Beam-beam effects

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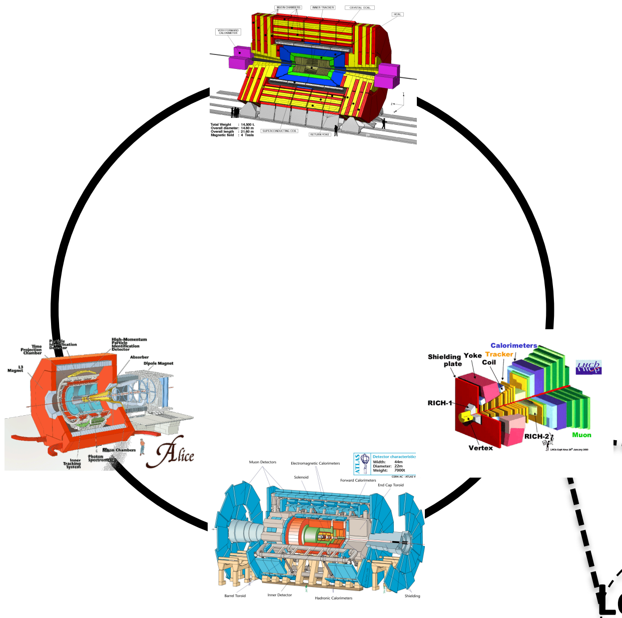
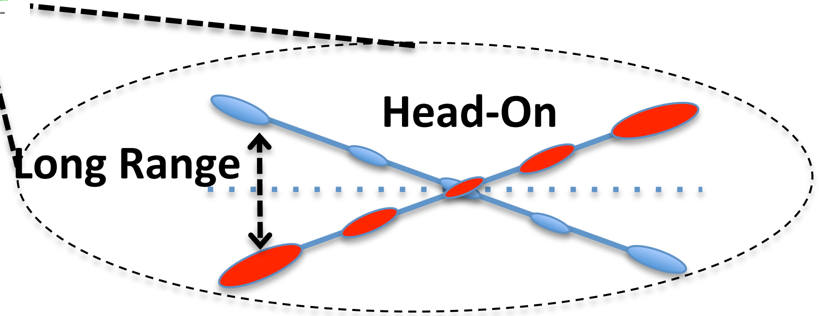
Electromagnetic interactions of two charged beams: colliders

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

$$F \propto N_p \cdot \frac{1}{r} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$





Nominal LHC case 4 head-on collisions and 120 long-range interactions

Beam-beam problem

$$\Delta U = -\frac{1}{\epsilon_0}\rho(x,y,z)$$

Derive potential from Poisson equation for charges with distribution ρ

Solution of Poisson equation

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\int\int\int \frac{\rho(x_0,y_0,z_0)dx_0dy_0dz_0}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2)}}$$

$$\vec{E} = -\nabla U(x,y,z,\sigma_x,\sigma_y,\sigma_z)$$

Then compute the fields

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

From Lorentz force one calculates the force acting on test particle with charge q

Making some assumptions we can simplify the problem and derive analytical formula for the force…

Round Gaussian distribution:

Gaussian distribution for charges:
Round beams:
Very relativistic, Force has only radial component :

$$\sigma_x = \sigma_y = \sigma$$

$$\beta \approx 1 \qquad r^2 = x^2 + y^2$$

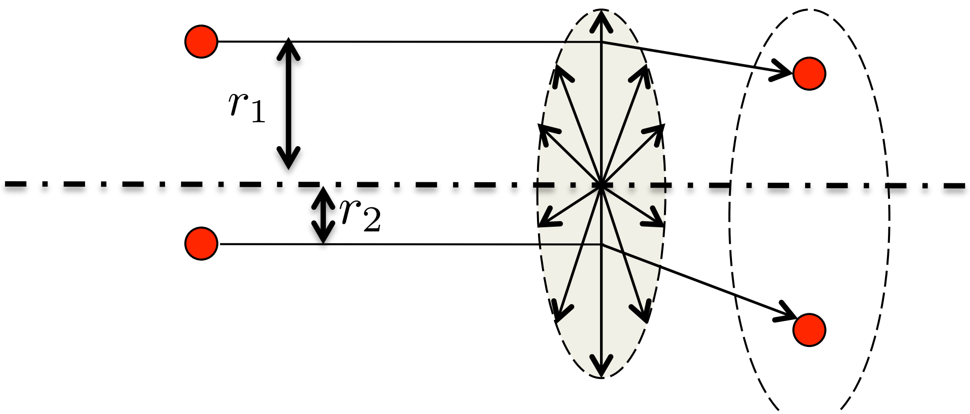
$$\mathrm{F} \propto N_p \cdot \frac{1}{r} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$

