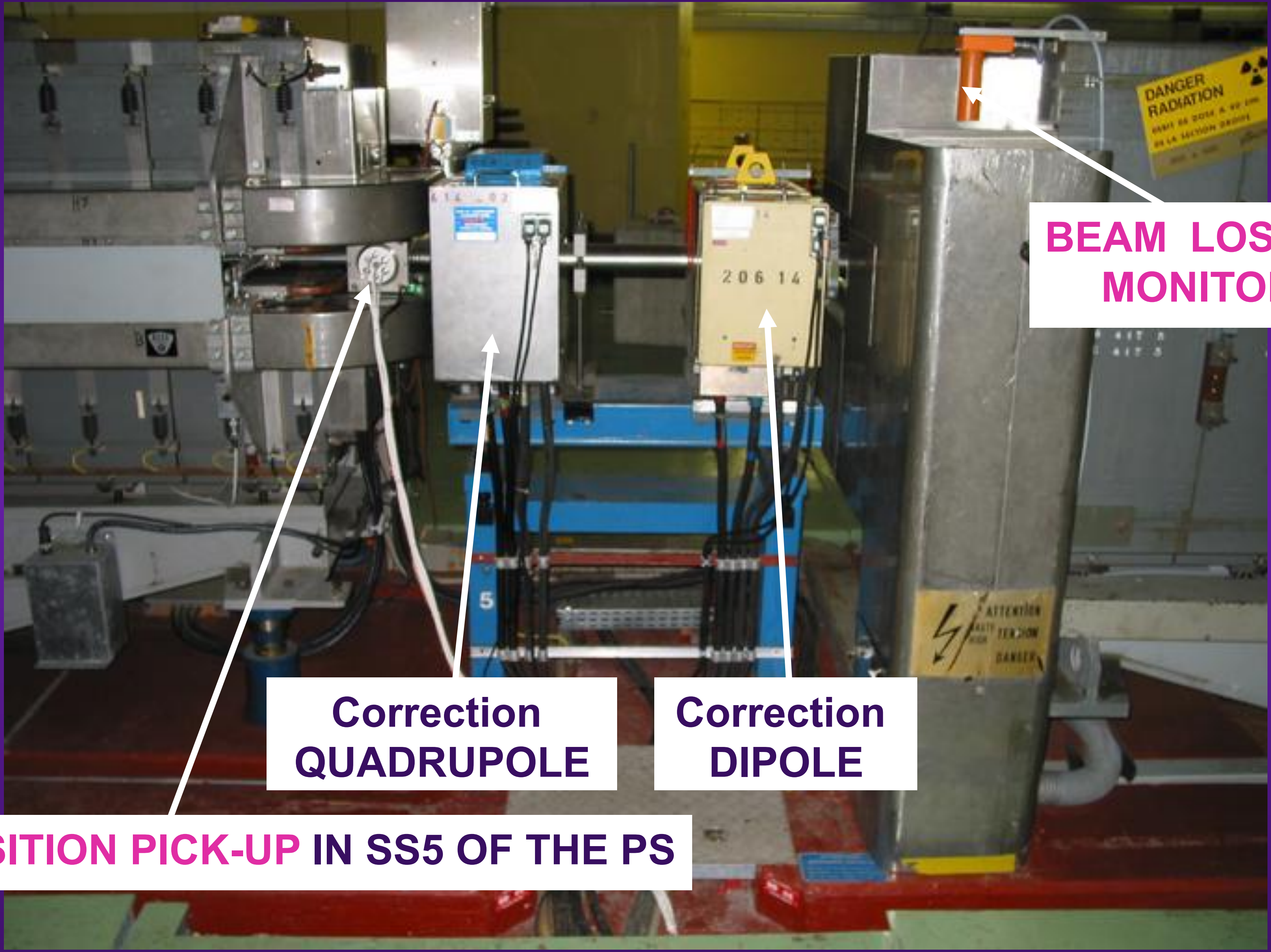
Courtesy H. Koziol

$$x = \frac{w}{2} \frac{U_R - U_L}{U_R + U_L}$$

⇒ **Horizontal beam orbit measurement in the PS**



6 spikes observed as $Q_x \approx 6.25$



BEAM LOSS MONITOR

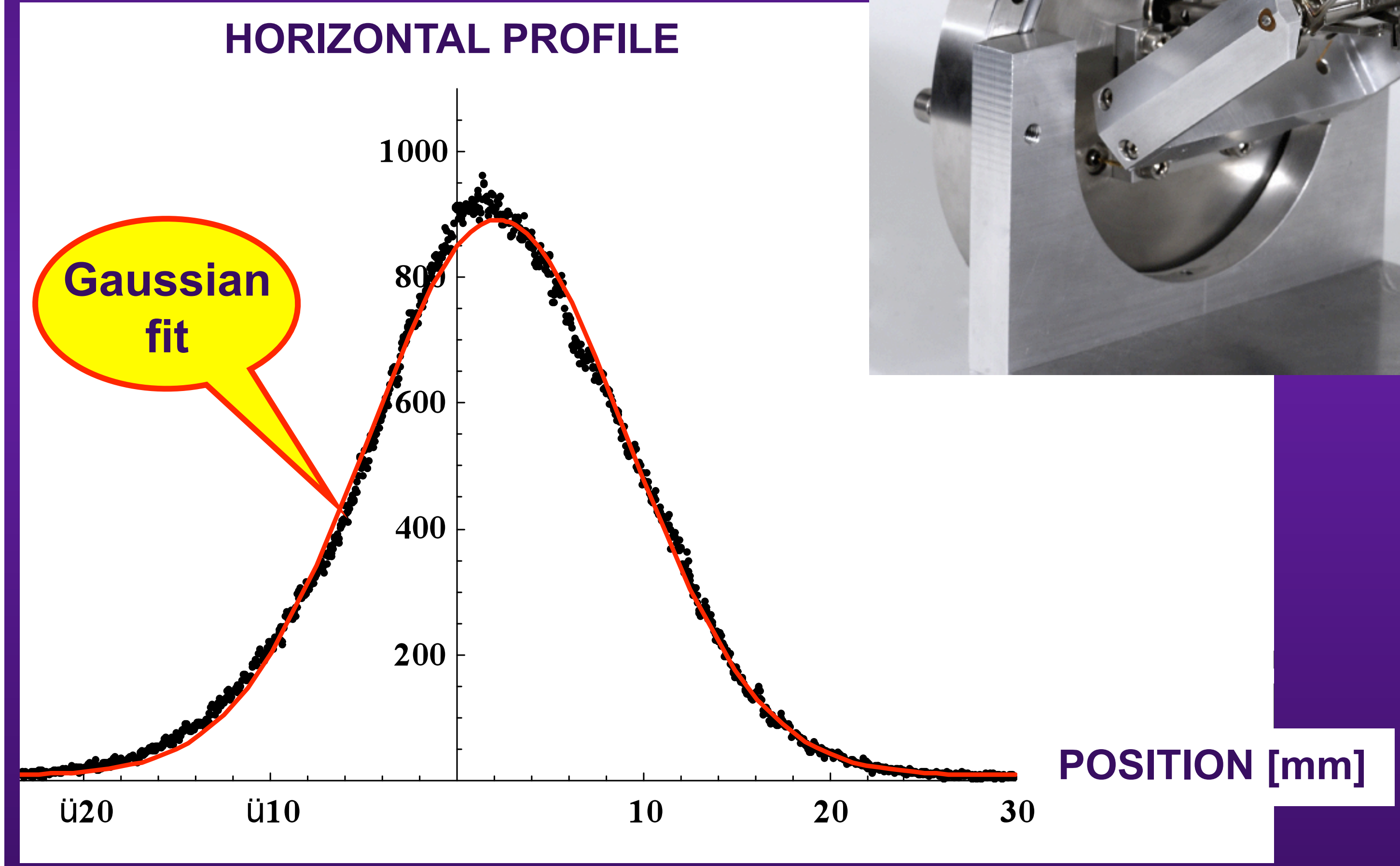
Correction QUADRUPOLE

