COLLECTIVE EFFECTS

Elias Métral

- Introduction
- Space charge
- Beam-beam
- Impedances and wake fields
- E-cloud

INTRODUCTION

- See Chapter 4 (on Impedance and Collective Effects) of a Handbook (Elementary Particles - Accelerators and Colliders), which has been recently published (Landolt-Börnstein, Springer) and which we wrote with Giovanni Rumolo and Werner Herr => http://emetral.web.cern.ch/emetral/ImpedanceAndCollectiveEffects_Chap4_HandbookFor%20ElementaryParticles_Final_EM.pdf

- As the beam intensity increases, the beam can no longer be considered as a collection of non-interacting single particles: in addition to the "single-particle phenomena", "collective effects" become significant

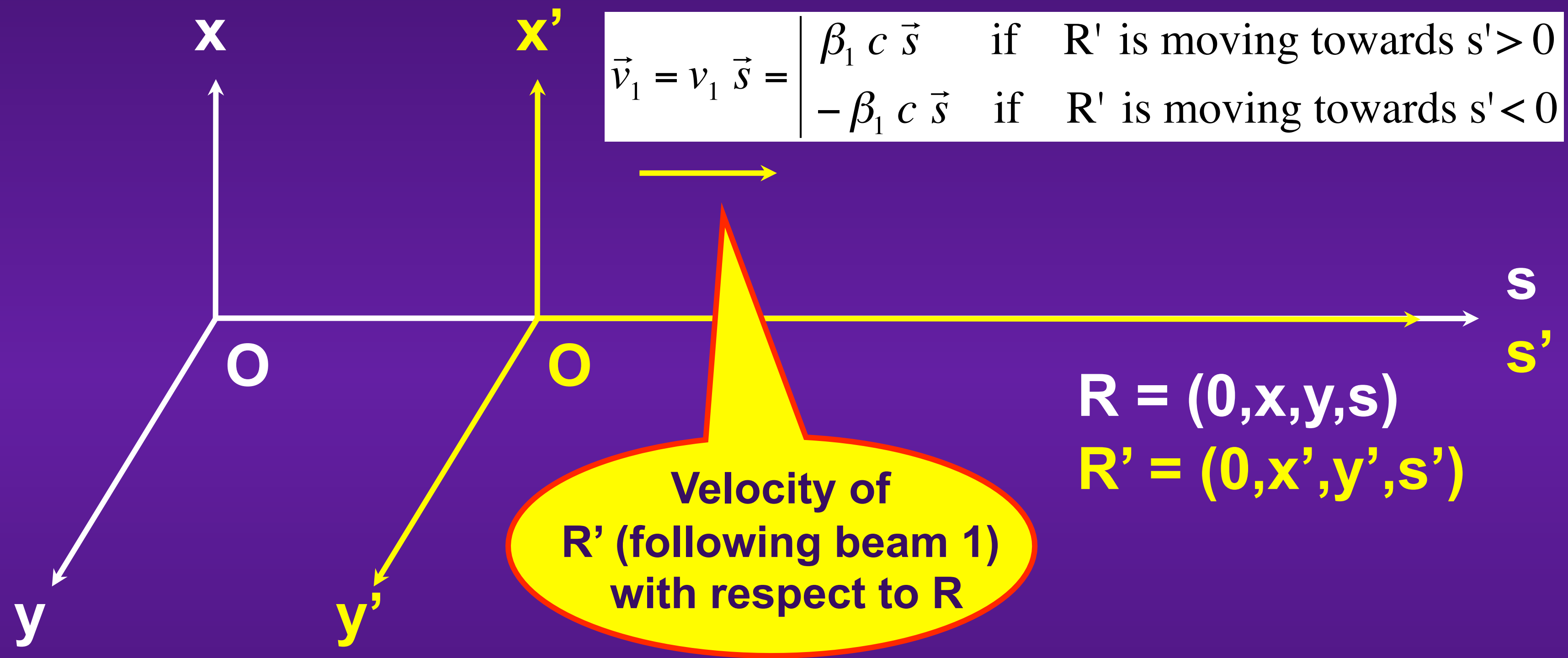
- They can lead to both incoherent (i.e. of a single particle) and coherent (i.e. of the centre of mass) effects, in the longitudinal and in one or both transverse directions, leading to beam quality degradation or even partial or total beam losses. Fortunately, stabilizing mechanisms exist

SPACE CHARGE (1/31)

- **2 space charge effects are distinguished**
  - **Direct space charge** => Comes from the interaction between the particles of a single beam, without interaction with the surrounding vacuum chamber
  - **Indirect space charge** => In the case of a beam off-axis in a perfectly conducting circular beam pipe, a coherent (or dipolar, i.e. of the centre of mass) force arises, which can be found by using the method of the images (to satisfy the boundary condition on a perfect conductor, i.e. of a vanishing tangential electrical field). A similar analysis can be done for asymmetric chambers (e.g. 2 parallel plates)
    => This is in fact a particular case of impedances (assuming perfect conductivity for the beam pipe)

SPACE CHARGE (2/31)

- **Reminder: Relativistic transformation of the EM fields**



$$\vec{v}_1 = v_1 \, \vec{s} = \left| \begin{array}{ll} \beta_1 \, c \, \vec{s} & \text{if} \quad \text{R' is moving towards s'} > 0 \\ -\beta_1 \, c \, \vec{s} & \text{if} \quad \text{R' is moving towards s'} < 0 \end{array} \right.$$

Velocity of R' (following beam 1) with respect to R

R = (0,x,y,s)

R' = (0,x',y',s')

$$E'_x = \gamma_1 \left(E_x - v_1 \, B_y \right)$$

$$E'_y = \gamma_1 \left(E_y + v_1 \, B_x \right)$$

$$E'_s = E_s \qquad B'_s = B_s$$

$$B'_x = \gamma_1 \left(B_x + \frac{v_1}{c^2} \, E_y \right)$$

$$B'_y = \gamma_1 \left(B_y - \frac{v_1}{c^2} \, E_x \right)$$

SPACE CHARGE (3/31)

- Lorentz force on the particle 2 moving with velocity $\vec{v}_2 = v_2 \, \vec{s}$

$$\vec{F} = e \left(\vec{E} + \vec{v}_2 \times \vec{B} \right)$$

- Beam 1 produces only an electric field in its rest frame R’

$$B'_x = B'_y = B'_s = 0$$

=> $B_x = - \frac{v_1}{c^2} \, E_y$ $B_y = \frac{v_1}{c^2} \, E_x$ $B_s = 0$

Space charge

=> $$F_{x,y} = e \, E_{x,y} \left| \begin{array}{ll} \left(1 - \beta_1 \, \beta_2 \right) & \text{if 2 moves in same direction as 1} \\ \left(1 + \beta_1 \, \beta_2 \right) & \text{if 2 moves in oppo. direction as 1} \end{array} \right.$$

Beam beam

SPACE CHARGE (4/31)

- **Let's assume SC regime and $\beta_1 = \beta_2 = \beta$**

=>

$$\boxed{F_{x,y} = e\, E_{x,y} \left(1 - \beta^2\right) = e \, \frac{E_{x,y}}{\gamma^2}}$$

Electric part

Magnetic part

and

$$E'_{x,y} = \frac{E_{x,y}}{\gamma}$$

$$B_x = -\frac{\beta}{c} E_y$$

$$E'_s = E_s$$

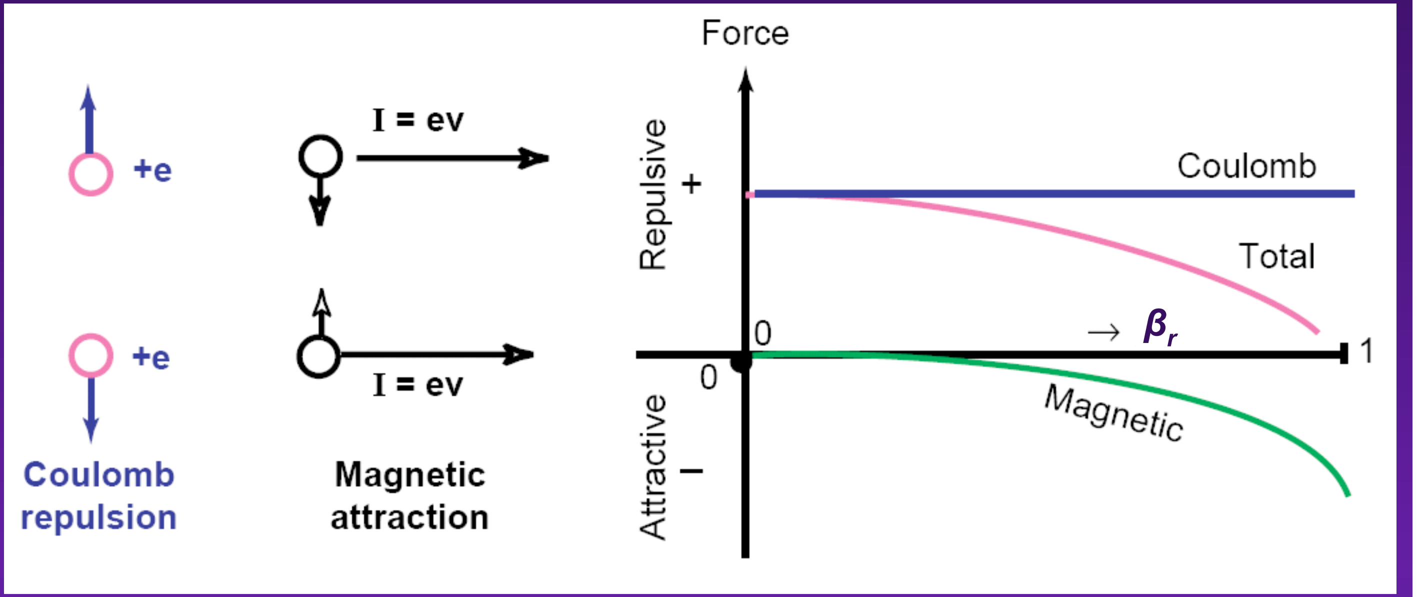
$$B_y = \frac{\beta}{c} E_x$$

$$B'_x = B'_y = B'_s = 0$$

$$B_s = 0$$

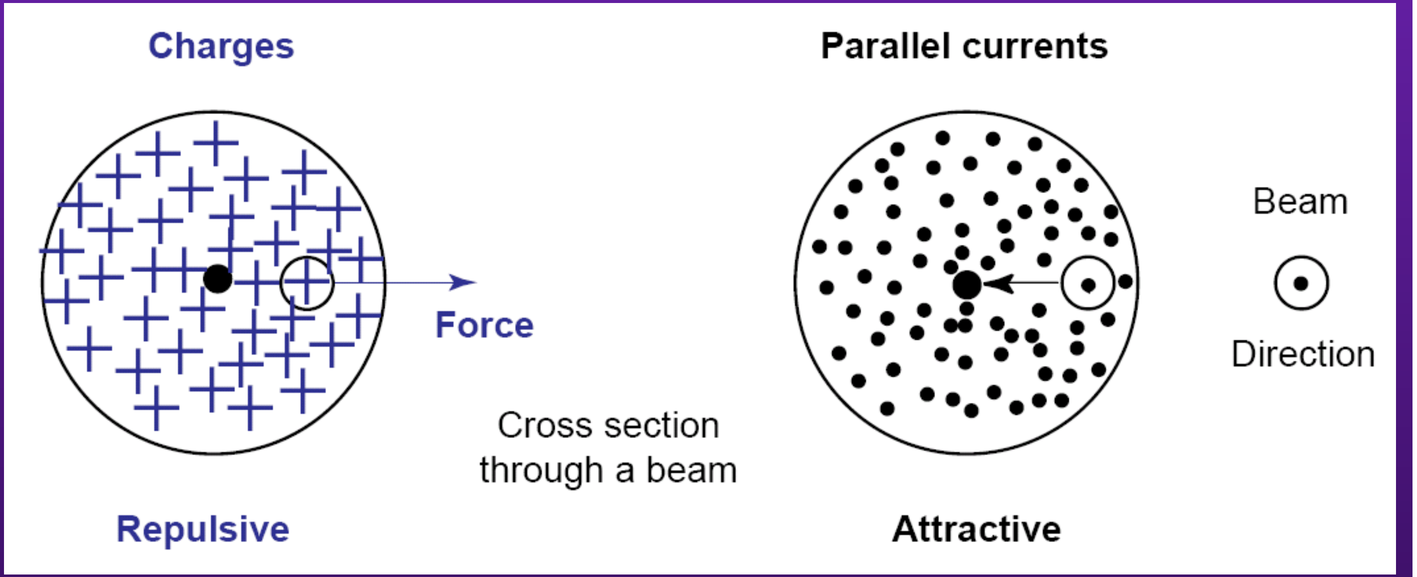
SPACE CHARGE (5/31)

2 particles at rest or travelling



Courtesy K. Schindl

Many charged particles travelling in an unbunched beam with circular cross-section

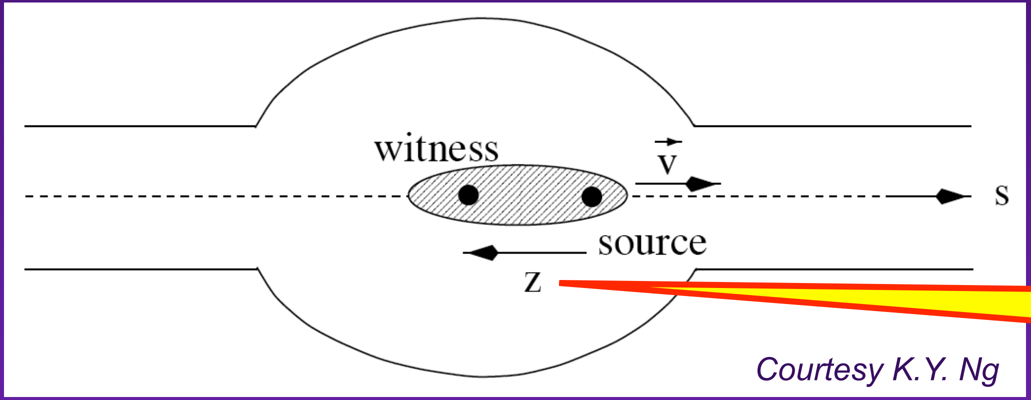


SPACE CHARGE (6/31)

- **Proof of the Panofsky-Wenzel theorem**

$$\boxed{\vec{\nabla}_\perp F_s = \frac{\partial \vec{F}_\perp}{\partial s}}$$

$$\vec{\nabla}_\perp \equiv \left| \begin{matrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{matrix} \right.$$



Courtesy K.Y. Ng

Rigid-beam approximation

$$\overrightarrow{rot}\, \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \left| \begin{matrix} \dfrac{\partial E_s}{\partial y} - \dfrac{\partial E_y}{\partial s} \\ \dfrac{\partial E_x}{\partial s} - \dfrac{\partial E_s}{\partial x} \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} \end{matrix} \right. = \left| \begin{matrix} -\dfrac{\partial B_x}{\partial t} \\ -\dfrac{\partial B_y}{\partial t} \\ -\dfrac{\partial B_s}{\partial t} \end{matrix} \right.$$

$$z = s - v\, t \quad \Rightarrow \quad \frac{d}{dz} = \frac{d}{ds} \qquad \frac{d}{dt} = -v \frac{d}{dz}$$

$$\Rightarrow \quad \frac{\partial E_s}{\partial x} = \frac{\partial}{\partial z}\left(E_x - v\, B_y \right) \quad \text{and} \quad \frac{\partial E_s}{\partial y} = \frac{\partial}{\partial z}\left(E_y + v\, B_x \right)$$

SPACE CHARGE (7/31)

- Computation of the electric field (in cylindrical coordinates)

$$\vec{\nabla}_{\perp} F_s = \frac{\partial \vec{F}_{\perp}}{\partial s}$$

=>

$$\frac{\partial E_s}{\partial r} = \frac{\partial}{\partial z}\left(E_r - v\, B_{\vartheta} \right)$$

$$B_{\vartheta} = \frac{\beta}{c} E_r$$

=>

$$\boxed{\frac{\partial E_s}{\partial r} = \frac{1}{\gamma^2}\frac{\partial E_r}{\partial z}}$$

=>

$$\boxed{E_s\left(r = 0 \right) = -\frac{1}{\gamma^2}\frac{\partial}{\partial z}\int_0^b E_r\, dr}$$

$E_s\left(r = b \right) = 0$ for a Perfectly Conducting (PC) beam pipe

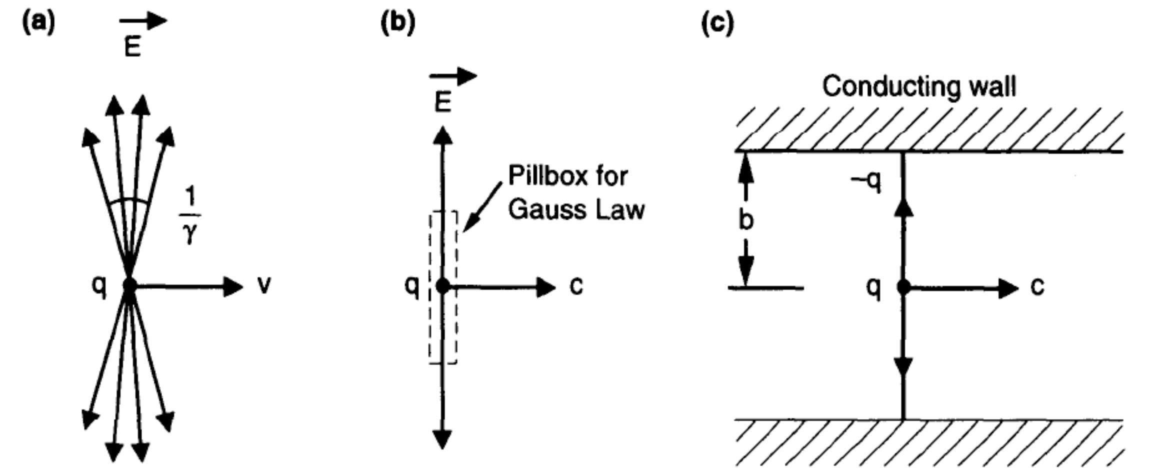


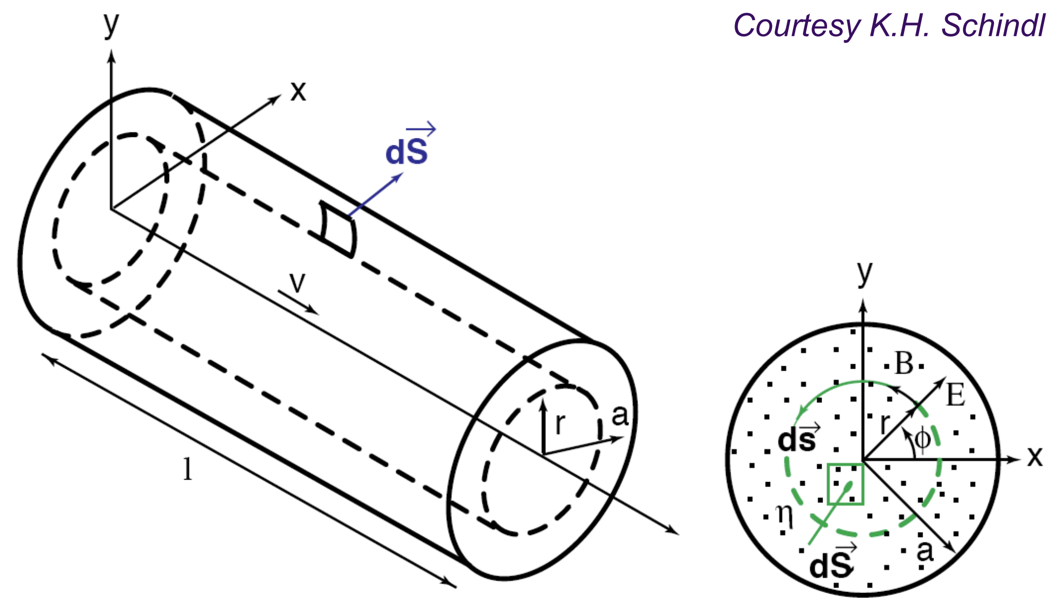
Figure 1.2. Electromagnetic field carried by an ultrarelativistic point charge: (a), (b) in free space; (c) in a perfectly conducting smooth pipe.

