Nested Head Tail Vlasov Solver:

Impedance, Damper, Radial Modes, Coupled Bunches, Beam-Beam and more…

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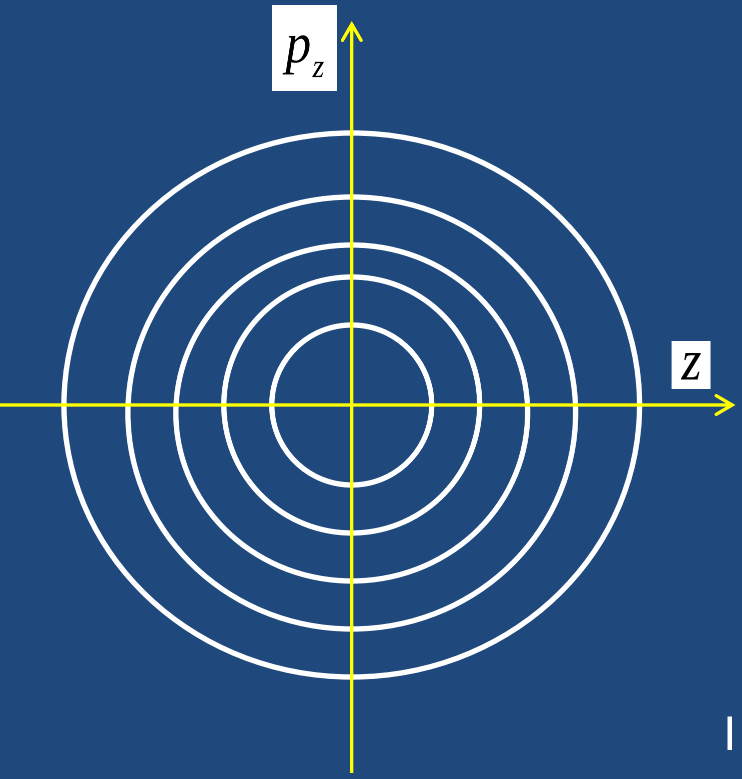
Fermilab-LARP

special thanks - to V.Danilov, E.Metral, N.Mounet, S.White, X.Buffat, and T.Pieloni

CERN, AP Forum Dec 4, 2012

Nested Head-Tail Basis



$$\psi_{l\alpha} \propto \exp(il\phi + i\chi_\alpha \cos\phi - i\Omega_l t)\ ;$$

$$\chi_\alpha = \frac{Q'\omega_0 r_\alpha}{c\eta}\ ;$$

$$\Omega_l = \omega_b + l\omega_s\ .$$

I am using n_r equally populated rings which radii r_α are chosen to reflect the phase space density.

Starting Equation, single bunch

- In the air-bag single bunch approximation, beam equations of motion can be presented as in Ref [A. Chao, Eq. 6.183]:

$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X$$

where X is a vector of the HT mode amplitudes,

$$(\hat{S} + \hat{Z})_{lm\alpha\beta} = -il\delta_{lm}\delta_{\alpha\beta} - i^{l-m}\frac{\kappa}{n_r}\int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau_\alpha - \chi_\alpha) J_m(\omega\tau_\beta - \chi_\beta)$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l}\frac{d}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta)$$

d is the damper gain in units of the damping rate, $\kappa = \dfrac{N_b r_0 R_0}{8\pi^2 \gamma Q_b Q_s}$

time is in units of the angular synchrotron frequency.

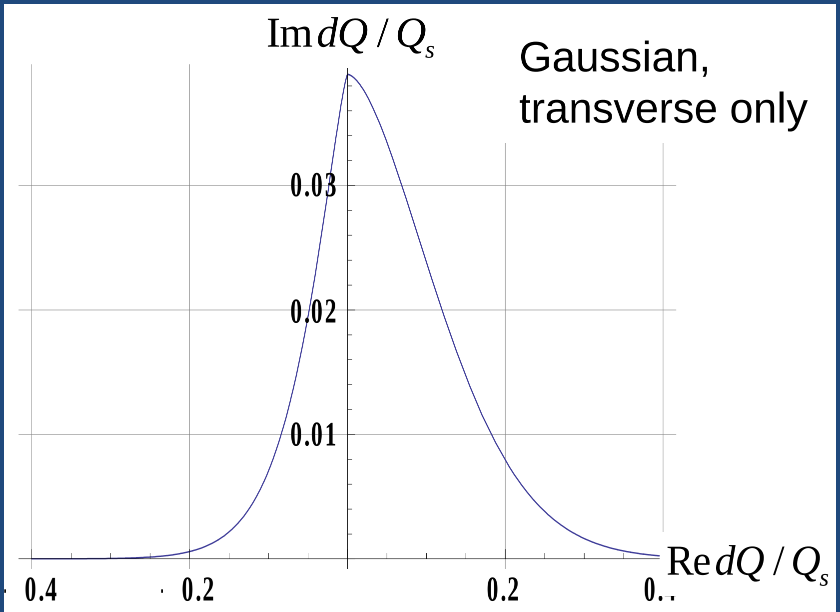
Analysis of solutions

1. For every given gain and chromaticity, the eigensystem is found for the LHC impedance table (N. Mounet).

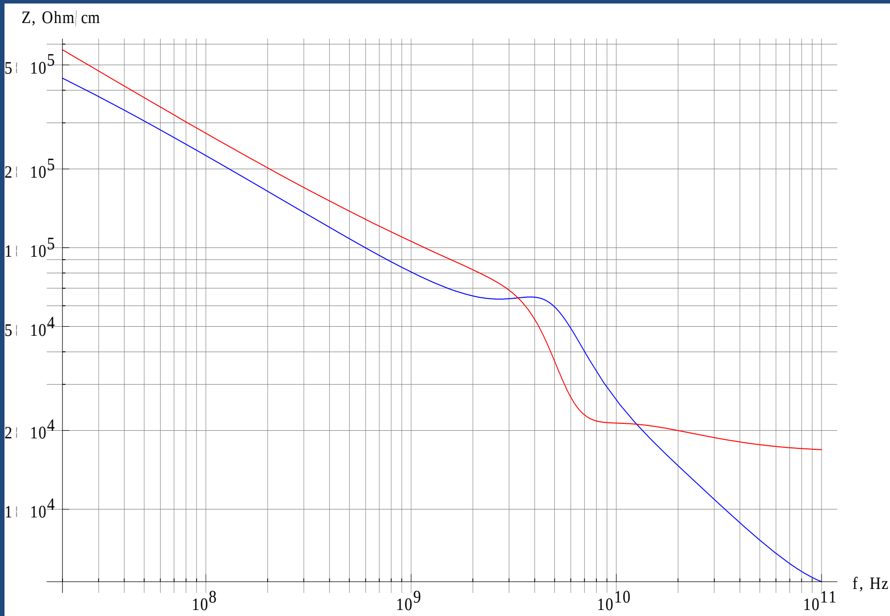
2. The complex tune shifts are found from the eigenvalues $\Delta\Omega_{l\alpha} = \Omega_{l\alpha} - l$

3. The stabilizing octupole current is found from the stability diagram for every mode, then max is taken.

Stability diagram at +200 A of octupoles



Impedances



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Coupled Equidistant Bunches

Main idea:

For LHC, wake field of preceding bunches can be taken as flat within the bunch length.

The only difference between the bunches is CB mode phase advance, otherwise they are all identical.

Thus, the CB kick felt by any bunch is proportional to its own offset, so the CB matrix $\hat{C}$ has the same structure as the damper matrix $\hat{D}$:

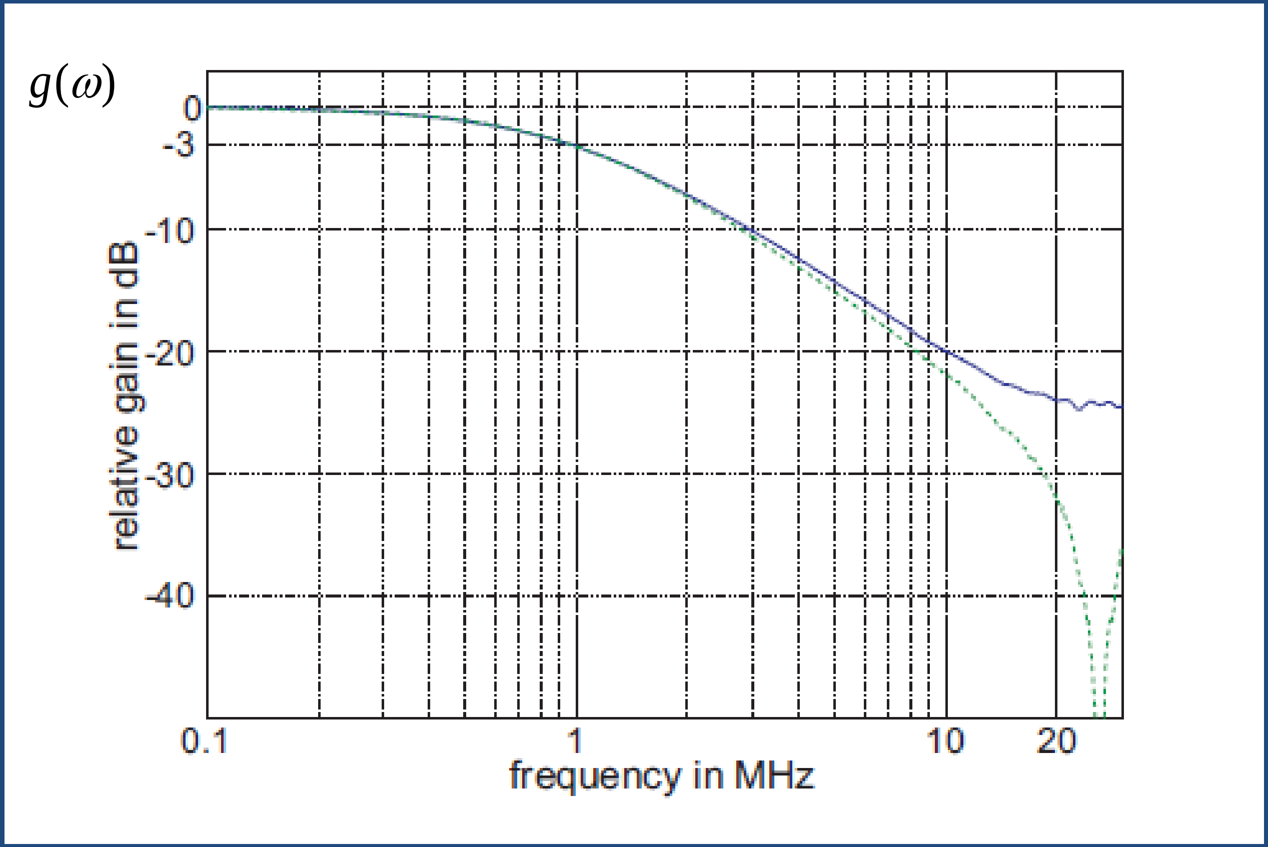
$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X + \hat{C} \cdot X;$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l} \frac{d_\mu}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta); \qquad \hat{C} = 2\pi i \kappa W(\varphi_\mu) \hat{D} / d_\mu;$$

$$W(\varphi_\mu) = \sum_{k=1}^{\infty} W(-ks_0) \exp(-ik\varphi_\mu); \qquad \varphi_\mu = 2\pi(1 - \{Q_x\}) + \frac{2\pi\mu}{M_b}; \qquad |\mu| \le \frac{M_b}{2}.$$

Wake and impedance are determined according to A. Chao book.

Old damper gain



Old narrow-band ADT gain profile (W. Hofle, D. Valuch) .
At 10 MHz it drops 10 times. The new damper is bbb for 50ns beam.

Below gain is measured in omega_s units, max gain=1.4 is equivalent to 50 turns of the damping time.