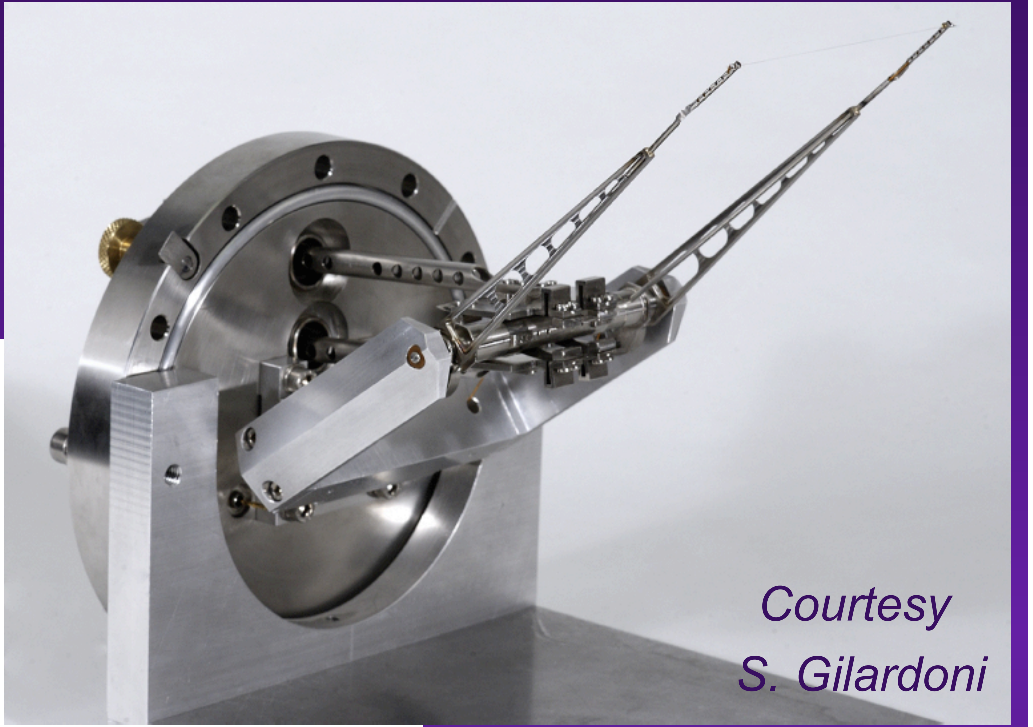
Correction DIPOLE

POSITION PICK-UP IN SS5 OF THE PS

FAST WIRE SCANNER

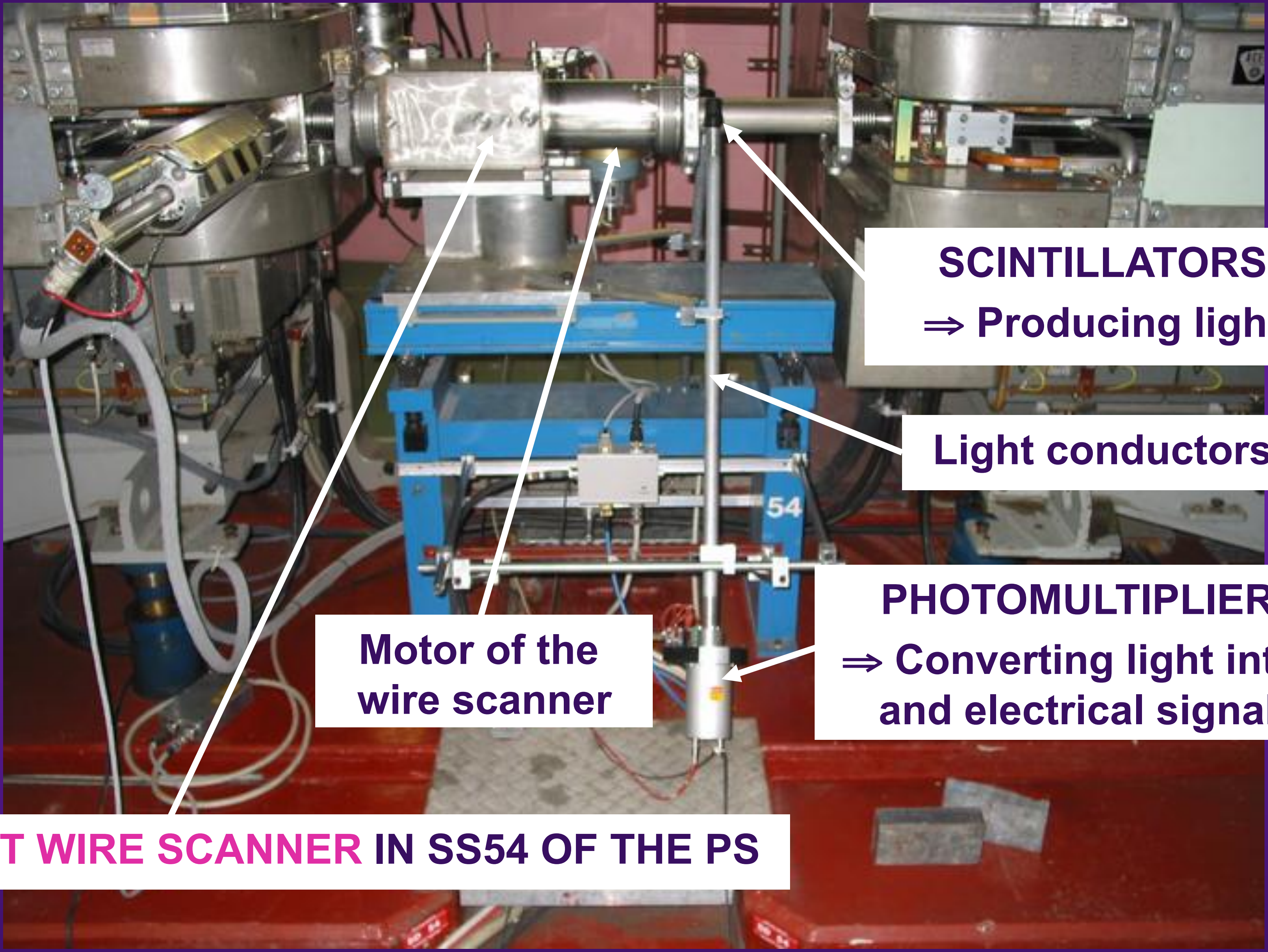
⇒ **Measures the transverse beam profiles by detecting the particles scattered from a thin wire swept rapidly through the beam**





