Elias Métral
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Corrections of the Longitudinal Beam Dynamics Examination

1.

- $\alpha_p = \frac{1}{\gamma_{tr}^2} \Rightarrow \gamma_{tr} = \frac{1}{\sqrt{\alpha_p}} = \frac{1}{\sqrt{3{,}225 \cdot 10^{-4}}} \simeq 55{,}7.$

- $\gamma_i = \frac{E_{ti}}{E_0} = \frac{\sqrt{E_0^2 + p_i^2 c^2}}{E_0} = \frac{\sqrt{0{,}938^2 + 450^2}}{0{,}938} \simeq 479{,}745$

$\Rightarrow \eta_i = \frac{1}{\gamma_i^2} - \alpha_p \simeq -3{,}18 \cdot 10^{-4} \longrightarrow$ Already very close to $-\alpha_p$.

- $\gamma_c = \frac{E_{tc}}{E_0} = \frac{\sqrt{E_0^2 + p_c^2 c^2}}{E_0} = \frac{\sqrt{0{,}938^2 + 7000^2}}{0{,}938} \simeq 7462{,}69$

$\Rightarrow \eta_c = \frac{1}{\gamma_c^2} - \alpha_p \simeq -3{,}22 \cdot 10^{-4}$

- $\eta_i < 0 \Rightarrow \eta$ is always < 0 and therefore LHC does not cross transition.
- As the LHC operates above transition, it means that an accelerated particle will take more time to travel and therefore the revolution period will increase.

2.

- $\gamma_i = \frac{1}{\sqrt{1-\beta_i^2}} \Rightarrow 1 - \beta_i^2 = \frac{1}{\gamma_i^2} \Rightarrow \beta_i = \sqrt{1 - \frac{1}{\gamma_i^2}} \simeq 0{,}999998 \simeq 1$

- $\gamma_c = \frac{1}{\sqrt{1-\beta_c^2}} \Rightarrow \beta_c = \sqrt{1 - \frac{1}{\gamma_c^2}} \simeq 1$

$\Rightarrow$ One can conclude that $\beta \simeq 1$ from injection till collision, and that one can approximate the velocity of the particles by c (the speed of light) and that the revolution frequency (and revolution period) is almost constant.

- $v = \beta . c = c = R . \omega_{rev} = 2\pi R . f_{rev}$ with $\beta \approx 1$ and $2\pi R = Circ$ $\Rightarrow f_{rev} \approx \frac{c}{Circ} = \frac{2{,}997925 . 10^8}{26658{,}883}$

$= 11{,}2455$ kHz

and $T_{rev} = \frac{1}{f_{rev}} \simeq 88{,}92\,\mu s$

- $f_{RF} = h . f_{rev} = 35640 \times 11245{,}5 = 400{,}79$ MHz

(3) • $\Delta E^{1s}_{gain} = \frac{\Delta E_k}{\underset{20min}{\Delta t}} = \frac{E_k - E_i}{\Delta t} = \frac{\sqrt{0{,}938^2 + 7000^2} - \sqrt{0{,}938^2 + 450^2}}{20 \times 60}$

$= 5{,}46$ GeV/s

$\Rightarrow \Delta E^{1\,turn}_{gain} = \Delta E^{1s}_{gain} \times T_{rev} \simeq 485{,}38$ keV/turn.

- $\Delta E^{1\,turn}_{gain} = e . V_{RF} . \sin\phi_s \Rightarrow \sin\phi_s = \frac{\Delta E^{1\,turn}_{gain}}{e . V_{RF}} \simeq 0{,}03$

As we are above transition, beam stability requires $\underset{<0}{\eta} . \cos\phi_s > 0$
$\Rightarrow \cos\phi_s < 0$

$\Rightarrow$ If we would have been below transition (BT), one would have had $\phi_s^{BT} = \text{Arcsin}(0{,}03) \simeq 0{,}03$ rad
$\simeq 1{,}74$ degrees

But the LHC is operating above transition, therefore $\phi_s = \pi - \phi_s^{BT} \simeq 3{,}11$ rad
$\simeq 178{,}26$ degrees.

(4) • $\theta_d = \frac{2\pi}{N_d} = \frac{2\pi}{1232} \simeq 5{,}1$ mrad
$\simeq 0{,}29$ degree

- $L_d = \rho_d . \theta_d \Rightarrow \rho_d = \frac{L_d}{\theta_d} = \frac{14,3}{0,0051} \simeq 2803,93$

- $\Delta E_{gain}^{1\,turn} = e.\rho_d . \underset{\underset{\frac{dB}{dt}}{\shortparallel}}{\dot{B}} . \underbrace{2\pi R}_{Circ} \Rightarrow \dot{B} = \frac{dB}{dt} = \frac{\Delta E_{gain}^{1\,turn}}{e.\rho_d . Circ}$

$\simeq 6,5\ mT/s$

- $\underset{(T.m)}{B_i . \rho_d} = 3,3356 . p_i\ [GeV/c] \Rightarrow B_i \simeq 0,535\ T$

- $\longrightarrow \underset{(T.m)}{B_c . \rho_d} = 3,3356 . p_c\ [GeV/c] \Rightarrow B_c \simeq 8,3\ T$

  $\longrightarrow B_c = B_i + \dot{B} . \Delta t = 0,535 + 6,5 . 10^{-3} \times \underbrace{60 \times 20}_{20\,min} \simeq 8,3\ T$

$\longrightarrow$ Same result obtained as foreseen.

