HEAD-TAIL INSTABILITY MODE NUMBERS (2) IN TIME AND FREQUENCY DOMAIN

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- **Reminder on the general (theoretical) approach**
  - Eigenvalue system to solve => Find the eigenvalues and eigenvectors of an infinite complex matrix
  - The result is an infinite number of modes *mq* (-∞ < *m* , *q* < +∞) of oscillation (as there are 2 degrees of freedom: amplitude and phase)
  - To each mode, one can associate
    - 1) a complex coherent tune shift (which is the *q*th eigenvalue),
    - 2) a coherent spectrum (which is the *q*th eigenvector) σ_{mq}
    - 3) and a perturbation distribution $\hat{X}_{mq}\left(\hat{\tau}\right)$

azimuthal

radial

$$\Delta Q_{mq} = Q_{mq} - \left(Q_0 + m\, Q_s \right)$$

synchrotron tune

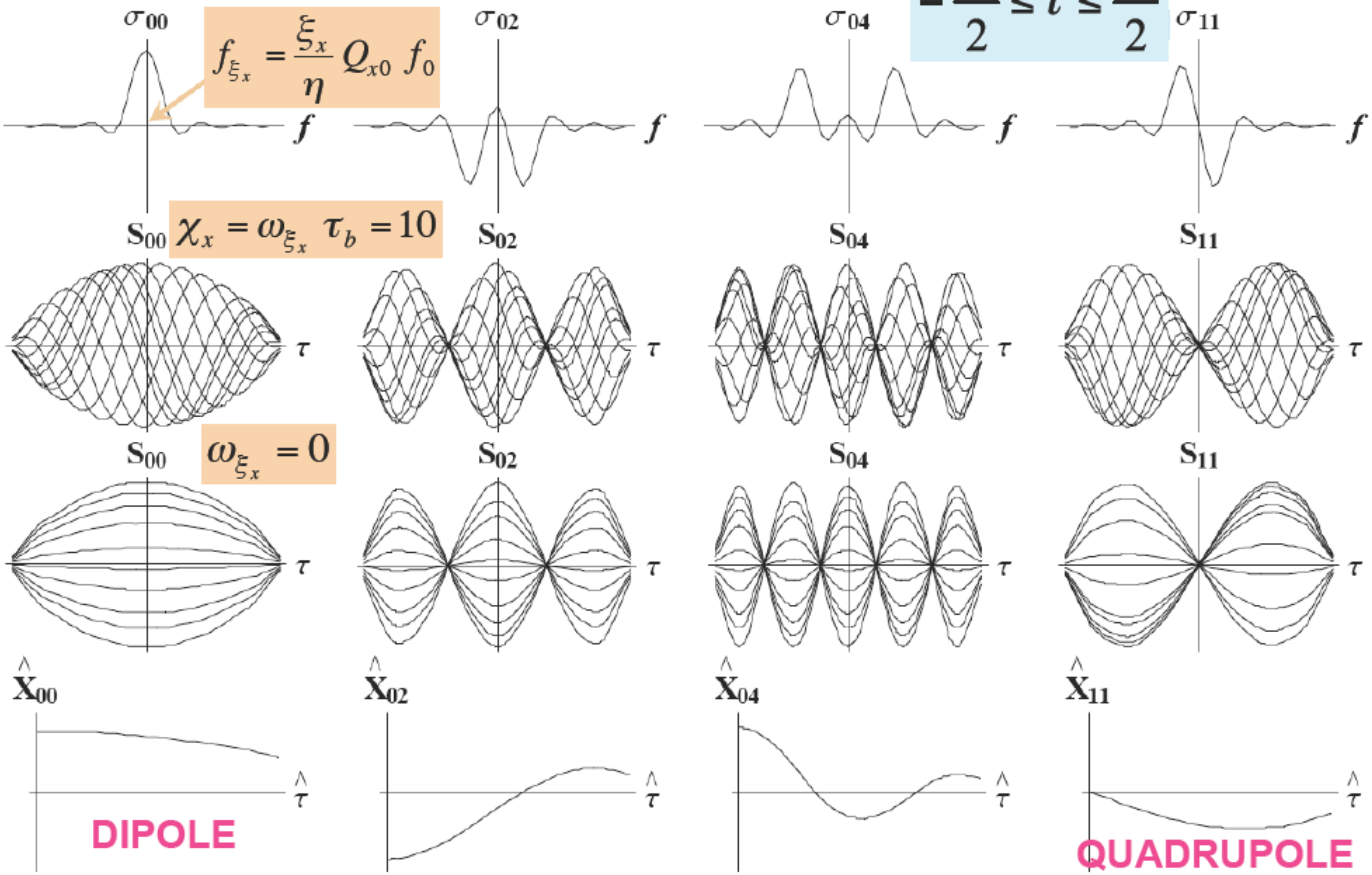
synchrotron amplitude

$q \equiv m + 2\,k$ $\quad$ $0 \leq k < +\infty$

Ex. of “water-bag” bunch interacting with a constant inductive impedance (1/2)

$$-\frac{\tau_b}{2} \leq \tau \leq \frac{\tau_b}{2}$$

$$Q_{x0} = x.13$$



$$f_{\xi_x} = \frac{\xi_x}{\eta} Q_{x0} \, f_0$$

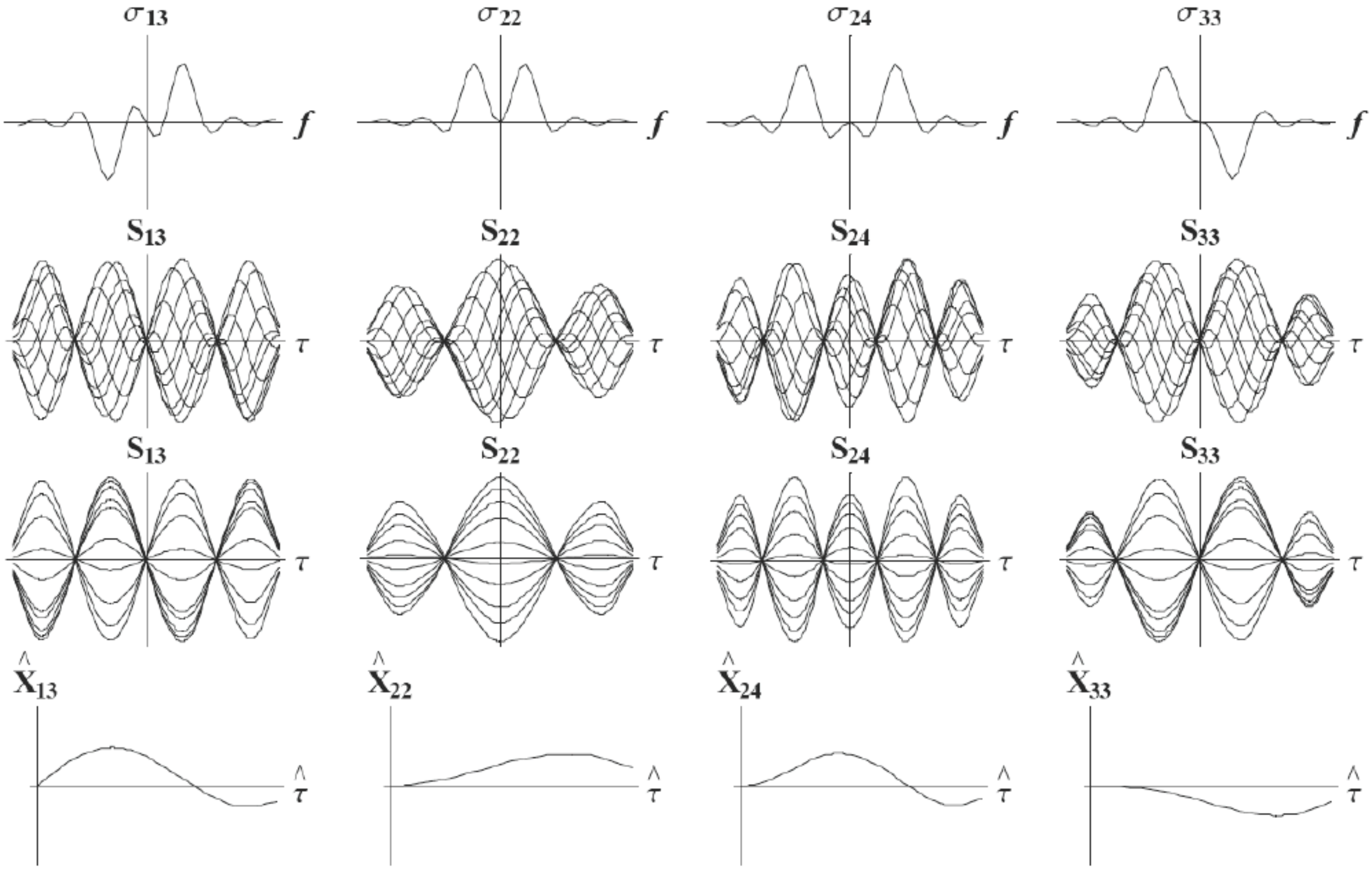
$$\chi_x = \omega_{\xi_x} \, \tau_b = 10$$

