FINAL EXERCISE FOR THE ONE-WEEK TRAINING COURSE IN ACCELERATOR PHYSICS (LUND 2013)

As can be seen in Fig. 1, the LHC (Large Hadron Collider) ring is composed of 8 arcs (called ARC) separated by 8 LSS (Long Straight Sections), which are available for beam interactions (in IP - Interaction Points - 1, 2, 5 and 8) or utility insertions (in points 3, 4, 6 and 7).

In the optics version discussed here, each of the 8 arcs is made of 23 FODO cells and all the arc cells are made of 2 identical half-cells with a length of 53.45 m (i.e. the total length of the arc cell is 106.9 m). The layout of an arc half-cell is shown in Fig. 2.

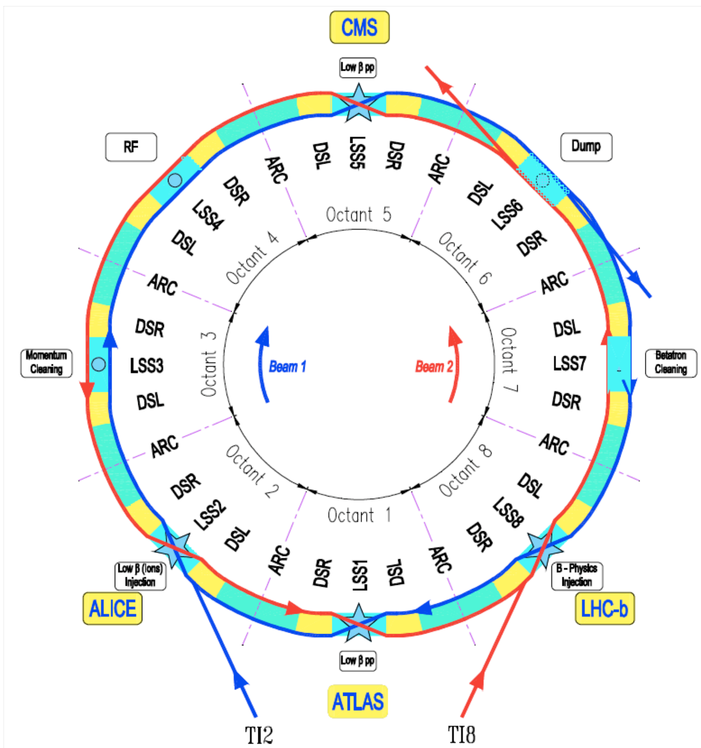

Figure 1: Layout of the LHC.

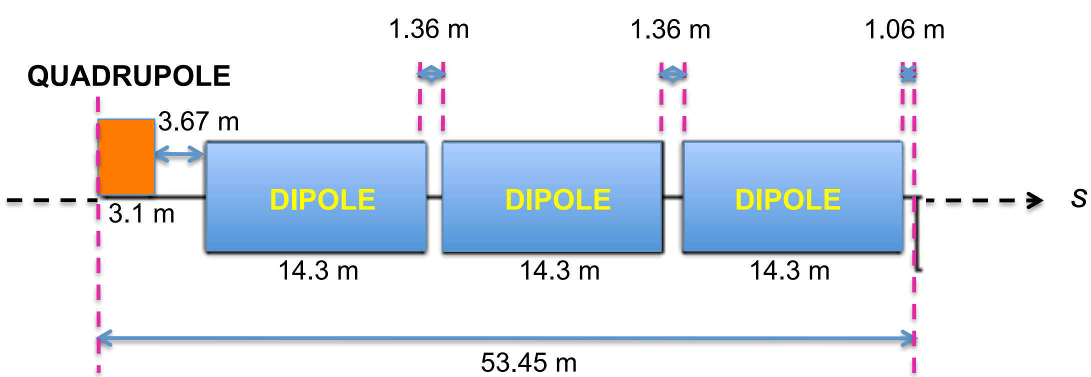

Figure 2: Layout of an arc half-cell (with lengths of the quadrupole, dipole and spacings between the different elements).

There are 2 lattice modules, called dispersion suppressor cells (DSL and DSR), located at either end of the arcs for the reduction of the horizontal dispersion. Each dispersion suppressor cells consists of 4 quadrupoles interleaved with 4 strings of 2 dipoles each. The dispersion suppressor dipoles are the same as those in the arcs.

1) What is the total number of dipoles in the LHC?

2) Deduce the bending angle and radius of curvature of a dipole.

3) Assuming that the focusing / defocusing arc quadrupole gradient is ± 205 T/m at the beam collision momentum of 7 TeV/c, compute the normalized quadrupole gradient (in m^{-2}) and deduce the focal length of the arc quadrupoles.