Courtesy A.W. Chao

Due to symmetry (cylindrical beam pipe) => Only E_r, E_s and B_θ

SPACE CHARGE (8/31)

- **EM fields of a cylinder with uniform density (with radius a) inside a beam pipe of radius b**



Courtesy K.H. Schindl

- **Charge density [C/m³]** $\rho = \frac{q}{\pi a^2 l}$ $q = N_b e$
- **Current density [A/m²]** $J = \rho v$
- **Line density [C/m]** $\lambda_0 = \frac{q}{l}$
- **Total Current [A]** $I = \lambda_0 v$

SPACE CHARGE (9/31)

- $$\iiint div\ \vec{E}\ dV = \iint \vec{E}\,.\,d\vec{S} = \frac{1}{\varepsilon} \iiint \rho\ dV$$

=> $$\left| \begin{array}{ll} E_r\ 2\,\pi\, r\, l = \dfrac{1}{\varepsilon_0}\, \rho\, \pi\, r^2\, l & \text{for} \quad \mathrm{r} < \mathrm{a} \\ E_r\ 2\,\pi\, r\, l = \dfrac{1}{\varepsilon_0}\, \rho\, \pi\, a^2\, l & \text{for} \quad a < \mathrm{r} < \mathrm{b} \end{array} \right.$$

=> $$\boxed{\left| \begin{array}{ll} E_r = \dfrac{\lambda(z)}{2\,\pi\,\varepsilon_0}\, \dfrac{r}{a^2} & \text{for} \quad \mathrm{r} < \mathrm{a} \\ E_r = \dfrac{\lambda(z)}{2\,\pi\,\varepsilon_0}\, \dfrac{1}{r} & \text{for} \quad a < \mathrm{r} < \mathrm{b} \end{array} \right.}$$

Generalization

$$\lambda_0 \rightarrow \lambda(z)$$

=> The (radial) Lorentz force on a particle of charge e inside the uniform cylinder is

$$F_r = \frac{e}{\gamma^2}\, E_r = \frac{e}{2\,\pi\,\varepsilon_0\,\gamma^2}\, \lambda(z)\, \frac{r}{a^2}$$

SPACE CHARGE (10/31)

- **The (longitudinal) Lorentz force on a particle of charge e inside the uniform cylinder (on r = 0) is**

$$F_s\left(r=0\right)=-\frac{e}{2\pi\varepsilon_0\gamma^2}\frac{d\lambda\left(z\right)}{dz}\left(\int_0^a\frac{r}{a^2}dr+\int_a^b\frac{1}{r}dr\right)$$

=>

$$\boxed{F_s\left(r=0\right)=-\frac{e}{4\pi\varepsilon_0\gamma^2}\frac{d\lambda\left(z\right)}{dz}\left[1+2\ln\left(\frac{b}{a}\right)\right]}$$

SPACE CHARGE (11/31)

- **EM fields and associated Lorentz force (for r < a) for a non-uniform bunch with Gaussian densities in r and s**

$$\rho\left(r,z\right) = \frac{1}{2\pi\sigma_r^2}\, e^{-\frac{r^2}{2\sigma_r^2}}\, \lambda\left(z\right)$$

$$\lambda\left(z\right) = \frac{q}{\sqrt{2\pi}\;\sigma_z}\, e^{-\frac{z^2}{2\sigma_z^2}}$$

- $$\iiint div\;\vec{E}\;dV = \iint \vec{E}\,.\,d\vec{S} = \frac{1}{\varepsilon}\iiint \rho\;dV$$

=> $$E_r\;2\,\pi\,r\;ds = \frac{\lambda\left(z\right)dz}{\varepsilon_0}\int_{\vartheta=0}^{2\pi}\int_{r'=0}^{r}\frac{e^{-\frac{r'^2}{2\sigma_r^2}}\;r'\,dr'}{2\pi\sigma_r^2}\,d\vartheta$$

=> $$\boxed{F_r = \frac{e}{\gamma^2}\,E_r = \frac{e\,\lambda\left(z\right)}{2\,\pi\,\varepsilon_0\,\gamma^2}\left(\frac{1-e^{-\frac{r^2}{2\sigma_r^2}}}{r}\right)}$$

Same result as for uniform case with $a=\sqrt{2}\;\sigma_r$

$$F_r \approx \frac{e\,\lambda\left(z\right)}{2\,\pi\,\varepsilon_0\,\gamma^2}\,\frac{r}{2\sigma_r^2} \quad \text{for} \quad \text{r} << \sigma_r$$

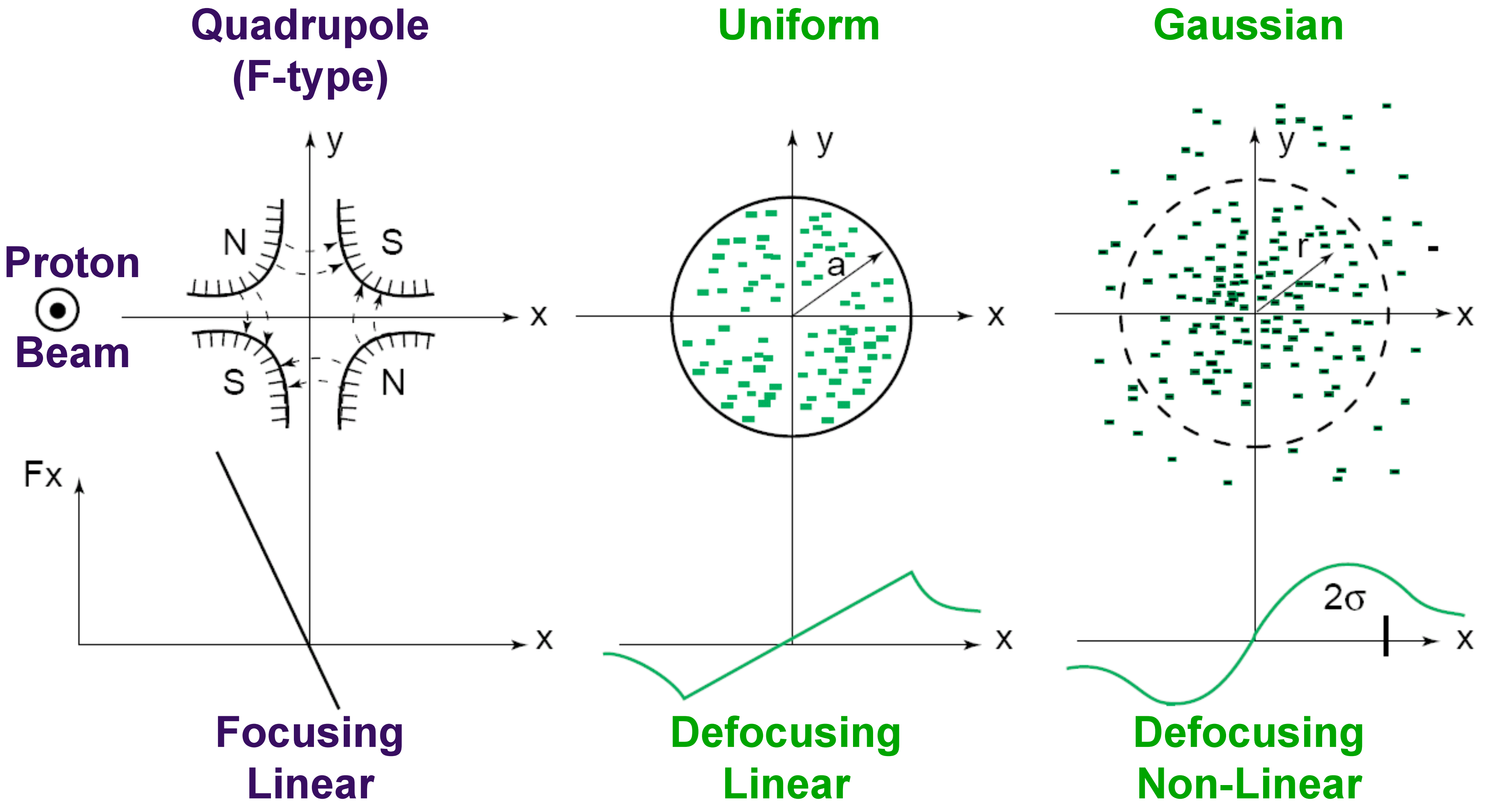
SPACE CHARGE (12/31)

- **The associated (longitudinal) Lorentz force on a particle of charge e is**

$$F_s(r) = -\frac{e}{2\pi\varepsilon_0\gamma^2}\frac{d\lambda(z)}{dz}\int_{r'=r}^{b}\frac{1-e^{-\frac{r'^2}{2\sigma_r^2}}}{r'}dr'$$

Using $\frac{\partial E_s}{\partial r} = \frac{1}{\gamma^2}\frac{\partial E_r}{\partial z}$

SPACE CHARGE (13/31)



Quadrupole (F-type)

Uniform

Gaussian

Proton ⊙ Beam

Focusing Linear

Defocusing Linear

Defocusing Non-Linear

Courtesy K. Schindl

SPACE CHARGE (14/31)

- **Transverse incoherent tune shift induced by the "direct" SC**
  - **Equation of motion**

$$\boxed{\frac{d^2x}{ds^2} + K_x(s)\,x = \frac{F_x}{\beta^2 E_{total}}}$$

F_x^{pert}

  - **Linearizing (for a transversally Gaussian bunch)**

$$F_x = \frac{e\,\lambda(z)}{2\,\pi\,\varepsilon_0\,\gamma^2}\,\frac{x}{2\sigma_x^2} \quad \text{for} \quad x << \sigma_x$$

=>

$$\frac{d^2x}{ds^2} + \left[K_x(s) + K_{SC,x}(z)\right] x = 0$$

with

$$K_{SC,x}(z) = -\frac{e\,\lambda(z)}{4\,\pi\,\varepsilon_0\,E_{total}\,\beta^2\,\gamma^2\,\sigma_x^2}$$

$$\Delta Q_x \equiv \frac{1}{4\,\pi}\int_{s=0}^{2\,\pi\,R} K_{SC,x}(z)\,\beta_x(s)\,ds$$

=>

$$\Delta Q_x = -\frac{e\,R\,\lambda(z)}{8\,\pi\,\varepsilon_0\,E_{total}\,\beta\,\gamma\,\varepsilon_{x,rms}^{norm}}$$

SPACE CHARGE (15/31)

with $\varepsilon_{x,rms}^{norm} = \beta\gamma\,\varepsilon_{x,rms}$ $\qquad \varepsilon_{x,rms} = \dfrac{\sigma_x^2}{\beta_x}$

At a place of 0 dispersion

- **Assuming then** $\lambda(z) = \dfrac{N_b\,e}{\sqrt{2\pi}\;\sigma_z}\,e^{-\frac{z^2}{2\sigma_z^2}}$

=> $$\Delta Q_x^{\max} = -\frac{N_b\,R\,r_p}{2\sqrt{2\pi}\;\beta\gamma^2\,\sigma_z\,\varepsilon_{x,rms}^{norm}}$$

=> $$\boxed{\Delta Q_x^{\max} = -\frac{N_b\,r_p}{4\pi\,\beta\,\gamma^2\,\varepsilon_{x,rms}^{norm}\,B}}$$

SC is always defocusing in the transverse planes

where *B* is the bunching factor defined by

$$B = \frac{I_{average}}{I_{peak}} = \frac{\sqrt{2\pi}\;\sigma_z}{2\pi R}$$

***B* = 1 for a coasting beam**

SPACE CHARGE (16/31)

- **Another way to deduce the transverse incoherent tune shift induced by the "direct" SC**
  - **Equation of motion** $$\frac{d^2 x}{ds^2} + K_x(s)\,x = \frac{F_x}{\beta^2 E_{total}}$$
  - **Smooth approximation** $$K_x(s) = \left(\frac{Q_{x0}}{R}\right)^2$$

=> $$\frac{d^2 x}{ds^2} + \frac{1}{R^2}\left(Q_{x0}^2 - \frac{e\,R^2\,\lambda(z)}{4\,\pi\,\varepsilon_0\,E_{total}\,\beta^2\,\gamma^2\,\sigma_x^2}\right)x = 0$$

$$(Q_{x0} + \Delta Q_x)^2 \approx Q_{x0}^2 + 2\,Q_{x0}\,\Delta Q_x$$

=> $$\Delta Q_x = -\frac{e\,R^2\,\lambda(z)}{4\,\pi\,\varepsilon_0\,E_{total}\,\beta^2\,\gamma^2\,\sigma_x^2} \times \frac{1}{2\,Q_{x0}}$$

New tune: $Q_x = Q_{x0} + \Delta Q_x$

It is the same result as before

SPACE CHARGE (17/31)

- **Case of an elliptical beam**

  - **Particle density** $n(x,y) = n\left(\frac{x^2}{x_m^2} + \frac{y^2}{y_m^2}\right)$ **, i.e. elliptical symmetry, and 0 outside the ellipse** $\frac{x^2}{x_m^2} + \frac{y^2}{y_m^2} = 1$

=> It can be shown that

$$\boxed{E_x = \frac{e\, x_m\, y_m\, x}{2\,\varepsilon_0} \int_{s=0}^{s=+\infty} n\left(\frac{x^2}{x_m^2+s} + \frac{y^2}{y_m^2+s}\right)\left(x_m^2+s\right)^{-3/2}\left(y_m^2+s\right)^{-1/2} ds}$$

$$\boxed{E_y = \frac{e\, x_m\, y_m\, y}{2\,\varepsilon_0} \int_{s=0}^{s=+\infty} n\left(\frac{x^2}{x_m^2+s} + \frac{y^2}{y_m^2+s}\right)\left(x_m^2+s\right)^{-1/2}\left(y_m^2+s\right)^{-3/2} ds}$$

SPACE CHARGE (18/31)

- **Let's assume first a constant density** $n\,(x,y) = \dfrac{N_1}{\pi\, a\, b}$

N_1 is the number of particles / unit length
(= N / 2 π R for a continuous beam)

=> $$E_x = \frac{e\,N_1}{\pi\,\varepsilon_0}\,\frac{x}{x_m\left(x_m + y_m\right)} \qquad E_y = \frac{e\,N_1}{\pi\,\varepsilon_0}\,\frac{y}{y_m\left(x_m + y_m\right)}$$

- **Reminder: For the case of a circular beam ($x_m = y_m = a$) with constant density we found (e.g. in y-plane)** $$E_y^{Const} = \frac{e\,N_1}{\pi\,\varepsilon_0}\,\frac{y}{2\,y_m^2}$$

=> **To go from a round to an elliptical beam, replace $2\,y_m^2$ by $y_m\left(x_m + y_m\right)$ (and $2\,x_m^2$ by $x_m\left(x_m + y_m\right)$)**

SPACE CHARGE (19/31)

- Let's assume now a parabolic density

$$n\,(x,y) = \frac{2\,N_1}{\pi\,a\,b}\left(1 - \frac{x^2}{x_m^2} - \frac{y^2}{y_m^2}\right)$$

N$_1$ is the number of particles / unit length
(= *N* / 2 π R for a continuous beam)

The integrals can be evaluated by changing the variable, using *u* given by

$$u^2 = b^2 + s$$

=>

$$E_x = \frac{2\,e\,N_1}{\pi\,\varepsilon_0}\left[x\,\frac{1}{x_m\,(x_m + y_m)} - x^3\,\frac{2\,x_m + y_m}{3\,x_m^3\,(x_m + y_m)^2} - x\,y^2\,\frac{1}{x_m\,y_m\,(x_m + y_m)^2}\right]$$

$$E_y = \frac{2\,e\,N_1}{\pi\,\varepsilon_0}\left[y\,\frac{1}{y_m\,(x_m + y_m)} - y^3\,\frac{2\,y_m + x_m}{3\,y_m^3\,(x_m + y_m)^2} - y\,x^2\,\frac{1}{x_m\,y_m\,(x_m + y_m)^2}\right]$$

SPACE CHARGE (20/31)

- Linearizing, we obtain (for instance in the y-plane)

$$E_y \approx \frac{e N_1}{\pi \varepsilon_0} \frac{2 y}{y_m \left(x_m + y_m \right)}$$

✧ Reminder: For the case of a bi-Gaussian beam, we had

$$E_y^{G,lin} \approx \frac{e N_1}{\pi \varepsilon_0} \frac{y}{\left(2 \sigma_y \right)^2}$$

=> The same result is obtained for the case of a round beam ($x_m = y_m$) if $y_m = 2 \sigma_y$

SPACE CHARGE (21/31)

Opposite convention to the course of LBD (<0 BT and >0 AT)!