$$\omega_{\xi_x} = 0$$

DIPOLE

QUADRUPOLE

Ex. of “water-bag” bunch interacting with a constant inductive impedance (2/2)



METHOD TO DETERMNE *m*

- **Frequency analysis**

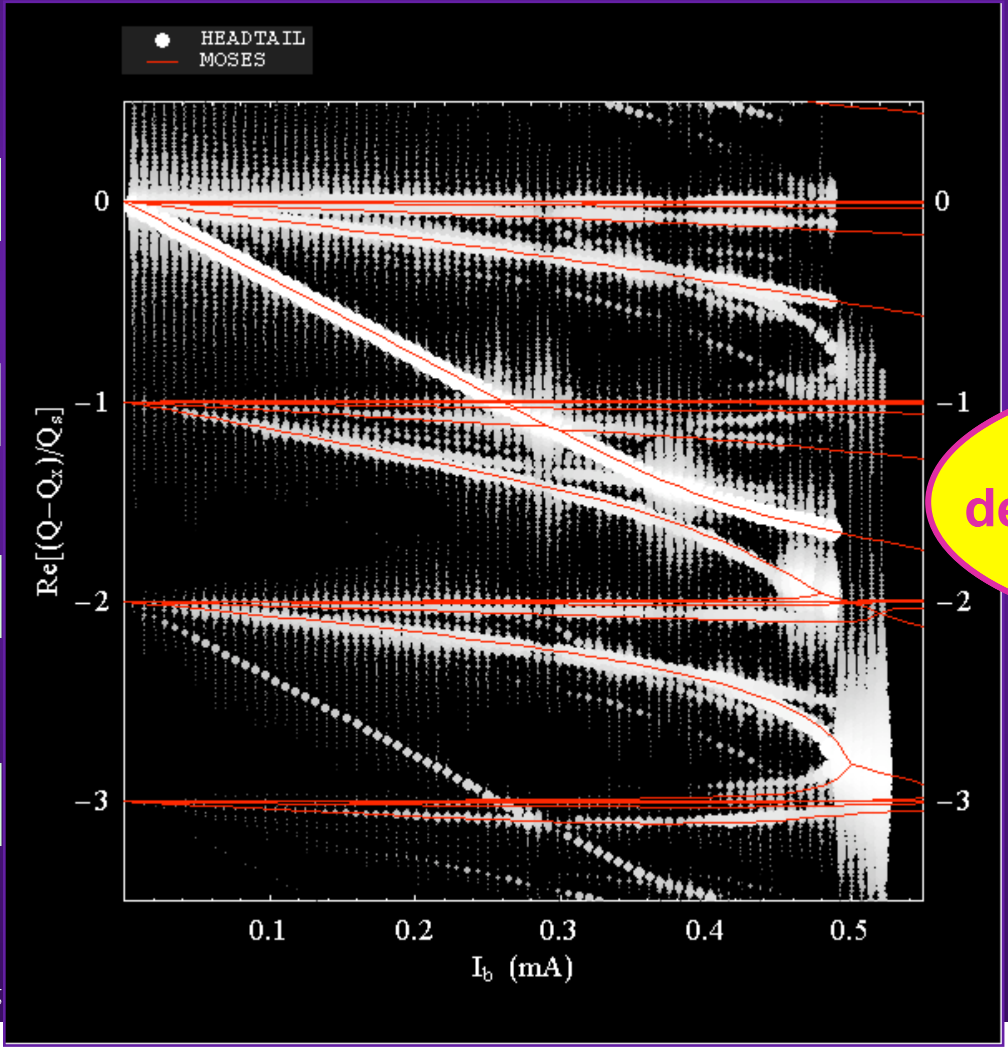
$$m = \frac{\mathrm{Re}\left(Q_{mq}\right) - Q_0 - \mathrm{Re}\left(\Delta Q_{mq}\right)}{Q_s}$$