CB Mode Damping Rate

With $g(\omega)$ as the frequency response function of the previous plot, the time-domain damper's "wake" is

$$G(\tau) = \int_0^\infty g(\omega)\cos(\omega\tau)\,d\omega/\pi \, ,$$

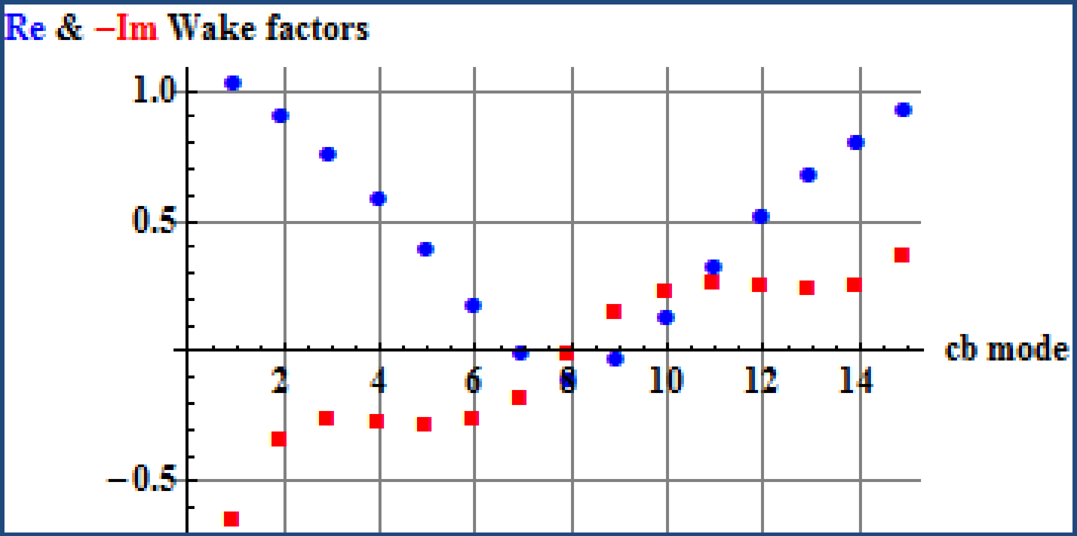
assuming this response to be even function of time (no causality for the damper!).

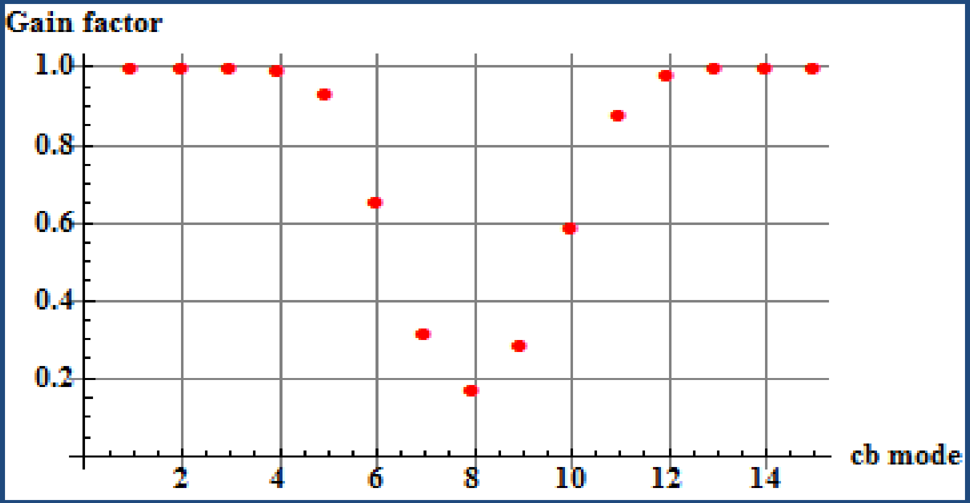
From here (equidistant bunches!):

$$d_\mu = d\,\frac{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)\cos(k\varphi_\mu)}{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)};$$

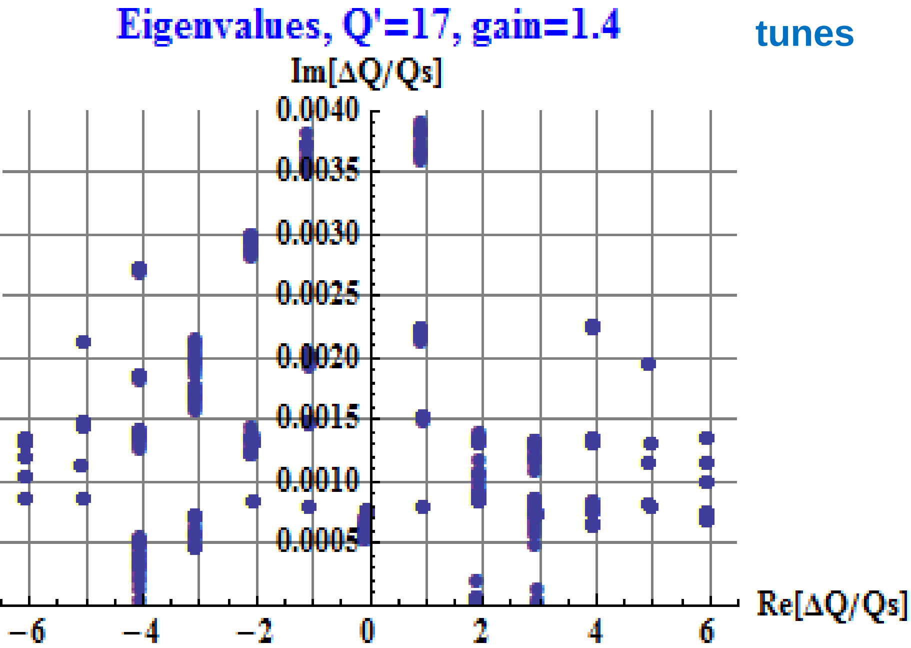
where d is the rate provided for low-frequency CB zero-head-tail modes at zero chromaticity.

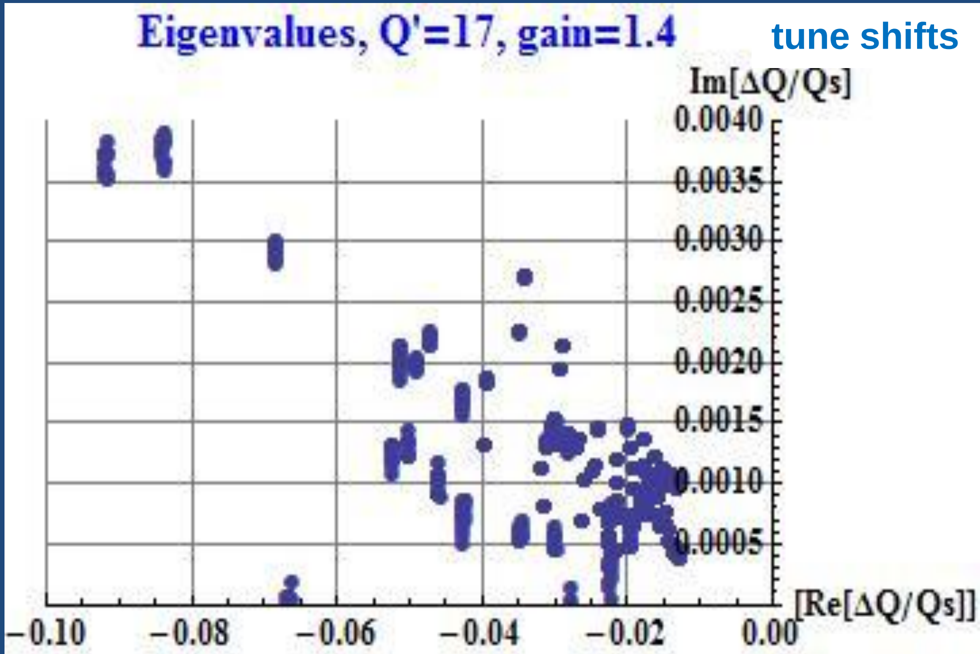
CB Wake and Gain Factors for the Old ADT

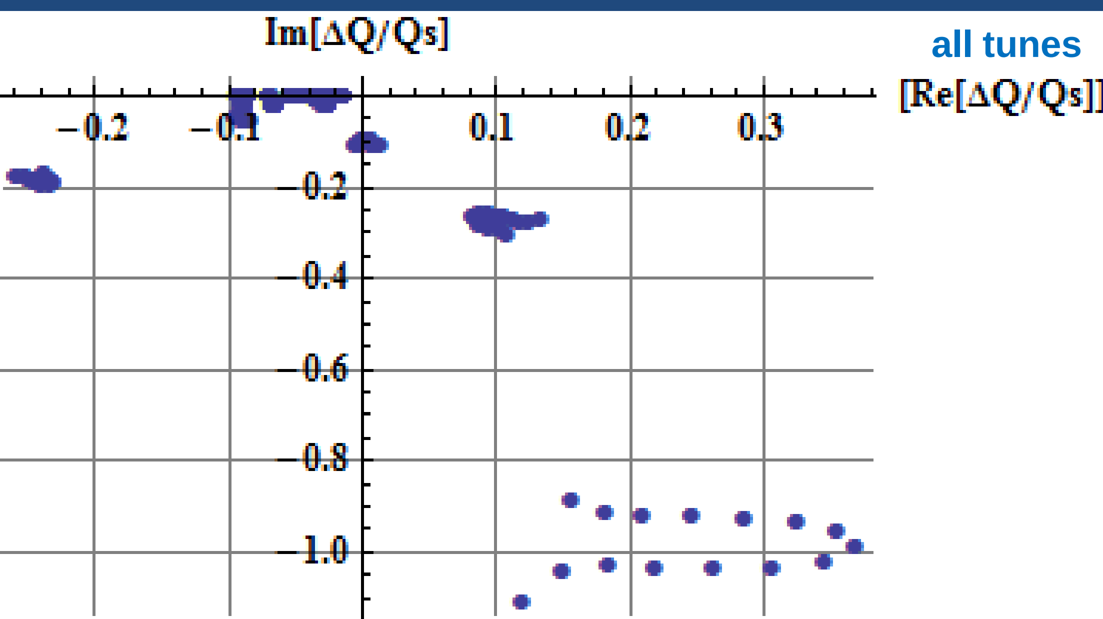




2⊗(SB and CB), flat ADT, Tunes at the Plateau







- All unstables -0.1<Re[dQ/Qs]<0.
- Weak head-tail is justified at the plateau.
- Mode with max rate (MUM) has ~max tune shift as well.
- For unstables -Re[dQ]/Im[dQ]~20-30.

