Nested Head Tail Vlasov Solver:

Impedance, Damper, Radial Modes, Coupled Bunches, Beam-Beam and more…

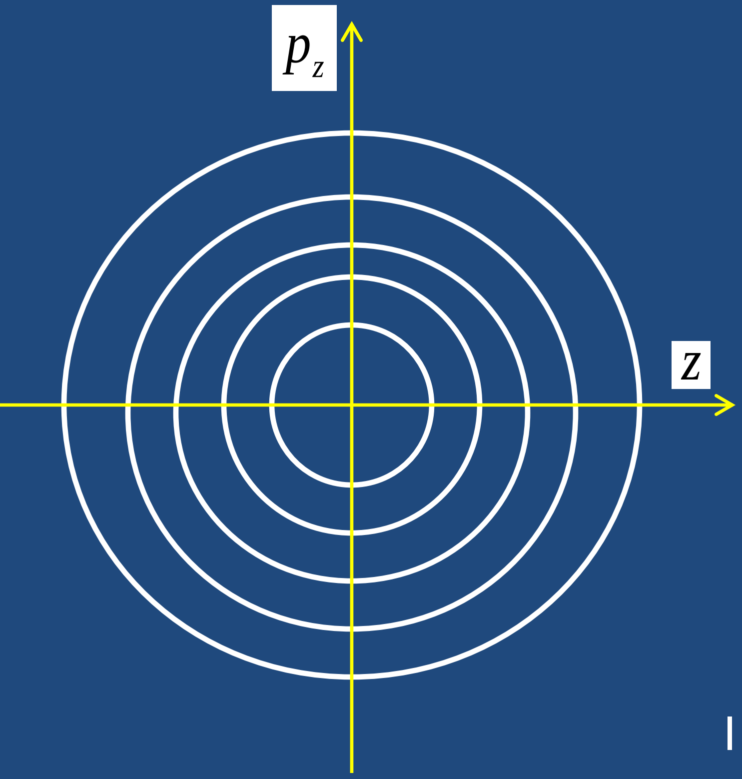
A. Burov

Fermilab-LARP

special thanks - to V.Danilov, E.Metral, N.Mounet, S.White, X.Buffat, and T.Pieloni

CERN, ICE Nov 21, 2012

Nested Head-Tail Basis



$$\psi_{l\alpha} \propto \exp(il\phi + i\chi_\alpha \cos\phi - i\Omega_l t)\ ;$$

$$\chi_\alpha = \frac{Q'\omega_0 r_\alpha}{c\eta}\ ;$$

$$\Omega_l = \omega_b + l\omega_s\ .$$

I am using n_r equally populated rings which radii r_α are chosen to reflect the phase space density.

AB

Starting Equation, single bunch

- In the air-bag single bunch approximation, beam equations of motion can be presented as in Ref [A. Chao, Eq. 6.183]:

$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X$$

where X is a vector of the HT mode amplitudes,

$$(\hat{S} + \hat{Z})_{lm\alpha\beta} = -il\delta_{lm}\delta_{\alpha\beta} - i^{l-m}\frac{\kappa}{n_r}\int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau_\alpha - \chi_\alpha) J_m(\omega\tau_\beta - \chi_\beta)$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l}\frac{d}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta)$$

d is the damper gain in units of the damping rate, $\kappa = \frac{N_b r_0 R_0}{8\pi^2 \gamma Q_b Q_s}$

time is in units of the angular synchrotron frequency.