$$Q_0$$

$$Q_0 - Q_s$$

$$Q_0 - 2\,Q_s$$

$$Q_0 - 3\,Q_s$$



Can be tricky to determine as depends on intensity…

Courtesy of B. Salvant (example)

POSSIBLE METHODS TO DETERMNE *q*

- **Time-domain analysis**
  - Superimpose several (~ 10) consecutive traces
  - The number of nodes gives *q*
- **Frequency-domain analysis**
  - Peak of power spectrum

Extension of ~ $\frac{1}{\tau_b}$

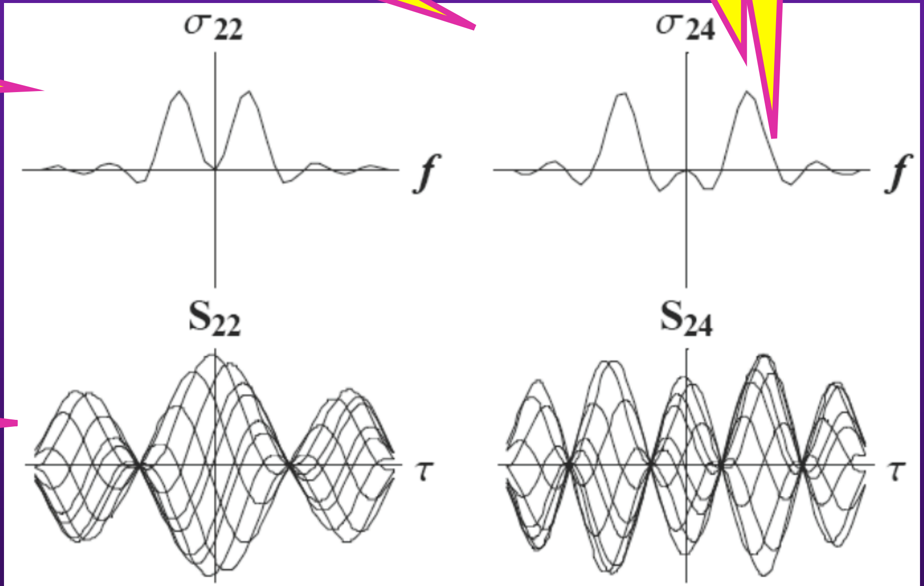
Peaked at ~ $f_q \approx f_{\xi_x} \pm \frac{q+1}{2\,\tau_b}$

Plots centered here at the chromatic frequency

Frequency-domain analysis (peak of the bunch spectrum)

Equivalent methods

Time-domain analysis (# of nodes)



SIMPLE EXAMPLE FOR THE LHC (1/2)

- An approximate fit of the power spectrum $\left|\sigma_{mq}\right|^2$ of the previous figure is obtained by the following function