Courtesy S. Gilardoni

$$\varepsilon_x^{(\sigma_x)} \equiv \frac{\sigma_x^2}{\beta_x}$$



FAST WIRE SCANNER IN SS54 OF THE PS

Motor of the wire scanner

SCINTILLATORS

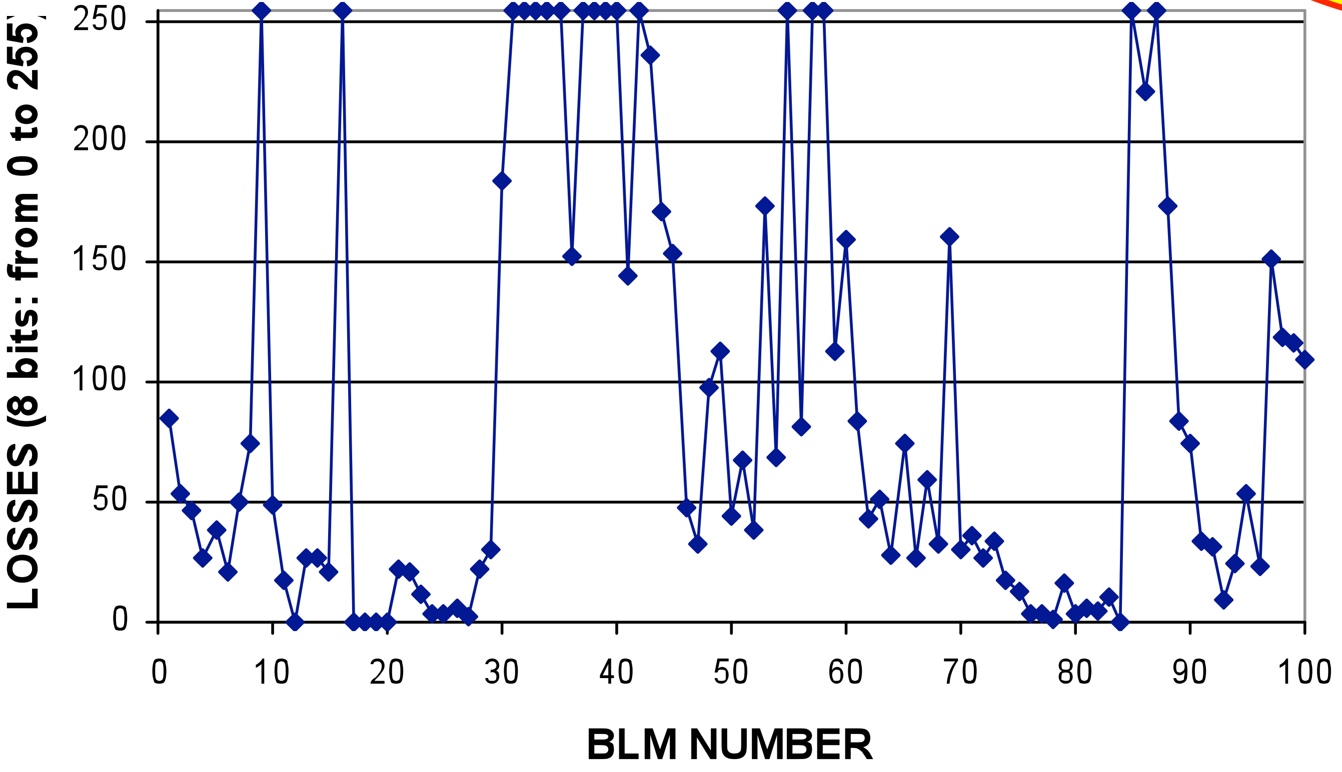
⇒ Producing light

Light conductors

PHOTOMULTIPLIER

⇒ Converting light into and electrical signal

BEAM LOSS MONITOR



$3.42\ 10^{13}$ ppp @ 14 GeV/c
(Monday 27/09/04 12:47)

PS record intensity (CNGS beam)

Saturation at 255 => Could be much higher

Detector = ACEM (Aluminum Cathod Electron Multiplier), placed at the beginning of the SS
⇒ Similar to a Photomultiplier and it is cheap, robust and radiation resistant

SUMMARY OF LECTURE ON TRANSVERSE BEAM DYNAMICS

- Design orbit in the centre of the vacuum chamber
- Lorentz force $\vec{F} = e\left(\cancel{\vec{E}} + \vec{v} \times \vec{B}\right)$
- Dipoles (constant force) ⇒ Guide the particles along the design orbit
- Quadrupoles (linear force) ⇒ Confine the particles in the vicinity of the design orbit
- Betatron oscillation in x (and in y) ⇒ Tune Q_x (and Q_y) >> 1
- Twiss parameters define the ellipse in phase space (x, $x' = d\,x \,/\, d\,s$)
- β-function reflects the size of the beam and depends only on the lattice
- Beam emittance must be smaller than the mechanical acceptance
- Higher order multipoles from imperfections (nonlinear force) ⇒ Resonances excited in the tune diagram and the working point (Q_x, Q_y) should not be close to most of the resonances
- Nonlinearities reduce the acceptance ⇒ Dynamic aperture
- Injection and extraction (septum and kicker)
- Betatron and dispersion matching (between a circular accelerator and a transfer line)

