JUAS session 2019 February 6th 2019

Longitudinal beam dynamics examination

(1h30 – Free access to lecture notes and paper documents)

This exam is composed of two independent exercises totalling 82 points. The marks will be normalized to 20

Exercise 1: The Large Electron Positron Collider (LEP) 42 pts

The Large Electron Positron collider (LEP) at CERN reached a beam energy of 104.5 GeV in 2001, before being shut down. The main parameters of LEP are described below:

- Circumference 26658.9 m
- Bending radius 3.026 km
- Momentum compaction factor 1.4e-4
- RF frequency 352 209 188 Hz

- Compute the relativistic factor gamma and the magnetic field in the LEP dipoles at 104.5 GeV (4 pts).

```
circumference = 26658.9
Etot = 104.5e9*e
E0=m_e*c**2
gamma=Etot/E0
print 'gamma=',gamma
rho=3.026e3
beta=np.sqrt(1 - gamma**-2)
p=beta*Etot/c
B=p/(e*rho)
print 'B=',B,'T'
```

```
gamma= 204501.400175
B= 0.115193152504 T
```

- What is the maximum number of bunches that could be stored in LEP? How is this number referred to in the course? (2 pts)

```
fRF=352209188
frev=beta*c/circumference
h=fRF/frev
print frev
print 'h=',h
```

```
11245.4924245
h= 31320.0324809
```

The maximum number of bunches is when all buckets are occupied, i.e. 31320 bunches. This number is also referred to as the harmonic number. Rounding errors can have a sizeable impact on this number.

- In the absence of synchrotron radiation and acceleration:
  - What is the phase of the synchronous particle? (4 pts)
  - Consider a particle with a higher momentum than the synchronous particle, is its revolution frequency larger or lower than the revolution frequency of the synchronous particle? (2 pts)
  - Compute the height of the RF bucket assuming a peak effective RF voltage of 100 MV (2 pts).
  - Draw qualitatively the bucket in that case, with a matched bunch that would have a bunch length of 12 mm (6 pts).
  - Is it reasonable to assume that all particles oscillate with the same synchrotron tune in these conditions? (2 pts)

It is a non-accelerating bucket, the synchronous phase is therefore 0 or π. We need first to compute the transition energy to know if we are below or above transition.

```
alpha_p=1.4e-4
gamma_tr=np.sqrt(1/alpha_p)
print 'the gamma at transition is ', gamma_tr
E_tr=gamma_tr*E0
print 'transition energy =',E_tr/e*1e-6,'MeV'
```

```
the gamma at transition is  84.5154254729
transition energy = 43.1872933601 MeV
```

We are therefore above transition and the phase of the synchronous particle is pi. The revolution frequency of the higher-momentum particle is then lower than that of the synchronous particle.

The height of the RF bucket is given in page 82 and one needs to compute the slippage factor

```
Vrf=100e6
eta=1/gamma**2-alpha_p
print 'slippage factor:',eta
deltaEsepMax=np.sqrt(2*beta**2*Etot*e*Vrf/(np.pi*h*-eta))
print 'bucket height',deltaEsepMax/e*1e-9,'GeV'
```

```
slippage factor: -0.000139999976088
bucket height 1.23175156876 GeV
```

```
bucket_length=1e9/fRF
print fRF
print 'the bucket_length is',bucket_length,'ns'
bunch_length=11e-3/(beta*c)*1e9
print 'the bunch length is',bunch_length,'ns'
```

```
352209188
the bucket_length is 2.83922178657 ns
the bunch length is 0.0366920504722 ns
```

Therefore the bunch is very small inside the bucket and occupies only 1% of the bucket area. Yes, the particles inside the bunch will remain in the small amplitudes, where the synchrotron motion is linear.