4) Assuming that the arc cells are thin lens FODO cells in which the dipoles occupy all the cell length (i.e. the 3 dipoles are expanded to cover the 53.45 m), compute:

4.1) The new length and bending angle of one dipole.
4.2) The betatron phase advance per arc cell.
4.3) The betatron functions at the thin arc quadrupole locations.
4.4) The horizontal and vertical dispersion functions at the thin arc quadrupole locations, after having shown that the dispersion functions can be written (starting from the equations of page 47 of the course on TBD)

$$D_{QF\,,\,QD} = L\,\vartheta_{half-cell}\,\frac{1 \pm \dfrac{\sin\left(\mu/2\right)}{2}}{\sin^2\left(\mu/2\right)}\quad,$$

with $\vartheta_{half-cell}$ the total bending angle of a half-cell (i.e. in the present case it is the total angle produced by the 3 dipoles).

5) Compute the total betatron phase advance for all the arc cells and the corresponding contribution to the transverse betatron tune (same in both transverse planes) of all the arc cells.

6) Knowing that the horizontal betatron tune is $Q_x = 64.28$ and the vertical one is $Q_y = 59.31$, what are thus the contributions of all the 8 insertions and of each insertion on average?

7) What are the horizontal and vertical beam sizes $(\sigma_{x,y})$ at the arc quadrupole locations if the normalized horizontal and vertical beam emittances are 3.75 μm?

8) Using the MAD-X code, plot the betatron and dispersion functions of an LHC arc cell for both the thin lens case computed before and the real thick lens case.

9) The goal of this exercise is to derive a simple rough estimate of the transition gamma γ_{tr} for machines made entirely of simple FODO cells with a betatron phase advance μ << 1:

9.1) Plot the betatron phase advance μ of a FODO cell vs. $L\,/\,(\,2\,f\,)$ from 0 to 1.

9.2) Plot the betatron function divided by 2 L (both QF and QD, i.e. max and min) and dispersion function divided 2 L θ (both QF and QD, i.e. max and min) vs the betatron phase advance μ between 0 and 180 deg.

9.3) Show that for μ << 1, the betatron and dispersion functions can be written

$$\frac{\beta_{\mu<<1}}{2\,L} \approx \frac{1}{\mu} \quad , \quad \frac{D_{\mu<<1}}{2\,L\,\vartheta} \approx \frac{2}{\mu^2} \quad , \quad D_{\mu<<1} \approx \frac{\beta^2_{\mu<<1}}{\rho} \quad .$$

9.4) Starting from the definition of the transverse betatron tune, show that in the "smooth approximation" (i.e. assuming the betatron function to be a constant over the circumference), the horizontal tune is given by (with $\beta_x = \text{constant}$)

$$Q_x = \frac{R}{\beta_x} \quad .$$

9.5) Assuming that the bending radius ρ is equal to the average machine radius R, show that the dispersion is given by

$$D_x \approx \frac{\rho}{Q_x^2} \quad .$$

9.6) Finally, starting from the definition of the momentum compaction factor and its relation to the transition gamma, show that the transition gamma is approximately given by

$$\gamma_{tr} \approx Q_x \quad .$$

9.7) Deduce what should be done if one wants to modify the transition gamma.

10) The goal of this exercise is to correct the chromaticities. Let's assume a machine made only of the LHC arc cells in the thin lens approximation, whose parameters (betatron functions, dispersion functions, and phase advance) have been computed previously (see Ex. 4).

10.1) Compute the transverse natural chromaticities.

10.2) We insert a thin sextupole into the center of each quadrupole (the sextupoles are powered in 2 families: family 1 at QFs and family 2 at QDs) to correct the chromaticities to 0. What are the required normalized sextupole strengths (in m^{-3}) S_{01} and S_{02} assuming sextupole lengths of 10 cm? Hint: equations of page 69 of the course on TBD with the sextupole strength distributed among all the sextupoles of a family. Note also that the transverse tunes in this case are given by Ex. 5 as we are considering only the arc cells.

11) In reality, the transition gamma of the LHC is $\gamma_{tr} = 55.68$. What is then the RF synchronous phase at injection (450 GeV/c) and collision (7 TeV/c), i.e. in the absence of acceleration?

12) In the LHC, the revolution frequency is 11.245 kHz and the harmonic number is 35640. What is therefore the frequency of the RF cavities and the stationary bucket length (in ns), knowing that the LHC circumference is 26658.883 m?

13) It takes 20 min to accelerate the beam from a momentum of 450 GeV/c to a momentum of 7 TeV/c. Assuming a linear increase of the beam momentum for the whole acceleration process, what is thus the rate of magnetic field increase (dB/dt)? Deduce the synchronous phase used during acceleration if the RF voltage is 16 MV.

14) Assuming now that the LHC is in collision mode and that after some time, due to a drift, the two counter-rotating beams are displaced transversally by 1 σ (i.e. 1 rms transverse beam size), what would happen to the luminosity?

15) During 2012, the LHC ran with an rms bunch length of ~ 10 cm due to beam-induced RF heating issues in some equipments. What was then the consequence for the peak luminosity?

16) Assuming a luminosity lifetime of 15 h, what should we do if we would like to have an average integrated luminosity equal to half the peak luminosity?