(5) . $\sin \phi_s = 0$ (as flat top)

$\underset{<0}{\eta} . \cos \phi_s > 0$ (for beam stability reason)

$\Rightarrow \cos \phi_s < 0$

$\Rightarrow \phi_s = \pi$ rad $= 180$ degrees

(6) . The angular synchrotron frequency is given by

$$\Omega_s = \sqrt{\frac{e.V_{RF} . \eta . h . c^2 . \cos \phi_s}{2\pi R_s^2\, E_s}}$$

$E_s \leftarrow$ synchronous particle

$R = \frac{Circ}{2\pi} \simeq 4242,89\ m$

$$= \sqrt{\frac{16.10^6 . 3,22.10^{-4} . 35640 . (2,997925.10^8)^2}{2\pi . 4242,89^2 . 7000}}$$ at top energy

$\simeq 144,478$ rad/s

- $f_s = \dfrac{\Omega_s}{2\pi} \simeq 23\ Hz$

- $T_s = \dfrac{1}{f_s} \simeq 43{,}5\ ms$

- $Q_s = \dfrac{f_s}{f_{rev}} \simeq 2 \cdot 10^{-3}$

- 1 synchrotron oscillation is performed in $\dfrac{1}{Q_s} \simeq 489$ turns of the LHC.

(7) 7.1) Clockwise, as beam 2 is moving anti-clockwise and it will go slower than beam 1 to perform 1 LHC turn as it is accelerated and we are above transition.

7.2) • $\eta_c = \dfrac{\frac{\Delta f_{rev}}{f_{rev}}}{\frac{\Delta p}{p}} \Rightarrow \Delta f_{rev} = \eta_c \cdot f_{rev} \cdot \dfrac{\Delta p}{p}$

$$= -3{,}22 \cdot 10^{-4} \cdot 11245{,}5 \cdot 10^{-4}$$

$$\simeq -0{,}36\ mHz$$

- $\Delta f_{RF} = \Delta f_{rev} \cdot h \simeq -12{,}9\ Hz$

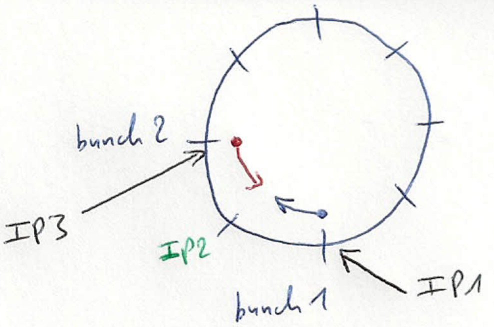
- $\alpha_p = \dfrac{\frac{\Delta Circ}{Circ}}{\frac{\Delta p}{p}} \Rightarrow \Delta Circ = \alpha_p \cdot Circ \cdot \dfrac{\Delta p}{p}$

$$= 3{,}225 \cdot 10^{-4} \times 26\,658{,}883 \times 10^{-4}$$

$$\simeq 859{,}7\ \mu m$$

$$\Rightarrow \Delta R = \frac{\Delta Circ}{2\pi} \simeq 136{,}8\ \mu m$$

7.3)



As bunch 1 is moving clockwise and bunch 2 is moving anti-clockwise with the same speed, when bunch 1 is at IP1, bunch 2 has to be at IP3 if they want to collide at IP2 $\Rightarrow$ It means that bunch 2 has to be shifted by a quarter of the LHC circumference compared to bunch 1 (and the initial situation where the 2 bunches collided in IP1).

7.4) • Let's call n the number of turns needed for bunch 2 to be shifted by a quarter of the LHC circumference and ΔT_{rev} the shift in revolution period for bunch 2 with the higher momentum ($\frac{\Delta p}{p} = 10^{-4}$ in the example)

$\Rightarrow n \cdot |\Delta T_{rev}| = \frac{T_{rev}}{4}$ if one wants bunch 2 to be shifted by a quarter of the LHC

$\Rightarrow n = \frac{1}{4 \cdot \frac{|\Delta T_{rev}|}{T_{rev}}}$ and $\left|\frac{\Delta T_{rev}}{T_{rev}}\right| = \left|\frac{\Delta f_{rev}}{f_{rev}}\right| = \left|\eta \cdot \frac{\Delta p}{p}\right|$

$\Rightarrow n = \frac{1}{4\left|\eta \frac{\Delta p}{p}\right|}$ and $n = \frac{\Delta t_{IP2}}{T_{rev}} = \Delta t_{IP2} \cdot f_{rev}$

$$\Rightarrow \Delta t_{IP2} = \frac{1}{4 f_{rev}\left|\eta \frac{\Delta p}{p}\right|}$$

• $\left.\begin{array}{l}\frac{\Delta p}{p} = 10^{-4} \\ \eta = \eta_c\end{array}\right\} \Rightarrow \Delta t_{IP2} = \frac{1}{4 \cdot 11245{,}5 \cdot 3{,}22 \cdot 10^{-4} \cdot 10^{-4}} \simeq 11 \min 29\,s$

7.5) Bunch 2 would have moved anticlockwise and it would have taken

3 times more time to have bunches 1 and 2 colliding in IP2 only. The other (faster) solution would have been to decelerate bunch 2 by using a momentum offset of $\ominus \cdot 10^{-4}$ and in this case the time would have been the same (11 min 29s).

7.6) - To collide in IP8 it will take 3 times more time, i.e. 34 min 28 s (starting from the situation of collisions in IP1 and IP5)

- The other method is to decelerate bunch 2 (momentum offset of $\ominus 10^{-4}$) and in this case it takes the same time as for bunches 2 and 1 to collide in IP2 $\Rightarrow$ 11 min 29 s
- If we start from the situation where bunches 2 and 1 collide in IP2, it takes 2 × 11 min 29 s = 22 min 58 s in both cases (+ and − 10^{-4}).

7.7) - To go faster we need to use a higher momentum offset.

- The problem is that it leads to a larger ΔR and therefore possible particle losses (due to interaction with the vacuum chamber or other equipments close to the beam.