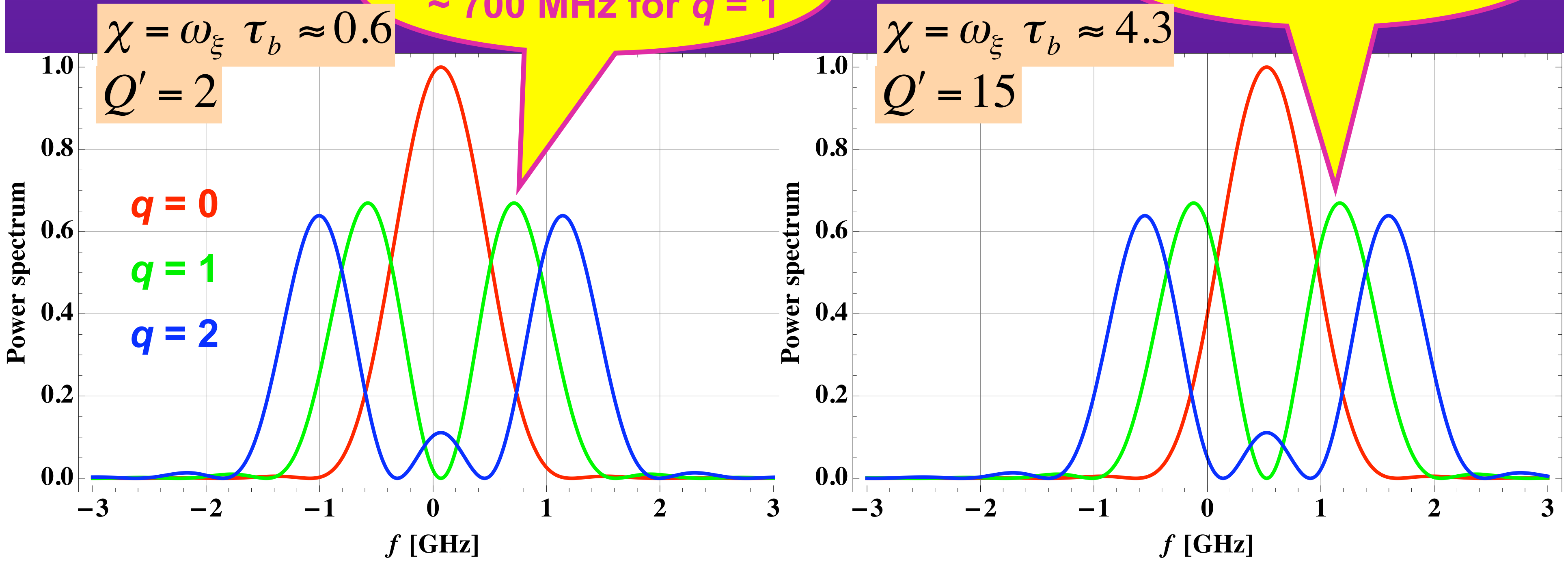
$$h_{m,q}(f) = \frac{\tau_b^2}{2\pi^4}\left(|q|+1\right)^2 \frac{1+(-1)^{|q|}\cos\left(2\pi f \tau_b\right)}{\left[\left(2 f \tau_b\right)^2-\left(|q|+1\right)^2\right]^2}$$

