Correction Longitudinal Beam Dynamics Exam
11th Feb. 2015

1) $\varphi_d = \dfrac{2\pi}{N_d} = \boxed{0{,}0628 \text{ rad}} \left(\times \dfrac{180^\circ}{\pi} = 3{,}6^\circ \right)$

$L_d = \varphi_d \rho_d = \boxed{4.398 \text{ m}}$

2) $p_{inj} \cancel{c} = \dfrac{1}{c}\sqrt{(E_{k_{inj}} + E_0)^2 - E_0^2} = 2.14 \text{ GeV}/c$

$B_{inj} = \dfrac{p_{inj}\,[\text{GeV}/c]}{0{,}3\,\rho_d} = \boxed{0{,}10199 \text{ T}}$

3) $\gamma_{inj} = \dfrac{E_{k_{inj}} + E_0}{E_0} = 2{,}49$

$\beta_{inj} = \sqrt{1 - \dfrac{1}{\gamma_{inj}^2}} = 0{,}916$

$f_{rev,inj} = \left(\dfrac{2\pi R}{\beta_{inj}\,c}\right)^{-1} = \boxed{436'902 \text{ Hz}}$

$f_{rev,ultrarel} = \left(\dfrac{2\pi R}{c}\right)^{-1} = 476'987 \text{ Hz}$

$\dfrac{f_{rev,ultra} - f_{rev,inj}}{f_{rev,inj}} \times 100 = \boxed{9{,}17\,\%}$

4) $h = 8 \Rightarrow f_{RF,inj} = h \times f_{rev,inj} = \boxed{3{,}495 \text{ MHz}}$

$B_{l,inj} = \dfrac{2\pi R}{h} = \boxed{78{,}54 \text{ m}}$

$\boxed{\dot{B}_{inj} = 0}$ (no acceleration ...)

5) $$Q_{s,inj} = \sqrt{\frac{\cancel{e} V_{RF,inj} \left| \eta_{inj} \cdot \cos\phi_{s,inj} \right| \cdot h_{inj}}{2\pi \beta_{inj}^2 \left(E_{k_{inj}} + E_0\right)_{\text{in eV}}}}$$

$$\eta_{inj} = \frac{1}{\gamma_{inj}^2} - \alpha_c = 0{,}134 \quad (>0 \Rightarrow \text{below transition})$$

$$\phi_{s,inj} = 0 \quad (\text{since there is no acceleration})$$

$$\Rightarrow Q_{s,inj} = \boxed{0{,}00148}$$

6) $$p_{ej} = 0{,}3\, B_{ej} \rho_d \Rightarrow (\beta\gamma)_{ej} = \frac{p_{ej}\,[\text{GeV}/c]}{E_0\,[\text{GeV}]} \Rightarrow \gamma_{ej} = \sqrt{(\beta\gamma)_{ej}^2 - 1} \Rightarrow$$

$$\Rightarrow E_{k,ej} = \gamma_{ej} E_0 - E_0 = \boxed{25{,}44\ \text{GeV}}$$

(or by using ultrarelativistic approx: $p_{ej}c \approx E_{k_{ej}} \approx E_{ej} \approx 26\ \text{GeV}$)

$$\Delta E/_{turn} = \frac{E_{k,ej} - E_{k,inj}}{t_{acc}} \times f_{rev,ultrarel}^{-1} = \boxed{100{,}8\ \text{keV}/_{turn}}$$

$$\dot{B} = \frac{\dot{p}}{0{,}3\rho_d} \approx \left(\frac{E_{k_{ej}} - E_{k_{inj}}}{t_{acc}}\right) \Big/ 0{,}3\rho_d = \boxed{2{,}289\ \text{T}/\text{s}}$$

Otherwise you can use: $\dot{B} = \frac{\Delta E/_{turn}}{e\rho_d\, 2\pi R}$

7) $$\gamma_t = \sqrt{\frac{1}{\alpha_c}} = 6{,}086$$

$\gamma_{inj} = 2{,}49$; $\gamma_{ej} = 28{,}12$ (computed in Ex. 3, 6) $\Big\}$ **Yes, it crosses transition!**

<u>Otherwise</u>:

$\eta_{inj} = 0{,}134 > 0 \Rightarrow$ below transition

$\eta_{ej} = \frac{1}{\gamma_{ej}^2} - \alpha_c = -0{,}0257 < 0 \Rightarrow$ above transition $\Big\}$ **Yes!**

- What is ϕ_s at 3.5 GeV and at 16 GeV ?