- We now account for synchrotron radiation. From the JUAS synchrotron radiation course, the energy loss per turn $(\Delta E)_{turn}$ due to synchrotron radiation can be expressed as $(\Delta E)_{turn} = \frac{e^2\gamma^4}{3\varepsilon_0\rho}$, where ε_0 is vacuum permittivity, γ is the relativistic factor and ρ is the bending radius.
    - How much energy does a LEP electron lose per turn? How much does it represent in percentage of its total energy? (2 pts)

```
eps_0=8.85e-12
deltaEjuas=e**2*gamma**4/(3*eps_0*rho)
print 'the energy loss per turn is',deltaEjuas/e/1e9, 'GeV'
print 'the ratio to the total energy is', deltaEjuas/Etot*100,'%'
```

```
the energy loss per turn is 3.48788441596 GeV
the ratio to the total energy is 3.33768843633 %
```

- Assuming that the energy loss per turn from synchrotron radiation needs to be compensated by the RF cavities, express the RF voltage as a function of gamma and the phase of the synchronous particle (for the case of no acceleration). (4 pts)

One needs to match the energy loss per turn from synchrotron radiation to the energy gain per turn given by the RF cavities:

$$eVsin(\varphi_s) = \frac{e^2\gamma^4}{3\varepsilon_0\rho}$$

$$V = \frac{e\gamma^4}{3\varepsilon_0\, \rho\, sin(\varphi_s)}$$

- What is the consequence on the synchronous phase φ_s? (2 pts)

The synchronous phase cannot be π as it would for a stationary bucket above transition, since otherwise the RF voltage would need to be infinite. Above transition, the synchronous phase will need to shift down to compensate for the loss of energy ($sin(\varphi_s)$ needs to be positive).

- The peak voltage of a LEP copper cavity is 3 MV, and its accelerating gap is 277 mm. Compute the transit time factor of the LEP cavity. How many such cavities do you need to produce an effective voltage of 3.6 GV? What is the phase of the synchronous particle in that case (in degrees)? (6 pts)

```
gap=0.277
Ta=np.sin(np.pi*fRF*gap/(beta*c))/(np.pi*fRF*gap/(beta*c))
print 'Ta=',Ta
```

```
Ta= 0.834673241416
```

The transit time factor is 0.83.

```
Ncav=3.6e9/(3e6*Ta)
print Ncav
```

```
1437.6883557
```

One needs 1438 such cavities.

```
Vrftot=3.6e9
phisSR=np.arcsin(e*gamma**4/(3*eps_0*rho*Vrftot))
print 'the synchronous phase should be',(np.pi-phisSR)*180/np.pi,'degrees'
```

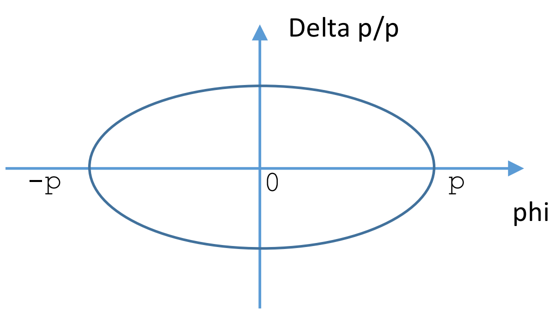
```
the synchronous phase should be 104.336824647 degrees
```

- Assuming that the power dissipated in a cavity is $P_{diss}=V^2/R_s$ with V the peak RF voltage in the cavity, and R_s the shunt impedance of the cavity (which is assumed independent of the RF voltage). How does this dissipated power vary with the beam energy? Is this dependence favourable for increasing the energy in a lepton machine? What machine parameter would you change to reach higher energy? (6 pts)

