LONGITUDINAL BEAM DYNAMICS EXAMINATION

The CERN PS Booster (PSB) is the first circular accelerator in the Large Hadron Collider (LHC) injector chain for protons. The PSB is actually composed of 4 superimposed synchrotron rings, which receive beam from the linear accelerator Linac2, at a kinetic energy of 50 MeV, and accelerate it to 1.4 GeV (kinetic) for injection in the PS.

The relevant characteristics of the four rings (which are almost equal and independent) are specified in this Table:

Ring Radius	R = 25 m
Injection Kinetic Energy	Ek_{inj} = 50 MeV
Extraction Kinetic Energy	Ek_{ej} = 1.4 GeV
Harmonic number	h = 1
RF maximum voltage (h=1)	$V_{h=1}$= 8kV
Number of dipoles	N_d = 32
Length of a dipole	L_d = 1.6177 m
Momentum compaction factor	α_c = 0.0579
Future injection energy for the Upgrade	Ek_{LIU} = 160 MeV
Future ramp rate at injection for the Upgrade	$d(B\rho)/dt)@_{160MeV}$ = 10 T m/s

N.B.: The proton rest energy is E_0 = 938.26 MeV and the velocity of light is c=~3 10^8 m/s

2 1) Fill in the table with the relativistic parameters at injection and at extraction.

	Ek	Etotal	p	β	γ	$B\rho$
Injection	50 MeV					
Extraction	1.4 GeV					

2 2) Can we say that the PSB is an ultra-relativistic machine? Compute the revolution frequency at injection and extraction.

1 3) What "h=1" implies in terms of the cavity RF frequency? Is the RF frequency constant during the cycle? What is the maximum RF bucket length (in meters)?

2 4) Does the PSB cross transition? Motivate your answer. Explain what "being below/above the transition energy" means.

1 5) What is the synchronous phase at the extraction magnetic flat top? What would have been if we were above transition?

Measuring the Dispersion in the extraction line

After the beam is extracted from the PSB, it is transported via a transfer line to the downstream machine, the PS. In order to measure the Horizontal and Vertical Dispersion in the line, the PS (transverse) beam dynamics guys need your help! ☺. Since the Dispersion couples the transverse with the longitudinal plane:

$$\Delta x_{BPM} = D_{x,BPM} \frac{\Delta p}{p_0}$$ (a similar formula holds for the vertical plane)

by measuring the beam position Δx_{BPM} (or Δy_{BPM}) at one or more Beam Position Monitors (BPM) located along the line, as a function of the variation of beam momentum Δp with respect to the nominal value at extraction p_0 , they can compute the value of the Dispersion at the BPM locations and then eventually interpolate it to any position in the line.

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6) What is the maximum (and minimum) relative momentum change you can apply $\pm\left(\frac{\Delta p}{p_0}\right)_{MAX}$ in order to keep the beam excursion smaller than $|\Delta x| < 3$ cm (the max value of the dispersion in the line is $D_{x,MAX} = 3$m)?

In operation, in order to change the momentum of the beam at extraction, you act on the RF frequency.

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7) What are the 2 new RF frequency values you need to enter, if you want to apply this change in the beam momentum $\pm\left(\frac{\Delta p}{p_0}\right)_{MAX}$?

The PSB Upgrade

The LHC Injector Upgrade (LIU) Project, which is actually ongoing at CERN, plans at increasing the PSB injection energy from 50 MeV to 160 MeV (kinetic). The aim is to produce beams of higher brightness (i.e. the bunch intensity within a given transverse normalized emittance) by overcoming the limits of Space-Charge at injection.

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8) What is the gain in terms of number of protons per bunch (N_b), assuming that the bunch length and the normalized transverse emittances stay the same and only the injection energy is changed?

(N.B. The idea is to keep the same Space Charge tune spread ΔQ^{SC} , which you can compute with this approximated formula:

$$\Delta Q^{SC} = \frac{r_p N_b R}{\sqrt{2\pi}\,\sigma_z\,\beta\gamma^2\varepsilon_N}$$ (assuming Gaussian shape, round beam, no dispersion)

Where: r_p=1.54e-18 m is the classical proton radius, N_b is the number of protons per bunch, R is the machine radius, σ_z is the rms bunch length and ε_N is the normalized transverse emittance.)

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9) What is the magnetic field at the present injection energy and at the future one?

Normally the injection process is done on a magnetic plateau (i.e. with no acceleration). However the plan for the LIU Project is to inject directly on a ramp starting at 160 MeV (kinetic).

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10) Knowing that (d(Bρ)/dt) is 10 Tm/s (PAY ATTENTION to the definition and the units!), what is the synchronous phase, for a pure h=1 beam? What is the maximum bucket length?

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11) Let's assume that the injection process lasts for 100 revolution periods, and that the momentum from the Linac is increased together with the magnetic field in order to inject and maintain the beam on the same closed orbit.

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a. What is the error on the RF frequency if it is kept constant all along the injection process?

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b. What is the beam kinetic energy at the end of the injection?