SUMMARY OF LECTURE ON LONGITUDINAL BEAM DYNAMICS

- RF cavities are used to accelerate (or decelerate) the particles
- Transition energy and sinusoidal voltage ⇒ $\vec{F} = e\left(\vec{E} + \cancel{\vec{v} \times \vec{B}}\right)$
- Harmonic number = Number of RF buckets (stationary or accelerating)
- Bunched beam (instead of an unbunched or continuous beam)
- Synchrotron oscillation around the synchronous particle in z
  ⇒ Tune $Q_z << 1$
- Stable phase Φ_s below transiton and $\pi - \Phi_s$ above transition
- Ellipse in phase space (Δt, ΔE)
- Beam emittance must be smaller than the bucket acceptance
- Bunch splittings and rotation very often used

SUMMARY OF LECTURE ON COLLECTIVE EFFECTS (1/2)

- (Direct) space charge = Interaction between the particles (without the vacuum chamber) ⇒ Coulomb repulsion + magnetic attraction
  - Tune footprint in the tune diagram ⇒ Interaction with resonances
  - Disappears at high energy
  - Reduces the RF bucket below transition and increases it above

- Wake fields = Electromagnetic fields generated by the beam interacting with its surroundings (vacuum pipe, etc.) ⇒ Impedance = Fourier transform of the wake field
  - Bunched-beam coherent instabilities
    - Coupled-bunch modes
    - Single-bunch or Head-Tail modes (low and high intensity)
  - Beam stabilization
    - Landau damping
    - Feedbacks
    - Linear coupling between the transverse planes

SUMMARY OF LECTURE ON COLLECTIVE EFFECTS (2/2)

- Beam-Beam = Interaction between the 2 counter-rotating beams
  ⇒ Coulomb repulsion + magnetic repulsion
  - Crossing angle, head-on and long-range interactions
  - Tune footprint in the tune diagram ⇒ Interaction with resonances
  - Does not disappear at high energy
  - PACMAN effects ⇒ Alternate crossing scheme
  - Coherent modes ⇒ Possible loss of Landau damping

- Electron cloud
  - Electron cloud build-up ⇒ Multi-bunch single-pass effect
  - Coherent instabilities induced by the electron cloud
    - Coupled-bunch
    - Single-bunch
  - Tune footprint in the tune diagram ⇒ Interaction with resonances
  - Does not disappear at high energy

REMINDERS: (1) RELATIVISTIC EQUATIONS

$$E_{rest} = m_0 \, c^2$$

$$\gamma = \frac{E_{total}}{E_{rest}} = \frac{m}{m_0} = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{v}{c}$$

$$\vec{p} = m \, \vec{v}$$

$$E_{total}^2 = E_{rest}^2 + p^2 \, c^2$$

For a particle of charge e

$$\frac{d\,\vec{p}}{d\,t} = \vec{F} = e \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

(2) MOST IMPORTANT 4-VECTORS & INVARIANTS AND LORENTZ SCALAR PRODUCT

- **4-dimensional radius vector**

$$\vec{X} = (c\,t\,,\vec{x}\,)$$

- **4-velocity**

$$\vec{V} = c\,\frac{d\,\vec{X}}{d\,s} = \frac{d\,\vec{X}}{d\,\tau} = \gamma\,(c\,,\vec{v}\,)$$

Proper time

- **4-momentum (energy-momentum vector)**

$$\vec{P} = \left(\frac{E}{c}\,,\vec{p}\right) = \gamma\,m_0\,(c\,,\vec{v}\,) = m_0\,\vec{V}$$

- **Current vector**

$$\vec{J} = \rho\,(c\,,\vec{v}\,) = \frac{\rho}{\gamma}\,\vec{V}$$

with $\rho = \rho_0$ the density in the rest system of the volume element considered

- **Lorentz scalar product**

$$\left(u_1\,u_2\right)=u_{1\mu}\;u_2^{\mu}$$

$$=u_1^0\;u_2^0-u_1^1\;u_2^1\;-u_1^2\;u_2^2\;-u_1^3\;u_2^3$$

with $u^{\mu}=\left(u^0,u^1,u^2,u^3\right)$ the contravariant 4-vector

and $u_{\mu}=\left(u^0,-u^1,-u^2,-u^3\right)$ the covariant 4-vector

- **Invariants**

$$X^2=X_{\mu}\;X^{\mu}=\left(c\,t\right)^2-\vec{x}^{\,2}$$

$$V^2=c^2$$

$$P^2=m_0^2\;c^2$$

$$J^2=\left(\frac{\rho}{\gamma}\right)^2\;c^2=\rho_0^2\;c^2$$

(3) ENERGY, MOMENTUM AND VELOCITY OF ONE PARTICLE SEEN FROM THE REST SYSTEM OF ANOTHER ONE

- Consider 2 particles: 1 and 2, with rest mass m_{01} and m_{02}
- The 3 invariants are $P_1^2 = m_{01}^2 \, c^2$, $P_2^2 = m_{02}^2 \, c^2$

  and $P_1 \, P_2$ (or $\left(P_1 + P_2 \right)^2$ or $\left(P_1 - P_2 \right)^2$)
- Total Centre of Mass (CM) energy squared

$$s = c^2 \left(P_1 + P_2 \right)^2 = E_{CM}^2$$

=> $\sqrt{s} = E_{CM}$

- **Making the computation in the rest system of particle 1, one can show the 3 following invariant expressions**
  - **The energy of particle 2 seen from particle 1 is**

$$E_{21} = \frac{P_1 \, P_2}{m_{01}}$$

  - **The momentum of particle 2 seen from particle 1 is**

$$\vec{p}_{21}^{\,2} = \frac{E_{21}^2}{c^2} - m_{02}^2 \, c^2 = \frac{\left(P_1 \, P_2 \right)^2 - m_{01}^2 \, m_{02}^2 \, c^4}{m_{01}^2 \, c^2}$$

  - **The relative velocity (symmetric in 1 and 2) is**

$$v_{21}^2 = \frac{\vec{p}_{21}^{\,2} \, c^4}{E_{21}^2} = c^2 \, \frac{\left(P_1 \, P_2 \right)^2 - m_{01}^2 \, m_{02}^2 \, c^4}{\left(P_1 \, P_2 \right)^2}$$

- **It can also be shown (using the relation given in the Useful relations $\left(\vec{v}_1 \times \vec{v}_2\right)^2 = \vec{v}_1^{\,2}\, \vec{v}_2^{\,2} - \left(\vec{v}_1 \,.\, \vec{v}_2\right)^2$) that**

$$v_{21} = \frac{\sqrt{\left(\vec{v}_1 - \vec{v}_2\right)^2 - \dfrac{\left(\vec{v}_1 \times \vec{v}_2\right)^2}{c^2}}}{1 - \dfrac{\vec{v}_1 \,.\, \vec{v}_2}{c^2}}$$

