EXERCISES FOR THE COURSE ON TRANSVERSE BEAM DYNAMICS (LUND 2013)

1) Give the general definition of the:

- Beam rigidity.
- 2 relations linking the total number of dipoles, the bending angle of a dipole and the length of a dipole.
- Hill's equation and the form of its solution.
- Focal length of a quadrupole (focusing and defocusing).
- Betatron phase advance and betatron tune.
- Transfer matrix, Twiss matrix and Twiss parameters.
- Transfer matrix of a drift space.
- Determinant and Trace of a Twiss matrix, and general stability criterion for a Twiss matrix.
- Thin-lens approximation.
- Transfer matrix of a quadrupole (focusing and defocusing) in the thin-lens approximation.
- Betatron function and phase advance around an IP (Interaction Point).
- Transverse beam emittance, beam envelope and beam divergence.
- Dispersion function.
- Hill's equation with normalized (Floquet's coordinates) and the form of its solution.
- Chromaticity.
- General resonance conditions.

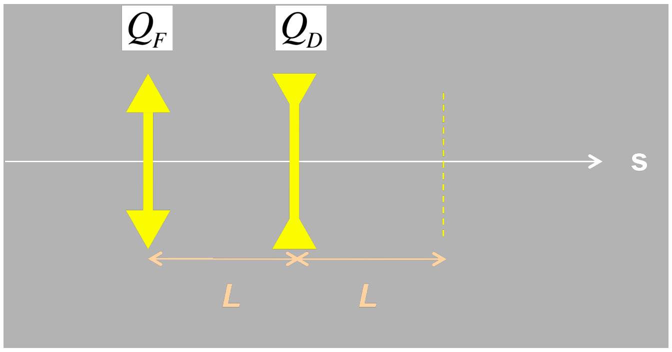
2) In the LHC, the radius of curvature of a dipole is 2803.95 m and the beam momentum at maximum energy is 7 TeV/c. What is then the maximum magnetic field? There are 1232 dipoles in total. What is the bending angle?

3) Compute the evolution of the betatron function and betatron phase advance around a LHC IP. Numerical applications for β^* = 55 cm and β^* = 5 cm (from s = - 50 m to + 50 m).

4) Derive the transfer matrix of a symmetric FODO cell (see Figure below) in the thin-lens approximation to recover the result of page 30. By comparison with the general form of a Twiss matrix, deduce the betatron functions at the location of the focusing quadrupole Q_F (maximum value) and at the defocusing quadrupole Q_D (minimum value). Compute also the betatron phase advance throughout the FODO cell and the betatron tune for a circular machine made of 2 such cells. Perform the numerical applications for the following case:

- Length of a quadrupole (F and D): l = 1 cm.
- Strength of a quadrupole (F and D): k = 20 m^{-2}.
- Total length of the FODO cell (2 L): 2 L = 10 m.

Compute the natural chromaticity of the machine.
What would happen if L is increased by a factor 4?



5) Derive the transfer matrix of a symmetric FODO cell (see Figure below) in the thin-lens approximation in which the drift spaces are replaced by dipole magnets of length L and bending radius ρ_0, to recover the result of page 47. Using the definition of the dispersion function, deduce the dispersion functions at the location of the focusing quadrupole Q_F and at the defocusing quadrupole Q_D. Perform the numerical applications for the following case:

- Same as in Ex. 4 + radius of curvature of the dipoles: $\rho_0 = 49.85$ m.