$$\tau_b = 1.3 \text{ ns}$$

$$\frac{f_\xi}{Q'} = \frac{f_{rev}}{\eta} \approx 35 \text{ MHz}$$

Peak activity around ~ 700 MHz for *q* = 1

Peak activity around ~ 1.2 GHz for *q* = 1

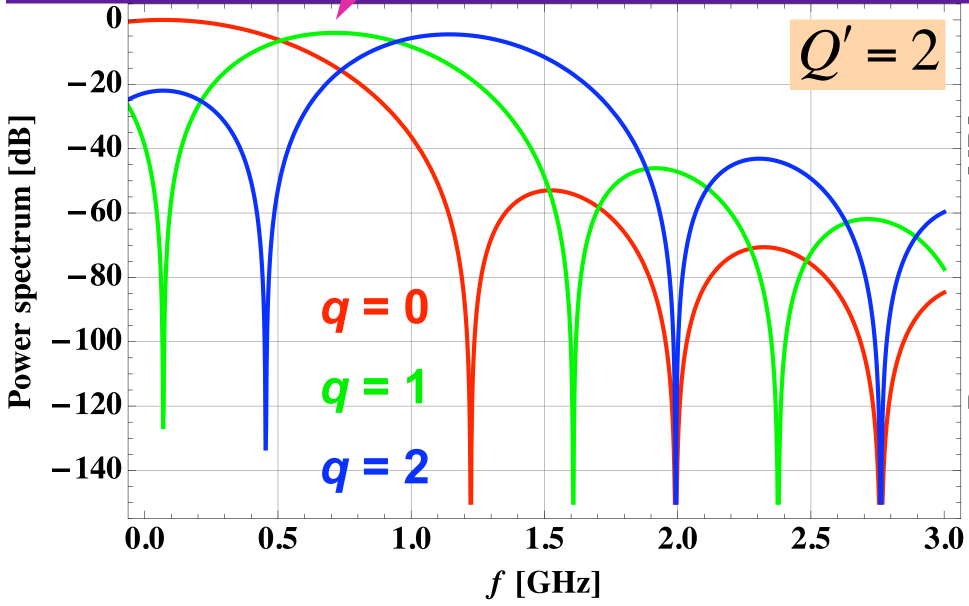


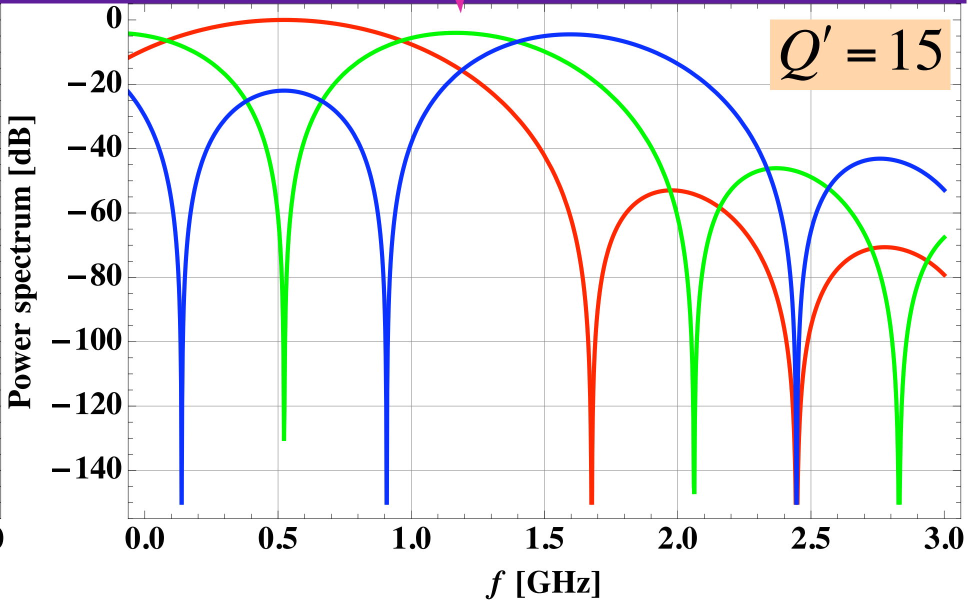
SIMPLE EXAMPLE FOR THE LHC (2/2)

- **Plotting things in Log Scale as done by Ralph Steinhagen for his Multiband-Instability-Monitor (MIM)**

Peak activity around ~ 700 MHz for *q* = 1

Peak activity around ~ 1.2 GHz for *q* = 1





CONCLUSIONS AND NEXT STEPS

- The number of nodes in time domain or the peak of the power spectrum in frequency domain gives information about the *radial mode number q (which can be equal to the azimuthal mode number but not necessarily)*
- In the general case, the power spectrum depends on the impedance and has to be deduced by solving the eigenvalue problem (finding the eigenvectors)
- More involved situation close to TMCI
- What happens in the presence of a transverse damper? Under study…
- What happens in the presence of other mechanisms?
- Etc.