- **Longitudinal tune shift from SC**
  - **Equation of motion**

$$\frac{d^2 z}{ds^2} + \left(\frac{Q_s}{R}\right)^2 z = -\eta \, \frac{F_s}{\beta^2 \, E_{total}}$$

$$\eta = \gamma_{tr}^{-2} - \gamma^{-2}$$

  - **Linearizing (for a transversally Gaussian bunch)**

$$F_s = -\frac{e}{2\pi\,\varepsilon_0\,\gamma^2}\,\frac{d\,\lambda(z)}{d\,z}\left(\int_r^{a=\sqrt{2}\,\sigma_r}\frac{r'}{2\sigma_r^2}\,dr' + \int_a^b \frac{dr'}{r'}\right)$$

As it is the same result as for uniform case with $a = \sqrt{2}\,\sigma_r$

=> $$F_s = -\frac{e}{4\pi\,\varepsilon_0\,\gamma^2}\,\frac{d\,\lambda(z)}{d\,z}\left[1 + 2\ln\left(\frac{b}{a}\right)\right]$$

=> $$\frac{d^2 z}{ds^2} + \left(\frac{Q_{s0}}{R}\right)^2 z = \frac{\eta\,e}{4\pi\,\varepsilon_0\,E_{total}\,\beta^2\,\gamma^2}\,\frac{d\,\lambda(z)}{d\,z}\left[1 + 2\ln\left(\frac{b}{a}\right)\right]$$

SPACE CHARGE (22/31)

- **Assuming then**

$$\lambda(z) = \frac{N_b\, e}{\sqrt{2\pi}\ \sigma_z}\, e^{-\frac{z^2}{2\sigma_z^2}}$$

=> $$\frac{d\lambda(z)}{d\,z} = -\frac{z}{\sigma_z^2}\,\lambda(z) \approx -z\,\frac{N_b\, e}{\sqrt{2\pi}\ \sigma_z^3} \quad \text{for} \quad z << \sigma_z$$

=> $$\frac{d^2 z}{ds^2} + \frac{1}{R^2}\left(Q_{s0}^2 + \frac{\eta\, N_b\, e^2\, R^2}{4\pi\sqrt{2\pi}\ \varepsilon_0\, E_{total}\, \beta^2\, \gamma^2\, \sigma_z^3}\left[1 + 2\ln\left(\frac{b}{a}\right)\right]\right) z = 0$$

$$\left(Q_{s0} + \Delta Q_s\right)^2 \approx Q_{s0}^2 + 2\, Q_{s0}\, \Delta Q_s$$

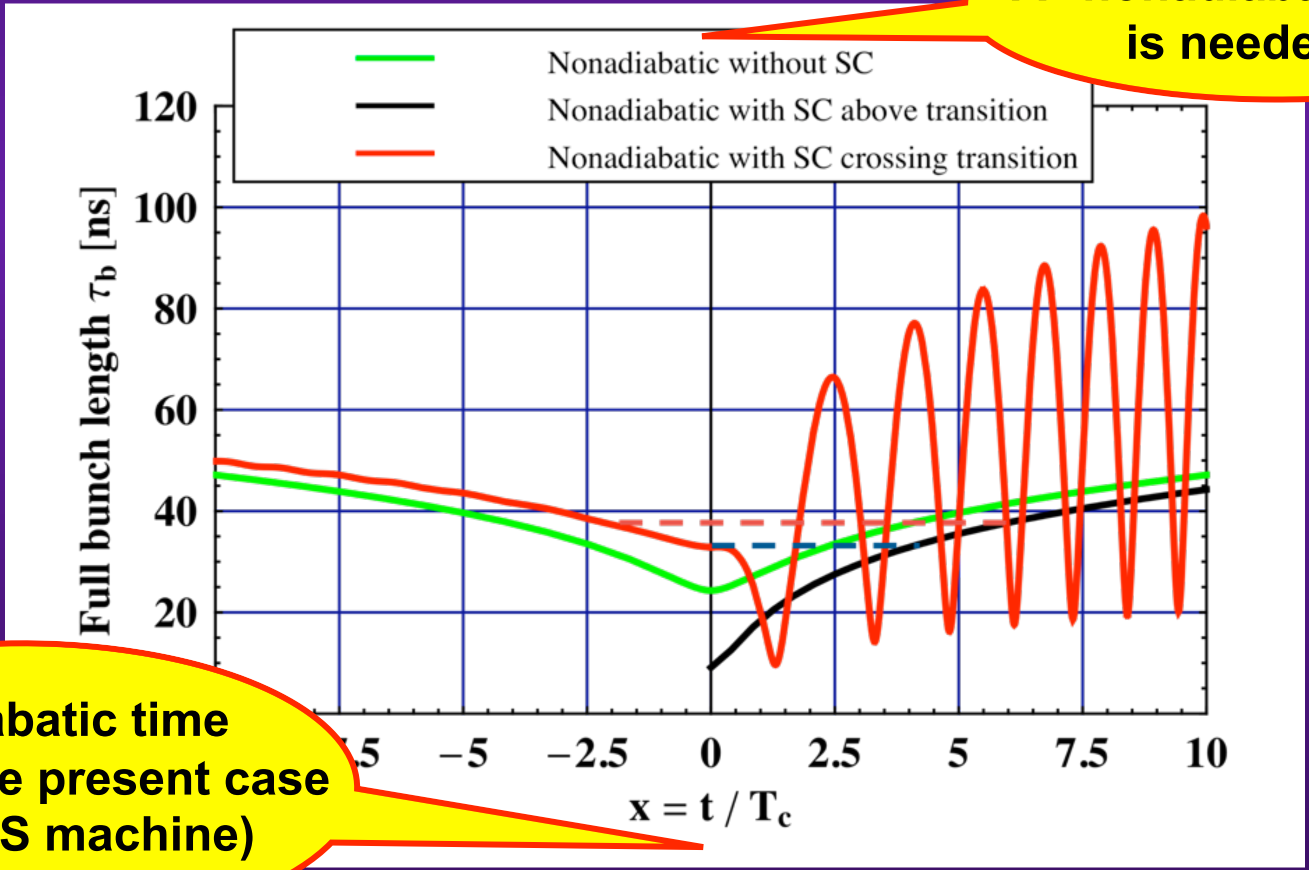
=> $$\boxed{\Delta Q_s = \frac{\eta\, N_b\, e^2\, R^2}{8\pi\sqrt{2\pi}\ \varepsilon_0\, E_{total}\, \beta^2\, \gamma^2\, \sigma_z^3\, Q_{s0}}\left[1 + 2\ln\left(\frac{b}{a}\right)\right]}$$

New tune: $Q_s = Q_{s0} + \Delta Q_s$

In the longitudinal plane, it is found that SC is defocusing Below Transition (BT) and focusing above (AT)

SPACE CHARGE (23/31)

- => One can therefore already anticipate some longitudinal mismatch issues when crossing transition with high-intensity bunches, i.e. the bunch length will not be in equilibrium anymore and will oscillate inside the RF buckets



A “nonadiabatic” theory is needed here

Nonadiabatic time
(~ 2 ms in the present case
for the PS machine)

SPACE CHARGE (24/31)

- Transverse tune spread (due to the nonlinear force)
  - Let's assume the following particle density (considering a round beam, $\sigma_x = \sigma_y = \sigma$)

$$x_m = \sqrt{10}\ \sigma \approx 3.2\,\sigma$$

$$n(x,y) = n_0 \left(1 - \frac{x^2 + y^2}{x_m^2} \right)^3$$

with

$$n_0 = \frac{2\,N_b}{B\,\pi^2 R\,x_m^2}$$

Assuming a Gaussian longitudinal profile

$$B = \sqrt{2\pi}\ \sigma_z / (2\pi R)$$

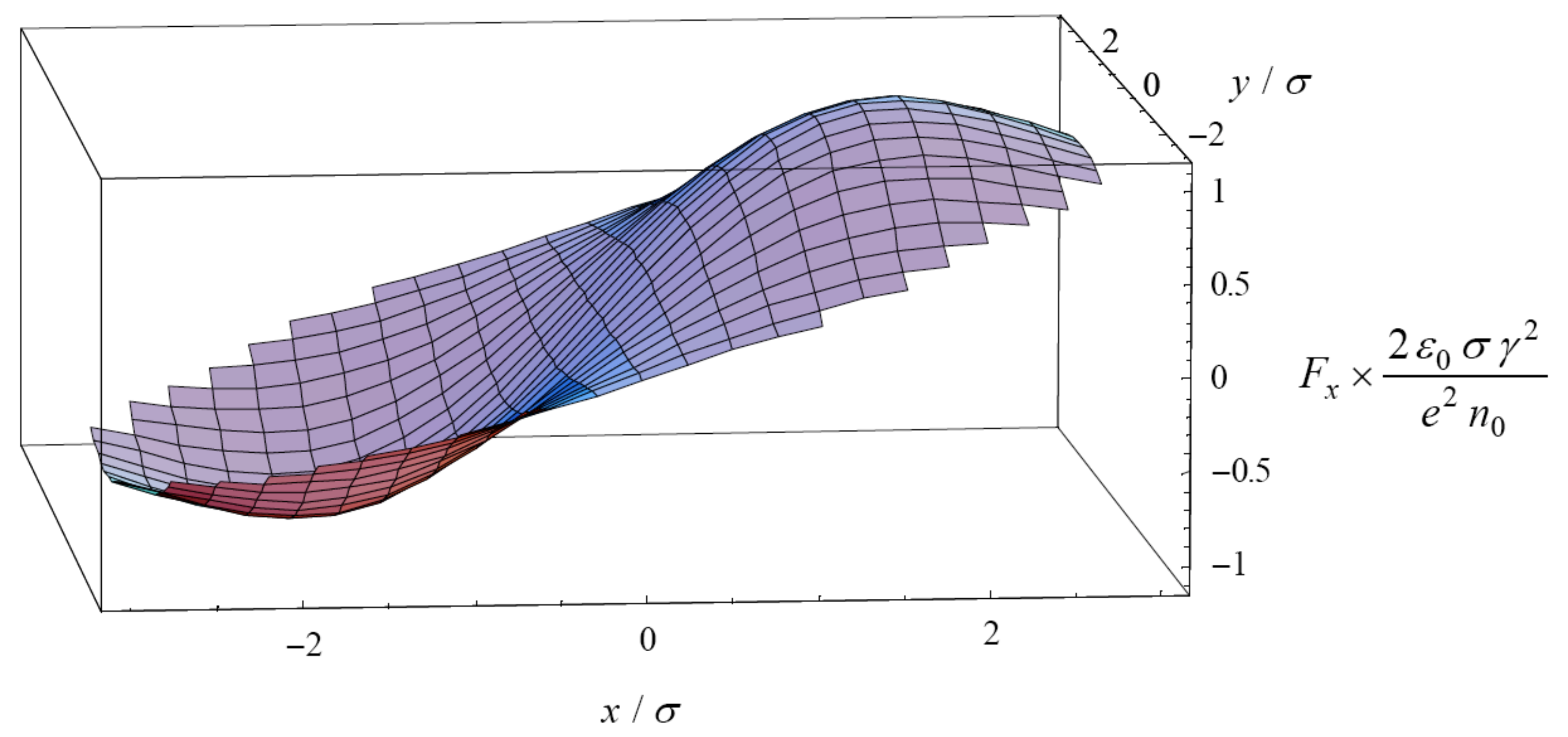
$$\int_x \int_y n(x,y) = \frac{N}{2\pi R}$$

For a coasting beam

=>

$$\boxed{F_x = \frac{e\,E_x}{\gamma^2} = \frac{e^2 n_0}{2\varepsilon_0\,\gamma^2}\left[x - \frac{3x\left(x^2+y^2\right)}{2\,x_m^2} + \frac{x\left(x^2+y^2\right)^2}{x_m^4} - \frac{x\left(x^2+y^2\right)^3}{4\,x_m^6} \right]}$$

SPACE CHARGE (25/31)



SPACE CHARGE (26/31)

- **(Non-linear) space-charge tune shift:** For an approximate solution, the non-linear dependence of the force is converted into an amplitude dependence of the particle's tune using the method of the harmonic balance, which is an averaging process over the incoherent betatron motions

Action variables

$x = x_0 \cos\varphi$ $\quad y = y_0 \cos\vartheta$ $\quad x_0 = \sqrt{2 J_x}$ $\quad y_0 = \sqrt{2 J_y}$

=>

$$< x^3 > \approx \frac{3}{4} x_0^2 \; x \,, \qquad < x\, y^2 > \approx \frac{1}{2} y_0^2 \; x \,, \qquad < x^5 > \approx \frac{5}{8} x_0^4 \; x \,,$$

$$< x^3 y^2 > \approx \frac{3}{8} x_0^2 \; y_0^2 \; x \,, \qquad < x\, y^4 > \approx \frac{3}{8} y_0^4 \; x \,, \qquad < x^7 > \approx \frac{35}{64} x_0^6 \; x \,,$$

$$< x^5 y^2 > \approx \frac{5}{16} x_0^4 \; y_0^2 \; x \,, \qquad < x^3 y^4 > \approx \frac{9}{32} x_0^2 \; y_0^4 \; x \,, \qquad < x\, y^6 > \approx \frac{5}{16} y_0^6 \; x \,,$$

SPACE CHARGE (27/31)

=> $$\Delta Q^x_{incoh}\left(j_x, j_y\right) = \Delta_0 \left[1 - \frac{9}{8} j_x - \frac{3}{4} j_y + \frac{5}{8} j_x^2 + \frac{3}{4} j_x \, j_y + \frac{3}{8} j_y^2 - \frac{35}{256} j_x^3 - \frac{15}{64} j_x^2 \, j_y - \frac{27}{128} j_x \, j_y^2 - \frac{5}{64} j_y^3 \right]$$

$$j_x = J_x / J_{\max}$$

$$j_y = J_y / J_{\max}$$

with $$\Delta_0 = - \frac{N_b \, r_p}{5 \, \pi \, B \, \beta \, \gamma^2 \varepsilon^{norm}_{rms}}$$

$$J_{\max} = 5 \, \sigma^2$$

$$\varepsilon^{norm}_{rms} = \beta \, \gamma \, \varepsilon$$

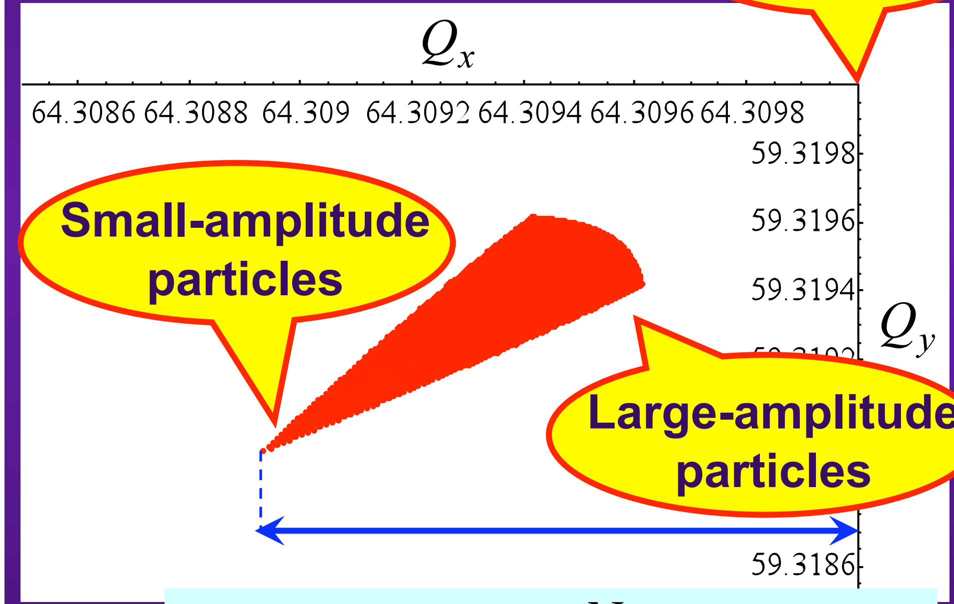
It was 4 in the case of a bi-Gaussian

SPACE CHARGE (28/31)