(4) LORENTZ FORCE

$$\vec{F} = e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- **Cartesian (x,y,s)**

$$F_x = e\left(E_x - v\,B_y\right)$$

$$F_y = e\left(E_y + v\,B_x\right)$$

$$F_s = e\,E_s$$

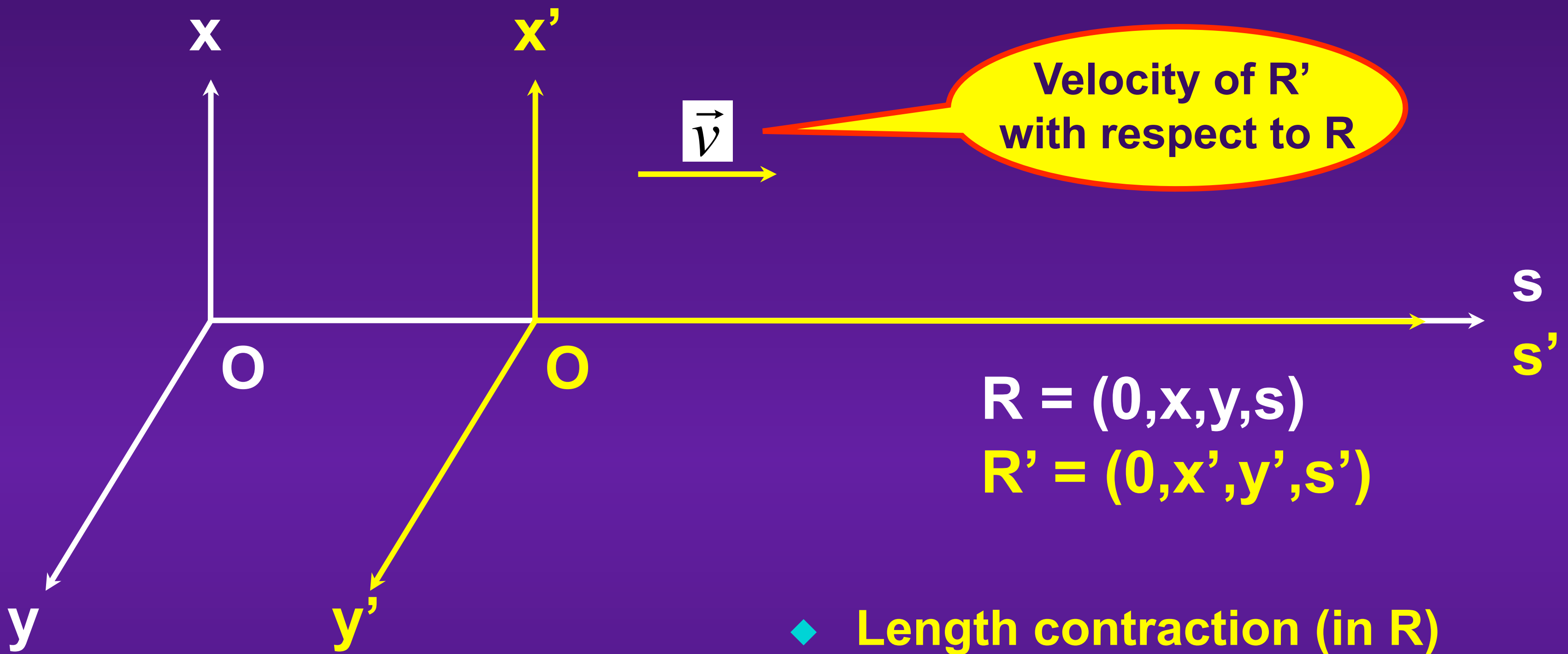
- **Cylindrical (r,θ,s)**

$$F_r = e\left(E_r - v\,B_\vartheta\right)$$

$$F_\vartheta = e\left(E_\vartheta + v\,B_r\right)$$

$$F_s = e\,E_s$$

(5) LORENTZ TRANSFORM



R = (0,x,y,s)

R' = (0,x',y',s')

$x = x'$ $y = y'$

$$s = \gamma \left(s' + v\, t' \right)$$

$$t = \gamma \left(\frac{v}{c^2} s' + t' \right)$$

- Length contraction (in R)

$$d\, s = \frac{d\, s'}{\gamma} \quad \text{for} \quad d\, t = 0$$

- Time dilatation (in R)

$$d\, t = \gamma\, d\, t' \quad \text{for} \quad d\, s' = 0$$

(6) MAXWELL EQUATIONS

- **Differential forms**

$$div\ \vec{E} = \frac{\rho}{\varepsilon}$$

Gauss's law for electric charge

$$div\ \vec{H} = 0$$

Gauss's law for magnetic charge

$$\overrightarrow{rot}\ \vec{E} = -\,\mu\,\frac{\partial \vec{H}}{\partial t}$$

Faraday's and Lenz law

$$\overrightarrow{rot}\ \vec{H} = \vec{J} + \varepsilon\,\frac{\partial \vec{E}}{\partial t}$$

Ampere's law

- **Integral forms**

$$\iiint div\ \vec{E}\ dV = \iint \vec{E}\,.\,d\vec{S} = \frac{1}{\varepsilon}\iiint \rho\ dV$$

$$\iiint div\ \vec{H}\ dV = \iint \vec{H}\,.\,d\vec{S} = 0$$

$$\iint \overrightarrow{rot}\,\vec{E}\,.\,d\vec{S} = \oint \vec{E}\,.\,d\vec{s} = -\,\mu \iint \frac{\partial \vec{H}}{\partial t}\,.\,d\vec{S}$$

$$\iint \overrightarrow{rot}\,\vec{H}\,.\,d\vec{S} = \oint \vec{H}\,.\,d\vec{s} = \iint \vec{J}\,.\,d\vec{S} + \varepsilon \iint \frac{\partial \vec{E}}{\partial t}\,.\,d\vec{S}$$

with $\vec{B} = \mu\,\vec{H}$

$$\vec{D} = \varepsilon\,\vec{E}$$

$$\vec{J} = \rho\vec{v} + \sigma\,\vec{E}$$

Maxwell equations valid
in homogeneous, isotropic,
continuous media

(7) NABLA, GRAD, ROT, DIV and LAPLACIAN OPERATORS

- **Cartesian (x,y,s)**

$$\vec{\nabla} \equiv \left| \begin{array}{c} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial s} \end{array} \right.$$