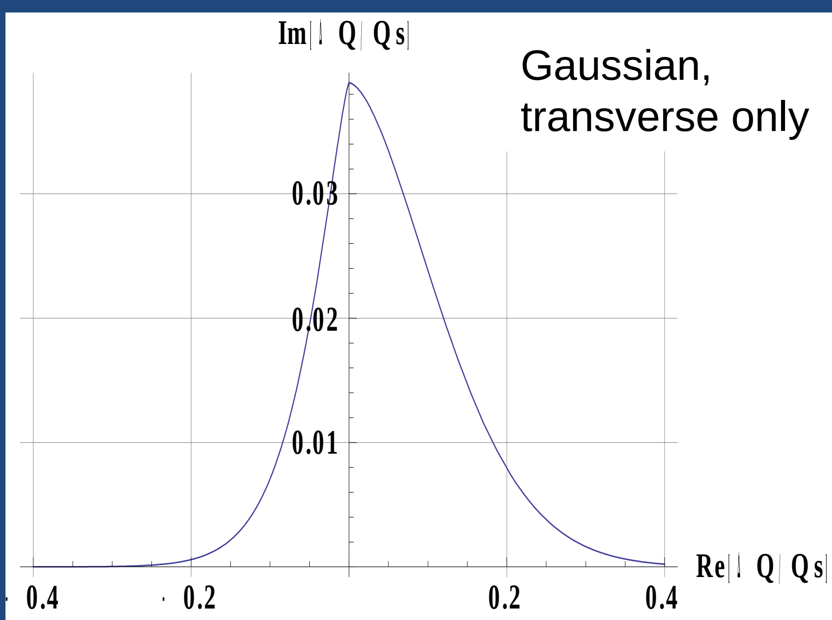
Analysis of solutions

1. For every given gain and chromaticity, the eigensystem is found for the LHC impedance table (N. Mounet).

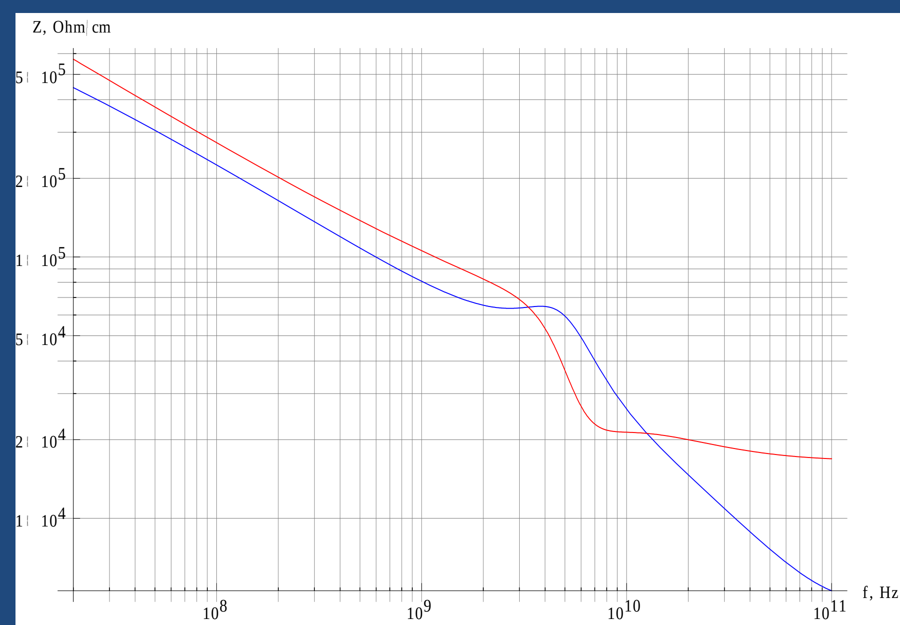
2. The complex tune shifts are found from the eigenvalues $\Delta\Omega_{l\alpha} = \Omega_{l\alpha} - l$

3. The stabilizing octupole current is found from the stability diagram for every mode, then max is taken.

Stability diagram at +200 A of octupoles



Impedances



Coupled Equidistant Bunches

Main idea:

For LHC, wake field of preceding bunches can be taken as flat within the bunch length.

The only difference between the bunches is CB mode phase advance, otherwise they are all identical.

Thus, the CB kick felt by any bunch is proportional to its own offset, so the CB matrix $\hat{C}$ has the same structure as the damper matrix $\hat{D}$:

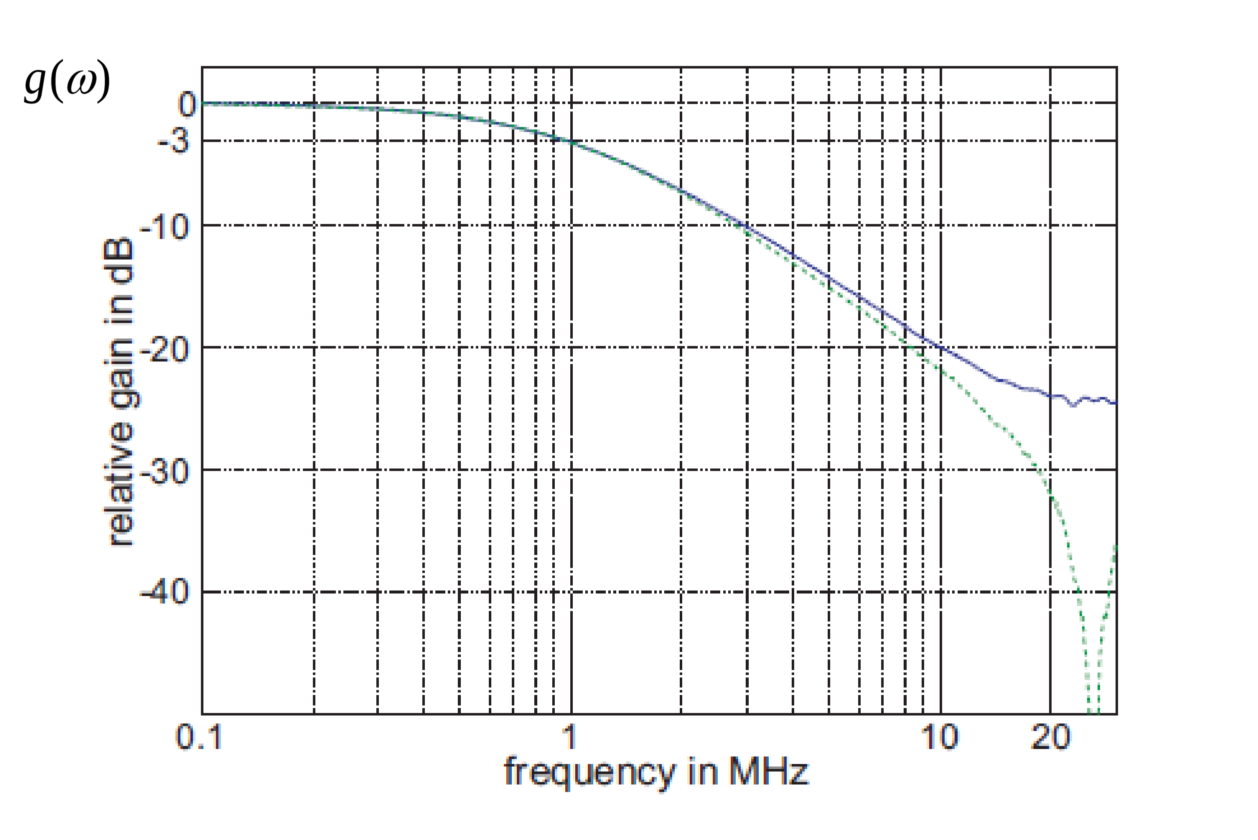
$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X + \hat{C} \cdot X;$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l} \frac{d_\mu}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta); \qquad \hat{C} = 2\pi i \kappa W(\varphi_\mu) \hat{D} / d_\mu;$$