- **2D TUNE FOOTPRINT**

  ⇒ **INCOHERENT**

  (single-particle) tunes



Low-intensity working point

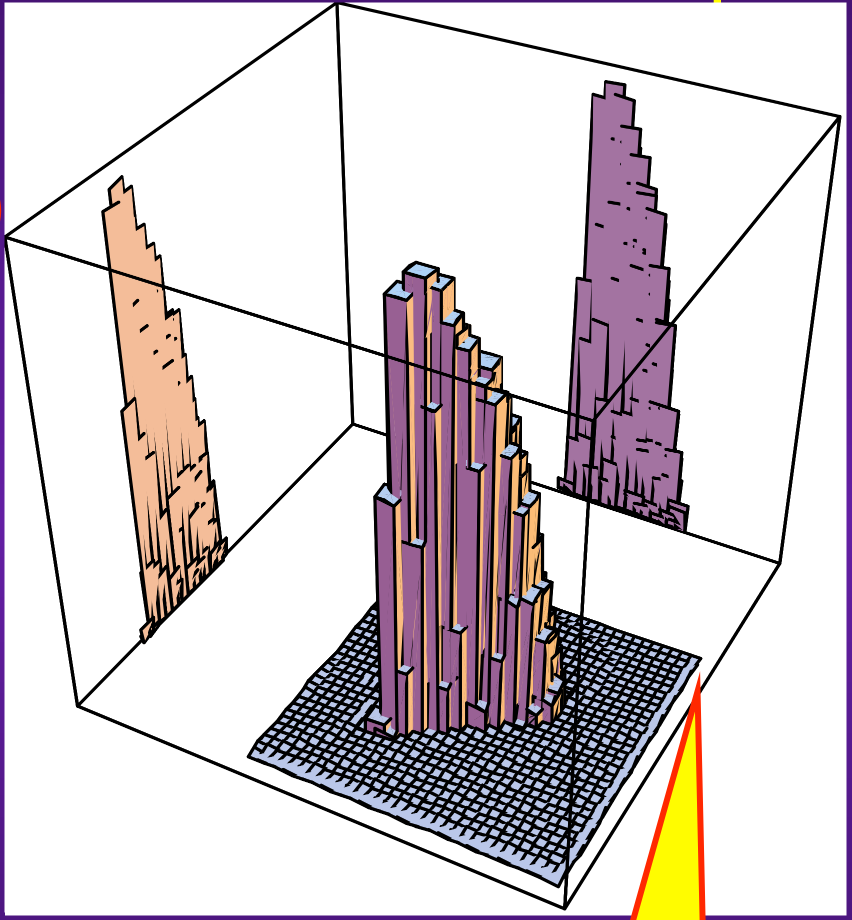
Small-amplitude particles

Large-amplitude particles

$$\Delta_0 \propto -\frac{N_b}{\beta\,\gamma^2\,\varepsilon_{rms}^{norm}}$$

= Linear space - charge tune shift

- **3D view of the 2D tune footprint**

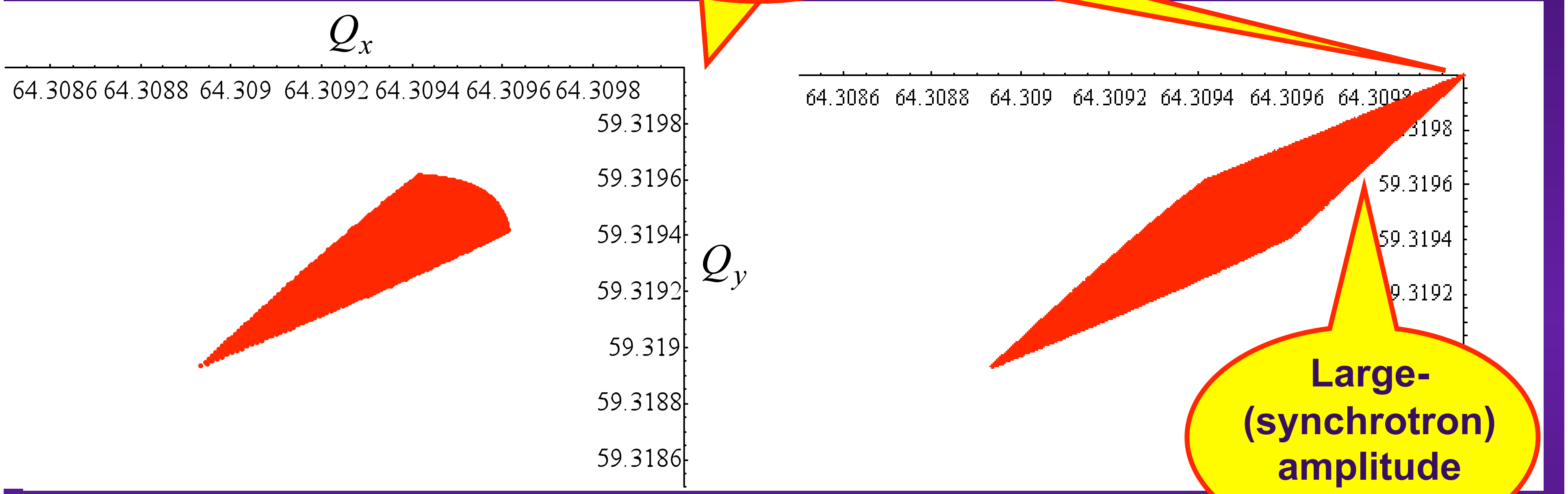


Low-intensity working point

SPACE CHARGE (29/31)

- **2D tune footprint**
- **3D tune footprint**

Low-intensity working point



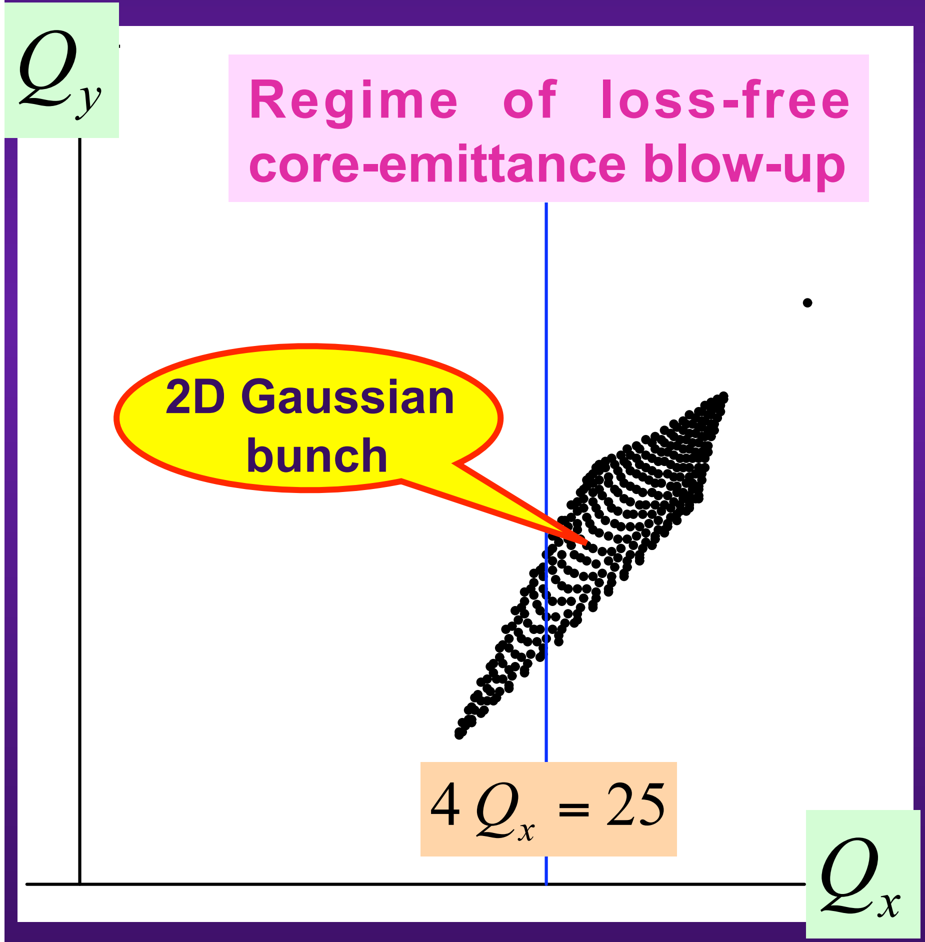
Large-(synchrotron) amplitude particles

Considering *B* (s) and not only *B* (0)

=> **The longitudinal variation (due to synchrotron oscillations) of the transverse space-charge force fills the gap until the low-intensity working point**

SPACE CHARGE (30/31)

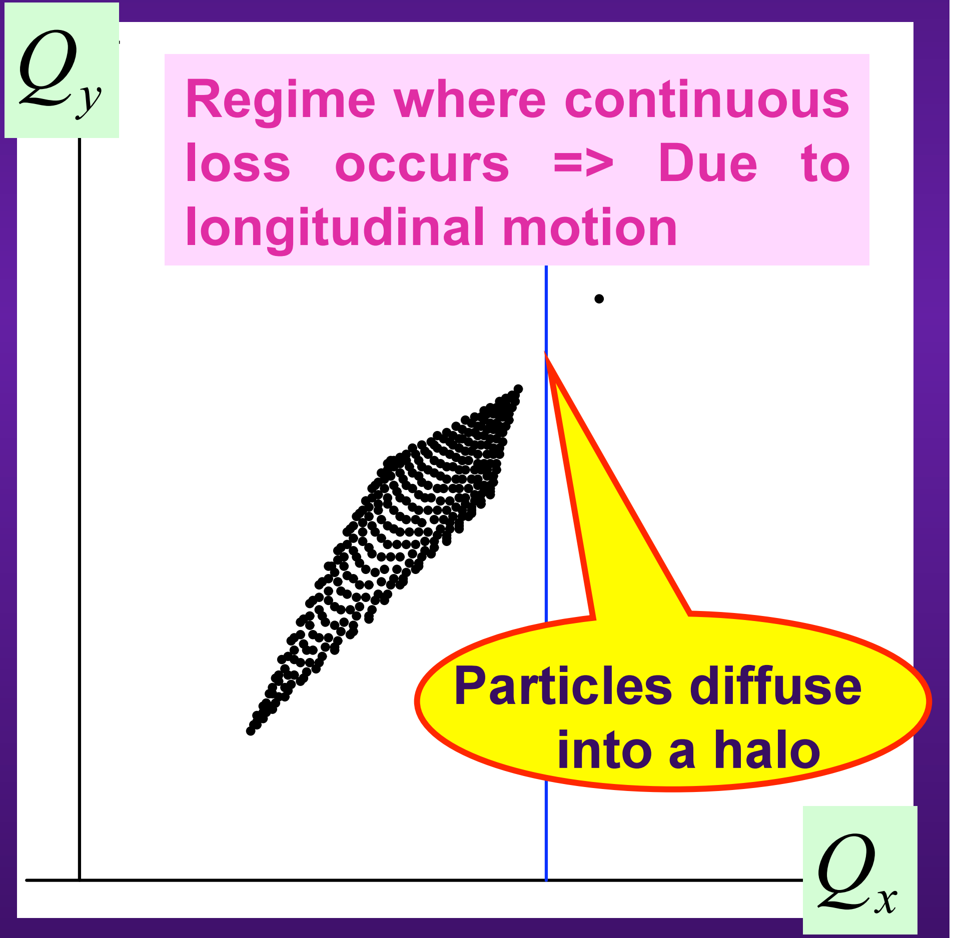
- **Examples of interaction with a lattice resonance**



Regime of loss-free core-emittance blow-up

2D Gaussian bunch

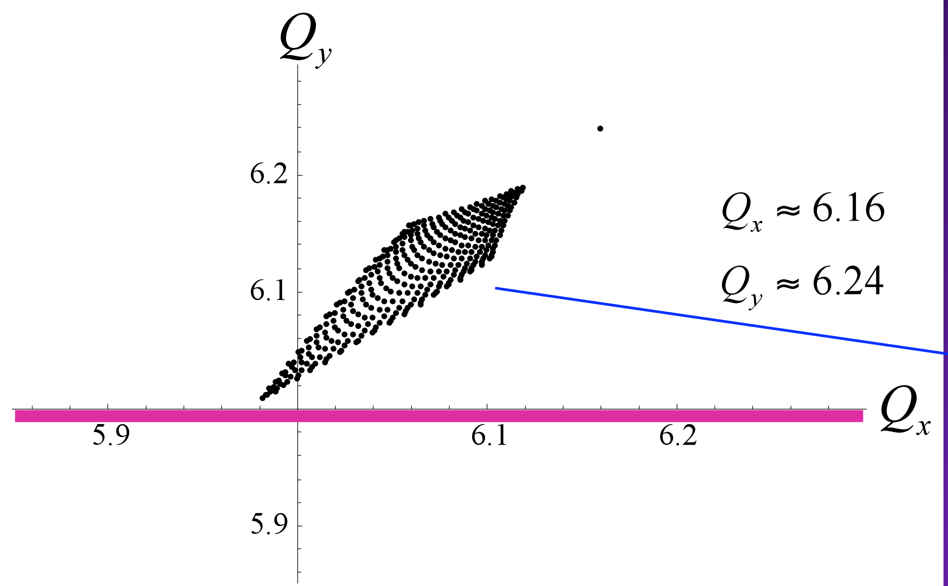
$4\,Q_x = 25$

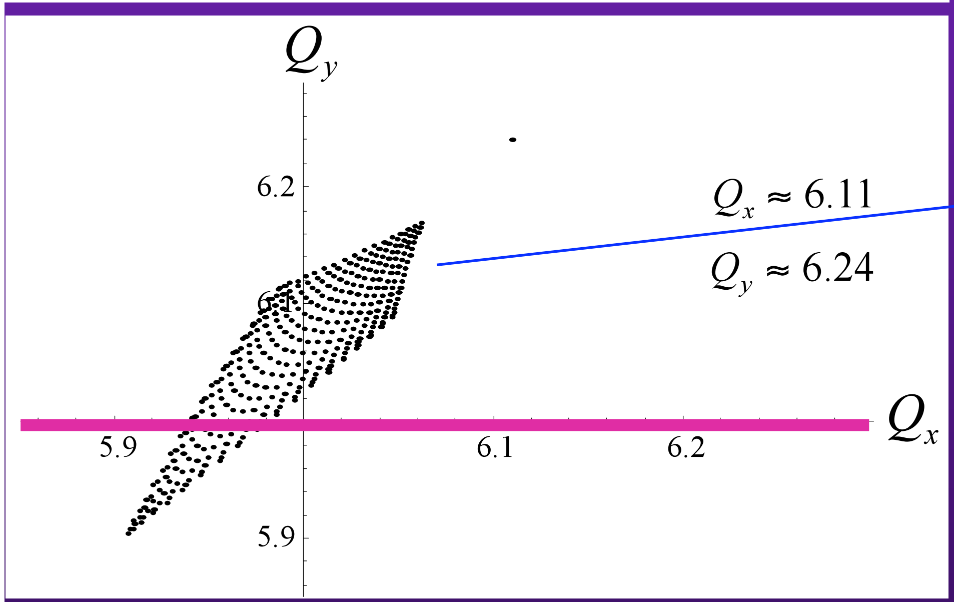


Regime where continuous loss occurs => Due to longitudinal motion

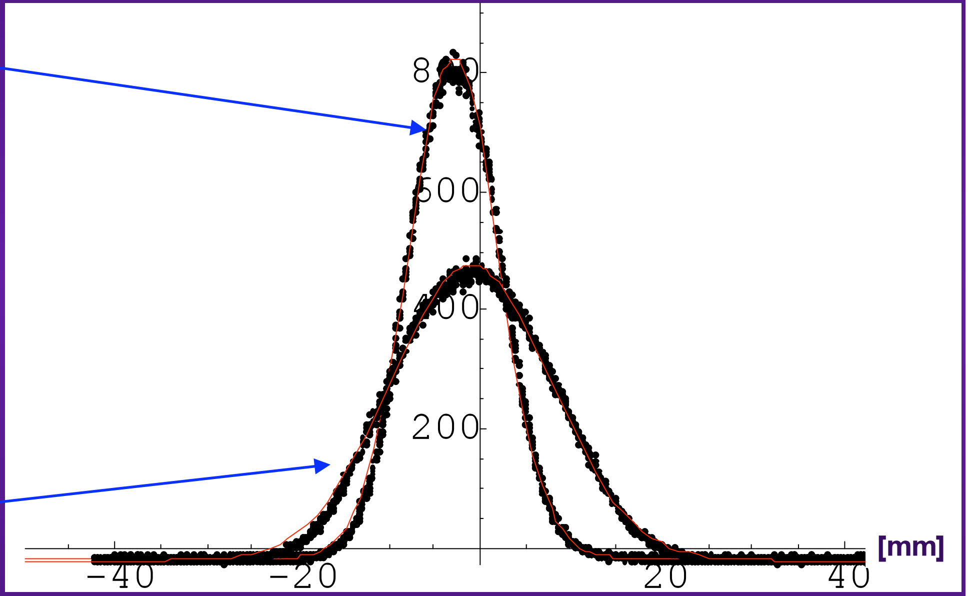
Particles diffuse into a halo

SPACE CHARGE (31/31)





Horizontal bunch profile
+ Gaussian fit



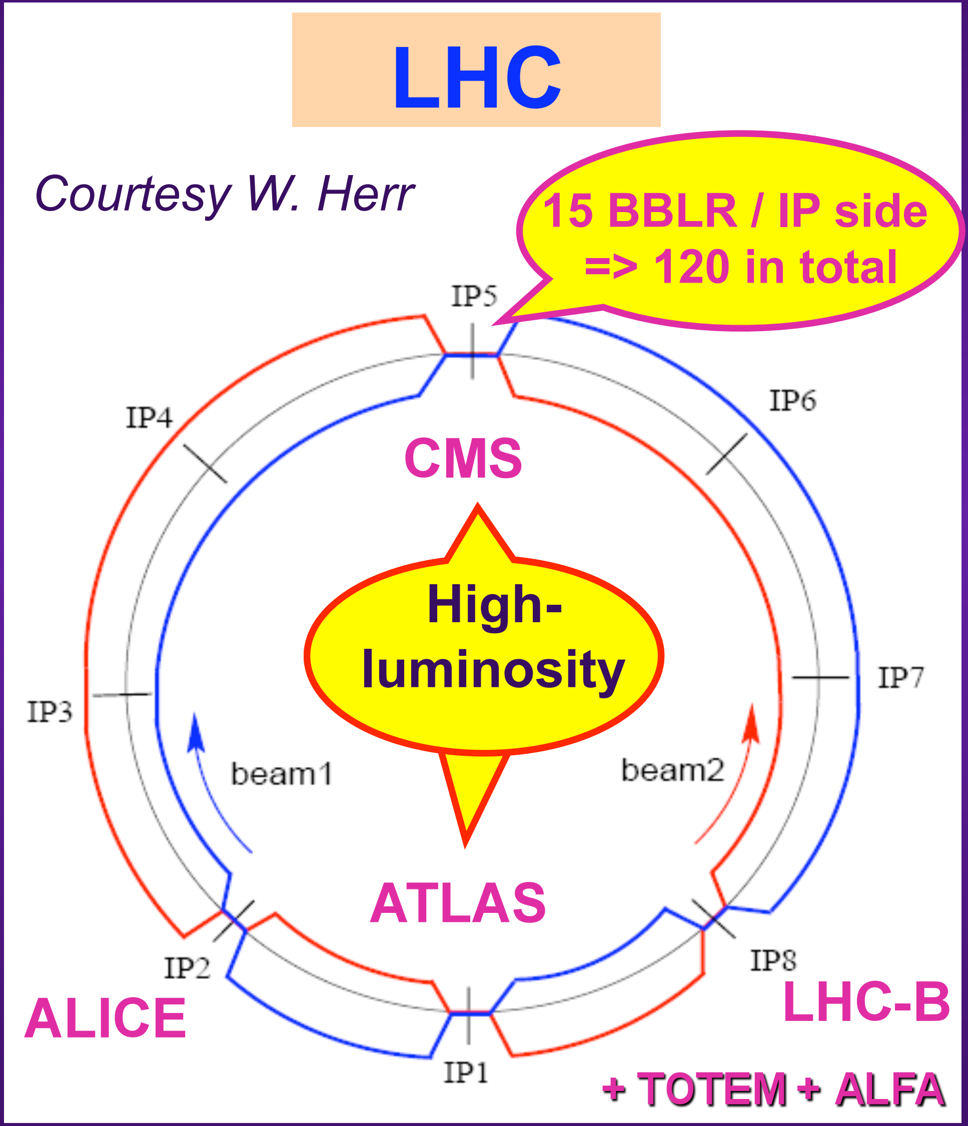
Regime of loss-free core-emittance blow-up