- **Cylindrical (r,θ,s)**

$$\vec{\nabla} \equiv \left| \begin{array}{c} \dfrac{\partial}{\partial r} \\ \dfrac{1}{r}\left(\dfrac{\partial}{\partial \vartheta}\right) \\ \dfrac{\partial}{\partial s} \end{array} \right.$$

Also noted
$\overrightarrow{curl}\,\vec{\mathrm{E}}$ or $\vec{\nabla} \wedge \vec{\mathrm{E}}$

$$\overrightarrow{grad}\;\rho \equiv \vec{\nabla}\;\rho = \left| \begin{array}{c} \dfrac{\partial \rho}{\partial x} \\ \dfrac{\partial \rho}{\partial y} \\ \dfrac{\partial \rho}{\partial s} \end{array} \right.$$

$$\overrightarrow{rot}\;\vec{E} \equiv \vec{\nabla} \times \vec{E} = \left| \begin{array}{c} \dfrac{\partial E_s}{\partial y} - \dfrac{\partial E_y}{\partial s} \\ \dfrac{\partial E_x}{\partial s} - \dfrac{\partial E_s}{\partial x} \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} \end{array} \right.$$

$$\overrightarrow{grad}\;\rho = \left| \begin{array}{c} \dfrac{\partial \rho}{\partial r} \\ \dfrac{1}{r}\left(\dfrac{\partial \rho}{\partial \vartheta}\right) \\ \dfrac{\partial \rho}{\partial s} \end{array} \right.$$

$$\overrightarrow{rot}\;\vec{E} = \left| \begin{array}{c} \dfrac{1}{r}\left(\dfrac{\partial E_s}{\partial \vartheta}\right) - \dfrac{\partial E_\theta}{\partial s} \\ \dfrac{\partial E_r}{\partial s} - \dfrac{\partial E_s}{\partial r} \\ \dfrac{1}{r}\left[\dfrac{\partial\left(r\,E_\theta\right)}{\partial r} - \dfrac{\partial E_r}{\partial \theta}\right] \end{array} \right.$$

$$div\;\vec{E} \equiv \vec{\nabla}\,.\,\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_s}{\partial s}$$

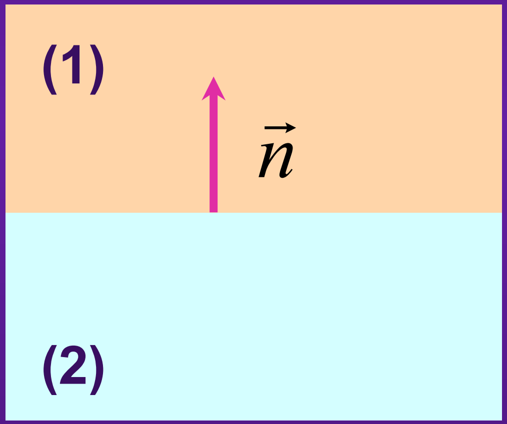
$$div\;\vec{E} = \frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,E_r\right) + \frac{1}{r}\,\frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_s}{\partial s}$$

$$\Delta\,\rho \equiv \nabla^2 \rho = \text{Laplacian operator}$$
$$= \frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial s^2}$$

$$\Delta\,\rho = \frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial \rho}{\partial r}\right) + \frac{1}{r^2}\,\frac{\partial^2 \rho}{\partial \theta^2} + \frac{\partial^2 \rho}{\partial s^2}$$

(8) GENERAL FIELD MATCHING CONDITIONS

Consider a surface separating two media "1" and "2". The following boundary conditions can be derived from Maxwell equations for the normal (⊥) and parallel (//) components of the fields at the surface



$$\vec{E}^1_{//} = \vec{E}^2_{//}$$

$$\vec{H}^1_{//} - \vec{H}^2_{//} = \vec{K}$$

$$D^1_{\perp} - D^2_{\perp} = \Sigma$$

$$B^1_{\perp} = B^2_{\perp}$$

where Σ is the surface charge density and $\vec{K}$ is the surface current density

(9) USEFUL RELATIONS / NOTIONS

- Gaussian distribution $\lambda(s) = \dfrac{q}{\sqrt{2\pi}\ \sigma_s} e^{-\frac{s^2}{2\sigma_s^2}}$

- Equation of motion (and solutions) of an harmonic oscillator (which will be very often used) => The best way to keep something under control (i.e. stable) is to make it oscillate!

- MKSA units are used here, whereas CGS units can be found in several books and publications => Conversion from CGS to MKSA

$$\frac{4\pi}{c} = Z_0 = 120\pi\ \Omega \qquad \frac{e^2}{m_0\, c^2} = r_0 = \text{Classical radius of the particle}$$

- The engineer convention is also adopted ($e^{j\omega t}$) instead of the physicist's one ($e^{-i\omega t}$)

- **Transposition of the product of 2 matrices** $(A\,B)^t = B^t\,A^t$

- **Inversion of a 2 × 2 matrix** $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

=> $$M^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- $$\int_{-\infty}^{+\infty} e^{-a\,t^2}\,dt = \sqrt{\frac{\pi}{a}}$$