$$W(\varphi_\mu) = \sum_{k=1}^{\infty} W(-ks_0) \exp(-ik\varphi_\mu); \qquad \varphi_\mu = 2\pi(1 - \{Q_x\}) + \frac{2\pi\mu}{M_b}; \qquad |\mu| \le \frac{M_b}{2}.$$

Wake and impedance are determined according to A. Chao book.

Old damper gain



Old narrow-band ADT gain profile (W. Hofle, D. Valuch) .
At 10 MHz it drops 10 times. The new damper is bbb for 50ns beam.

Below gain is measured in omega_s units, max gain=1.4 is equivalent to 50 turns of the damping time.

CB Mode Damping Rate

With $g(\omega)$ as the frequency response function of the previous plot, the time-domain damper's "wake" is

$$G(\tau) = \int_0^\infty g(\omega)\cos(\omega\tau)\, d\omega / \pi \, ,$$

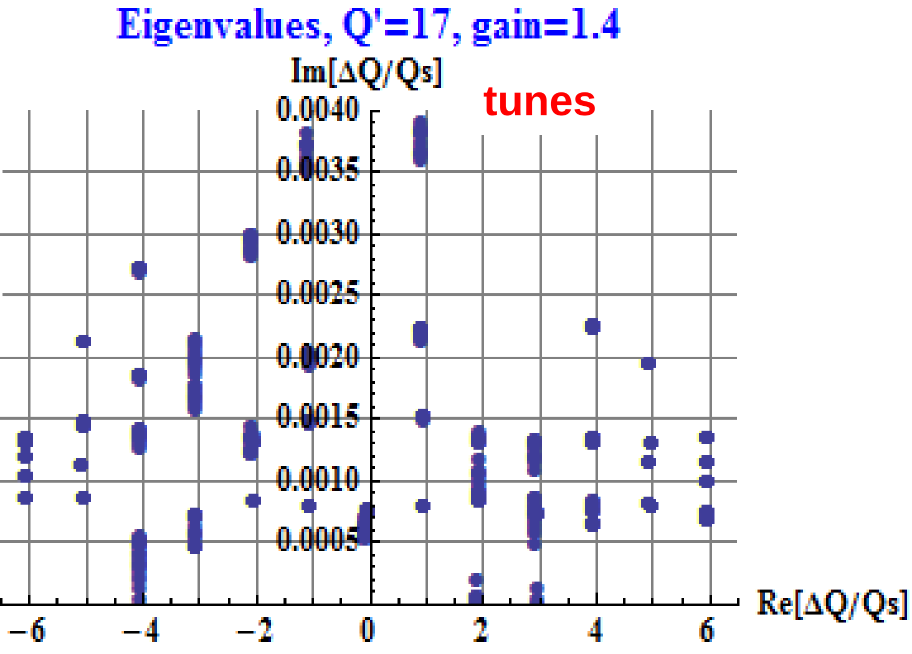
assuming this response to be even function of time (no causality for the damper!).

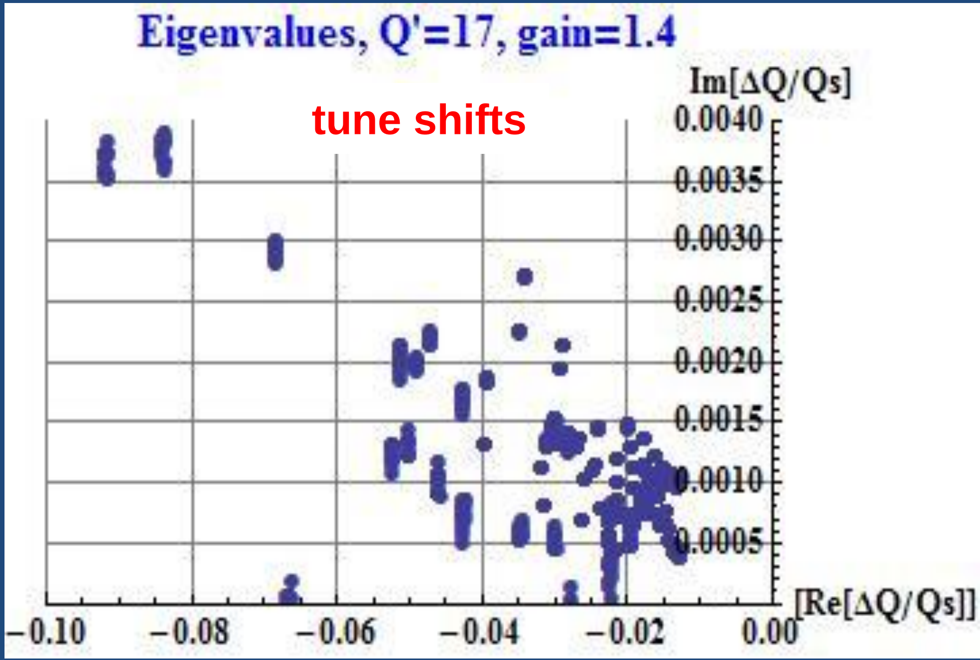
From here (equidistant bunches!):