BEAM-BEAM (1/9)

- Interaction between the 2 counter-rotating beams of a collider

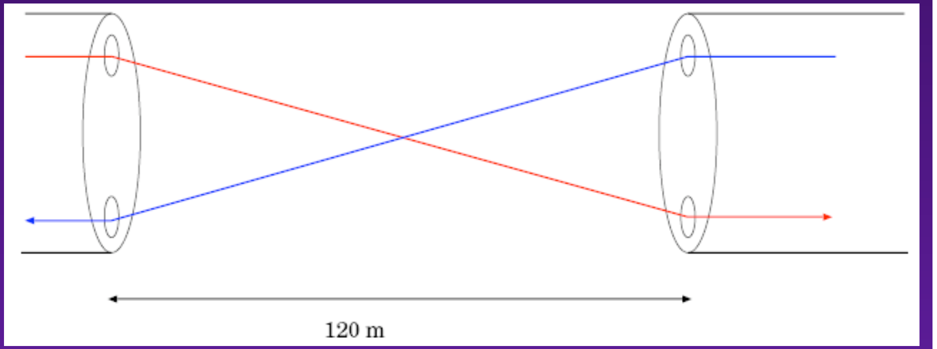
- This can lead to
  - Incoherent beam-beam effects ⇒ Lifetime + dynamic aperture
  - PACMAN effects ⇒ Bunch to bunch variation
  - Coherent beam-beam effects ⇒ Beam oscillations and instabilities

BEAM-BEAM (2/9)

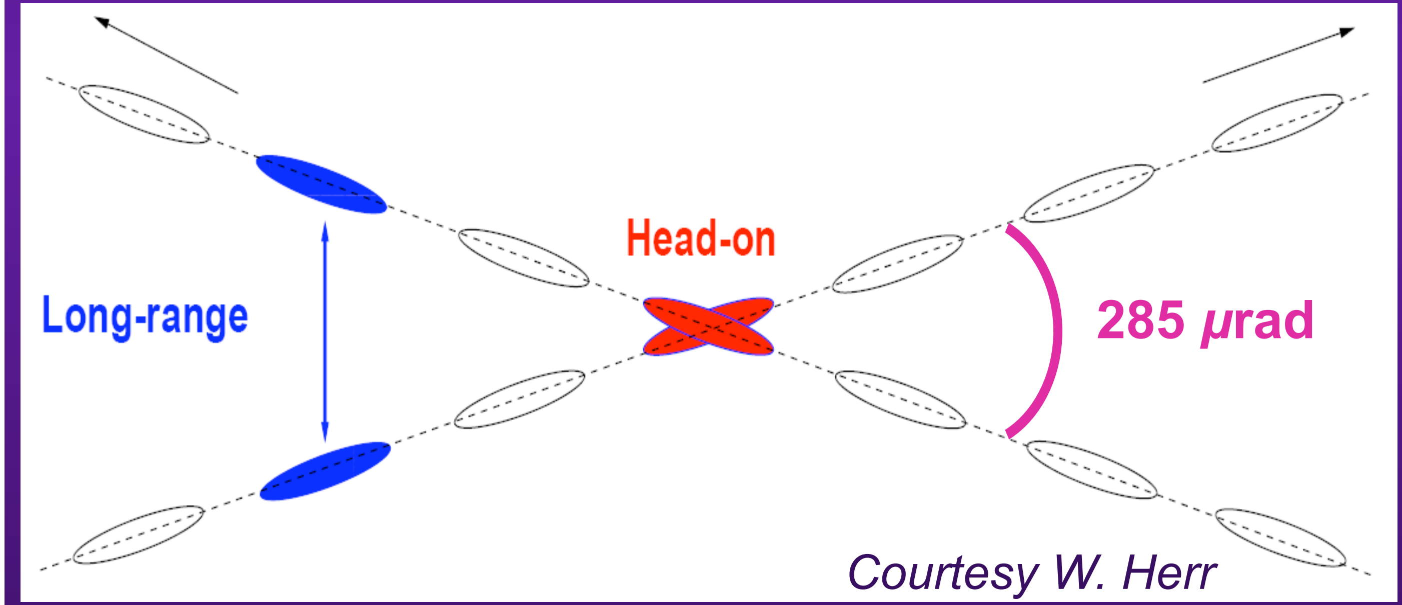

LHC
Courtesy W. Herr
15 BBLR / IP side
=> 120 in total
IP5
IP4
IP6
CMS
High-
luminosity
IP3
IP7
beam1
beam2
ATLAS
IP2
IP8
ALICE
LHC-B
IP1
+ TOTEM + ALFA

BEAM-BEAM (3/9)

CROSSING ANGLE ⇒ To avoid unwanted collisions, a crossing angle is needed to separate the 2 beams in the part of the machine where they share a vacuum chamber



Courtesy W. Herr

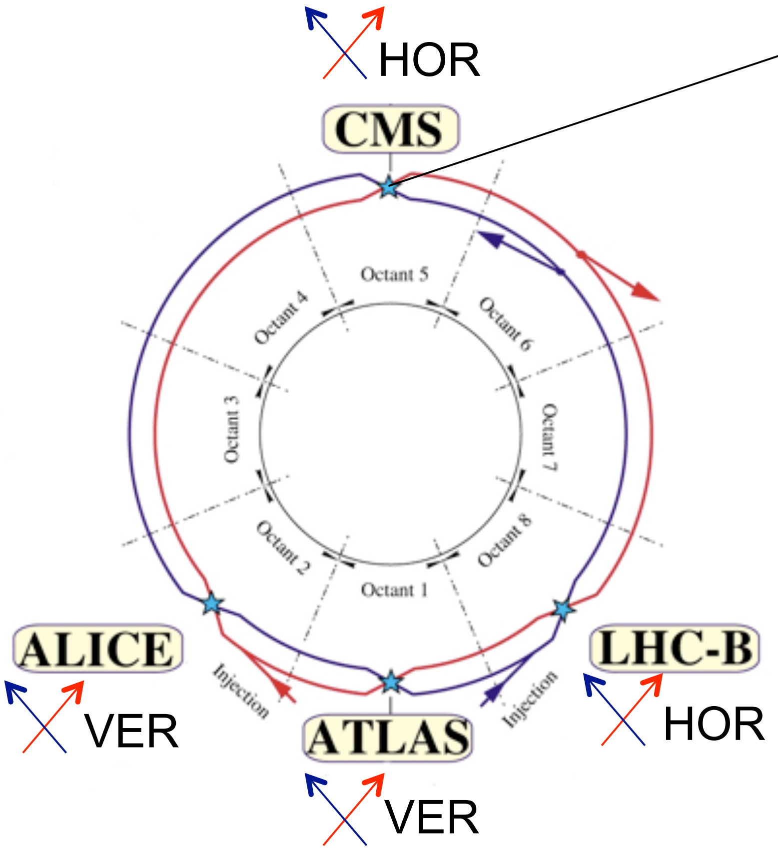


Courtesy W. Herr

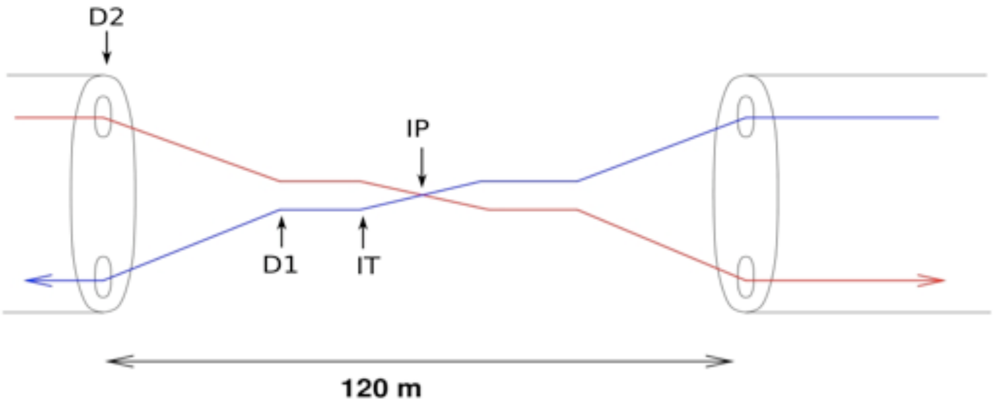
- **30 long-range interactions around each IP ⇒ 120 in total**
- **Separation: 9 σ**

LHC Crossing-Angle Production

Layout



Production of crossing-angle



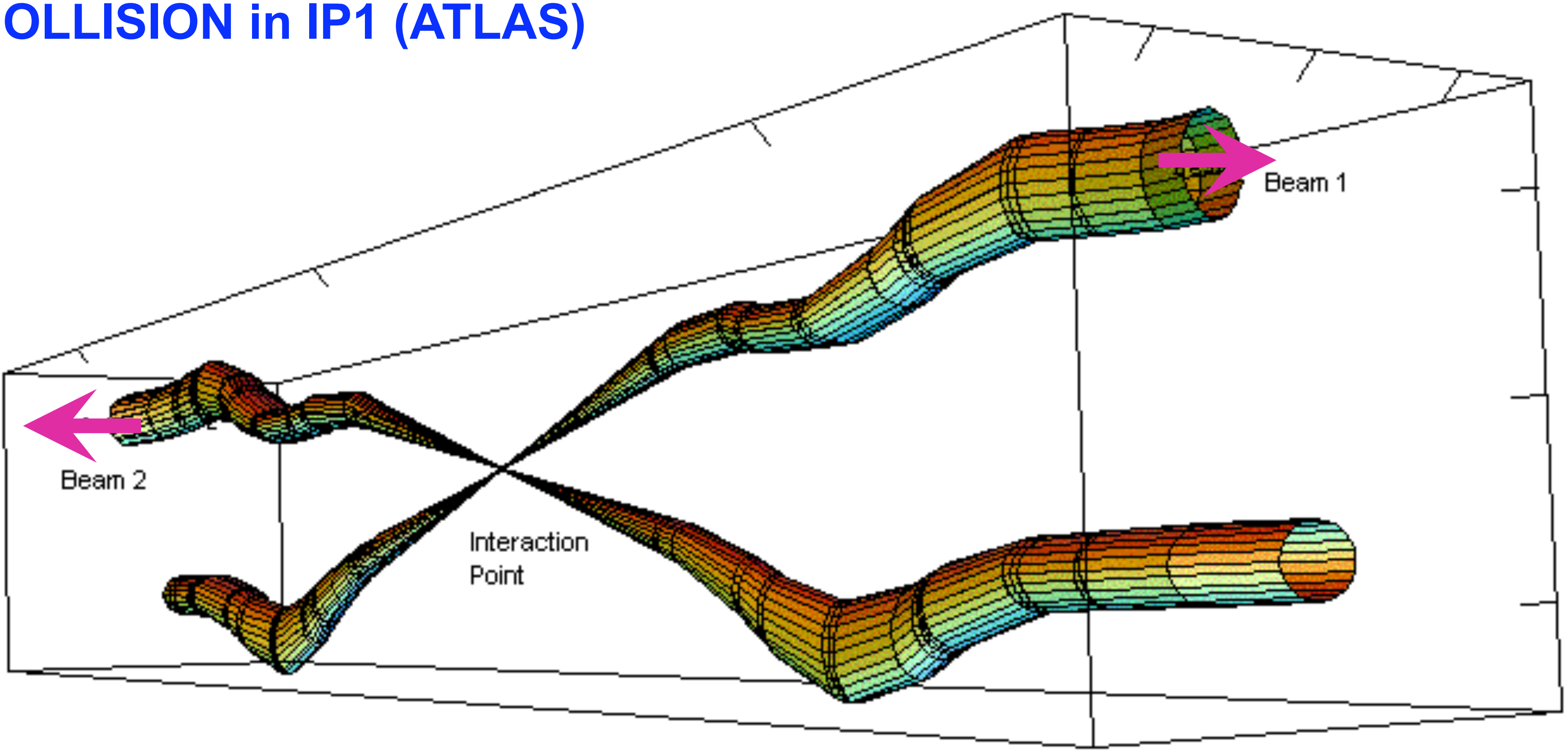
Cross over between inner and outer vacuum chamber in the LHC

IP = interaction point
IT = inner triplet magnets
D1 and D2 = separation-recombination dipoles

Courtesy M. Schaumann

BEAM-BEAM (5/9)

COLLISION in IP1 (ATLAS)

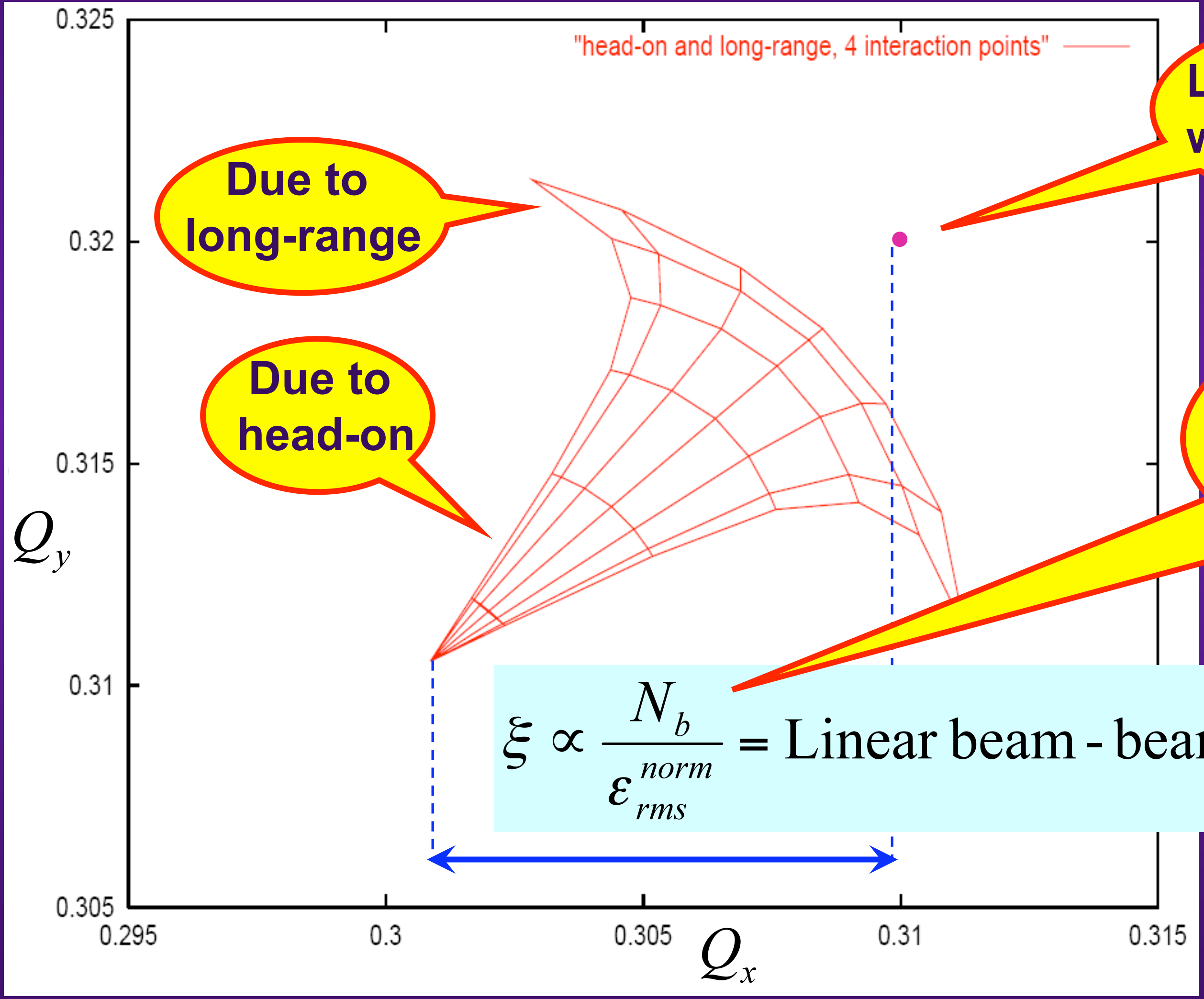


Relative beam sizes around IP1 (Atlas) in collision

⇒ Vertical crossing angle in IP1 (ATLAS) and horizontal one in IP5 (CMS)

BEAM-BEAM (6/9)

- **2D tune footprint for nominal LHC parameters in collision. Particles up to amplitudes of 6 σ are included**