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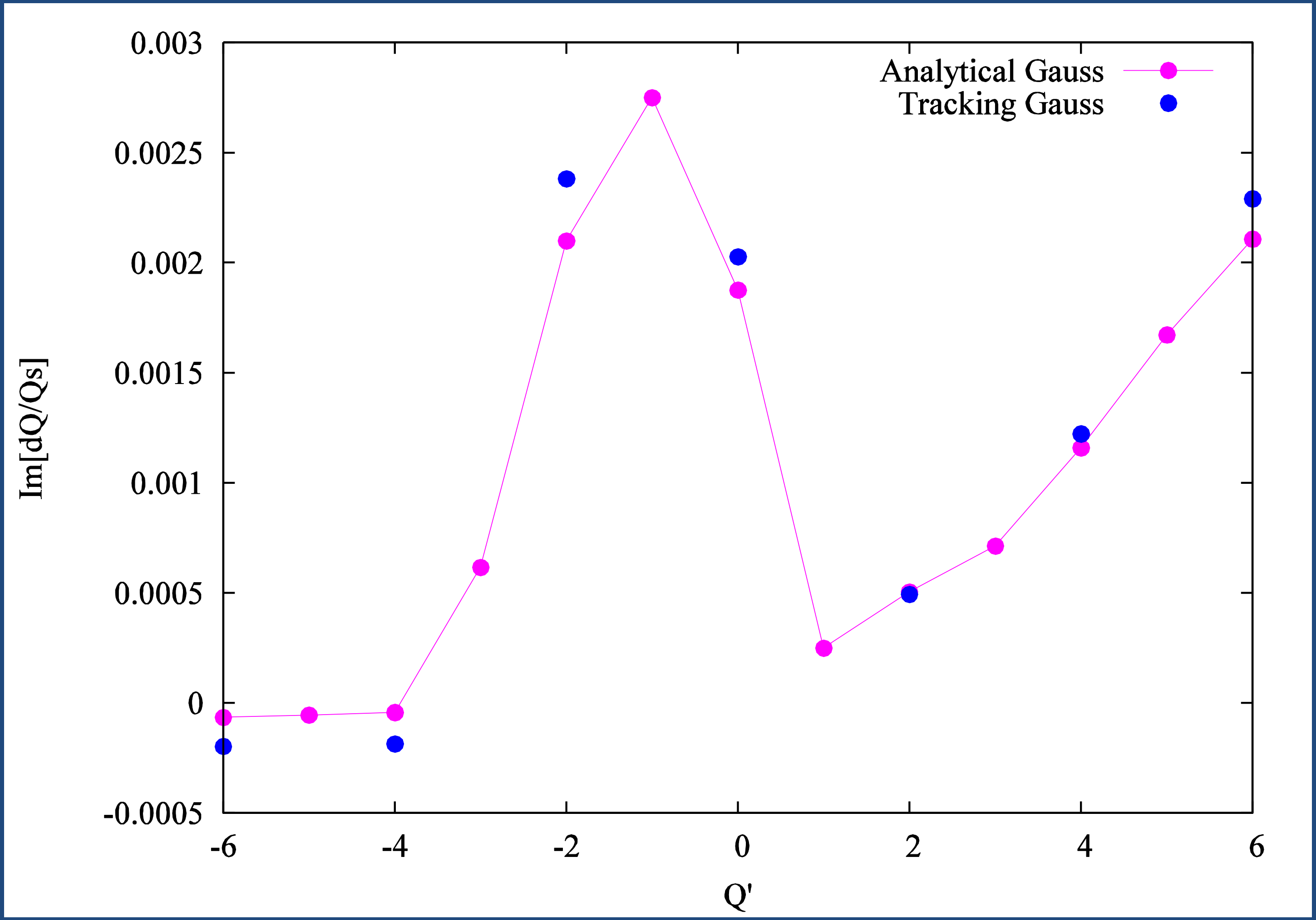
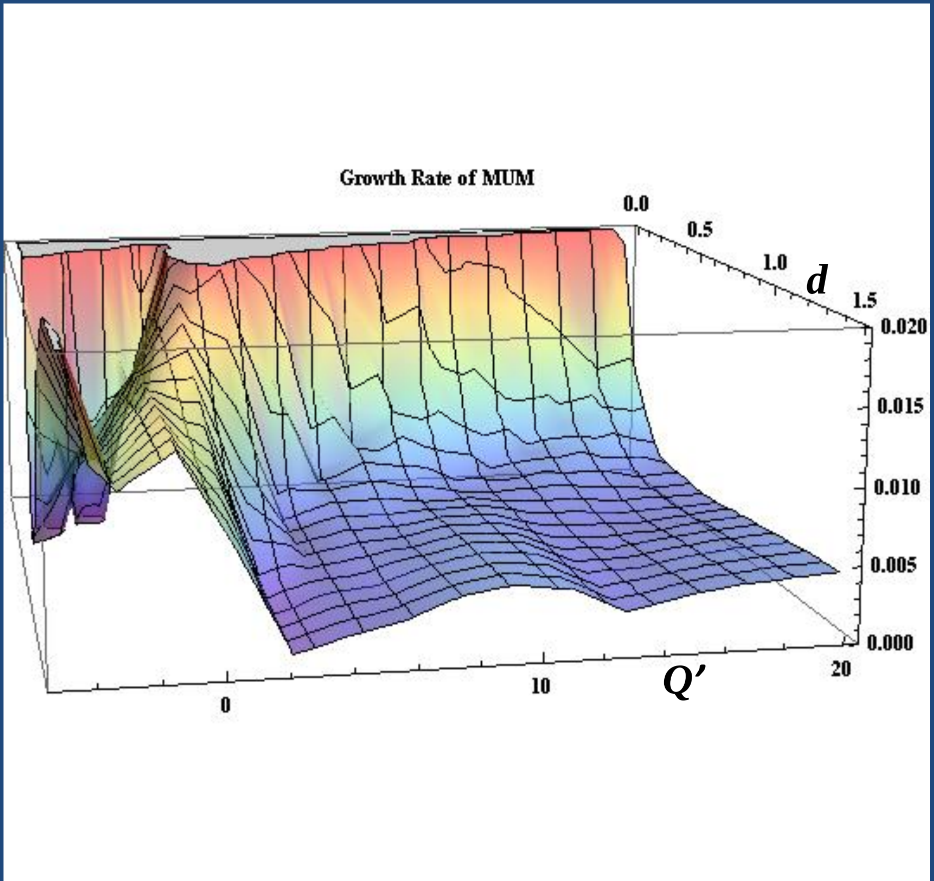
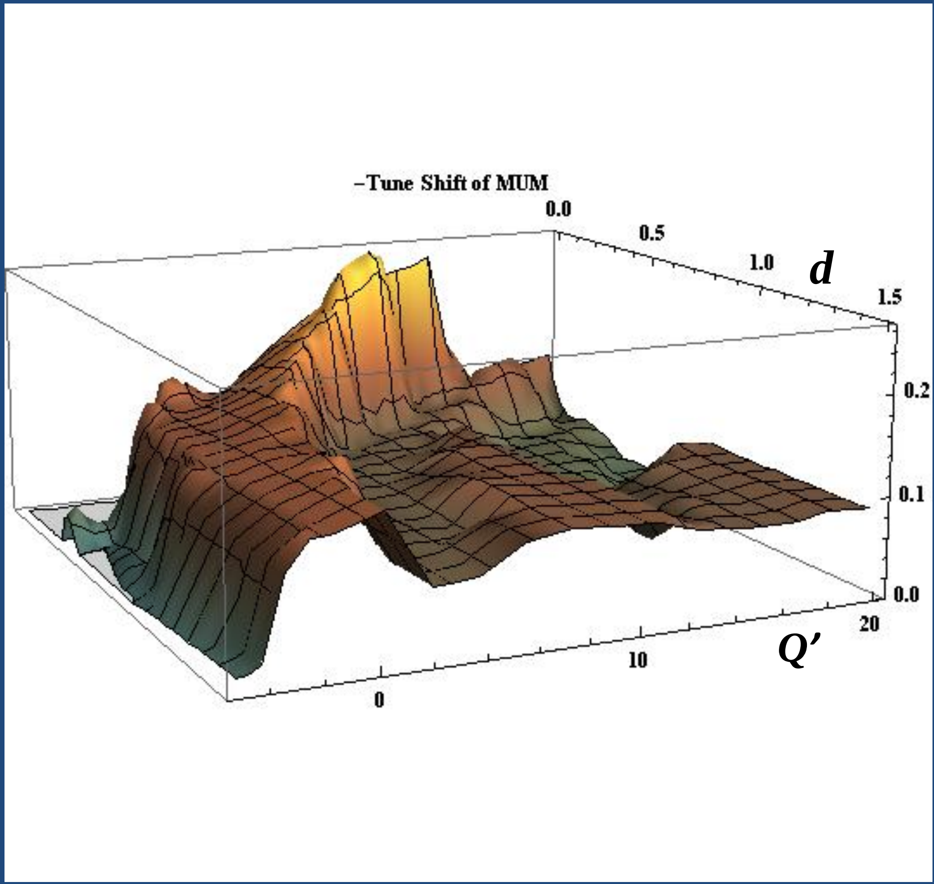
NHT vs BeamBeam3D tracking (S. White)



Highest growth rates for single bunch, gain=1.4 and nominal impedance

2⊗(SB and CB), flat ADT, MUM

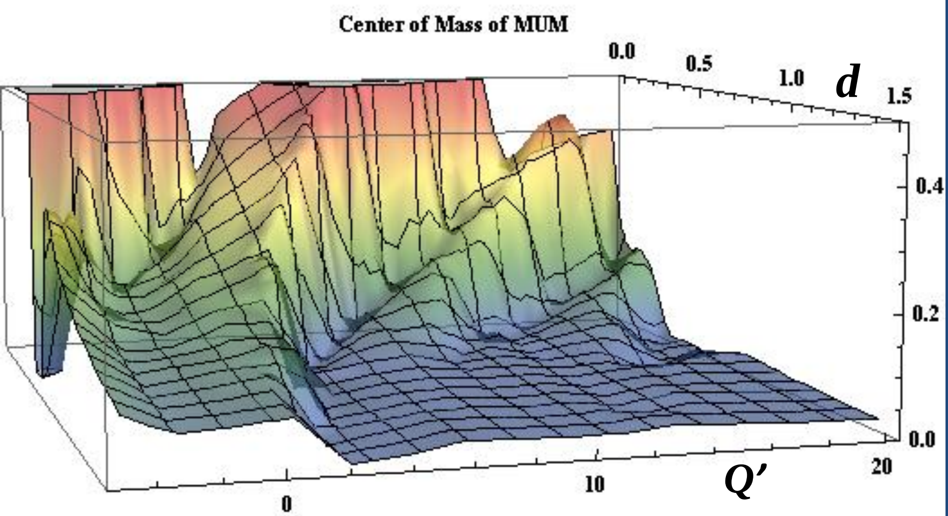


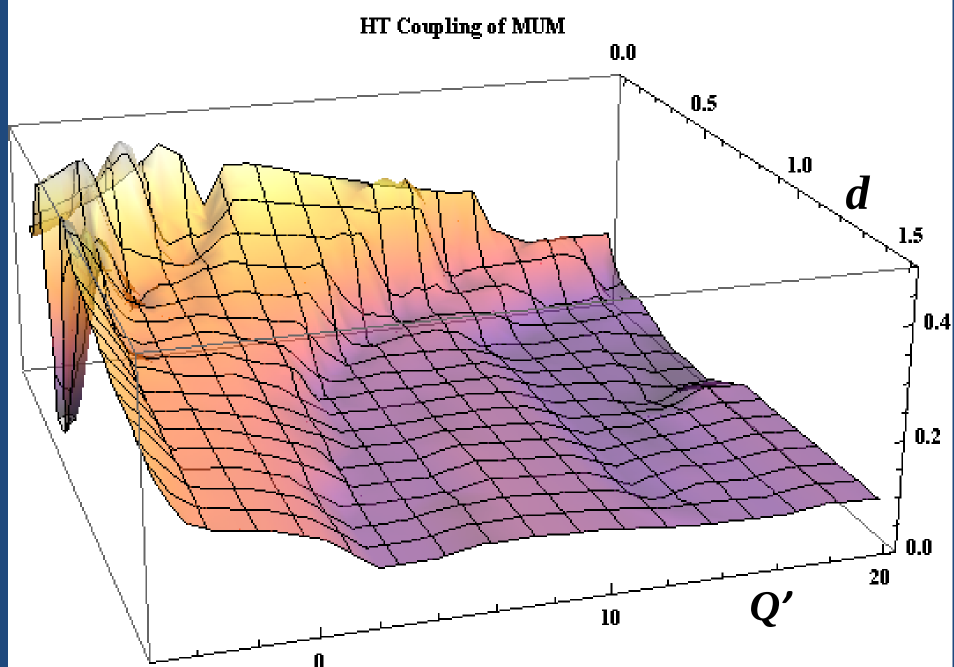


Growth rate and -tune shift of the most unstable mode (MUM) vs chroma and gain.
Both are in units of Qs.

Note that at the plateau the rate (Im[dQ_c]) is ~20-30 times smaller than the shift (Re[dQ_c]).

2⊗(SB and CB), flat ADT, MUM CM and Coupling





$$A_{l\alpha} = i^l J_l(\chi_\alpha) / \sqrt{n_r}; \quad \bar{x} = X \cdot A.$$

$$|X_l|^2 \equiv \sum_{\alpha=1}^{n_r} |X_{l\alpha}|^2; \quad \sum_l |X_l|^2 = 1;$$

$$l_m : |X_{l_m}|^2 = \max_l |X_l|^2; \quad \mathrm{HTC} = \sqrt{1 - |X_{lm}|^2}$$

Center of mass (CM) and head-tail coupling parameters for MUM.

Note strong suppression of CM at the plateau by the damper.
Note that at plateau the weak head-tail approximation is well-justified.