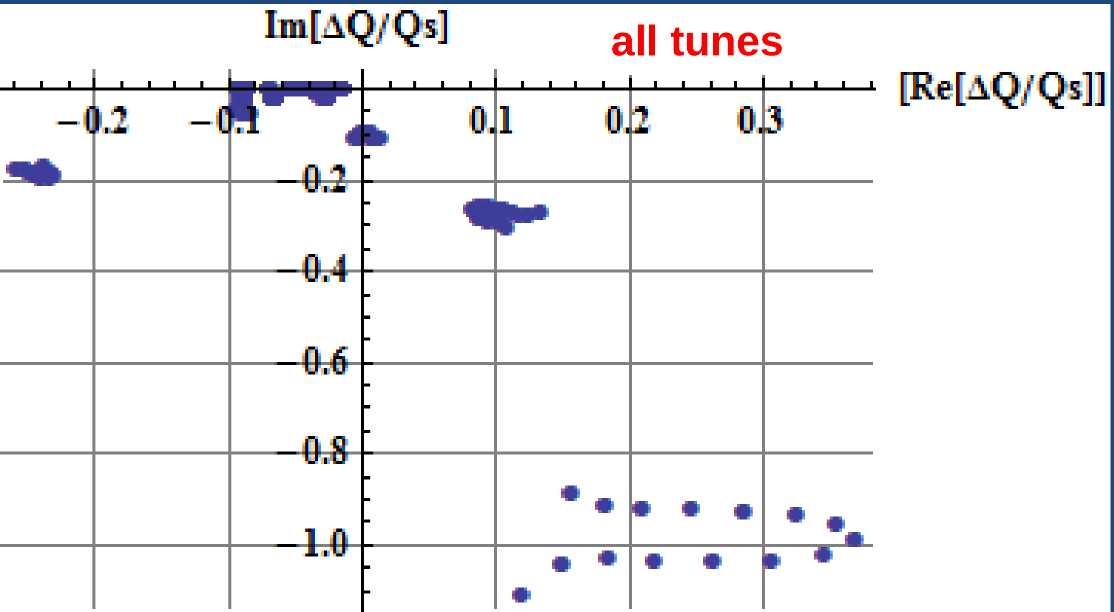
$$d_\mu = d \, \frac{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)\cos(k\varphi_\mu)}{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)} ;$$

where d is the rate provided for low-frequency CB zero-head-tail modes at zero chromaticity.

