LONGITUDINAL BEAM DYNAMICS EXAMINATION

Consider a proton accelerator with the following characteristics:

Ring radius	R = 100 m
Injection kinetic energy	Ek_{inj} = 1.4 GeV
Harmonic number at injection	h_{inj} = 8
Number of dipoles	N_d = 100
Bending radius	ρ_d = 70 m
Momentum compaction factor	α_c = 0.027
RF Voltage at injection	$V_{RF, inj}$=30 kV
Dipole magnetic field at extraction	B_{ej}=1.257 T
Maximum RF Voltage during acceleration	$V_{RF, acc}$=100 kV
Duration of the acceleration	t_{acc}=0.5 s

N.B.: The proton rest energy is E_0 = 938.26 MeV and the velocity of light is c=2.997 925 10^8 m/s

1) What are the bending angle φ_d and the effective length L_d of a dipole?
2) What is the magnetic field at injection B_{inj}?
3) Compute the revolution frequency at injection ($f_{rev,inj}$). What is the error (in %) that one would make if one assumes that the machine is ultra-relativistic?
4) What is the RF frequency ($f_{RF,inj}$) at injection and what is the bucket length, knowing that the injection takes place on a flat-bottom (i.e. constant magnetic field and no acceleration)? What is the value of Bdot ($\dot{B}_{inj}$) during the injection process?
5) What is the synchrotron tune $Q_{s,inj}$ during the injection flat-bottom?
6) After the injection process is completed, the beam is accelerated up to the extraction energy within t_{acc}. Compute the kinetic energy at extraction Ek_{ej}. Compute the energy gain per turn ΔE and the Bdot ($\dot{B}_{acc}$) assuming a linear increase of the momentum and using the ultra-relativistic assumption.
7) Does the accelerator cross the transition energy? What is the synchronous phase ϕ_s at 3.5 GeV and at 14 GeV? Make the sketch of the shape of the bucket at these two energies. Don't forget the axis titles. Write the names and values of all the points and lines that you know (NB: if numerical integration is required you are not expected to solve the equation, but just to write it down).
8) Explain the physical meaning of the slippage factor η and compute its value at transition energy.
9) In addition to proton acceleration, the same ring is also accelerating Lead ions Pb^{+54} (mass number A=207, charge Z=54). What is the final extraction energy of the ion beam, knowing that it should have the same rigidity ($B\rho$) of the extracted proton beam? *Hint: re-derive the expression for the magnetic rigidity of Slide #11, computing the Lorentz force for Lead Ions with charge (Ze).*