Due to long-range

Due to head-on

Low-intensity working point

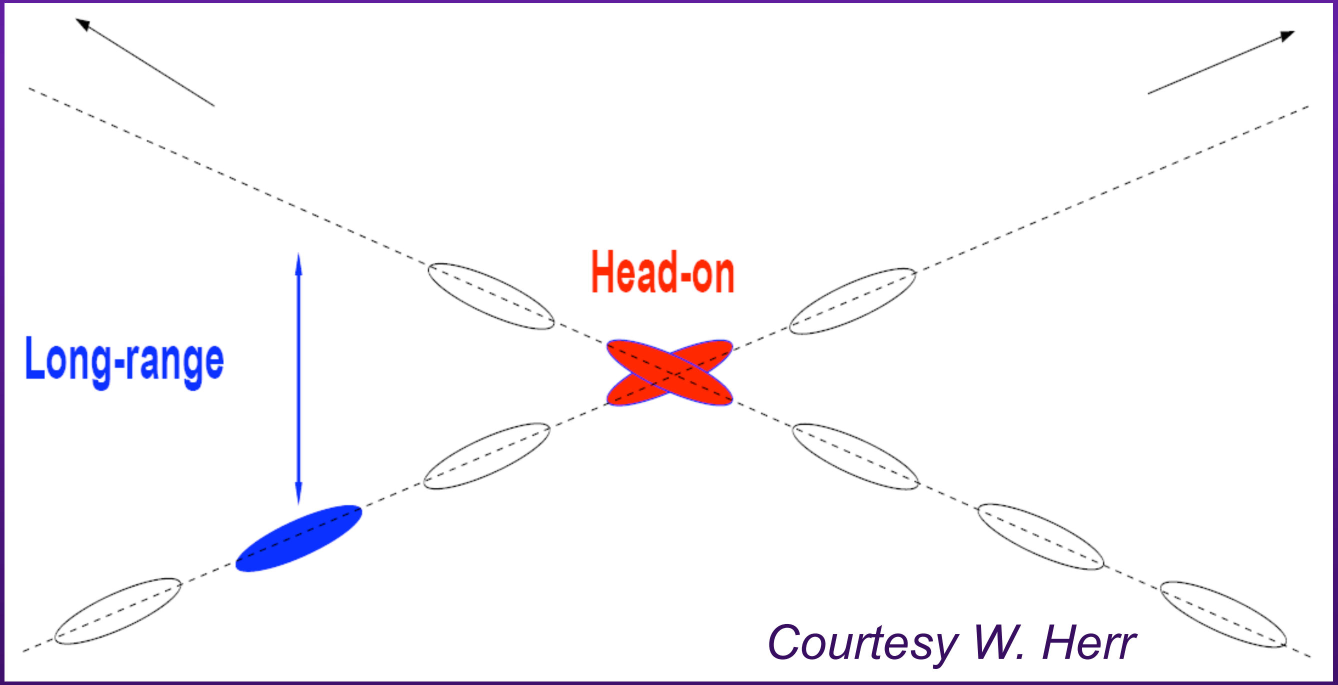
No 1 / γ_r^2 term as for space charge, as the 2 beams are moving in opposite direction

$$\xi \propto \frac{N_b}{\varepsilon_{rms}^{norm}} = \text{Linear beam - beam tune shift}$$

Courtesy W. Herr

BEAM-BEAM (7/9)

- PACMAN BUNCHES
  - LHC bunch filling not continuous: Holes for injection, extraction, dump…
  - 2808 bunches out of 3564 possible bunches ⇒ 1756 holes
  - Holes will meet holes at the IPs
  - But not always… a bunch can meet a hole at the beginning and end of a bunch train



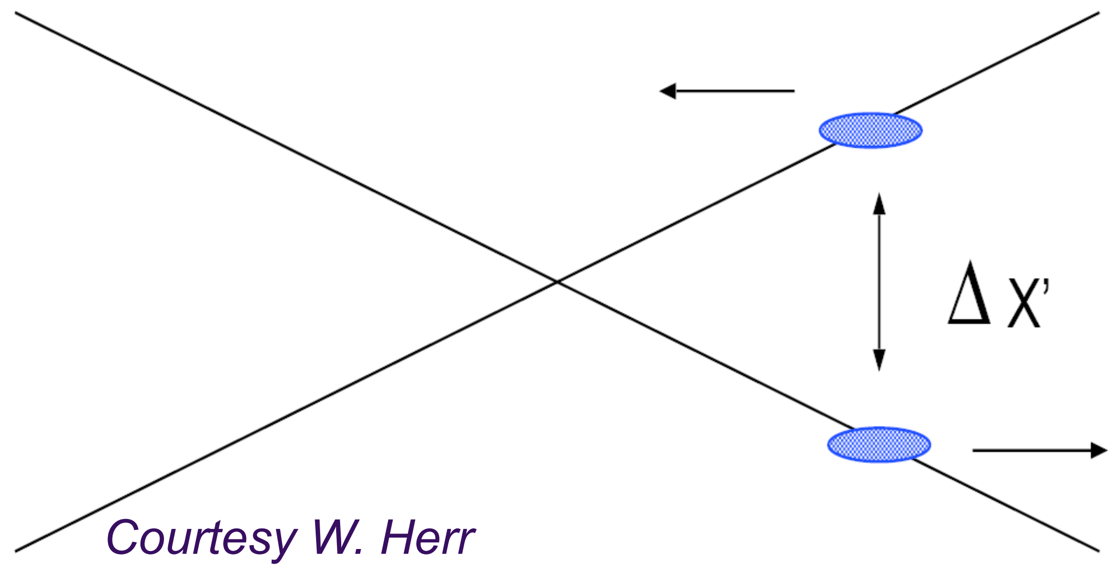
Courtesy W. Herr

BEAM-BEAM (8/9)

- Bunches which do not have the regular collision pattern have been named PACMAN bunches ⇒ ≠ integrated beam-beam effect
- Only 1443 bunches are regular bunches with 4 head-on and 120 long range interactions, i.e. about half of the bunches are not regular
- The identification of regular bunches is important since measurements such as tune, orbit or chromaticity should be selectively performed on them
- SUPERPACMAN bunches are those who will miss head-on interactions
  - 252 bunches will miss 1 head-on interaction
  - 3 will miss 2 head-on interactions
- ALTERNATE CROSSING SCHEME: Crossing angle in the vertical plane for IP1 and in the horizontal plane for IP5 ⇒ The purpose is to compensate the tune shift for the Pacman bunches

BEAM-BEAM (9/9)

- **COHERENT BEAM-BEAM EFFECT**



Courtesy W. Herr

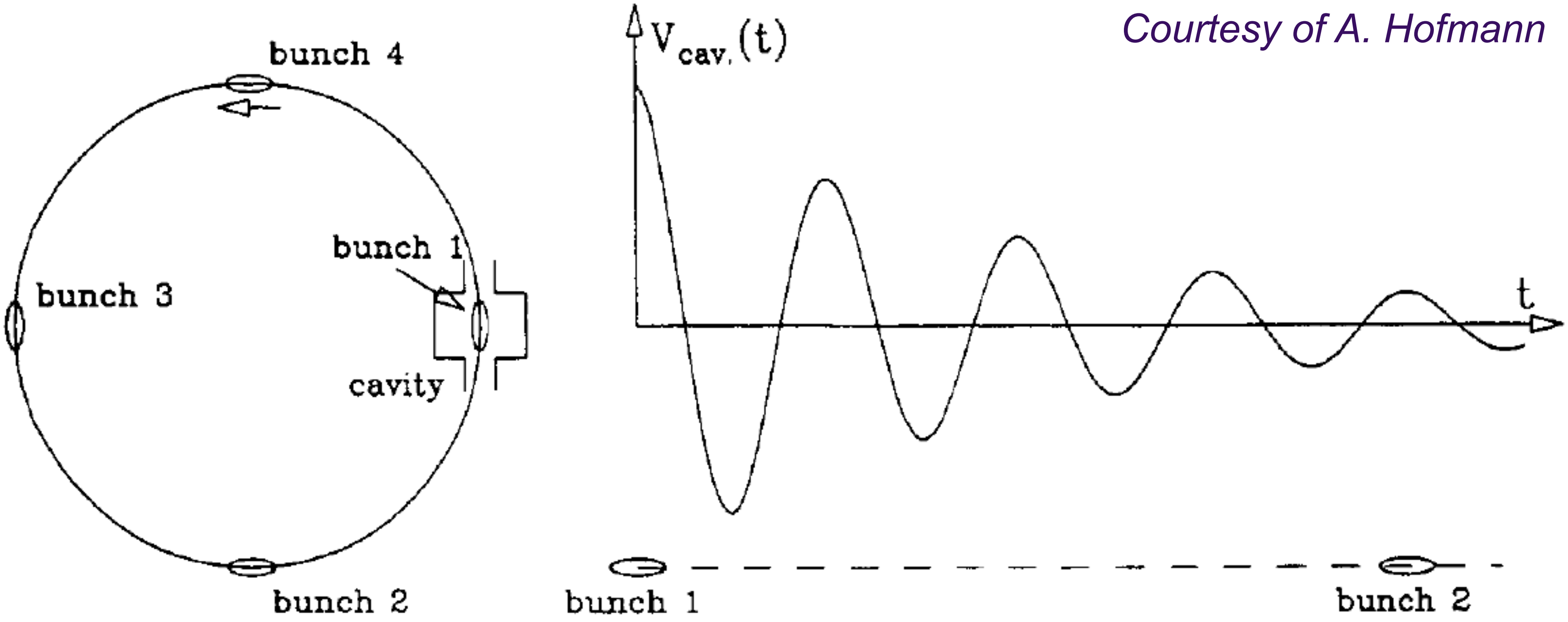
- **A whole bunch sees a (coherent) kick from the other (separated) beam ⇒ Can excite coherent oscillations**

- **All bunches couple together because each bunch "sees" many opposing bunches ⇒ Many coherent modes possible!**

IMPEDANCES AND WAKE FIELDS (1/10)

- Wake fields = Electromagnetic fields generated by the beam interacting with its surroundings (vacuum pipe, etc.)
  - Energy loss
  - Beam instabilities
  - Excessive heating
- For a collective instability to occur, the beam environment must not be a perfectly conducting smooth pipe
- Impedance = Fourier transform of the wake field
- As the conductivity, permittivity and permeability of a material depend in general on frequency, it is usually better (or easier) to treat the problem in the frequency domain

IMPEDANCES AND WAKE FIELDS (2/10)

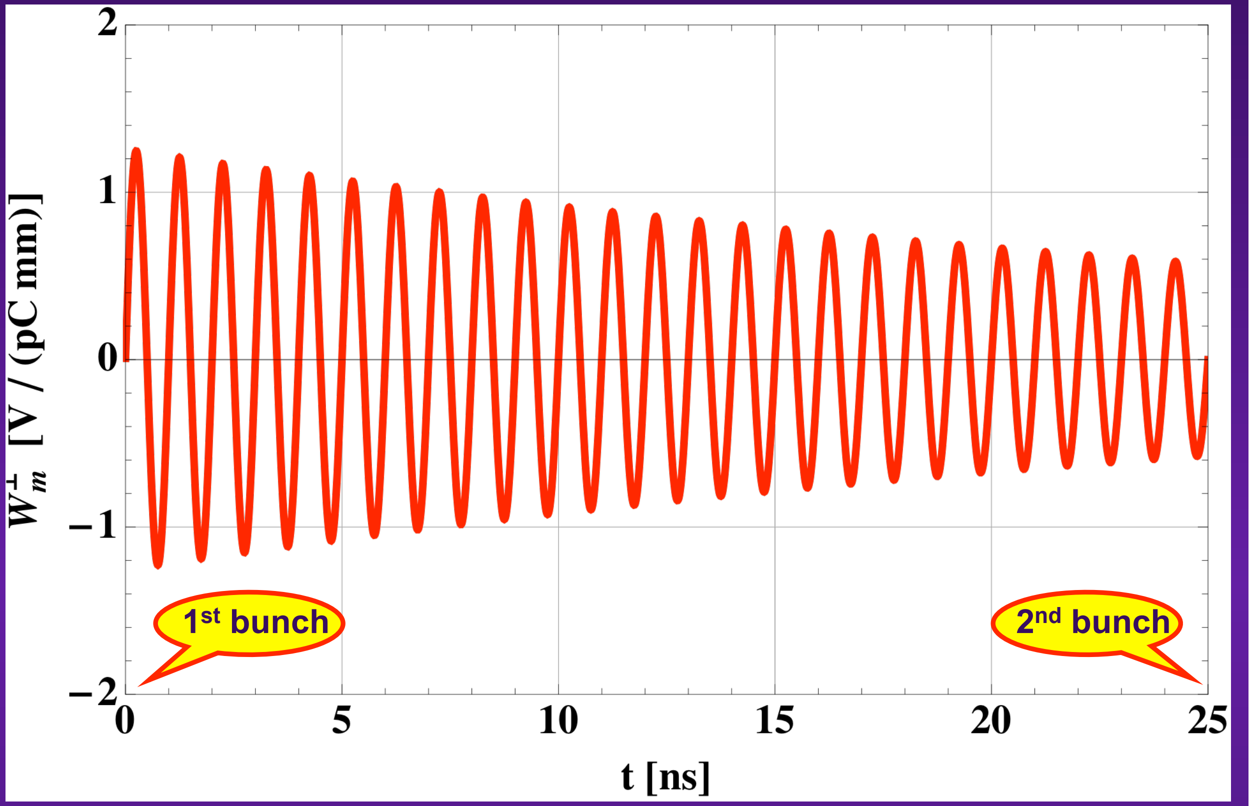


Courtesy of A. Hofmann

- **Origin of the impedance in this case is coming from a (abrupt) change of geometry (cavity, trapping some EM fields) => Usually computed using EM simulation codes**
- **Can come also from a smooth pipe due its finite conductivity => Available theories**

IMPEDANCES AND WAKE FIELDS (3/10)

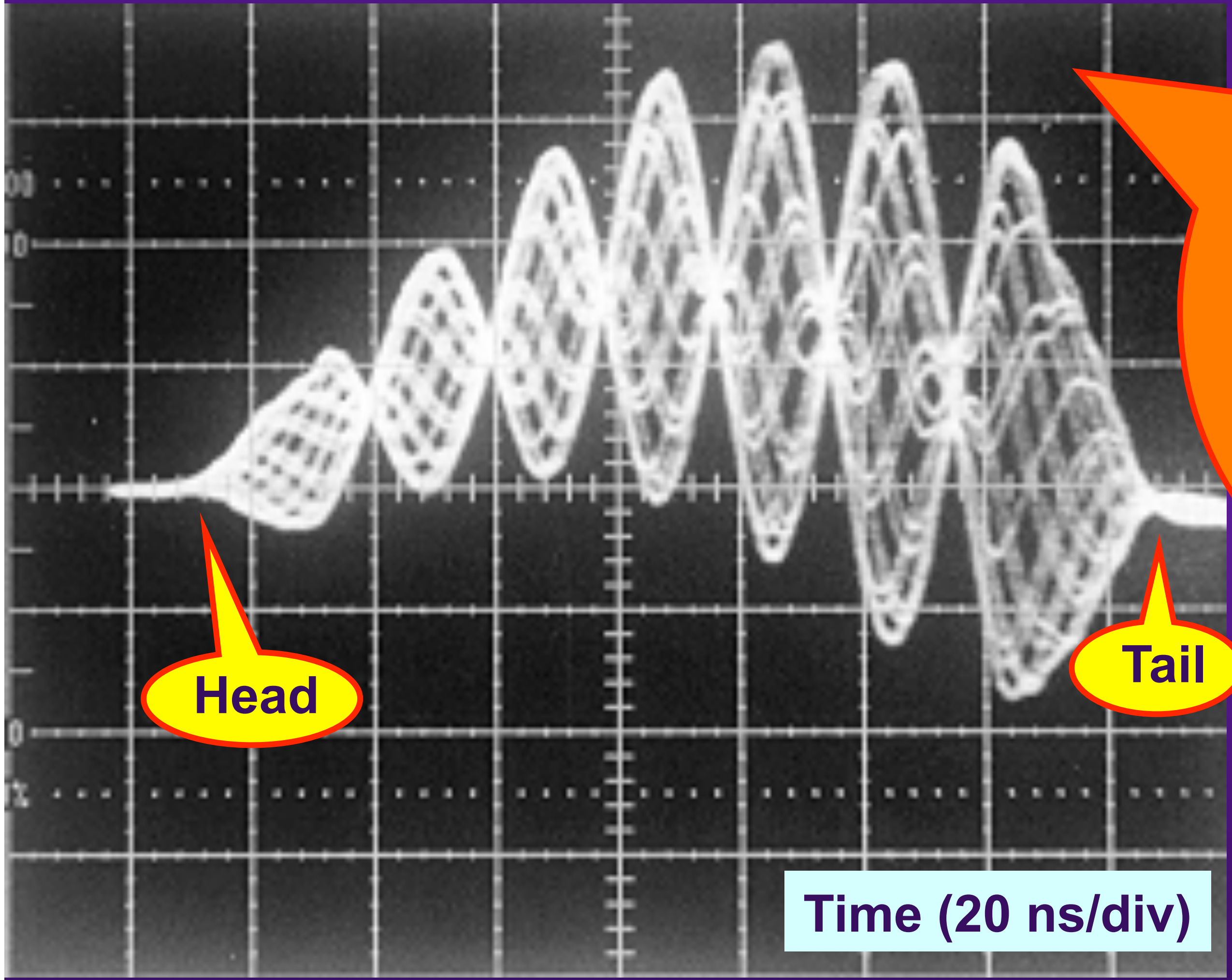
- Example of transverse wake field => Leads to



- Single-bunch effects if the wake field do not couple the consecutive bunches (i.e. decay rapidly) => Short-range wake field (corresponding to broad-band impedances)
- Coupled-bunch effects if it couples (as it is the case here) => Long-range wake field (corresponding to narrow-band impedances)

IMPEDANCES AND WAKE FIELDS (4/10)