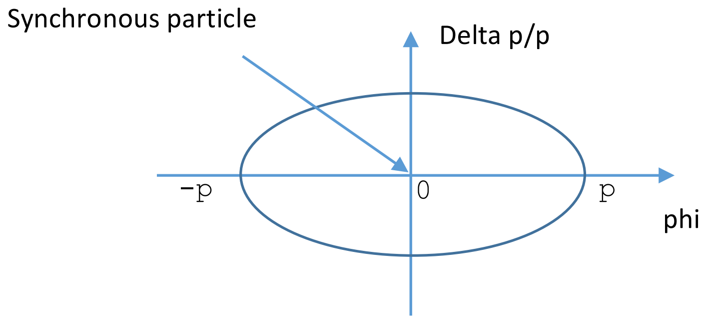
V varies with the fourth power of gamma (therefore of the energy), and the dissipated power varies with the eighth power of the beam energy. This dissipated power is one of the strongest limitations for lepton machines with normal conducting cavities. Increasing the bending radius would reduce the RF voltage (linearly with 1/rho) and the dissipated power (with 1/rho^2). Using superconducting cavities with a much higher Q factor would also reduce the power dissipation, even though this power would then need to be extracted at low temperature.

Exercise 2: answer the questions below (a: 14 pts, b: 12 pts, c: 14 pts) → 40 pts

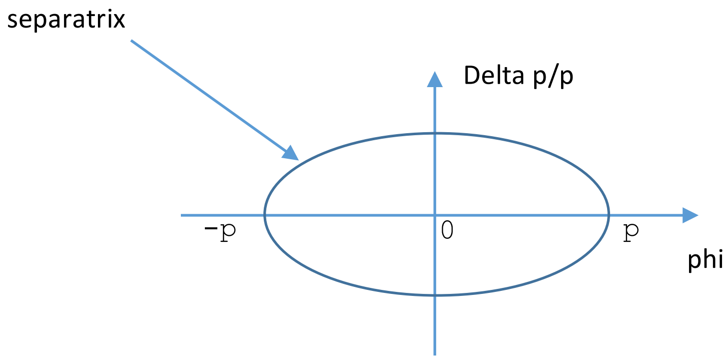
a. **Draw a stationary bucket below transition and:**
   - **Give the minimum and maximum phase for particles inside the bucket(2 pts)**



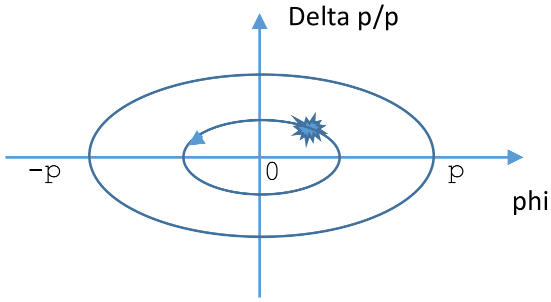
   - **Locate the synchronous particle (2 pts)**



- **Locate the separatrix (2 pts)**

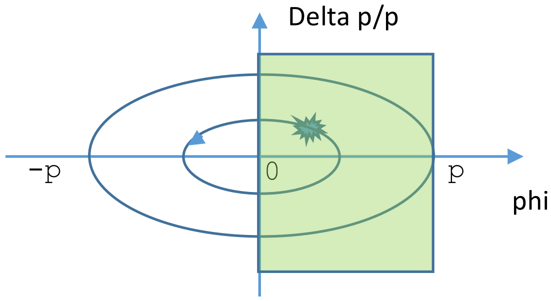


- **Draw the trajectory of a particle inside this bucket in phase space and its direction.**



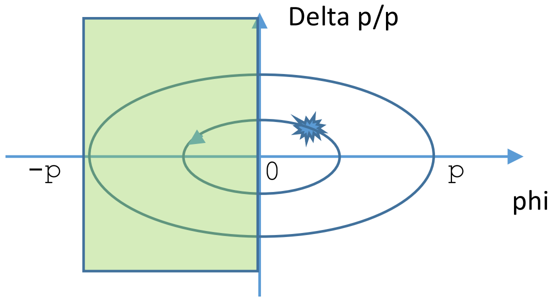
1. **In which parts of its trajectory does the particle arrive later than the synchronous particle at the RF cavity? (1 pt)**

If the particle arrives later, then $\Delta\theta < 0$, which means $\Delta phi > 0$ (see for instance p.68)