2⊗(SB and CB), flat ADT, Tunes at the Plateau



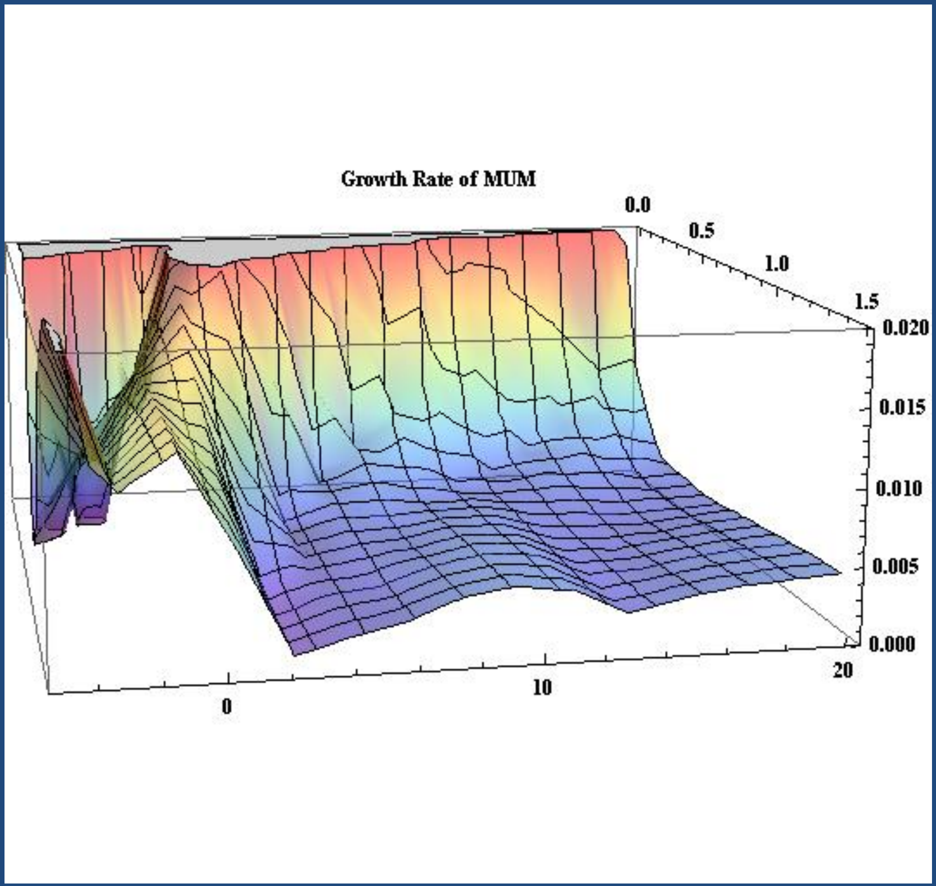


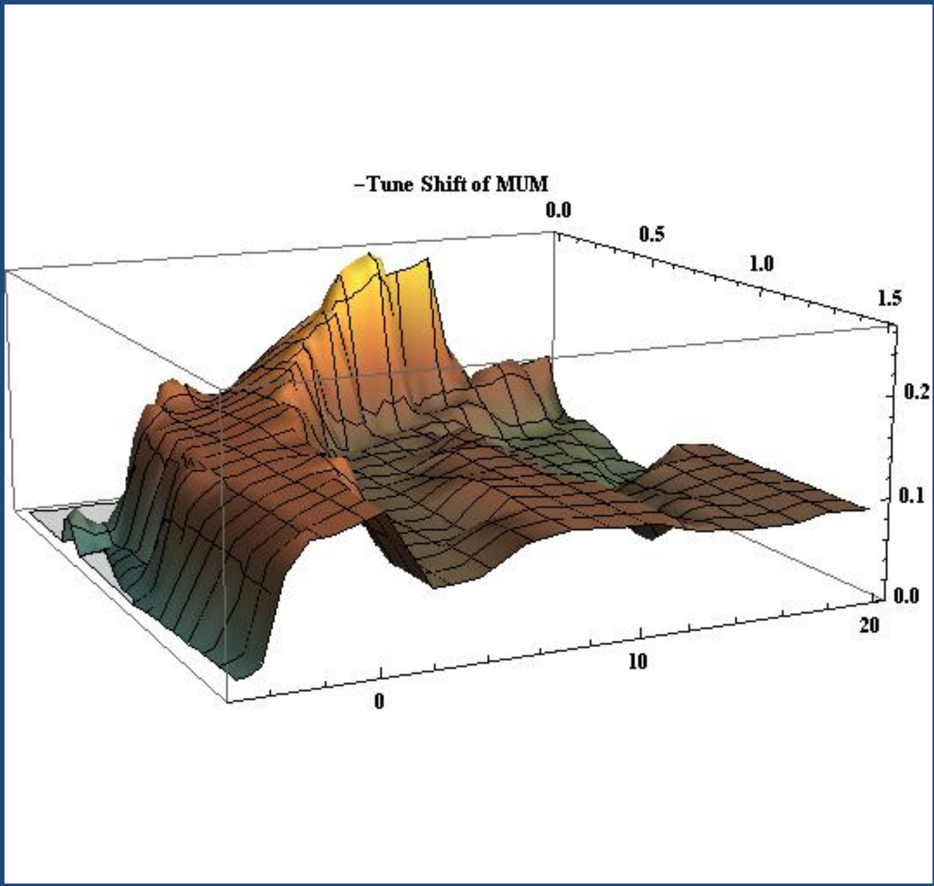


- All unstables -0.1<Re[dQ/Qs]<0.
- Weak head-tail is justified at the plateau.
- Mode with max rate (MUM) has ~max tune shift as well.
- For unstables -Re[dQ]/Im[dQ]~20-30.