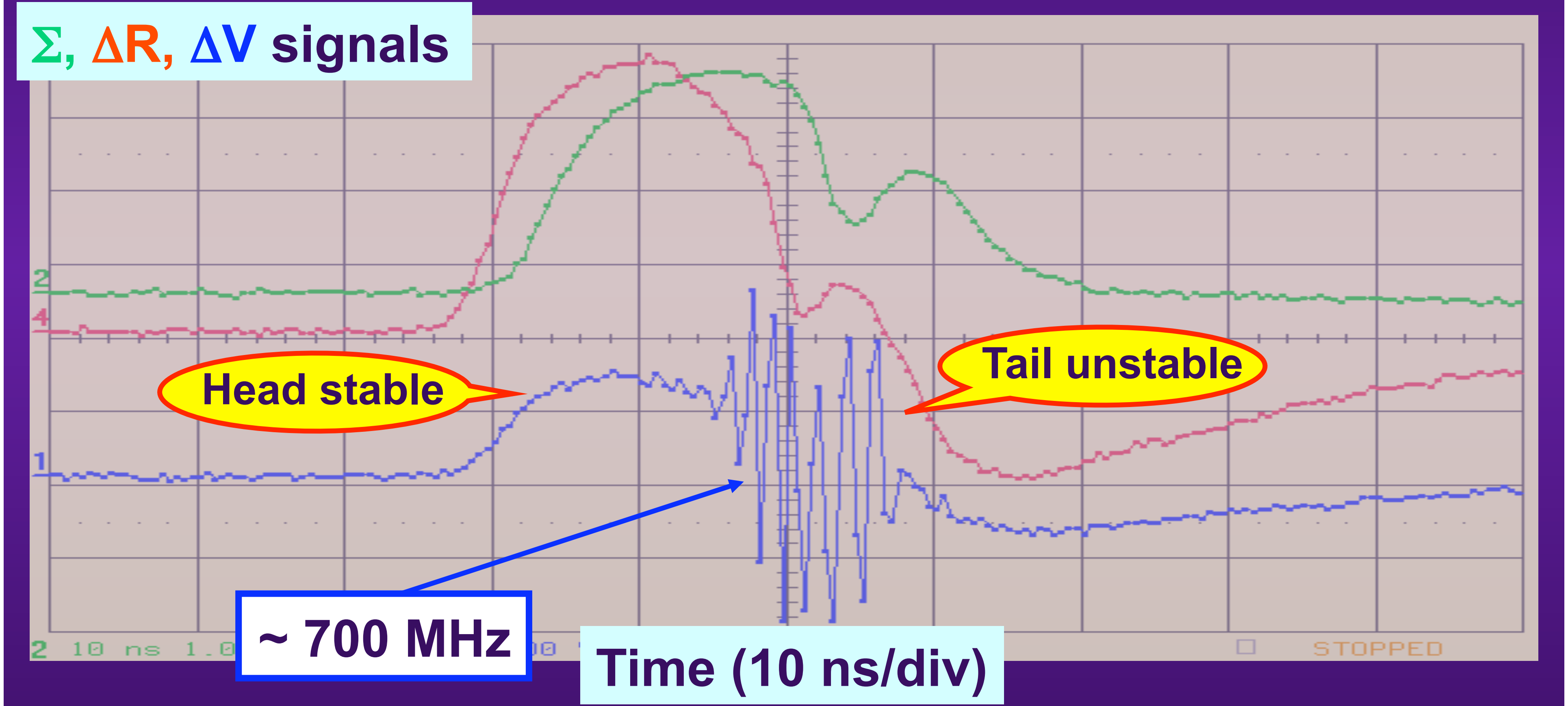
Observation of horizontal single-bunch instability in the PS at injection energy in 1999 (20 revolutions superimposed)



Mathematical explanation given by Sacherer and Laclare => See review of Laclare's theory at http://emetral.web.cern.ch/emetral/CAS09course/TransverseInstabilities.pdf

IMPEDANCES AND WAKE FIELDS (5/10)

Observation of another type of instability in the PS near transition energy in 2000



Σ, ΔR, ΔV signals

Head stable

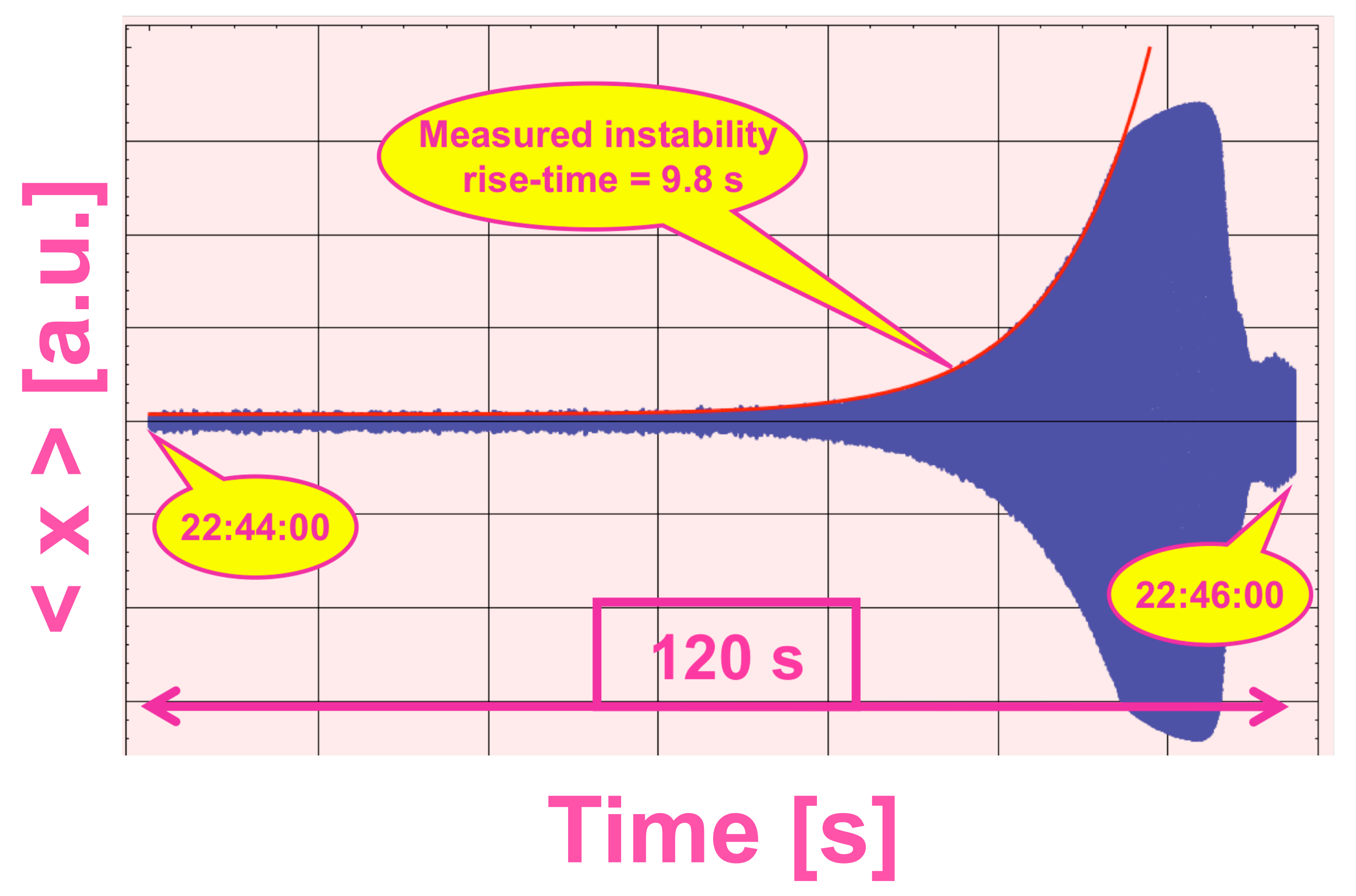
Tail unstable

~ 700 MHz

Time (10 ns/div)

IMPEDANCES AND WAKE FIELDS (6/10)

Observation of a horizontal single-bunch instability in the LHC in 2010 => Over a much longer time than in the previous 2 plots, to deduce the instability rise-time

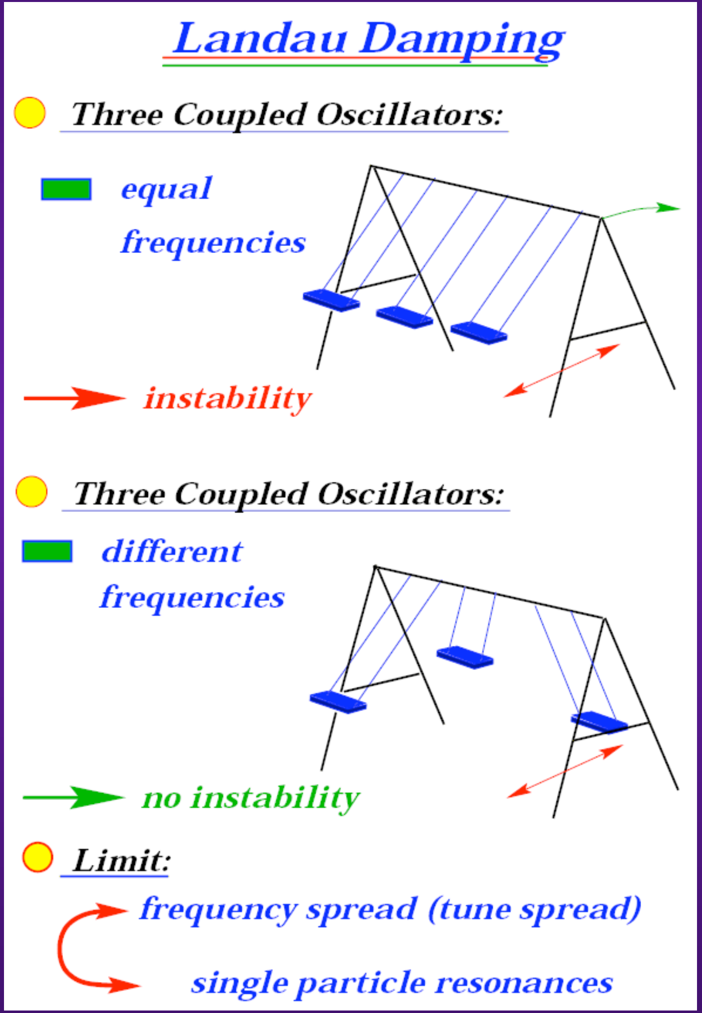


IMPEDANCES AND WAKE FIELDS (7/10)

- **Measurement of the rise-time of an instability**
  - Plot the transverse beam position vs. time
  - Look at the very beginning of the instability, where one should see an exponential growth (perturbative approach)
  - Do the fit: exponential or linear fit in log plot
  - The instability rise-time is defined by the time needed for the amplitude (of the envelope) to be multiplied by Exp[1] ≈ 2.7

IMPEDANCES AND WAKE FIELDS (8/10)

Courtesy O. Bruning



Landau Damping

○ *Three Coupled Oscillators:*

🟩 *equal frequencies*

→ *instability*

○ *Three Coupled Oscillators:*

🟩 *different frequencies*

→ *no instability*

○ *Limit:*

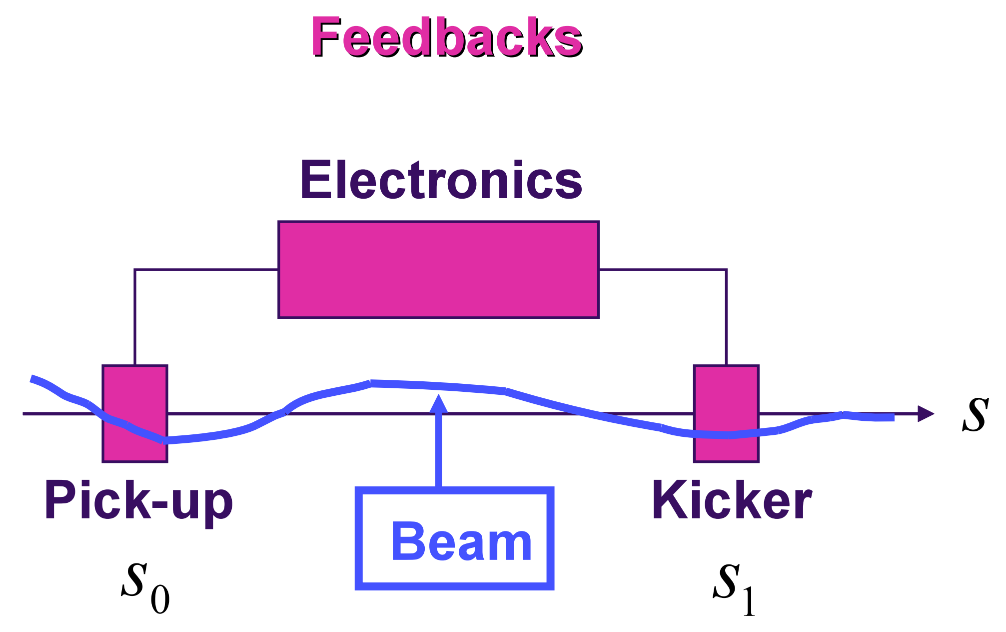
frequency spread (tune spread)

single particle resonances

- **Stabilization methods**
  - Landau damping

Stabilization when the coherent tune is inside the incoherent tune spread (due to octupoles….)

  - Feedbacks

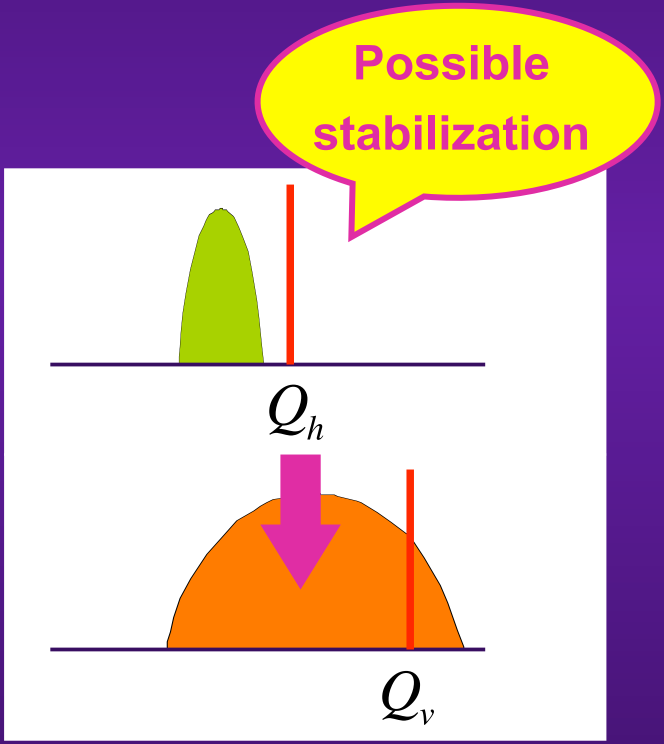


  - Linear coupling between the transverse planes (from skew quadrupoles)

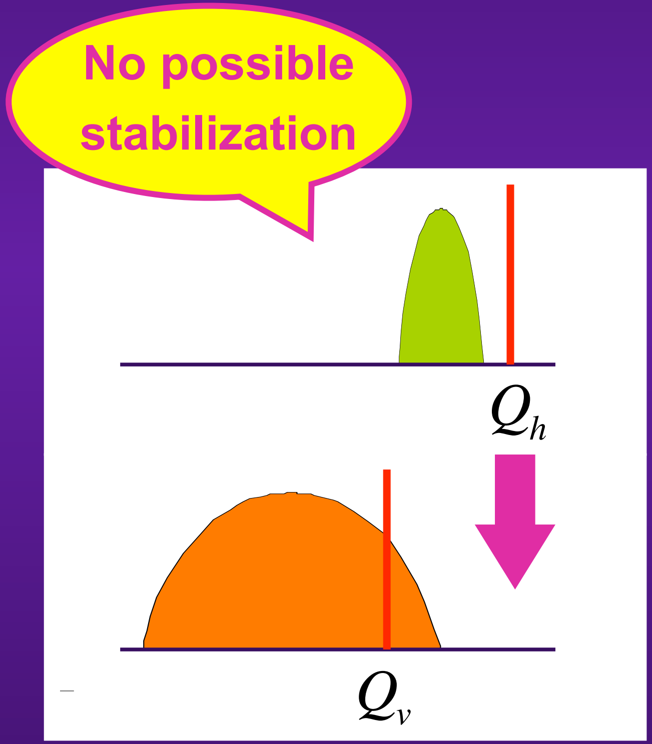
IMPEDANCES AND WAKE FIELDS (9/10)

=> 2 (stabilizing) effects predicted with linear coupling

- Transfer of instability growth rates (inverses of instability rise-times)
- Transfer of Landau damping



Possible stabilization



No possible stabilization

IMPEDANCES AND WAKE FIELDS (10/10)

- **Major concern in the LHC in 2011 and 2012 => Beam-induced RF heating!**

  => In the case of a narrow longitudinal resonance (with maximum value *R*$_s$)

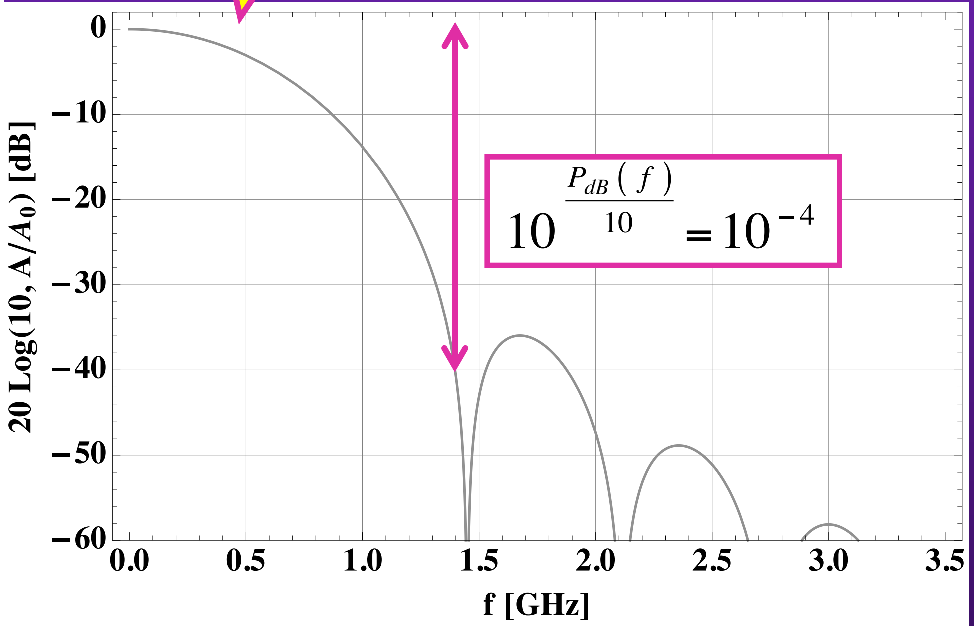
Total beam current (1 beam)