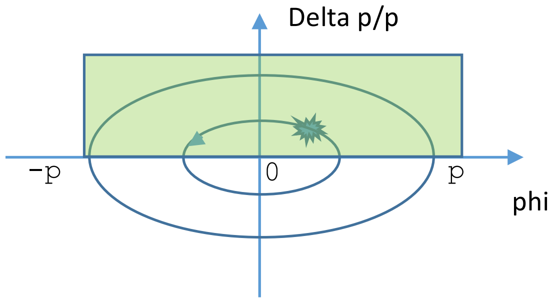
2. **In which parts of its trajectory does the particle arrive earlier than the synchronous particle at the RF cavity? (1 pt)**

If the particle arrives earlier, then $\Delta\theta > 0$, which means $\Delta phi < 0$ (see for instance p.68)



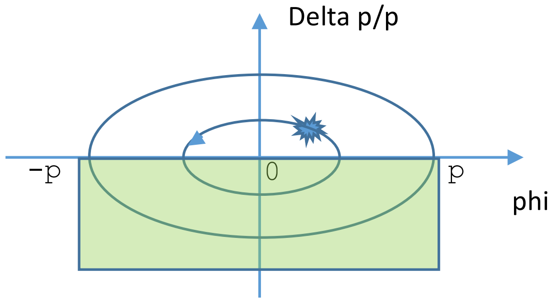
3. **In which parts of its trajectory does the particle need to travel a larger circumference than the synchronous particle? (1 pt)**

Larger circumference means higher momentum, so $\Delta p>0$.

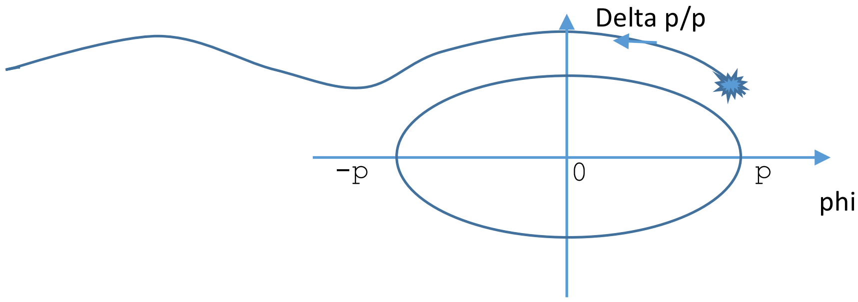


4. **In which parts of its trajectory does the particle need to travel a smaller circumference? (1 pt)**

Larger circumference means higher momentum, so Δp<0.



- **Draw the trajectory of a particle outside of the bucket and its direction. (2pts)**

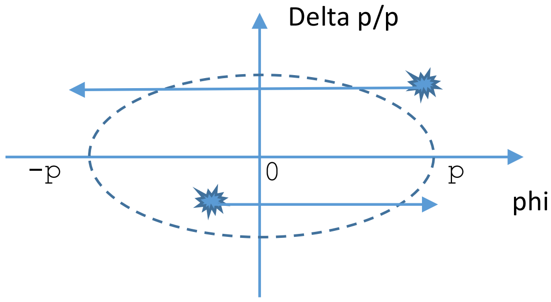


- **Is the synchrotron tune of a particle near the separatrix smaller or larger than the synchrotron tune of the synchronous particle? (2pts)**

It is smaller, particles travel slower in phase space when they are close to the separatrix due to the non-linearity of the bucket.

- **What happens to the beam if the RF voltage is switched off? (2 pts)**

It debunches: particles will keep the same momentum and drift in phase space.



b. A bunch of particles inside the bucket defined in (a) is accelerated up to transition

- **What happens to the particles outside of the bucket when acceleration starts? (2 pts)**

Their momentum difference with the synchronous particle increases until they are eventually lost at an aperture restriction (due to dispersion).

- **Describe qualitatively what happens to the bunch and to the bucket during acceleration and close to transition (4 pts)**

During acceleration, the bucket is not symmetric anymore and its phase acceptance decreases. The synchronous phase becomes positive (between 0 and 90 degrees). Close to transition, the bucket height increases a lot, so that the bunch length reduces, while its momentum spread increases (if adiabatic).