Beam-beam Force

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r, s, t)\ dt$$

Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)

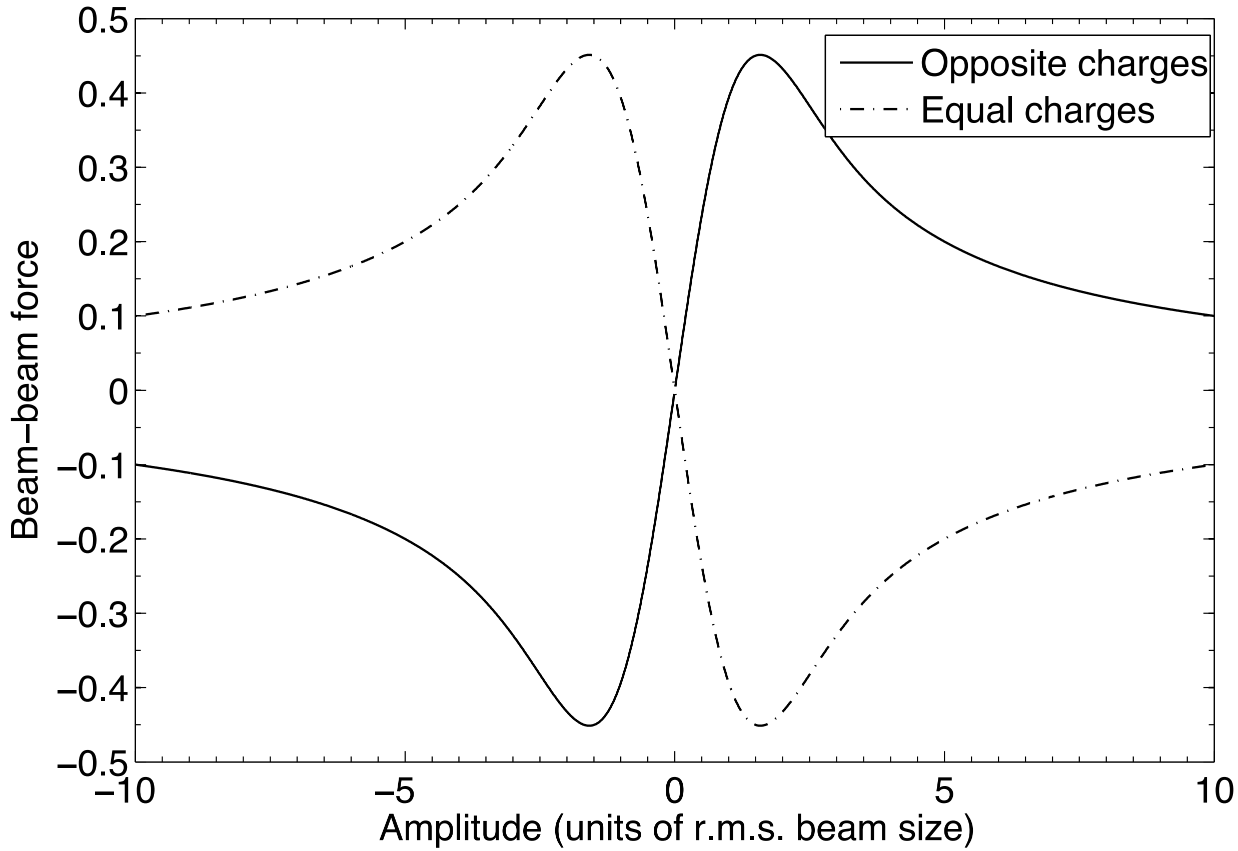
$$\Delta r' = -\frac{N_p r_0}{\gamma} \cdot \frac{r}{r^2} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$



Only radial component in relativistic case

How does this force look like?