Resonance frequency

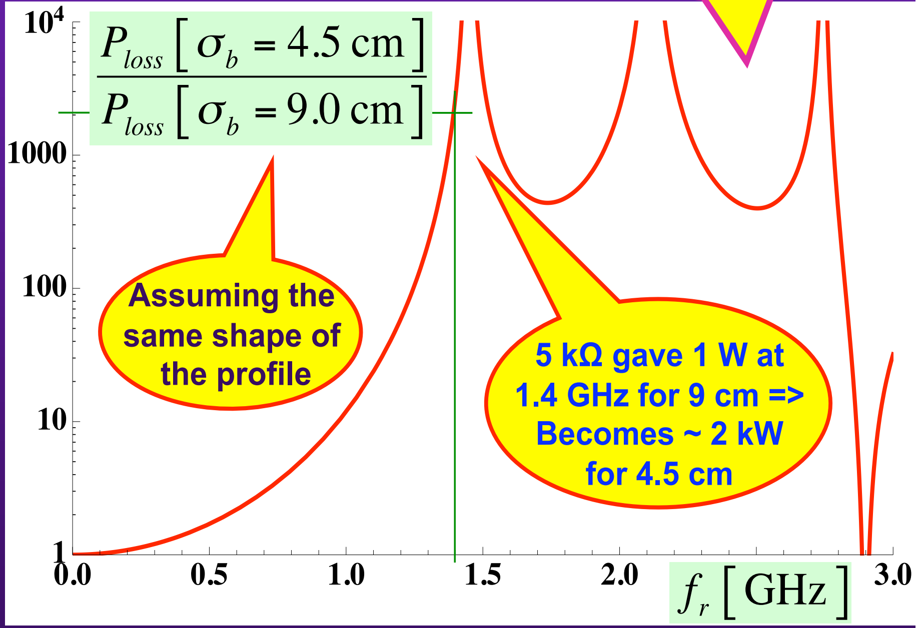
Beam power spectrum

1 A assumed here

$$P_{loss} = I_{total}^{2} \times 2\,R_s \times 10^{\frac{P_{dB}(f_r)}{10}}$$



$$10^{\frac{P_{dB}(f)}{10}} = 10^{-4}$$



$$\frac{P_{loss}\left[\sigma_b = 4.5\text{ cm}\right]}{P_{loss}\left[\sigma_b = 9.0\text{ cm}\right]}$$

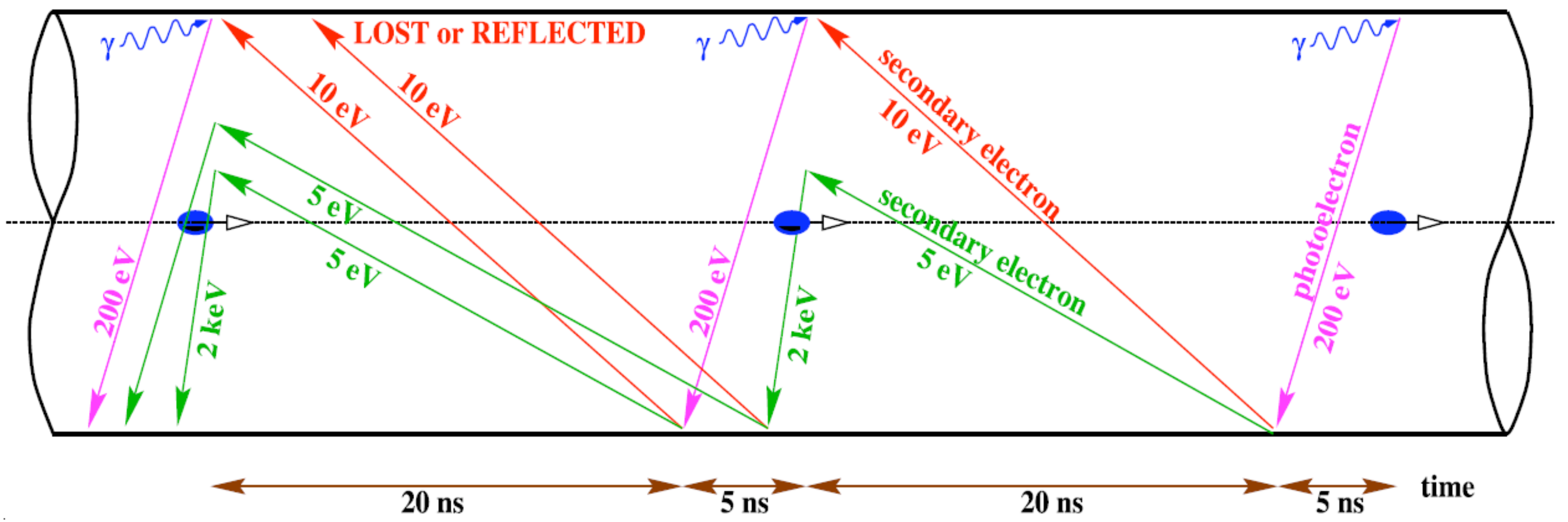
$$f_r\left[\text{GHz}\right]$$

Assuming the same shape of the profile

5 kΩ gave 1 W at 1.4 GHz for 9 cm => Becomes ~ 2 kW for 4.5 cm

ELECTRON CLOUD (1/4)

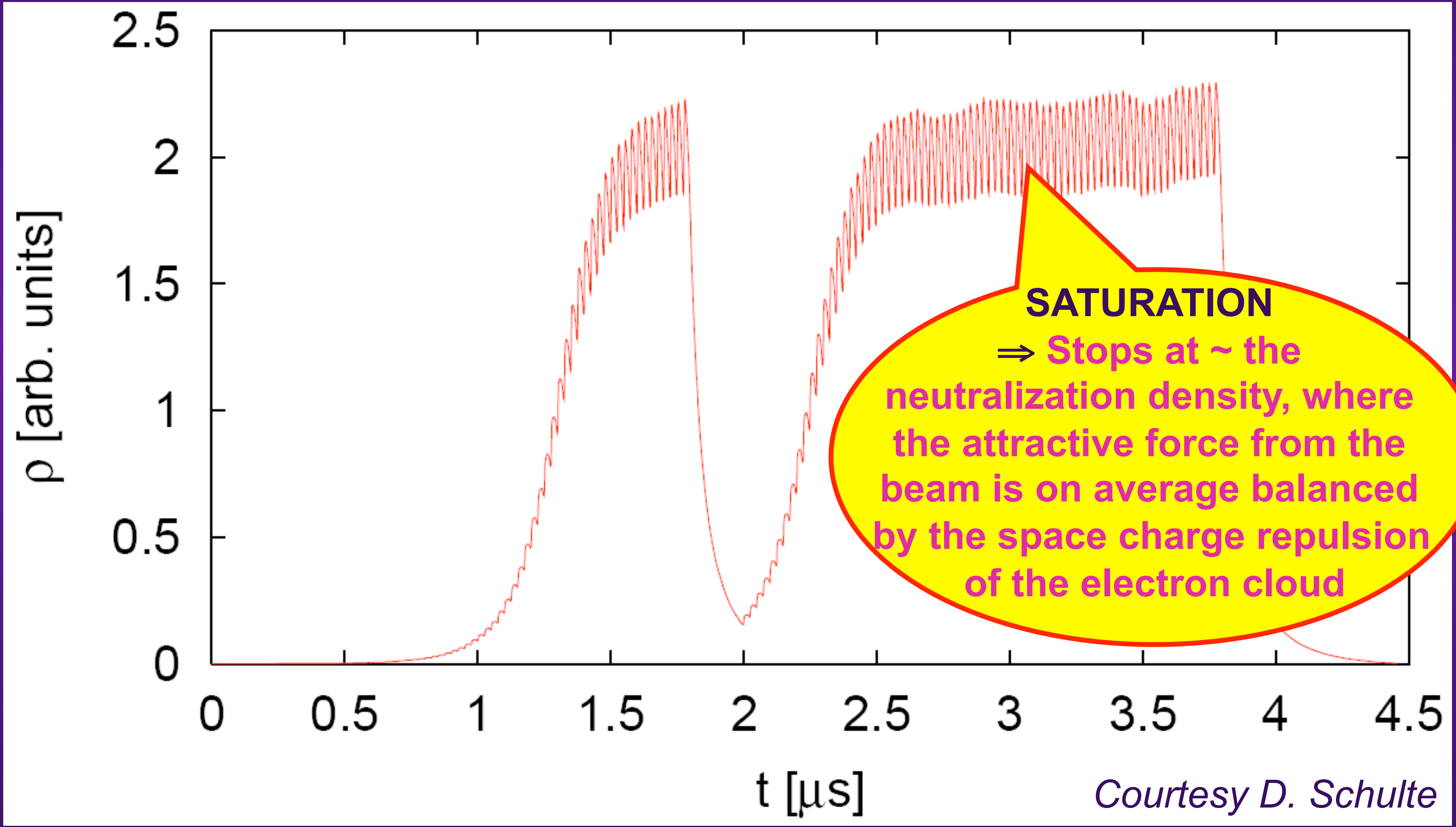
- The charged particles can also interact with other charged particles present in the accelerator (leading to two-stream effects, and in particular to electron cloud effects in positron/ hadron machines)
- Schematic of electron-cloud build up in the LHC beam pipe during multiple bunch passages, via photo-emission (due to synchrotron radiation) and secondary emission => Important parameter here is the **SEY = Secondary Emission Yield**



Courtesy F. Ruggiero

ELECTRON CLOUD (2/4)

- **Simulations of electron-cloud build-up along 2 bunch trains (= 2 batches of 72 bunches) of LHC beam in SPS dipole regions**

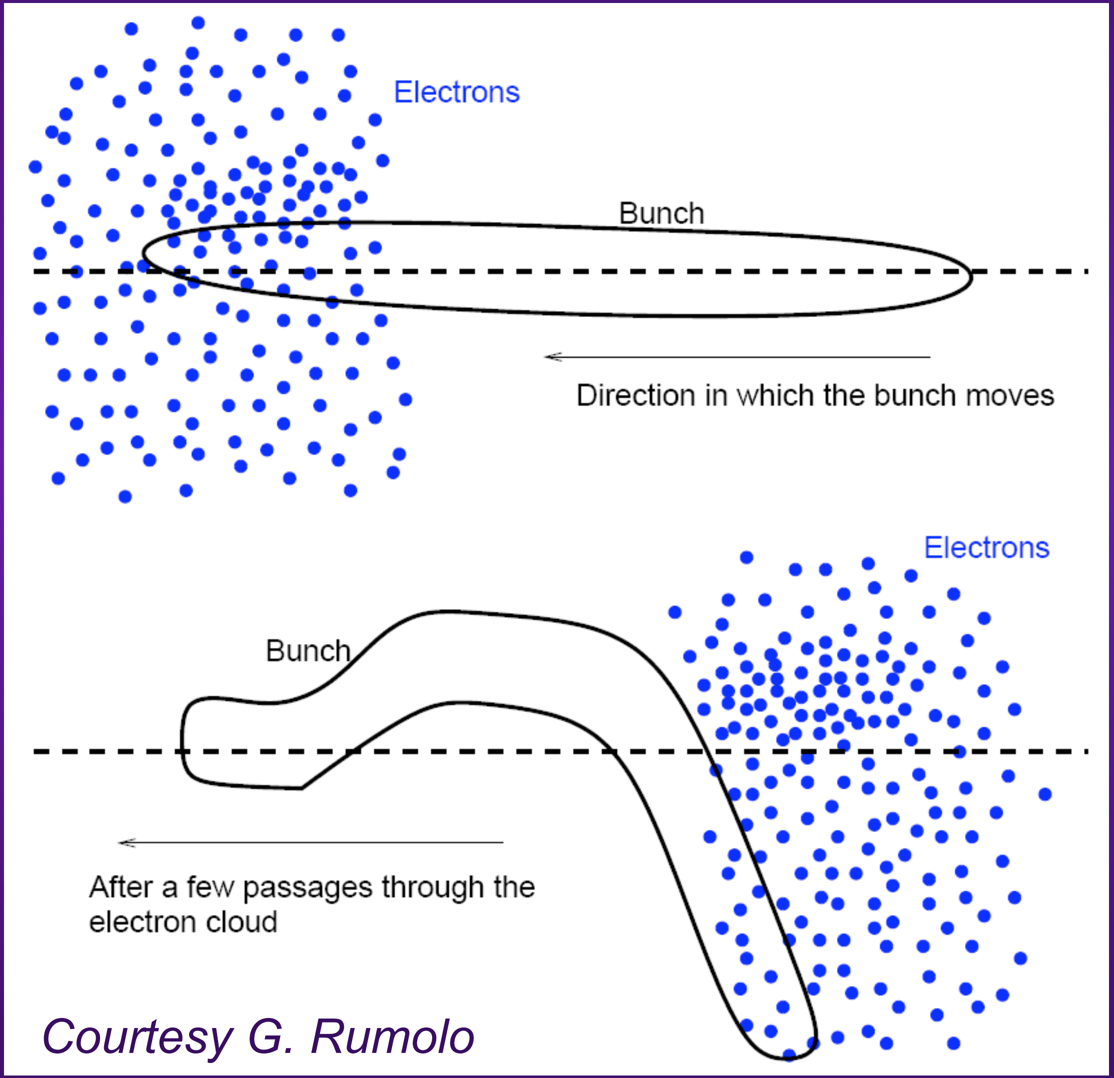


SATURATION
⇒ Stops at ~ the neutralization density, where the attractive force from the beam is on average balanced by the space charge repulsion of the electron cloud

Courtesy D. Schulte

ELECTRON CLOUD (3/4)

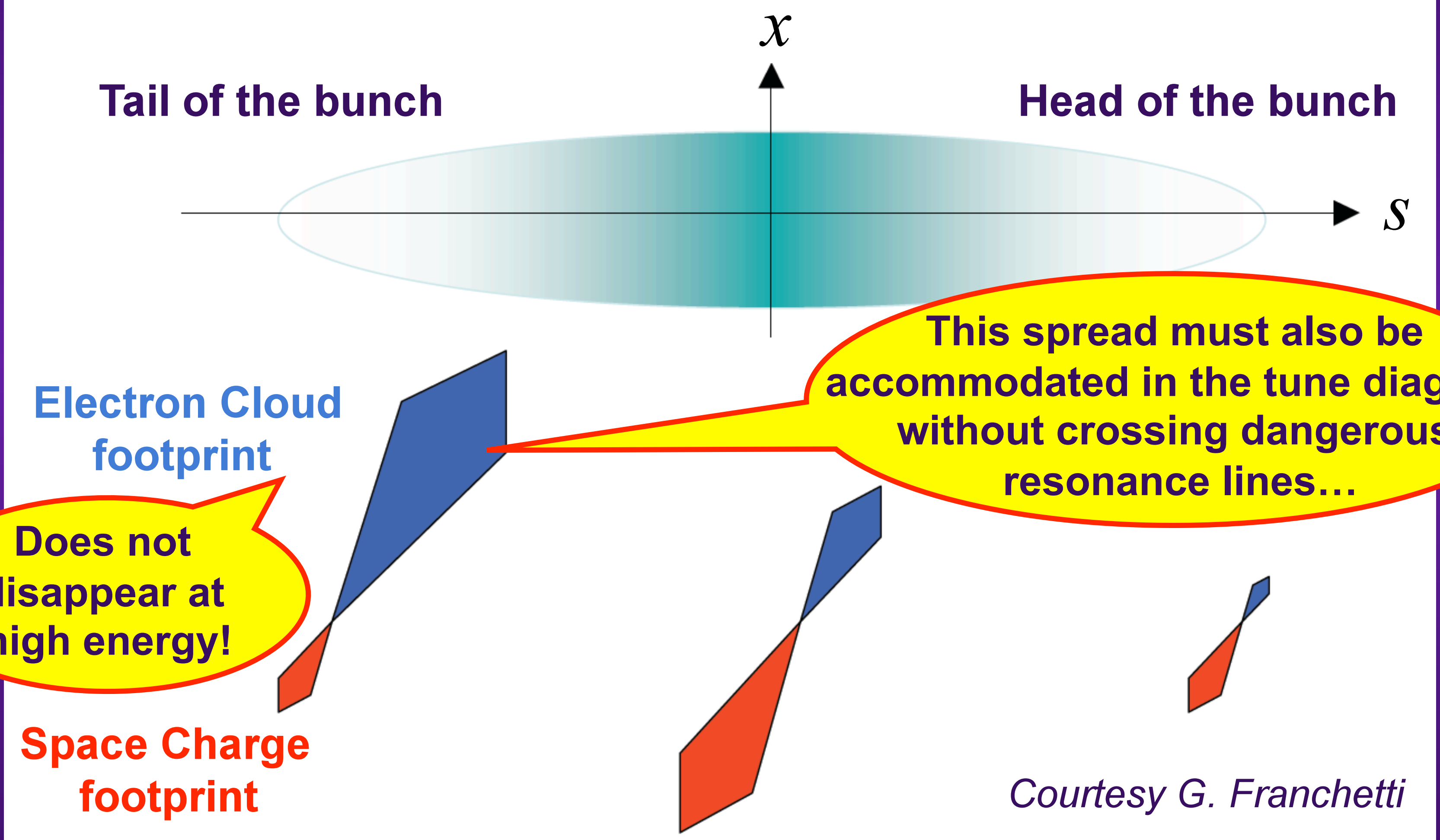
- **Schematic of the single-bunch (coherent) instability induced by an electron cloud**



Courtesy G. Rumolo

ELECTRON CLOUD (4/4)

- **Incoherent effects induced by an electron cloud**



This spread must also be accommodated in the tune diagram without crossing dangerous resonance lines…

Does not disappear at high energy!

Courtesy G. Franchetti