- **What needs to be done from RF point of view when crossing transition? What happens if nothing is done? Which parameter changes sign at transition energy? (4 pts)**

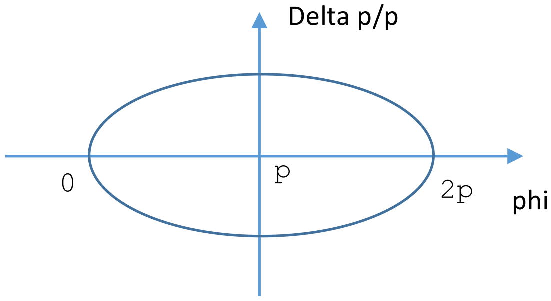
The phase of the RF needs to be changed from phi_s to p-phi_s. If this is not done, then the whole bunch sits outside of the bucket and is lost when acceleration continues after transition. The slippage factor changes sign at transition.

- **What happens to the beam if the RF voltage is switched off when the beam is at transition energy? (2 pts)**

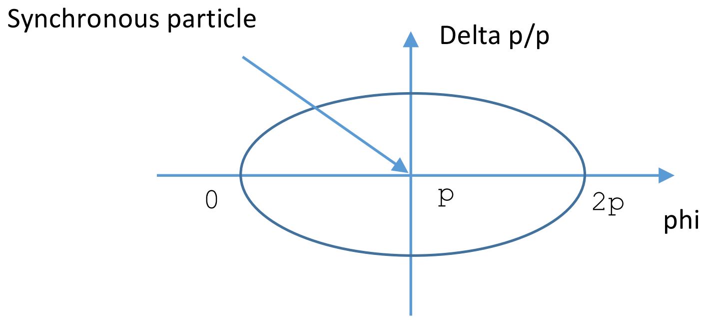
At transition energy, particles with different momenta have the same revolution frequency. Phases with respect to the RF remain constant and the beam therefore stays bunched without RF: its longitudinal motion is frozen.

c. The beam energy is now above transition. Draw a stationary bucket and:

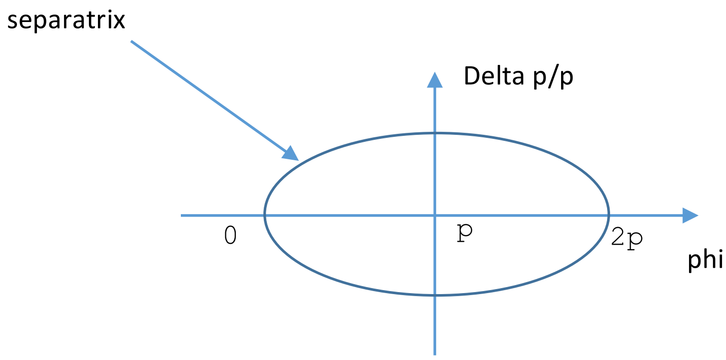
- **Give the minimum and maximum phase for particles inside the bucket(2 pts)**



- **Locate the synchronous particle (2 pts)**



- **Locate the separatrix (2 pts)**



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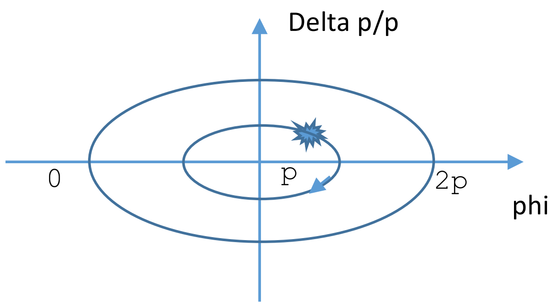

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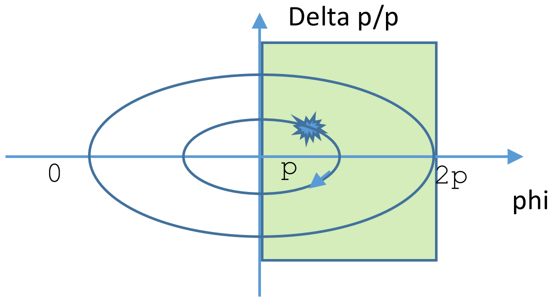