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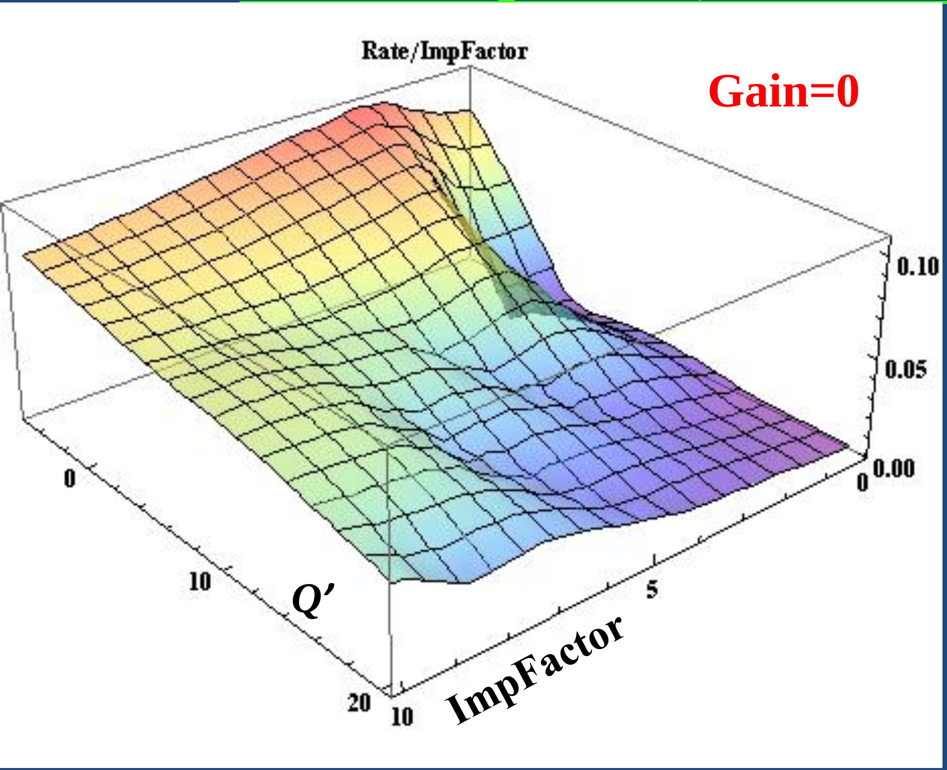
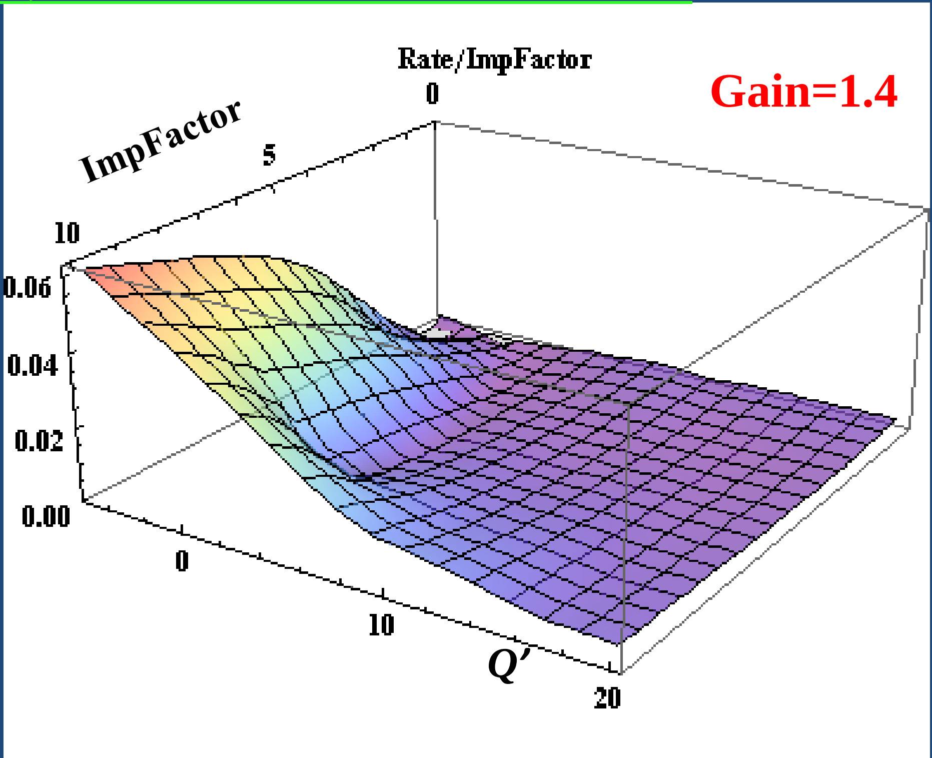
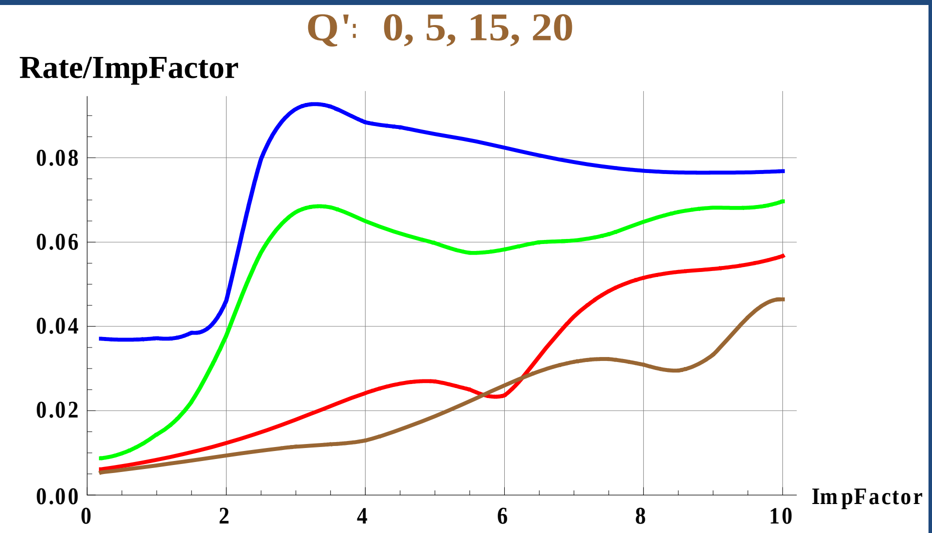
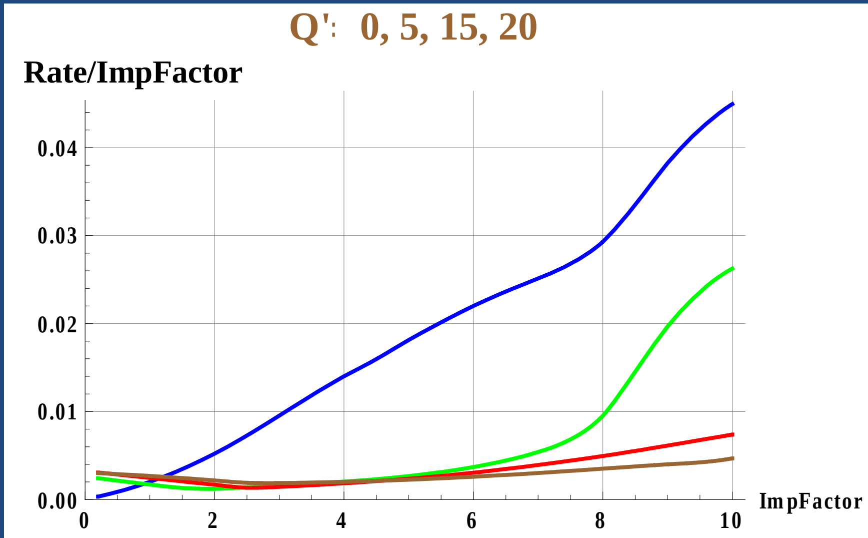
Intensity scan, flat ADT, MUM: where is TMCI?









Coherent Beam-Beam

Main assumption: bunch length << beta-function. For transversely dipolar modes, CBB is a cross-talk of bunch CM – thus, intra-bunch matrix structure is similar to the ADT and CB:

$$\dot{X}_1 = \hat{S}\cdot X_1 + \hat{Z}\cdot X_1 + \hat{D}\cdot X_1 + \hat{C}\cdot X_1 + b_{12}\hat{B}\cdot X_2;$$

$$\dot{X}_2 = \hat{S}\cdot X_2 + \hat{Z}\cdot X_2 + \hat{D}\cdot X_2 + \hat{C}\cdot X_2 + b_{21}\hat{B}\cdot X_1;$$

$$\hat{B} = -i\Delta\omega_{\mathrm{bb}}(\hat{D}/d_\mu)\left[1 + 2\frac{\rho_0^2}{\beta_0}\sum_{k=1}^{K}\frac{\beta_k}{\rho_k^2}\cos(k\phi_\mu)\right]/(2K+1);$$

$$b_{12} = b_{21}^* = 1 - \exp(-i\psi).$$

Here 2 identical opposite IRs are assumed (IR1 and IR5 for LHC) with $2K+1$ LR collisions for each, every one with its beta-function and separation β_k, ρ_k .

Alternating x/y collision for IR1/IR5 is assumed with ψ as a difference between the two phase advances, while $\Delta\omega_{\mathrm{bb}}$ is the incoherent beam-beam tune shift per IR.

Dispersion Equation

Let's consider a small fraction of the beam described by an NHT amplitude vector x_p :

$$\dot{x}_p = -i\delta\omega_p x_p + \hat{S}_p \cdot x_p + \hat{Z} \cdot X$$

$$\hat{Z} \cdot X = -i(\Omega_c \hat{I} - i\hat{S}) \cdot X$$

Due to the frequency spread eigenvalues are slightly changed, $\Omega_c \to \Omega$ but eigenvector at the first approximation are the same (similar to QM). From here

$$x_p = \left[(\Omega - \delta\omega_p)\hat{I} - i\hat{S}_p \right]^{-1} \left[\Omega_c \hat{I} - i\hat{S} \right] \cdot X$$

$$X = \left\langle \left[(\Omega - \delta\omega_p)\hat{I} - i\hat{S}_p \right]^{-1} \right\rangle \left[\Omega_c \hat{I} - i\hat{S} \right] \cdot X$$

$$1 = X^{\dagger} \cdot \left\langle \left[(\Omega - \delta\omega_p)\hat{I} - i\hat{S}_p \right]^{-1} \right\rangle \left[\Omega_c \hat{I} - i\hat{S} \right] \cdot X$$

$$1 = -\sum_l (\Omega_c - l\bar{\omega}_s) \int \frac{|X_l(J_s)|^2}{\Omega - l\omega_s - \delta\omega_x + io} \frac{J_x \partial F}{\partial J_x} d\Gamma$$

$$\int |X_l(J_s)|^2 F d\Gamma = \int F d\Gamma = 1$$

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Weak Head-Tail case

This derivation assumes frequency spread can be treated as a perturbation. This is justified when the resonant particles are at the tails of the distribution.

$$1 = -\sum_l (\Omega_c - l\bar{\omega}_s) \int \frac{|X_l(J_s)|^2}{\Omega - l\omega_s - \delta\omega_x + io} \frac{J_x \partial F}{\partial J_x} d\Gamma \,;$$

$$\int |X_l(J_s)|^2 \, F d\Gamma = \int F d\Gamma = 1$$

With the damper, weak HT approximation can be applied at many cases. If so (true for LHC), the DE is simplified:

$$1 = -(\Omega_c - l\bar{\omega}_s) \int \frac{|X_l(J_s)|^2}{\Omega - l\omega_s - \delta\omega_x + io} \frac{J_x \partial F}{\partial J_x} d\Gamma \,.$$

When the LD is provided by the far tails, the mode form-factor $|X_l|^2$ can be omitted with the logarithmic accuracy:

$$1 = -(\Omega_c - l\bar{\omega}_s) \int \frac{J_x \partial F / \partial J_x}{\Omega - l\omega_s - \delta\omega_x + io} d\Gamma \,.$$

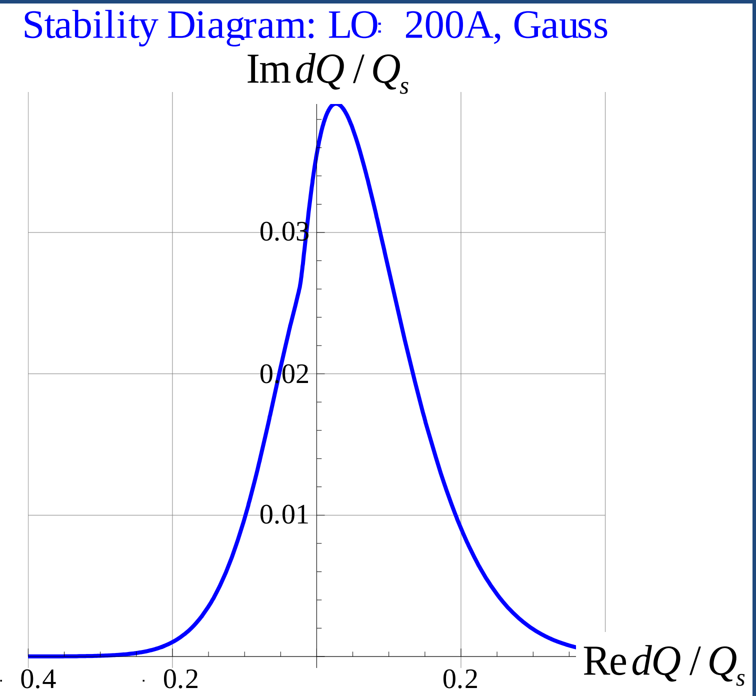
Stability Diagram

Stability diagram (SD) is defined as a map of real axes Ω on the complex plane:

$$D = \left(-\int \frac{J_x \partial F / \partial J_x}{\Omega - l\omega_s - \delta\omega_x + io} d\Gamma \right)^{-1}$$

$$D = \Omega_c - l\bar{\omega}_s$$

To be stable, the coherent tune shift has to be inside the SD.