AB

2⊗(SB and CB), flat ADT, MUM

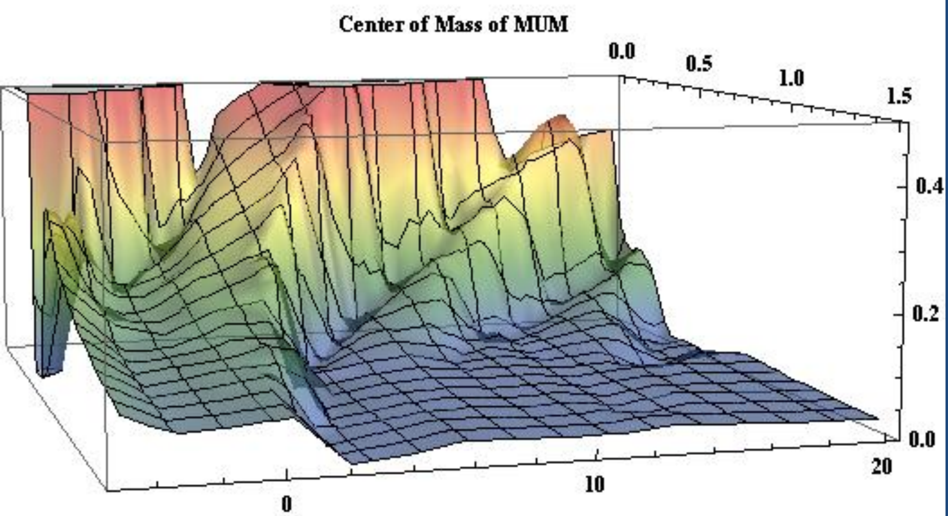




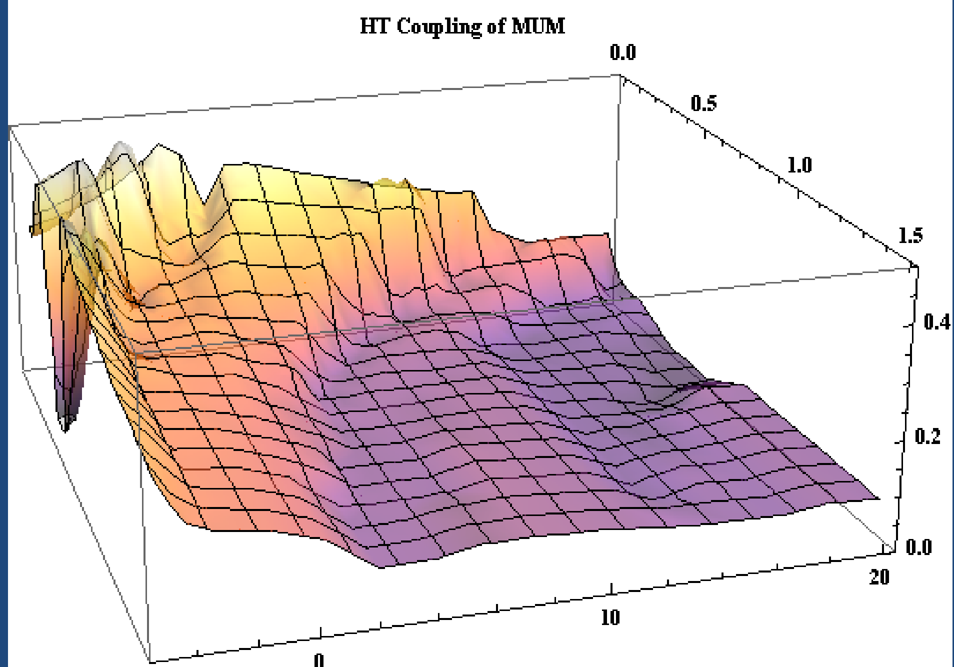
Growth rate and -tune shift of the most unstable mode (MUM) vs chroma and gain.
Both are in units of Qs.

Note that at the plateau the rate (Im[dQ_c]) is ~20-30 times smaller than the shift (Re[dQ_c]).

2⊗(SB and CB), flat ADT, MUM CM and Coupling



$$A_{l\alpha} = i^l J_l(\chi_\alpha)/\sqrt{n_r}; \quad \bar{x} = X \cdot A.$$



$$|X_l|^2 \equiv \sum_{\alpha=1}^{n_r} |X_{l\alpha}|^2; \quad \sum_l |X_l|^2 = 1;$$

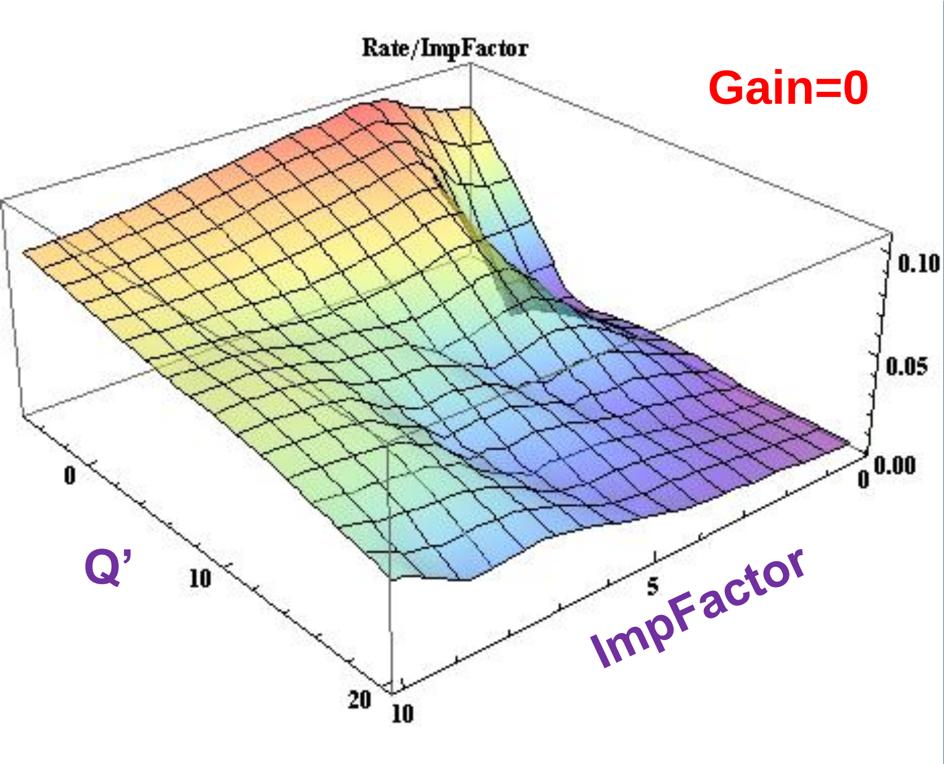
$$l_m : |X_{l_m}|^2 = \max_l |X_l|^2; \quad \mathrm{HTC} = \sqrt{1 - |X_{lm}|^2}$$