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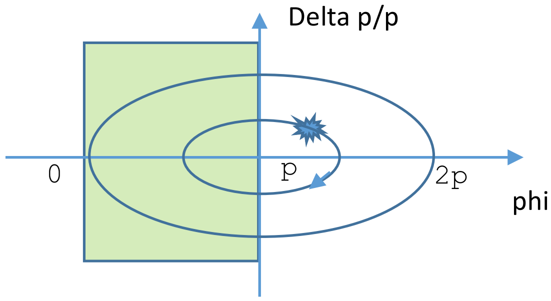

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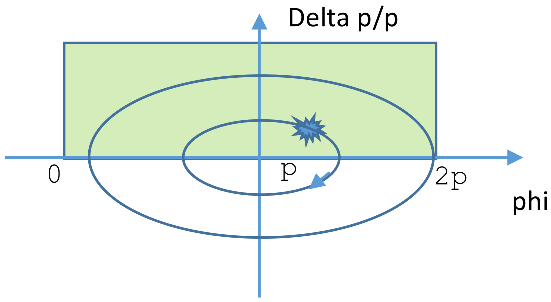

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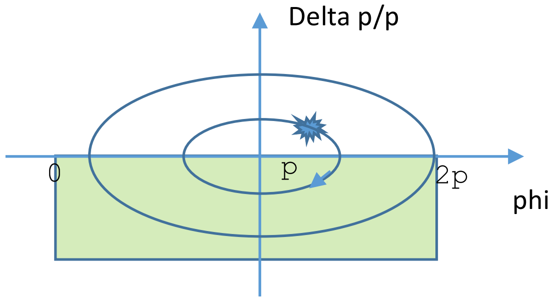

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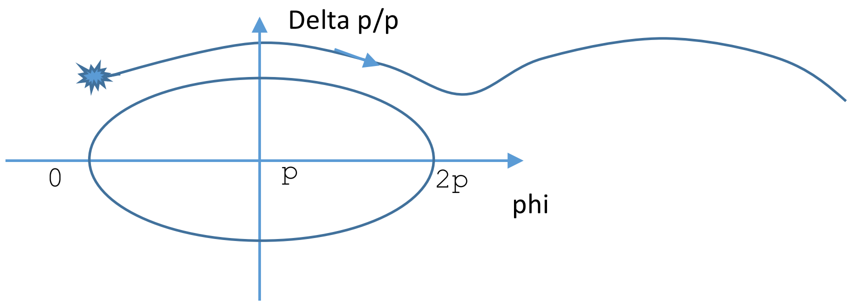

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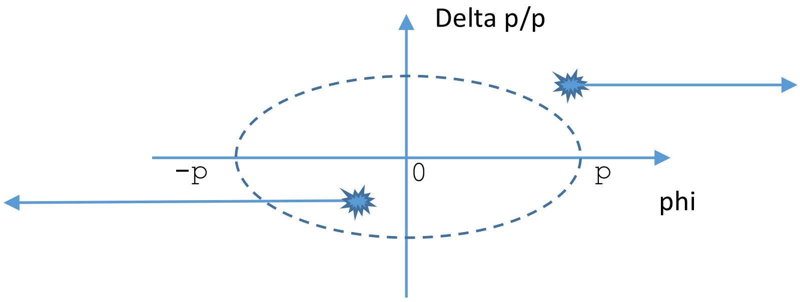

Physical constants:

- Elementary charge: $e = 1.60\ 10^{-19}$ C
- Electron mass: $m_e = 9.11\ 10^{-31}$ kg
- Proton mass: $m_p = 1.67\ 10^{-27}$ kg
- Speed of light: $c = 3.00\ 10^{8}$ m/s
- Vacuum permittivity: $\varepsilon_0 = 8.85\ 10^{-12}$ F/m