For LHC, with Landau octupoles (LO):

(E.Metral, N.Mounet, B.Salvant, 2010)

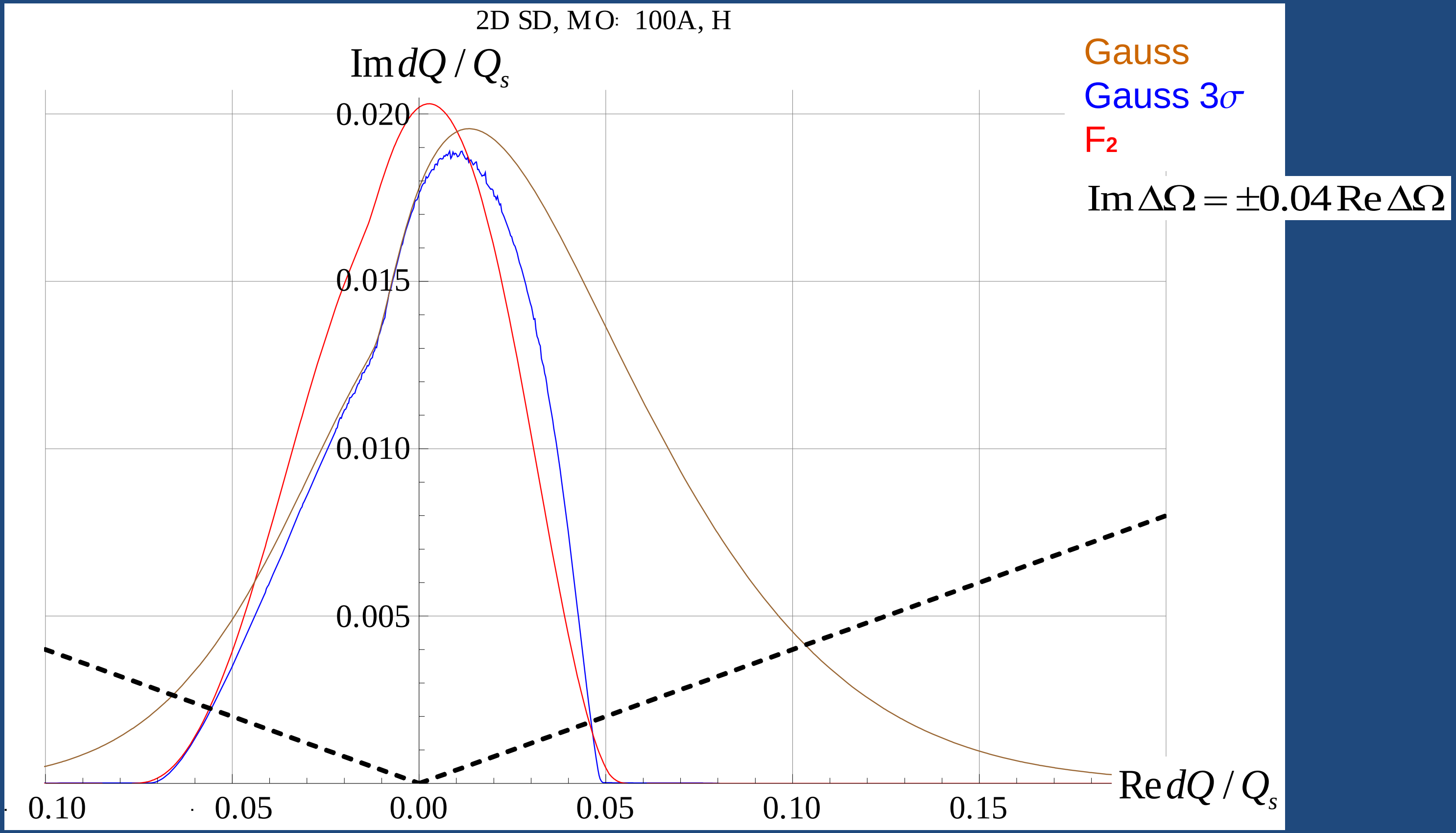
$$\Delta\mathbf{Q} \equiv \left(\Delta Q_x, \Delta Q_y\right)^{\mathrm{T}}; \quad \mathbf{J} \equiv (J_x, J_y)^{\mathrm{T}};$$

$$\Delta\mathbf{Q} = \hat{\mathbf{A}} \cdot \mathbf{J} / \varepsilon; \quad \hat{\mathbf{A}} = Q_s \frac{I_{\mathrm{LO}}}{100\mathrm{A}} \begin{pmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{pmatrix};$$

$$a_{xx} = a_{yy} = 1.8 \cdot 10^{-2} \varepsilon / (2\mu\mathrm{m});$$

$$a_{xy} = a_{yx} = -1.3 \cdot 10^{-2} \varepsilon / (2\mu\mathrm{m});$$

Stability Diagrams

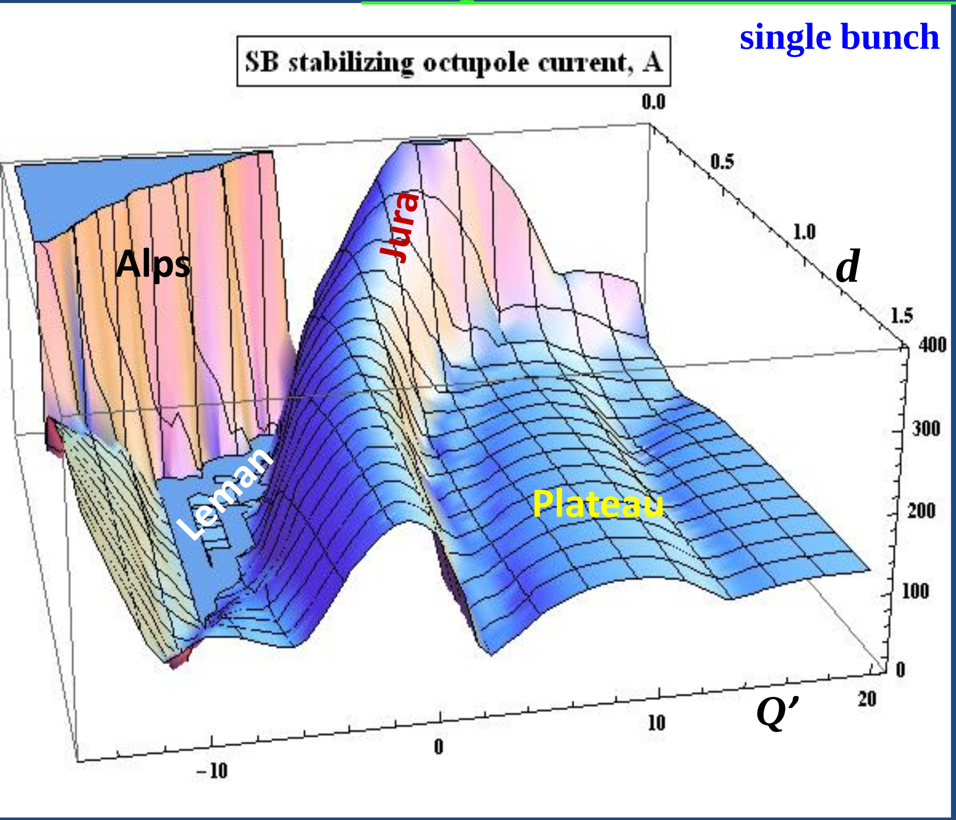


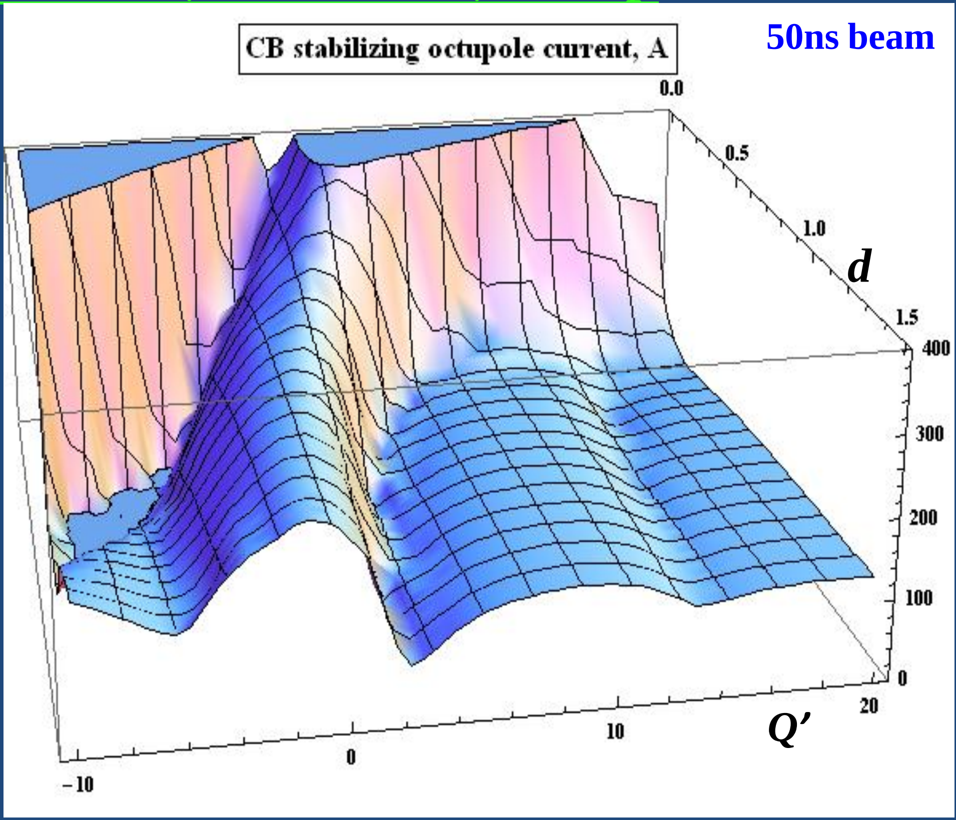
LHC stability diagrams for both emittances $2\mu\mathrm{m}$ and 100A of the octupole current.