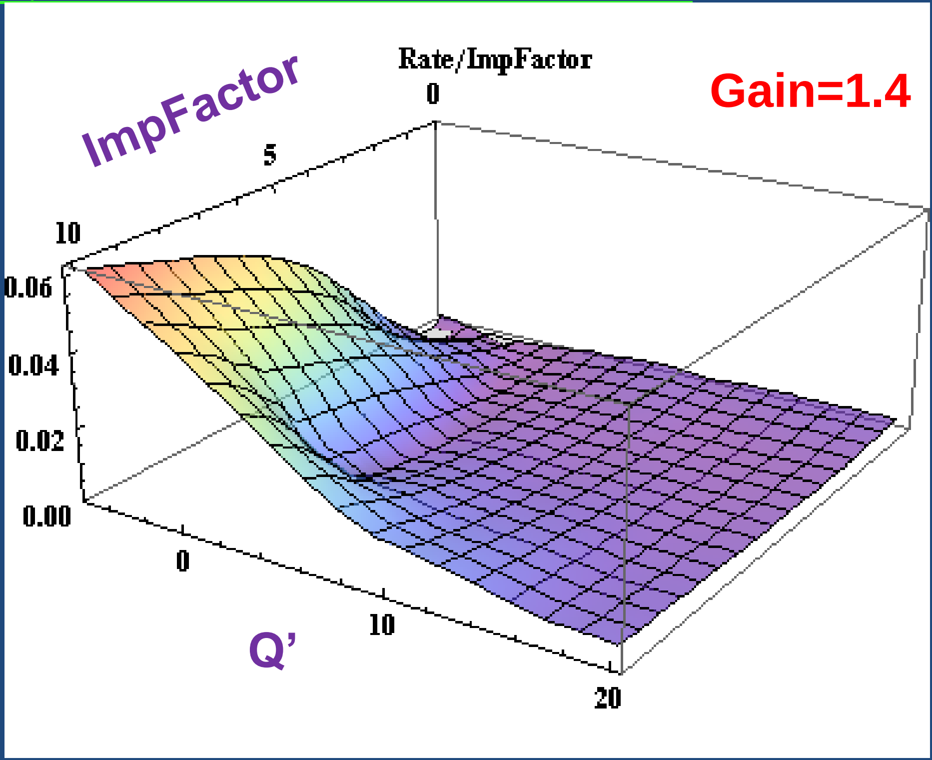
Center of mass (CM) and head-tail coupling parameters for MUM.

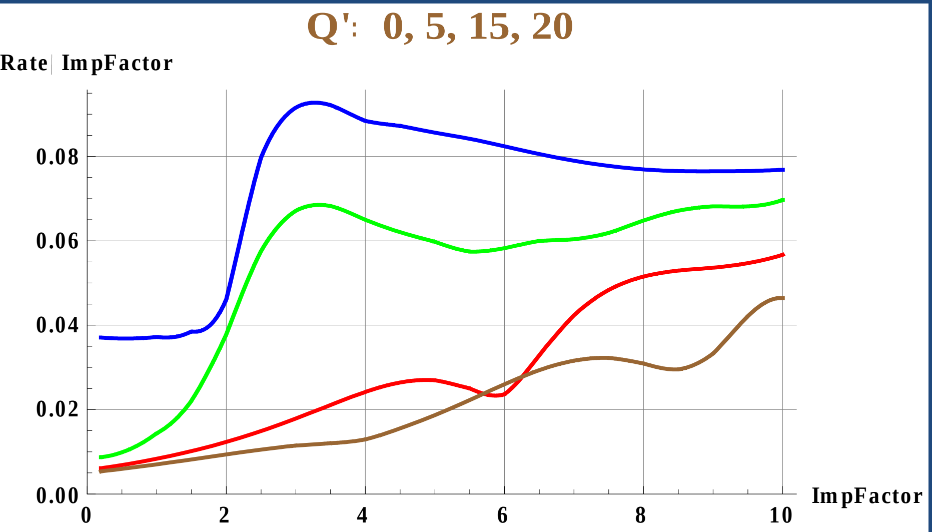
Note strong suppression of CM at the plateau by the damper.
Note that at plateau the weak head-tail approximation is well-justified.

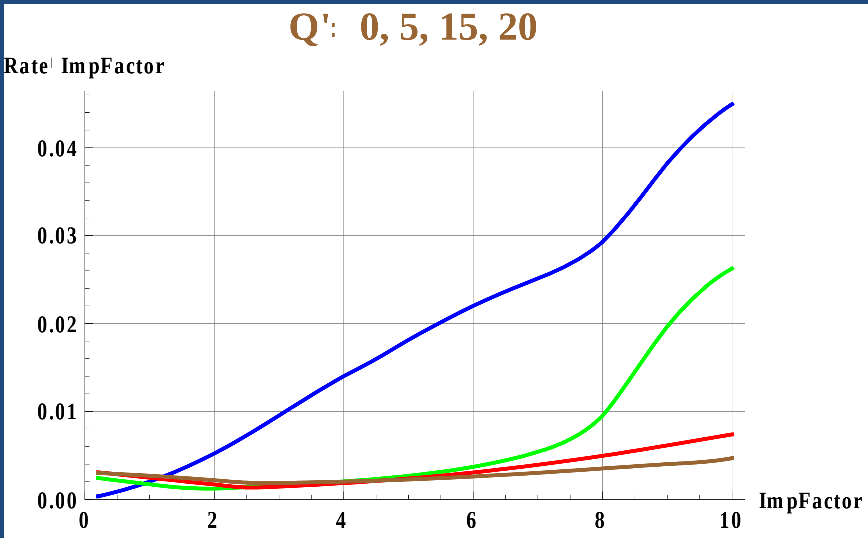
AB

Intensity scan, flat ADT, MUM: where is TMCI?