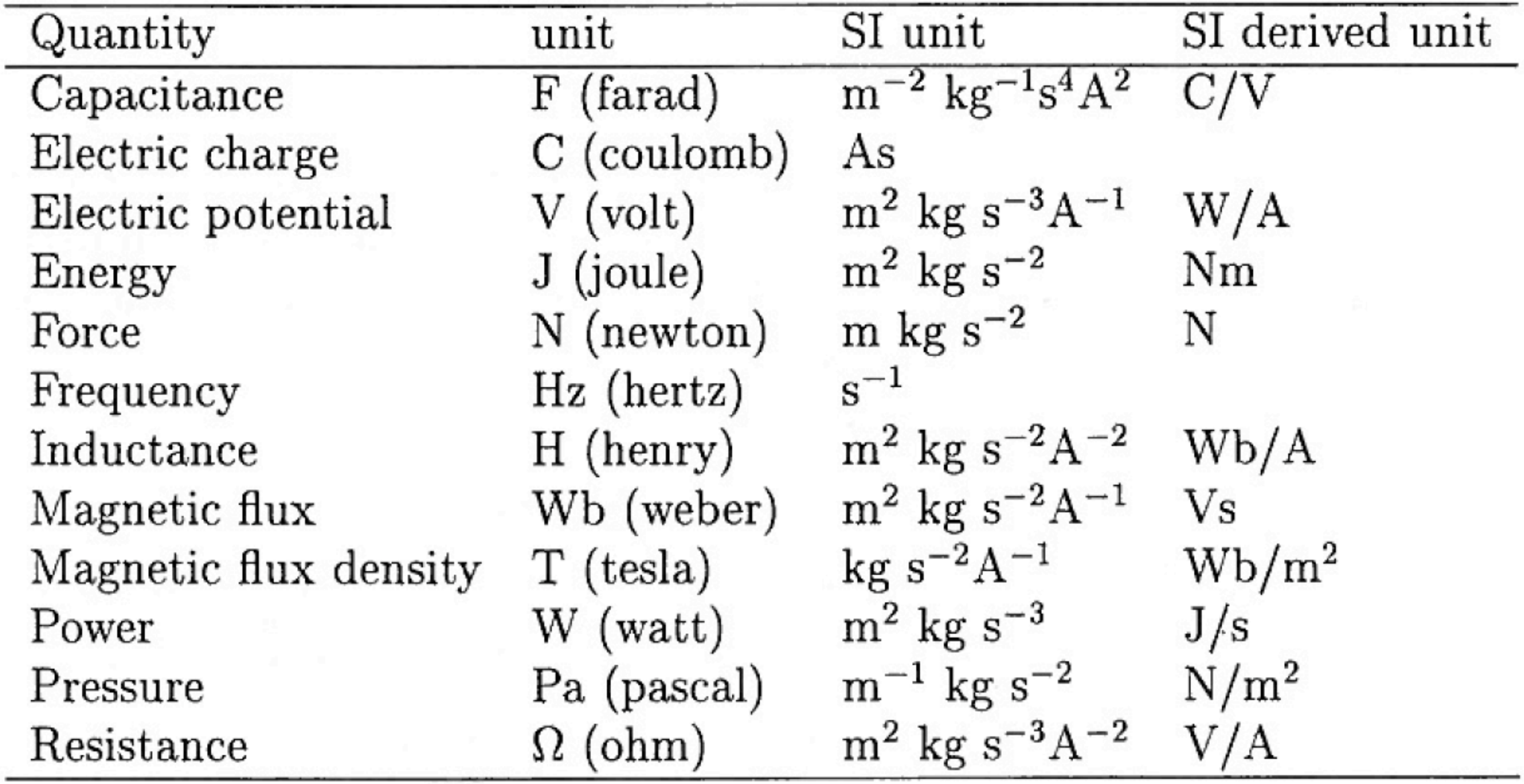
- $$\int_{-\infty}^{+\infty} e^{-\left(a\,t^2 + b\,t + c\right)}\,dt = \sqrt{\frac{\pi}{a}}\;e^{\frac{b^2}{4\,a} - c}$$

- $$\cos\left(a+b\right) = \cos a \cos b - \sin a \sin b$$
- $$\cos\left(a-b\right) = \cos a \cos b + \sin a \sin b$$
- $$\sin\left(a+b\right) = \sin a \cos b + \sin b \cos a$$
- $$\sin\left(a-b\right) = \sin a \cos b - \sin b \cos a$$
- **Rotation (by an angle + Φ / 2) matrix**

$$R = \begin{bmatrix} \cos\frac{\Phi}{2} & -\sin\frac{\Phi}{2} \\ \sin\frac{\Phi}{2} & \cos\frac{\Phi}{2} \end{bmatrix}$$

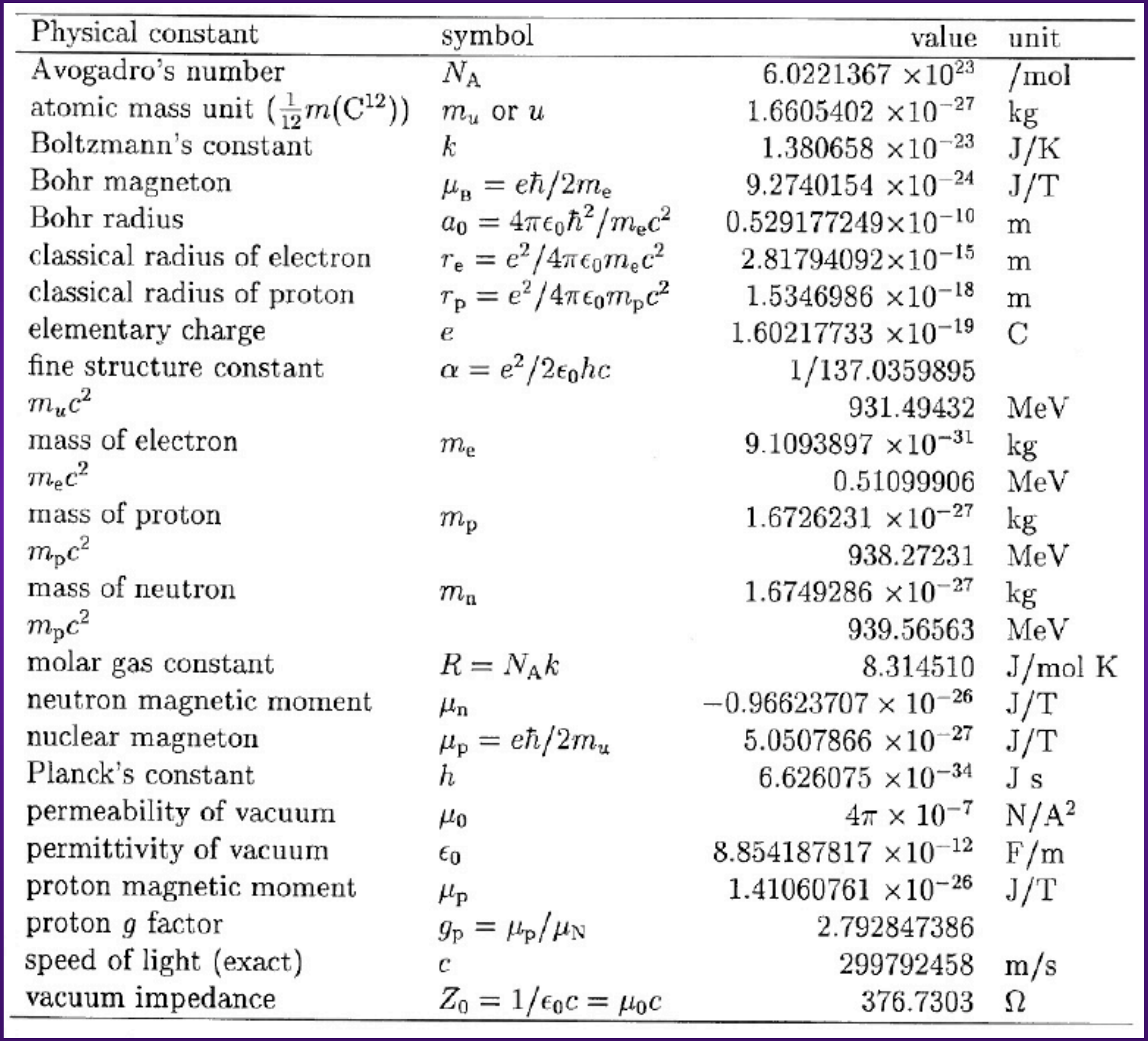
- $$\left(\vec{v}_1 \times \vec{v}_2\right)^2 = \vec{v}_1^{\,2}\,\vec{v}_2^{\,2} - \left(\vec{v}_1 \,.\, \vec{v}_2\right)^2$$
- $$\int_0^s \frac{d\,t}{1+t^2} = \arctan s$$