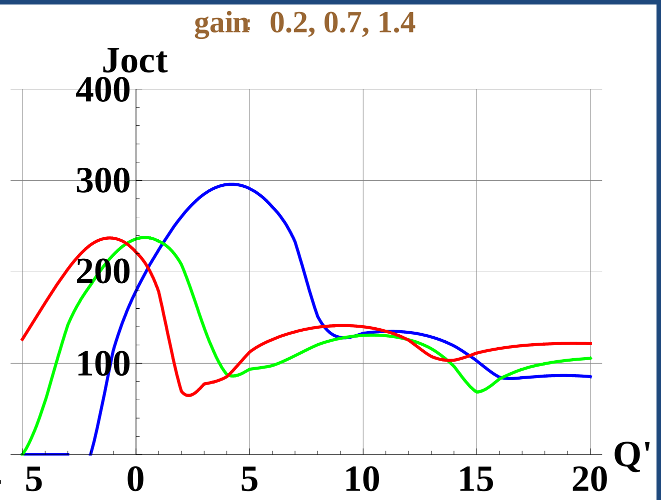
$$F_n(J_x, J_y) = a_n\left(1 - \frac{J_x + J_y}{b_n}\right)^n; \ \int F_n(J_x, J_y)\,dJ_x dJ_y = 1; \ \int J_x F_n(J_x, J_y)\,dJ_x dJ_y = 1.$$

see more on SD with F_n at E. Metral & A. Verdier, 2004

Couple Bunch Factor: LO+, bbb ADT, 2Imp

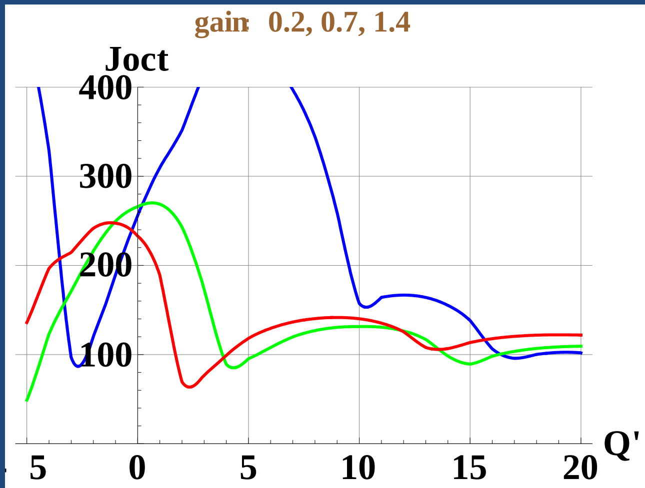




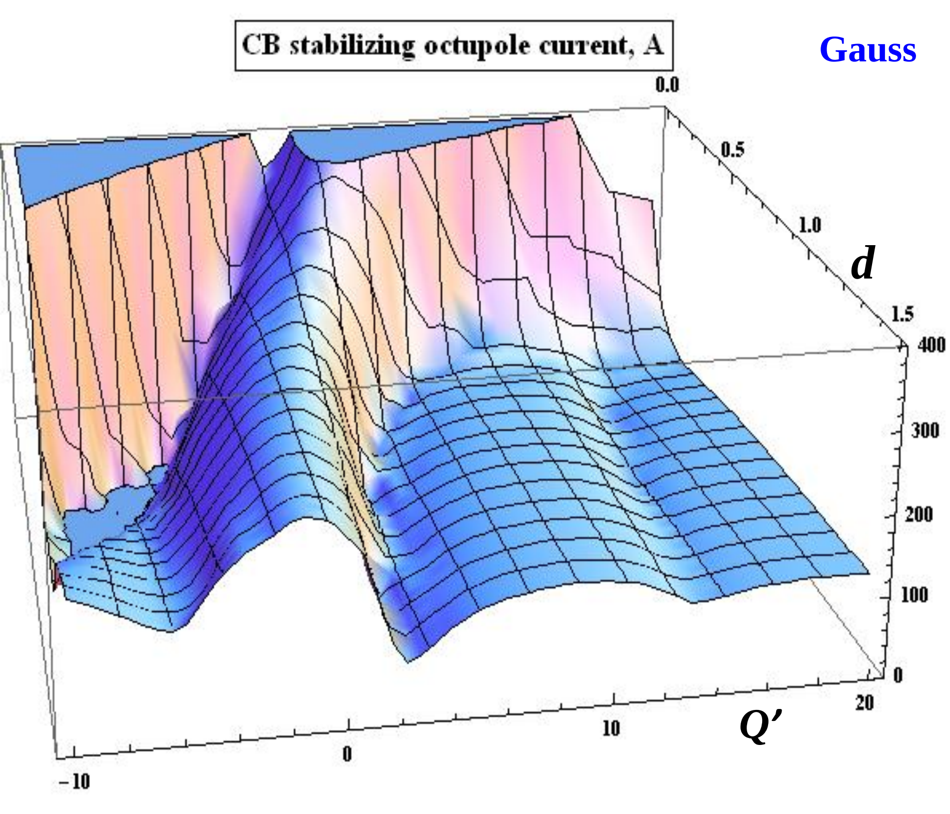


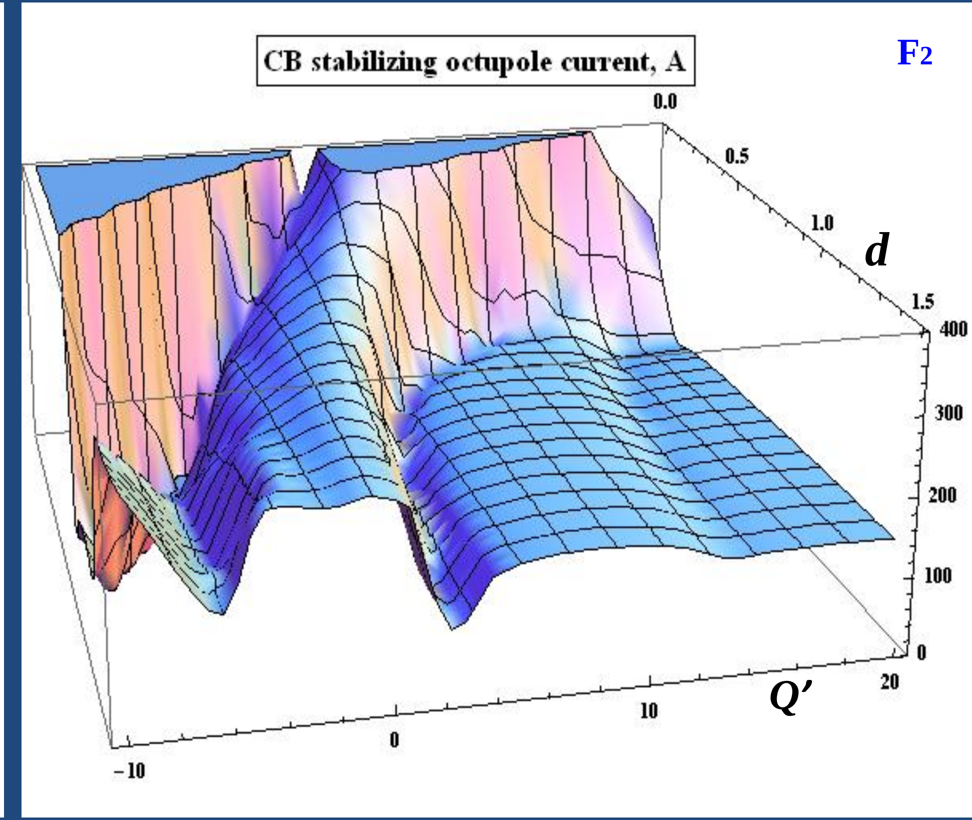
Almost no difference at Plateau.

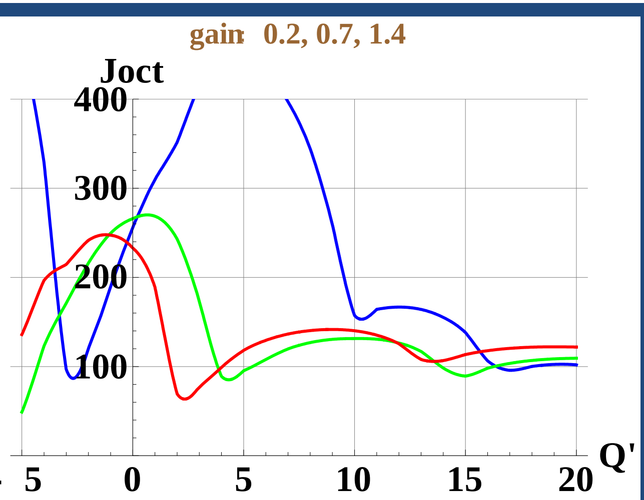
All the plots – for 1.5E11 p/b, emittances of 2μm, 50ns beam.



