Elias Métral
10/02/2012

Corrections of the Longitudinal Beam Dynamics Examination

(1) • $\varphi_d \cdot N_d = 2\pi \Rightarrow \varphi_d = \frac{2\pi}{N_d} = \frac{2\pi}{800} \simeq 0{,}0079$ rad $\simeq 0{,}45$ deg

• $\rho \cdot \varphi_d = L_d \Rightarrow \rho = \frac{L_d}{\varphi_d} = \frac{L_d \cdot N_d}{2\pi} = \frac{4 \times 800}{2\pi} = \frac{1600}{\pi} \simeq 509{,}3$ m

(2) $B_{inj} \cdot \rho = 3{,}3356\, p$ or $pc = \beta E_t \Rightarrow B_{inj} \cdot \rho = 3{,}3356 \cdot \beta_{inj} \cdot E_{inj} \times \frac{1}{10^9}$
[T.m] [GeV/c] [T.m]

$\Rightarrow B_{inj} = 3{,}3356 \cdot \beta_{inj} \cdot \frac{E_{inj}}{10^9} \times \frac{1}{\rho}$

or $E_t = E_0 + E_k$

$\gamma = \frac{E_t}{E_0}$, $\beta = \sqrt{1 - \frac{1}{\gamma^2}} \Rightarrow \beta_{inj} \simeq 0{,}99954 \Rightarrow$ We will approximate β in the rest of the cycle by $\beta \simeq 1$

$\Rightarrow B_{inj} \simeq 0{,}2$ T

(3) $\Delta E_{turn} = e \rho \dot{B}\, 2\pi R$ with $\dot{B} = \frac{dB}{dt}$

$\Rightarrow \dot{B} = \frac{\Delta E_{turn}}{e\rho\, 2\pi R} = \frac{600 \text{ keV}}{e \rho\, 2\pi R} = \frac{600 \times 1000}{509{,}3 \times 2\pi \times 5000} \simeq 0{,}0375$

(4) $V_{rf}^{min} = \frac{\Delta E_{turn}}{e} = 600$ kV $\Rightarrow$ Yes, the RF voltage given in the table is sufficient as 600 kV < 1 MV.

(5) $\Delta E_{turn} = e \cdot V_{rf} \cdot \sin\phi_s \Rightarrow \sin\phi_s = \frac{\Delta E_{turn}}{e \cdot V_{rf}} \simeq 0{,}6 \Rightarrow \phi_s \simeq 0{,}64$ rad $\simeq 36{,}9$ deg

(6) $B_{trans} = B_{inj} + \dot{B} \cdot \Delta t_{trans} \simeq 0{,}58$

$\Rightarrow E_{trans} = B_{trans} \cdot \frac{\rho}{3{,}3356} \simeq 88{,}6$ GeV

$\Rightarrow \gamma_t = \frac{E_{trans}}{E_0} \simeq 94$

(7) $\frac{dB}{B} = 10^{-3}$

$V_{rf} = 0$ (on a plateau $\Rightarrow dp = 0$)

$$\frac{dp}{p} = 0 \Rightarrow \gamma^2 \frac{df}{f} + \gamma^2 \frac{dR}{R} = 0 \Rightarrow \frac{df}{f} = -\frac{dR}{R}$$

$$\frac{dp}{p} = 0 \Rightarrow \gamma_t^2 \frac{dR}{R} + \frac{dB}{B} = 0 \Rightarrow \frac{df}{f} = \frac{1}{\gamma_t^2} \frac{dB}{B}$$

$$\Rightarrow df = \frac{f}{\gamma_t^2} \cdot \frac{dB}{B}$$

$$f = f_{rev} = \frac{c}{2\pi R} \text{ as } \beta \simeq 1 \Rightarrow df = \frac{c}{2\pi R \gamma_t^2} \cdot \frac{dB}{B} \simeq 1{,}1 . 10^{-3}$$

(8) $\phi_s^{A.T.} = \pi - \phi_s^{B.T.}$

A.T. = Above Transition
B.T. = Below Transition

$\simeq 2{,}5$ rad

$\simeq 143{,}1$ deg

Because the phase stability requires $\eta . \cos\phi_s \geqslant 0$.

(9) $\Delta t^{acc} = \frac{B_{ej} - B_{inj}}{\dot{B}} \simeq 26{,}6$ s.

(10) $E_{t\,inj} = \frac{B_{ej} . \rho}{3{,}3356} \simeq 183{,}2$ GeV.

(11) The revolution frequency decreases at injection, as it is below transition, and it increases at top energy, as it is above transition.

(12) $\eta_{inj} = \frac{1}{\gamma_{inj}^2} - \frac{1}{\gamma_t^2} \simeq 8{,}1 . 10^{-5}$

$\eta_t = 0$

$\eta_{ej} = \frac{1}{\gamma_{ej}^2} - \frac{1}{\gamma_t^2} \simeq -8{,}7 . 10^{-5}$

(13) $h = \frac{f_{rf}}{f_{rev}} = 1000$

$T_{rev} = \frac{1}{f_{rev}} = 104{,}7\ \mu s$

$$T_s^{inj} = T_{rev}\sqrt{\frac{2\pi E_{t\,inj}}{e.V_{rf}.h.|\cos\phi_{s\,inj}\,\eta_{inj}|}} \simeq 57{,}5 \text{ ms}$$

$$T_s^{ej} = T_{rev}\sqrt{\frac{2\pi E_{t\,ej}}{e.V_{rf}.h.|\cos\phi_{s\,ej}.\eta_{ej}|}} \simeq 426 \text{ ms}$$

$$T_s^{trans} = \text{infinity}$$

(14) $\Delta\theta = 2\pi f_{rev}.\tau_b = 6.10^{-4}$ rad

$\Delta\phi = h.\Delta\theta$ (in fact there is a minus sign but here we look at the extension only)

$= h.2\pi f_{rev}.\tau_b = 0{,}6$ rad

$$\Rightarrow \phi_{max}^{RF} = \frac{\Delta\phi}{2} = 0{,}3 \text{ rad}$$

$$p_{ej} = \sqrt{E_{t\,ej}^2 - E_0^2} \simeq 183{,}2 \text{ GeV/c}$$

$$\Delta p_{max} = \phi_{max}^{RF}.\frac{2\pi p_{ej} R}{T_s^{ej}|\eta_{ej}|hc}$$

$$\simeq 155{,}4 \text{ MeV/c}$$

$$\left(\frac{\Delta p}{p}\right)_{max}^{ej} = \frac{\Delta p_{max}}{p_{ej}} \simeq 8{,}5.10^{-4}$$

$$\Delta E_{max}^{ej} = E_{t\,ej}.\left(\frac{\Delta p}{p}\right)_{max}^{ej} = 155{,}4 \text{ MeV}$$

$$\varepsilon_l^{ej} = \pi.\frac{\tau_b}{2}.\Delta E_{max}^{ej} \simeq 2{,}44 \text{ eV.s}$$

(15) At any energy different from transition energy, if the RF voltage is switched off, then the bunch starts to debunch and after a certain time one ends up with a coasting beam.

At transition energy, with the RF voltage off, the bunch does not debunch: the longitudinal positions are frozen as in a Linac.