Beam-beam Force



$$F_r(r) = \pm \frac{ne^2(1+\beta_{rel}^2)}{2\pi\epsilon_0}\frac{1}{r}[1-\exp(-\frac{r^2}{2\sigma^2})]$$

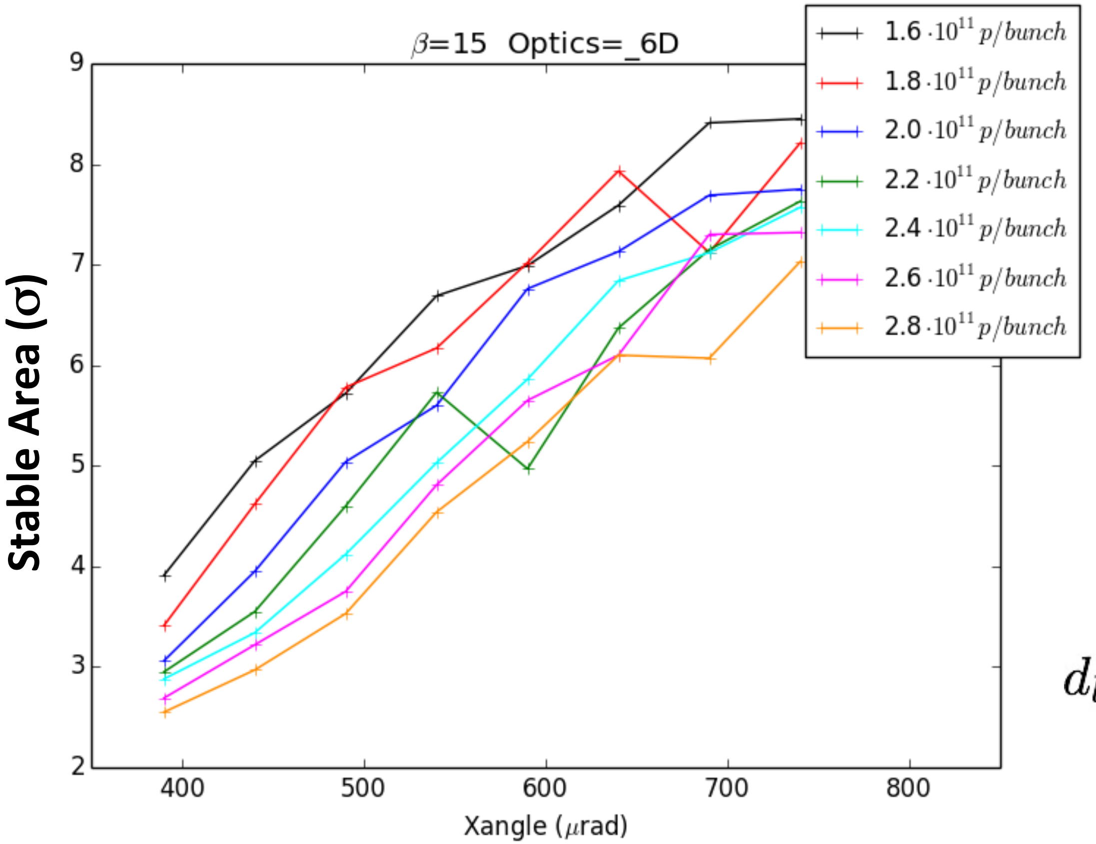
Problems

- Analytical solutions for Gaussians distributions and simple cases
- Semi-analytical studies
- Tracking single particles
- Tracking multi particles