(10) Units of physical quantities



Quantity	unit	SI unit	SI derived unit
Capacitance	F (farad)	$m^{-2}\ kg^{-1} s^4 A^2$	C/V
Electric charge	C (coulomb)	As	
Electric potential	V (volt)	$m^2\ kg\ s^{-3} A^{-1}$	W/A
Energy	J (joule)	$m^2\ kg\ s^{-2}$	Nm
Force	N (newton)	$m\ kg\ s^{-2}$	N
Frequency	Hz (hertz)	s^{-1}	
Inductance	H (henry)	$m^2\ kg\ s^{-2} A^{-2}$	Wb/A
Magnetic flux	Wb (weber)	$m^2\ kg\ s^{-2} A^{-1}$	Vs
Magnetic flux density	T (tesla)	$kg\ s^{-2} A^{-1}$	Wb/m^2
Power	W (watt)	$m^2\ kg\ s^{-3}$	J/s
Pressure	Pa (pascal)	$m^{-1}\ kg\ s^{-2}$	N/m^2
Resistance	Ω (ohm)	$m^2\ kg\ s^{-3} A^{-2}$	V/A

(11) Fundamental physical constants



Physical constant	symbol	value	unit
Avogadro's number	N_A	6.0221367×10^{23}	/mol
atomic mass unit ($\frac{1}{12}m(C^{12})$)	m_u or u	$1.6605402 \times 10^{-27}$	kg
Boltzmann's constant	k	1.380658×10^{-23}	J/K
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.2740154 \times 10^{-24}$	J/T
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_ec^2$	$0.529177249 \times 10^{-10}$	m
classical radius of electron	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.81794092 \times 10^{-15}$	m
classical radius of proton	$r_p = e^2/4\pi\epsilon_0 m_p c^2$	$1.5346986 \times 10^{-18}$	m
elementary charge	e	$1.60217733 \times 10^{-19}$	C
fine structure constant	$\alpha = e^2/2\epsilon_0 hc$	1/137.0359895	
m_uc^2		931.49432	MeV
mass of electron	m_e	$9.1093897 \times 10^{-31}$	kg
m_ec^2		0.51099906	MeV
mass of proton	m_p	$1.6726231 \times 10^{-27}$	kg
m_pc^2		938.27231	MeV
mass of neutron	m_n	$1.6749286 \times 10^{-27}$	kg
m_pc^2		939.56563	MeV
molar gas constant	$R = N_A k$	8.314510	J/mol K
neutron magnetic moment	μ_n	$-0.96623707 \times 10^{-26}$	J/T
nuclear magneton	$\mu_p = e\hbar/2m_u$	$5.0507866 \times 10^{-27}$	J/T
Planck's constant	h	6.626075×10^{-34}	J s
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
permittivity of vacuum	ϵ_0	$8.854187817 \times 10^{-12}$	F/m
proton magnetic moment	μ_p	$1.41060761 \times 10^{-26}$	J/T
proton g factor	$g_p = \mu_p/\mu_N$	2.792847386	
speed of light (exact)	c	299792458	m/s
vacuum impedance	$Z_0 = 1/\epsilon_0 c = \mu_0 c$	376.7303	Ω

REFERENCES (1/2)

[1] E. Wilson, An Introduction to Particle Accelerators, Oxford University Press, 252 p, 2001

[2] W. Herr and B. Muratori, Concept of Luminosity (in Particle Colliders), CAS 2007, Daresbury [http://cern.ch/lhc-beam-beam/talks/Daresbury luminosity.pdf]

[3] G. Guignard, Selection of Formulae Concerning Proton Storage Rings, CERN 77-10, ISR Division, 1977

[4] M.A. Furman, The Møller Luminosity Factor, LBNL-53553, CBP Note-543, 2003

[5] R. Hagedorn, Relativistic Kinematics, W.A. Benjamin, Inc., 1973

[6] M. Martini, An Introduction to Transverse Beam Dynamics in Accelerators, CERN/PS 96-11 (PA), 1996, [http://doc.cern.ch/archive/electronic/cern/preprints/ps/ps-96-011.pdf]

[7] L. Rinolfi, Longitudinal Beam Dynamics (Application to synchrotron), CERN/PS 2000-008 (LP), 2000, [http://doc.cern.ch/archive/electronic/cern/preprints/ps/ps-2000-008.pdf]

[8] Theoretical Aspects of the Behaviour of Beams in Accelerators and Storage Rings: International School of Particle Accelerators of the 'Ettore Majorana' Centre for Scientific Culture, 10–22 November 1976, Erice, Italy, M.H. Blewett (ed.), CERN report 77-13 (1977) [http://preprints.cern.ch/cgi-bin/setlink?base=cernrep&categ=Yellow_Report&id=77-13]

[9] CERN Accelerator Schools [http://cas.web.cern.ch/cas/]

[10] K. Schindl, Space Charge, CERN-PS-99-012-DI, 1999 [http://doc.cern.ch/archive/electronic/cern/preprints/ps/ps-99-012.pdf]

[11] A.W. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, New York: Wiley, 371 p, 1993 [http://www.slac.stanford.edu/~achao/wileybook.html]

REFERENCES (2/2)

[12] Web site on LHC Beam-Beam Studies [http://wwwslap.cern.ch/collective/zwe/lhcbb/]

[13] Web site on Electron Cloud Effects in the LHC [http://ab-abp-rlc.web.cern.ch/ab-abp-rlc-ecloud/]

[14] LHC Design Report [http://ab-div.web.cern.ch/ab-div/Publications/LHC-DesignReport.html]

[15] E. Métral and G. Rumolo, USPAS09 course on "Collective Effects in Beam Dynamics" [http://uspas.fnal.gov/materials/09UNM/CollectiveEffects.html]

[16] E. Métral's web page [http://emetral.web.cern.ch/emetral/] where several courses (with some exercises, exams and corrections) can be found