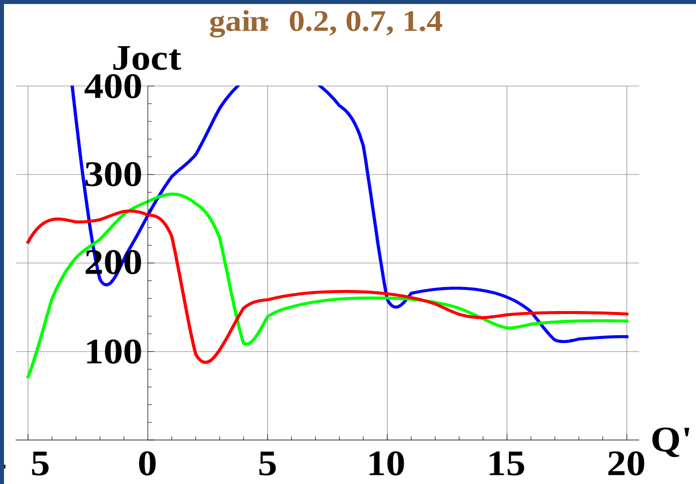
Tails Factor: LO+, CB, bbb ADT, 2Imp



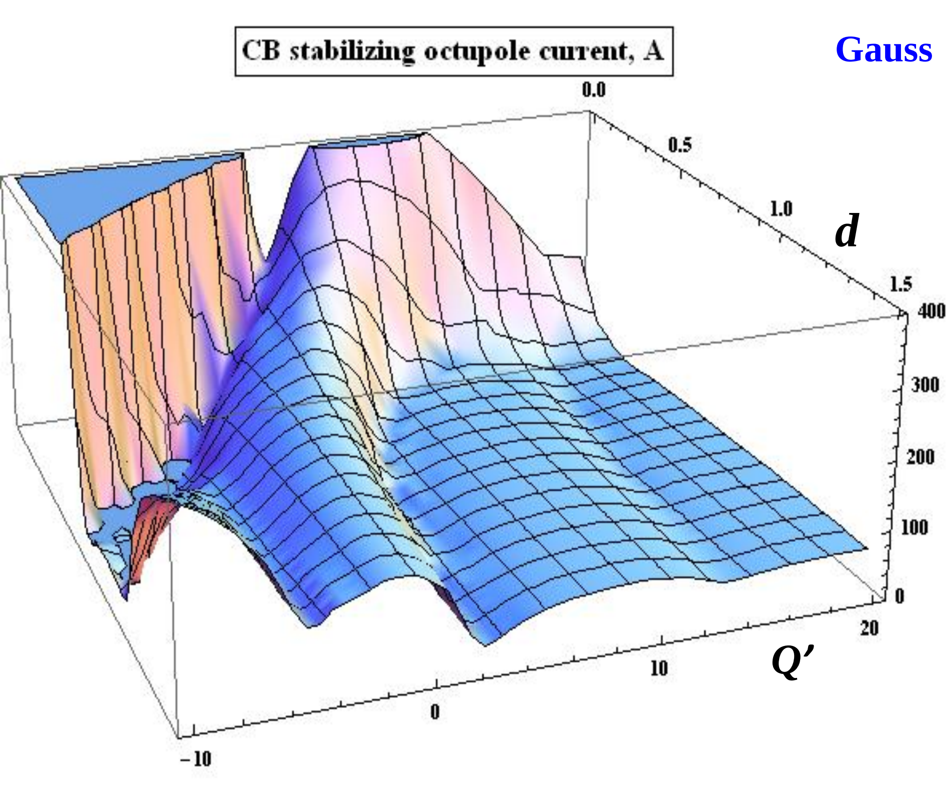


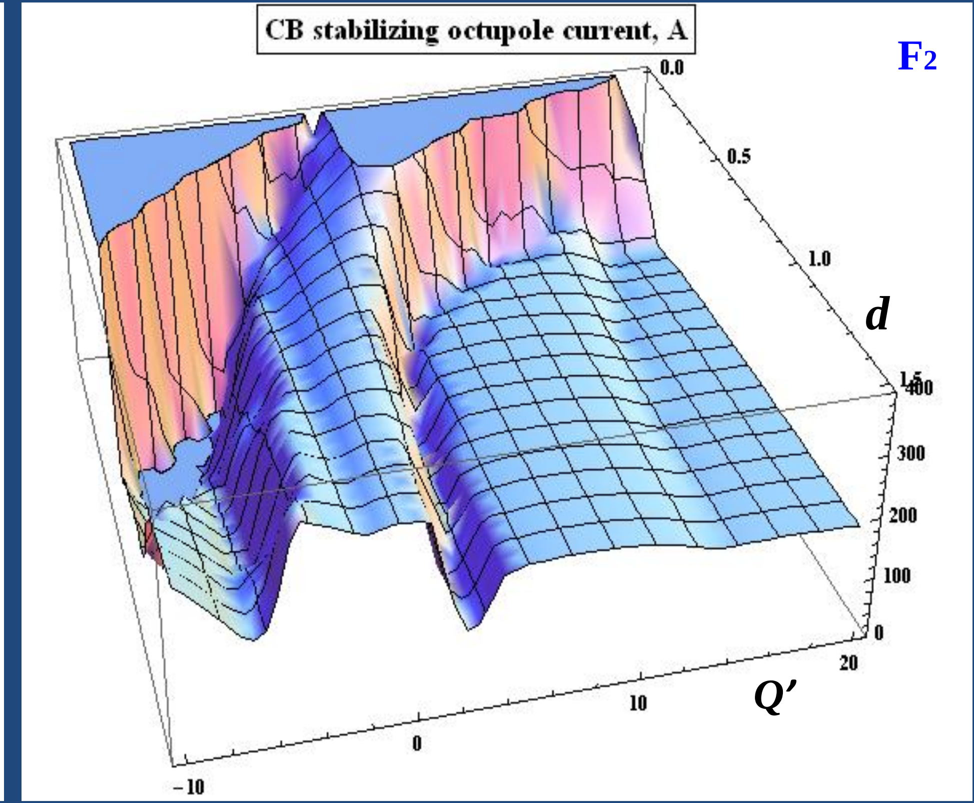


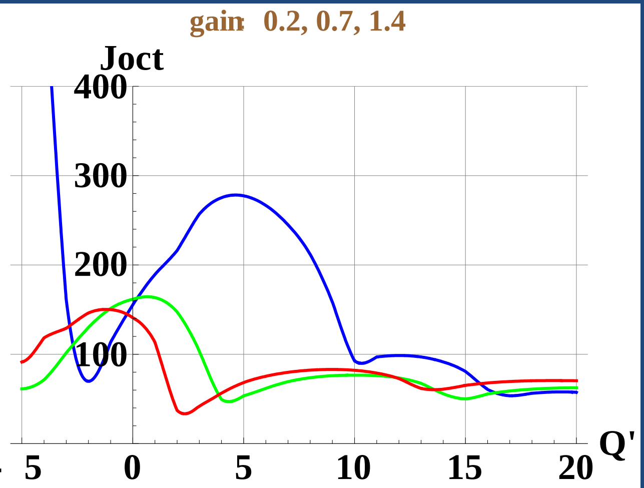
Almost no difference at this polarity.



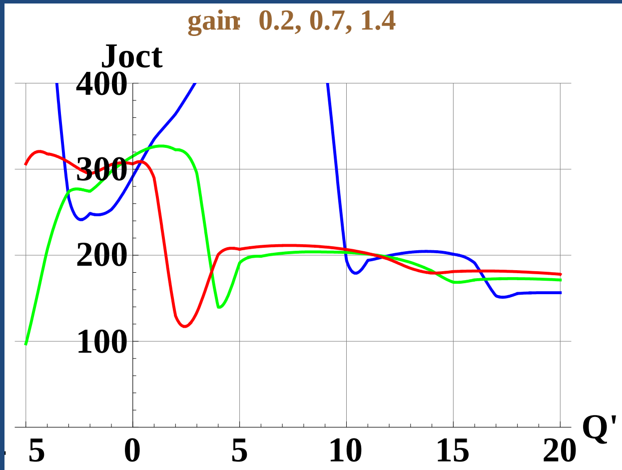
Tails Factor: LO-, bbb ADT, 2Imp



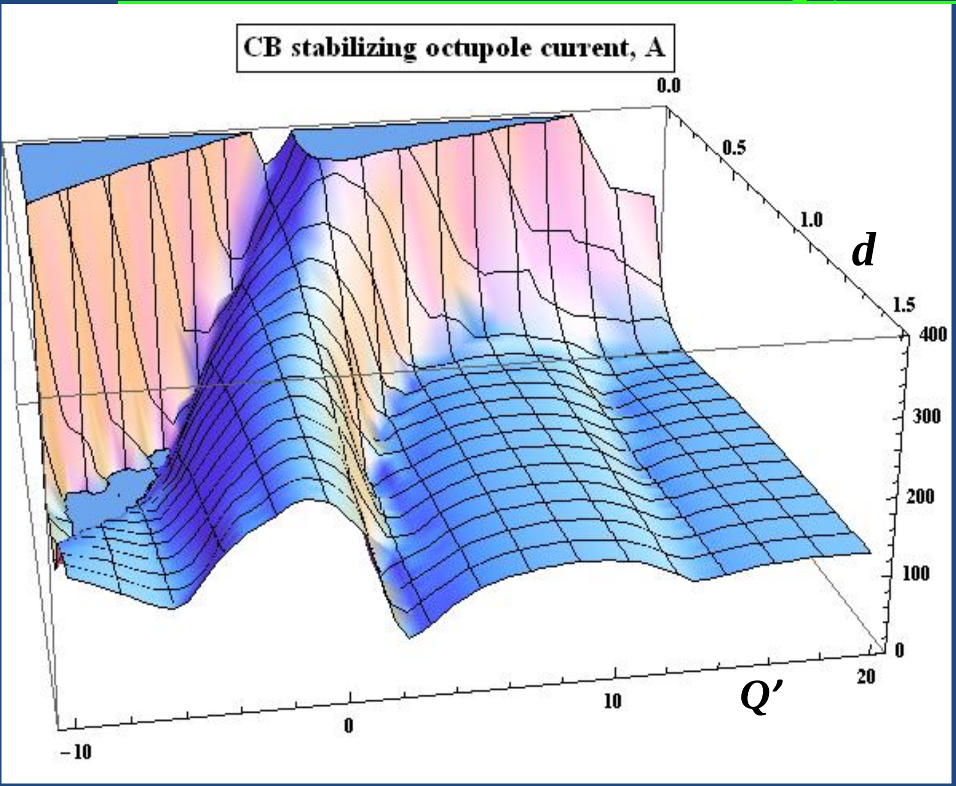


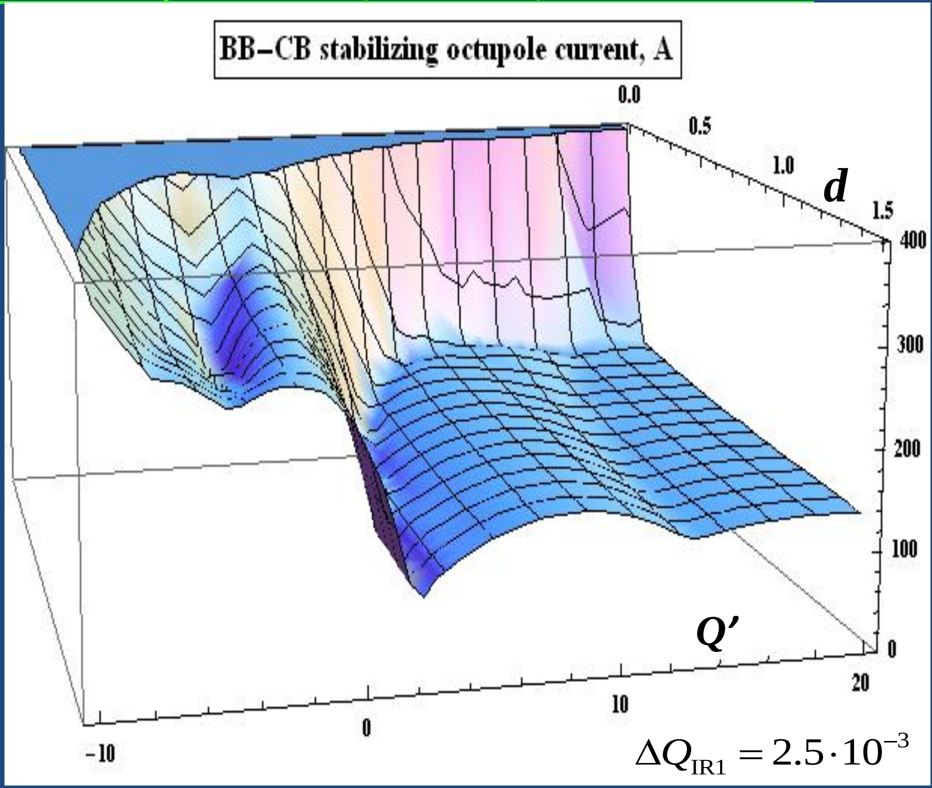


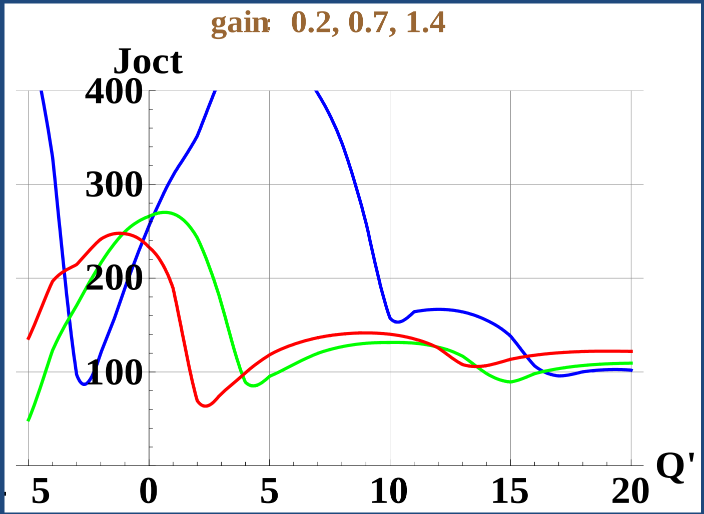
About a factor of 3 difference at the plateau!