→ For the stability condition: $\eta \cos\phi_s > 0$

- If we are below transition (@ 3.5 GeV : $\eta > 0$)

$$\Rightarrow \cos\phi_s > 0 \quad \Rightarrow \quad \boxed{0 < \phi_s < \frac{\pi}{2}}$$

- If we are above transition (@ ~~14~~ 16 GeV : $\eta < 0$)

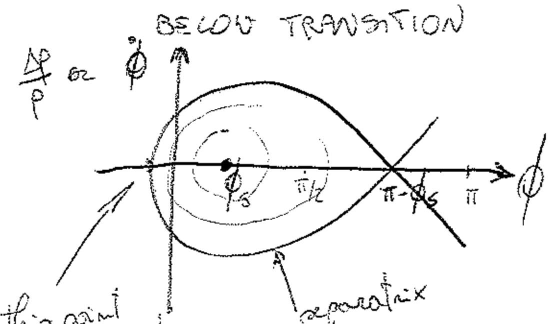
$$\Rightarrow \cos\phi_s < 0 \quad \Rightarrow \quad \boxed{\frac{\pi}{2} < \phi_s < \pi}$$

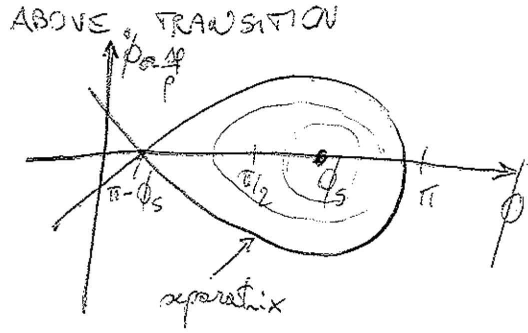
→ Formula to get ϕ_s : $\boxed{\Delta E/_{turn} = e V_{RF,acc} \sin\phi_s}$

→ However due to an error in the text of the exam:

$$\sin\phi_s = \frac{100.8 \text{ keV}}{e \times 100 \text{ kV}} > 1 \Leftarrow \boxed{\text{not possible !}}$$

→ Apologizes for the error! Most of you however noticed the problem and I have evaluated it in the correction !...and made anyway the sketch of a bucket, without putting numbers:





this point can be found by solving:

$$\begin{cases} \dot\phi = 0 \\ \dfrac{\dot\phi^2}{2} - \dfrac{\Omega_s^2}{\cos\phi_s}(\cos\phi + \phi\cos\phi_s) = \dfrac{\Omega_s^2}{\cos\phi_s}\left[\cos(\pi-\phi_s) + (\pi-\phi_s)\sin\phi_s\right] \end{cases} \text{(eq. of the separatrix)}$$

8) One of the possible answers:

$$\eta = \frac{\Delta f/f}{\Delta p/p}$$

the slippage factor represents the change in revolution frequency due to an offset in momentum e.g. with respect to the synchronous particle

- below transition ($\eta > 0$):
  $\Rightarrow$ a particle with a higher momentum than the synchronous particle will "travel faster"
- above transition ($\eta < 0$):
  $\Rightarrow$ a particle with a higher energy will have a longer revolution period, i.e. it will arrive later in the RF cavity

$$\boxed{\eta = 0 \text{ @ transition}}$$

9) $(Ze)\, v B = \frac{m v^2}{\rho} \Rightarrow \boxed{B\rho = \frac{p}{Ze} \approx \frac{p\,[\mathrm{GeV}/c]}{Z \times 0{,}3}}$

$$(B\rho)_{ions} = (B\rho)_{p^+, ej} = 87{,}99 \text{ Tm} \quad (\text{from Ex. } 6)$$

$$p_{ions} = 0{,}3\, Z\, (B\rho)_{ions} = 1425{,}44 \text{ GeV}/c$$

$$E_{ions} = \sqrt{p_{ions}^2 c^2 + \underbrace{(AE_0)^2}_{\approx \text{ approx}}} = \boxed{1438{,}61 \text{ GeV}}$$

Usually one is interested in the energy per nucleon:

$$E_{ions}/A = 6{,}95 \text{ GeV}/u$$