Single Particle stability

TRACKING single particles over many turns 10^6 to study chaotic on-set

Dynamic Aperture: area in amplitude space with stable motion

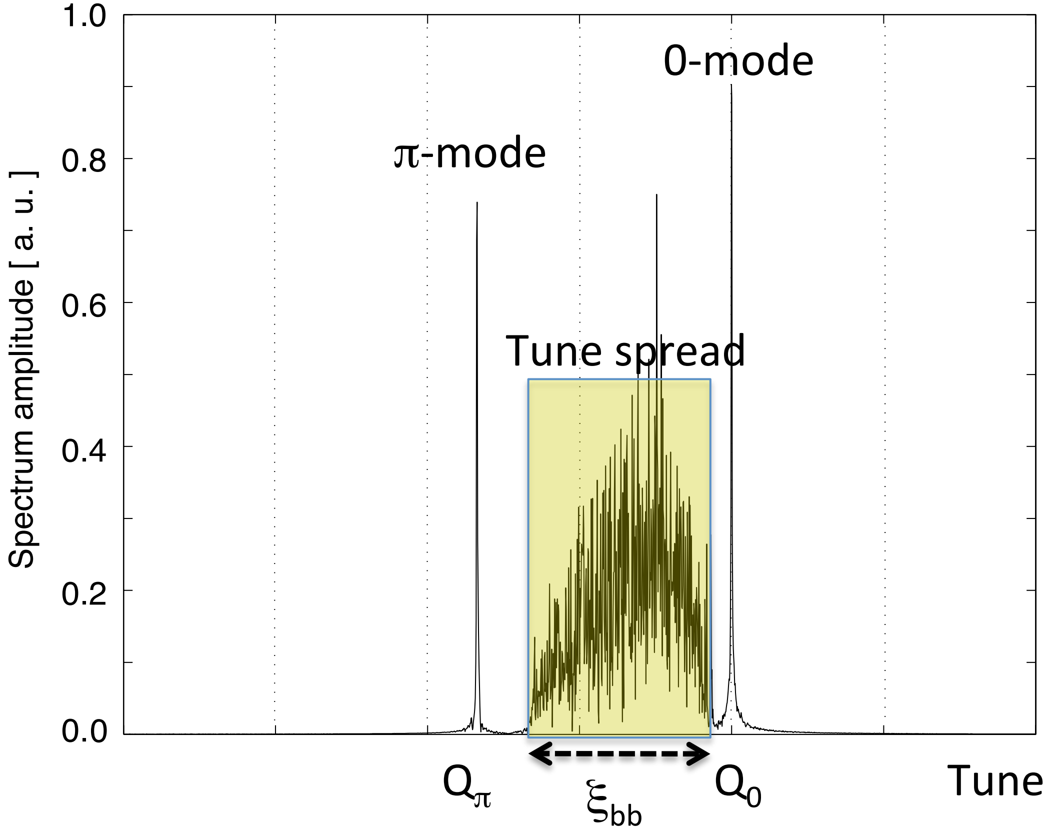


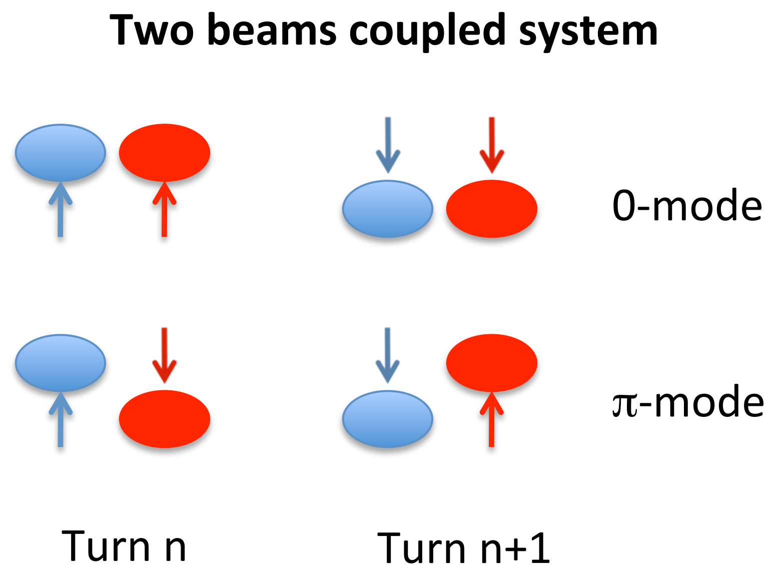
$$F_{bb} \propto N_p$$

$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$

Set accelerator parameters to avoid chaotic motion for too small amplitude particles!

Coherent beam-beam modes
Multi particle tracking self consistent





Coherent mode: two bunches are "locked" in a coherent oscillation

Head-on beam-beam coherent mode: LHC

BBQ Signals