Joint Universities Accelerator School *3rd February 2011*

LONGITUDINAL BEAM DYNAMICS EXAMINATION

THE CERN-PS BEAM FOR LHC

The PS machine as part of the LHC injector chain has to provide to the SPS a proton beam with specific characteristics. In the transverse domain, the main issue is to provide a beam of high brightness (i.e. intensity to emittance ratio).

In the longitudinal plane, the main problem is to generate at the exit of the PS a train of 84 very short bunches (~ 4 ns) spaced by 25 ns, starting from 8 long bunches (~ 200 ns) coming from the PSB.

The PSB is constituted of 4 independent equal rings with a radius of 25 m, each of them working on the harmonic number $h_{PSB} = 1$. Each bunch is extracted from the PSB at 1.4 GeV (kinetic energy). A pulse is composed by 4 consecutive bunches between the PSB and the PS. After 1.2 s, a second pulse (i.e. 4 bunches) is extracted from the PSB and sent to the PS. The PS harmonic number at injection energy is $h_{PS} = 8$. During a magnetic flat-top, the PS receives the first pulse, which fills half of the PS, and then the second pulse fills the whole machine.

The beam is then accelerated up to 3.5 GeV/c, where a magnetic flat-top of a few milliseconds allows some RF manipulations. Afterwards, the beam is accelerated again up to 26 GeV/c. At this momentum, a last magnetic flat-top is used to shorten the bunch length before the extraction towards the SPS.

During a basic cycle, which lasts for 3.6 s, the PS magnetic field starts from 0 and returns to 0 (a few ms after the extraction). Several RF gymnastics are made between injection and extraction, in order to obtain the required beam characteristics when the protons are extracted from the PS.

1) Draw schematically the variation of the PS magnetic field versus time during the basic cycle.

2) Draw schematically the variation of the PS beam intensity assuming no loss during the basic cycle.

3) Calculate the value of the peak RF voltage at injection in the PS in order to obtain a perfect longitudinal matching when the proton beam is injected at 1.4 GeV. We remind that the value of the longitudinal emittance is the same between 2 consecutive synchrotrons if it is expressed in [eV.s]. Therefore the conservation of the ratio $\Delta E_{\max} / \Delta t_{\max}$, between the PSB and PS buckets, provides a perfect longitudinal matching for a stationary bucket. We remind one of the expression of the bucket

height, $\Delta E_{\max} = \beta_s \sqrt{\frac{e\hat{V}_{RF}\,E_s}{\pi|\eta|h}F(\Phi_s)}$, where $F(\Phi_s) = 2\cos(\Phi_s) - (\pi - 2\Phi_s)\sin(\Phi_s)$, and the expression of the RF phase extension $\hat{\phi} = \omega_{RF}\,\Delta t_{\max}$.

4) From the RF voltage found in 3), calculate the PS synchrotron frequency f_s and the synchrotron tune Q_s at injection energy ? Does it verify $Q_s << 1$?

5) At injection, the maximum relative momentum spread is $\Delta p / p_0 = \pm\, 2\times 10^{-3}$ and the longitudinal emittance is $\varepsilon_l = 1$ eV.s . Calculate the value of the bunch length (assuming a perfect elliptic area). Compare with the value given in the first paragraph of the Introduction.

6) Calculate the magnetic field, both at injection and ejection energies in the PS.

7) Assuming that the orbit remains the same during the acceleration, in which direction does the RF frequency change between the injection and the ejection energies?

8) At 26 GeV/c, RF manipulations allow to debunch and rebunch adiabatically the beam in order to obtain the 84 bunches. The longitudinal emittance of each bunch is then 0.3 eV.s. Assuming no blow-up of the longitudinal emittance between PS and SPS, what should be the minimum value for the SPS acceptance?

9) What is the bucket length at PS ejection? What can be done to reduce the bunch length as much as possible before extraction?

10) Assuming that the variation of the magnetic field during acceleration is dB/dt = 1 T/s, how long will it take to accelerate from 3.5 GeV/c to 26 GeV/c? What should be the synchronous phases at 5 GeV/c and 10 GeV/c with a peak RF voltage of 100 kV?

Numerical values:

$R_{PS} = 100\,\text{m}$	PS radius
$R_{PSB} = R_{PS}/4$	PSB radius
$\hat{V}_{RF}^{PSB} = 8$ kV	Peak RF voltage at the ejection of the PSB
$\alpha_p^{PS} = \gamma_{tr,PS}^{-2} = 0.027$	PS momentum compaction factor
$\alpha_p^{PSB} = \gamma_{tr,PSB}^{-2} = 0.0617$	PSB momentum compaction factor
$\rho_{PS} = 70\,\text{m}$	PS curvature radius