Beam-Beam Factor: 2Imp, CB, CBB $\psi=\pi/2$, L0+, bbb ADT

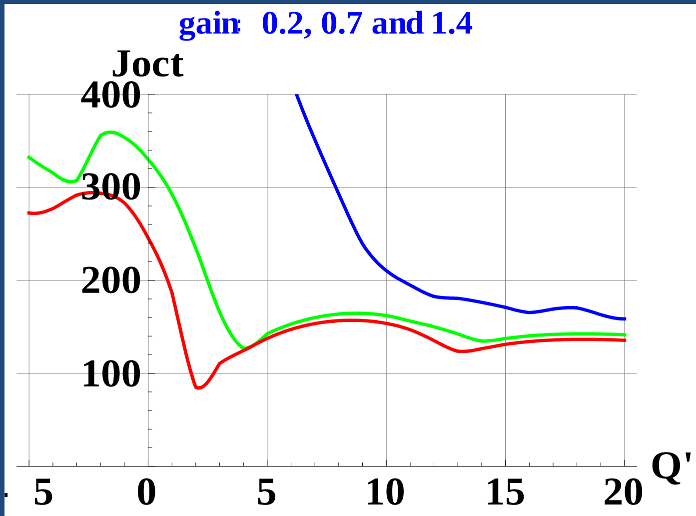




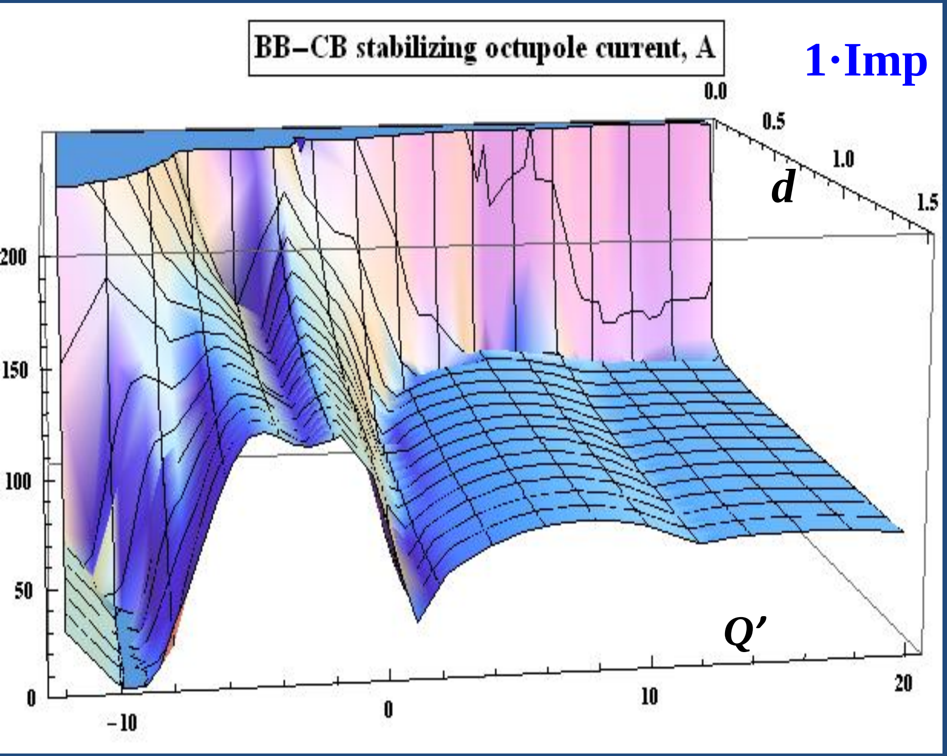


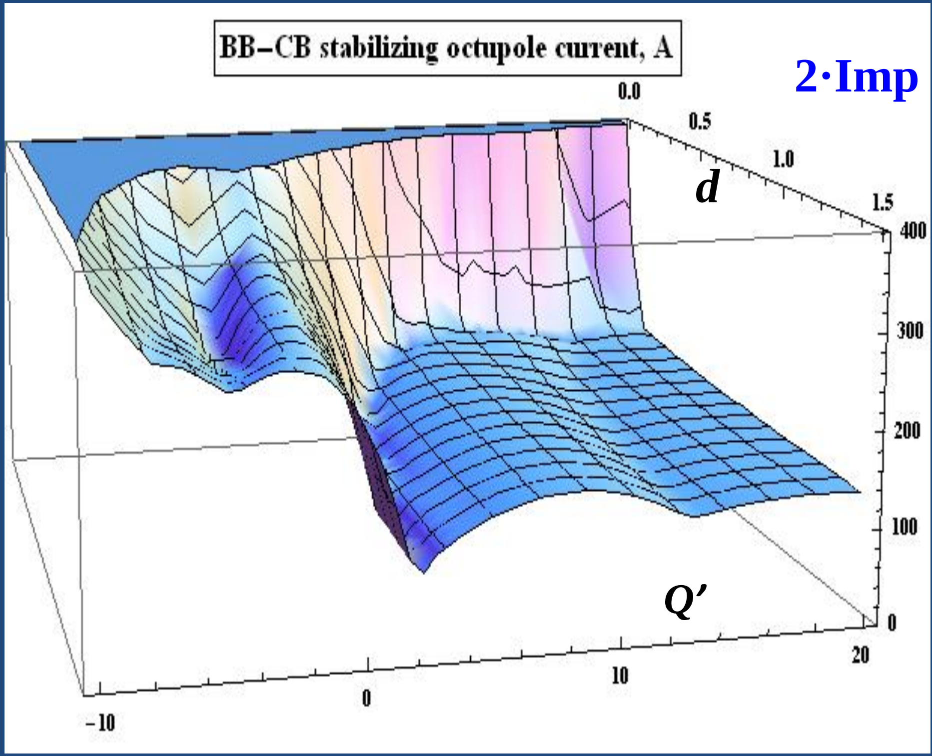
Almost no difference at the plateau:

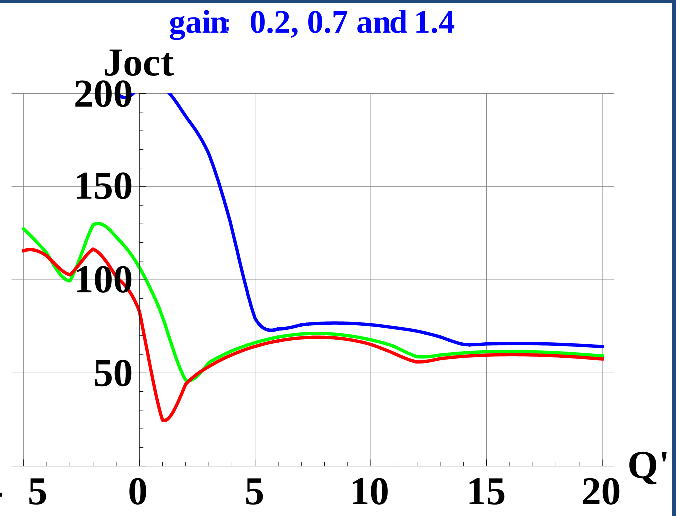
No CBB at Plateau.



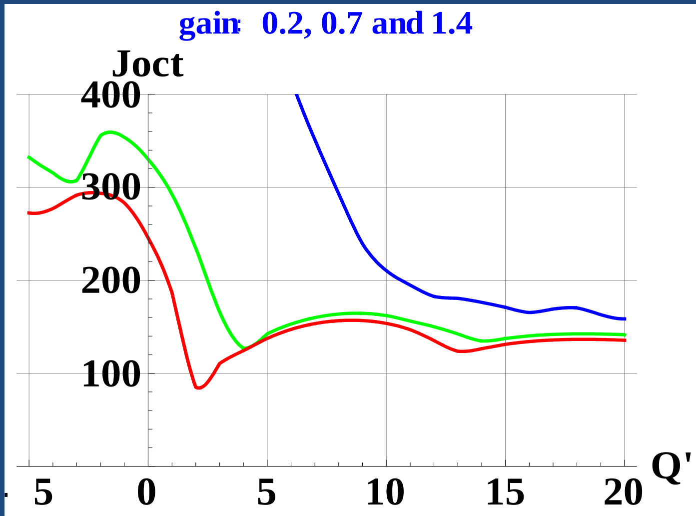
Impedance Factor: CB, CBB $\psi=\pi/2$, LO+, bbb ADT







At the Plateau it scales ~ linearly



Long-Range Beam-Beam Tune Spread

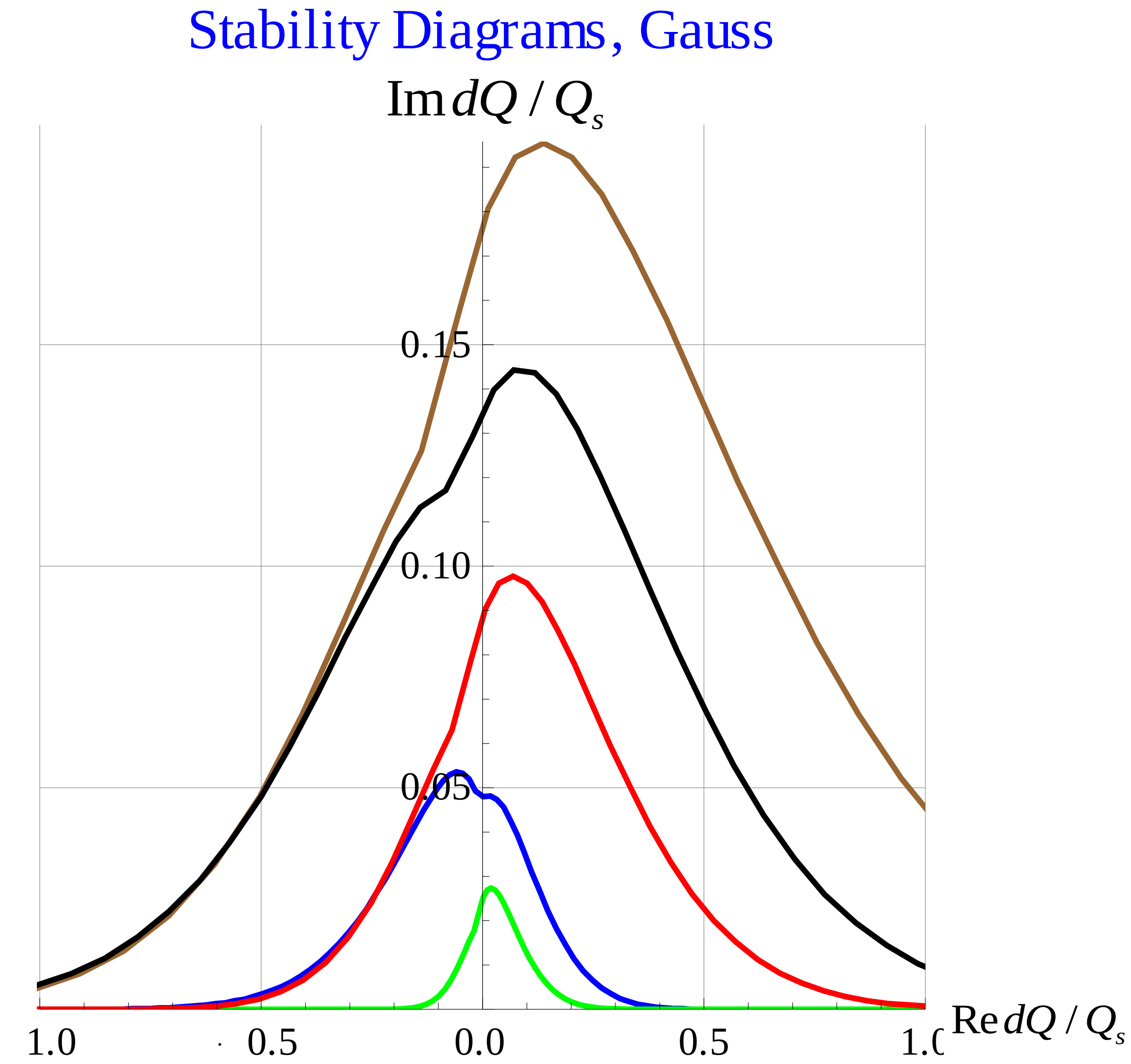
- For the alternating x/y IR1/IR5 collision scheme, the octupolar LR tune spread is

$$\Delta Q_{4x} = \frac{3\Delta Q_{\mathrm{bb}}}{r^2}\frac{J_x - 2J_y}{\varepsilon}.$$

Here ΔQ_{bb} is the linear LR bb tune shift per IR, $r \gg 1$ is beam separation in units of their rms size at that point. Round betas are assumed.

- For LHC at the end of the squeeze $\Delta Q_{\mathrm{bb}} = 2.5 \cdot 10^{-3}$, $r = 9.5$.

Stability Diagrams with Long Range Beam-Beam



LO=140A – computed threshold

BB only, LO=0

LO=500A, no BB

BB and LO=500A

LO=1000A, no BB

At the end of the squeeze and LO+, BB is equivalent to +500A of LO.

For the black curve, where we are now, we must be very stable, being 7 times in effective LO above the threshold.

However, we are unstable!
A big beast is still overlooked…

E-cloud in the IRs? Big drift of Q'? …? Any idea can be checked with NHT.

Summary: power of the model

- Method of nested head-tail modes (NHT) is implemented on a base of Mathematica. It allows to find coherent tunes for all the modes, solving the eigenproblem at its 4D set:

  azimuthal ⊗ radial ⊗ coupled-bunch ⊗ beam-beam.

- The external data: impedance/wake, ADT frequency profile, distribution functions and nonlinearities, beam-beam scheme.

- Based on that, all the coherent modes with all the details are computed.

- The LO parameter scan, with 5 radial, 21 azimuthal and 15 representative CB modes for 40 chromaticity points and 15 gain points takes only 15 min on my 3 years old laptop.

- With all the new features it is not going to take much longer.

AB

Next steps

- To include longitudinal plane into SD.
- To include train structure.
- To include detuning wakes/impedances.
- To make all that user friendly and public.

However powerful are our models – they are nothing but tools to see consequences of our ideas. Models cannot have more ideas than we put into them.

Many thanks for your attention!