AB

Coherent Beam-Beam

Main assumption: bunch length << beta-function. For transversely dipolar modes, CBB is a cross-talk of bunch CM – thus, intra-bunch matrix structure is similar to the ADT and CB:

$$\dot{X}_1 = \hat{S}\cdot X_1 + \hat{Z}\cdot X_1 + \hat{D}\cdot X_1 + \hat{C}\cdot X_1 + b_{12}\hat{B}\cdot X_2;$$

$$\dot{X}_2 = \hat{S}\cdot X_2 + \hat{Z}\cdot X_2 + \hat{D}\cdot X_2 + \hat{C}\cdot X_2 + b_{21}\hat{B}\cdot X_1;$$

$$\hat{B} = -i\Delta\omega_{\mathrm{bb}}(\hat{D}/d_\mu)\left[1 + 2\frac{\rho_0^2}{\beta_0}\sum_{k=1}^{K}\frac{\beta_k}{\rho_k^2}\cos(k\phi_\mu)\right]/(2K+1);$$

$$b_{12} = b_{21}^* = 1 - \exp(-i\psi).$$

Here 2 identical opposite IRs are assumed (IR1 and IR5 for LHC) with $2K+1$ LR collisions for each, every one with its beta-function and separation β_k, ρ_k .

Alternating x/y collision for IR1/IR5 is assumed with ψ as a difference between the two phase advances, while $\Delta\omega_{\mathrm{bb}}$ is the incoherent beam-beam tune shift per IR.

Dispersion Equation

Let's consider a small fraction of the beam described by an NHT amplitude vector x_p :

$$\dot{x}_p = -i\delta\omega_p x_p + \hat{S}_p \cdot x_p + \hat{Z} \cdot X \qquad \hat{Z} \cdot X = -i(\Omega_c \hat{I} - i\hat{S}) \cdot X$$

Due to the frequency spread eigenvalues are slightly changed, $\Omega_c \to \Omega$ but eigenvector at the first approximation are the same (similar to QM). From here

$$x_p = \left[(\Omega - \delta\omega_p)\hat{I} - i\hat{S}_p \right]^{-1} \left[\Omega_c \hat{I} - i\hat{S} \right] \cdot X$$

$$X = \left\langle \left[(\Omega - \delta\omega_p)\hat{I} - i\hat{S}_p \right]^{-1} \right\rangle \left[\Omega_c \hat{I} - i\hat{S} \right] \cdot X$$

$$1 = X^{\dagger} \cdot \left\langle \left[(\Omega - \delta\omega_p)\hat{I} - i\hat{S}_p \right]^{-1} \right\rangle \left[\Omega_c \hat{I} - i\hat{S} \right] \cdot X$$

$$1 = -\sum_l (\Omega_c - l\bar{\omega}_s) \int \frac{|X_l(J_s)|^2}{\Omega - l\omega_s - \delta\omega_x + io} \frac{J_x \partial F}{\partial J_x} d\Gamma$$

$$\int |X_l(J_s)|^2 F d\Gamma = \int F d\Gamma = 1$$

AB

Weak Head-Tail case

This derivation assumes frequency spread can be treated as a perturbation. This is justified when the resonant particles are at the tails of the distribution.

$$1 = -\sum_l (\Omega_c - l\bar{\omega}_s) \int \frac{|X_l(J_s)|^2}{\Omega - l\omega_s - \delta\omega_x + io} \frac{J_x \partial F}{\partial J_x} d\Gamma \, ;$$

$$\int |X_l(J_s)|^2 \, F d\Gamma = \int F d\Gamma = 1$$

With the damper, weak HT approximation can be applied at many cases. If so (true for LHC), the DE is simplified:

$$1 = -(\Omega_c - l\bar{\omega}_s) \int \frac{|X_l(J_s)|^2}{\Omega - l\omega_s - \delta\omega_x + io} \frac{J_x \partial F}{\partial J_x} d\Gamma \, .$$

When the LD is provided by the far tails, the mode form-factor $|X_l|^2$ can be omitted with the logarithmic accuracy:

$$1 = -(\Omega_c - l\bar{\omega}_s) \int \frac{J_x \partial F / \partial J_x}{\Omega - l\omega_s - \delta\omega_x + io} d\Gamma \, .$$

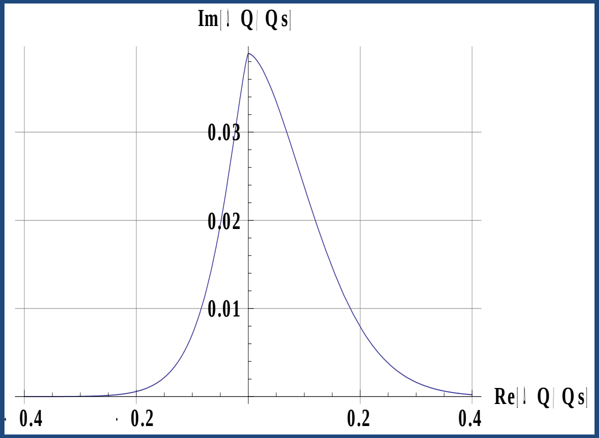
Stability Diagram

Stability diagram (SD) is defined as a map of real axes Ω on the complex plane:

$$\mathrm{D} = \left(-\int \frac{J_x \partial F / \partial J_x}{\Omega - l\omega_s - \delta\omega_x + io} d\Gamma \right)^{-1}$$

$$\mathrm{D} = \Omega_c - l\bar{\omega}_s$$

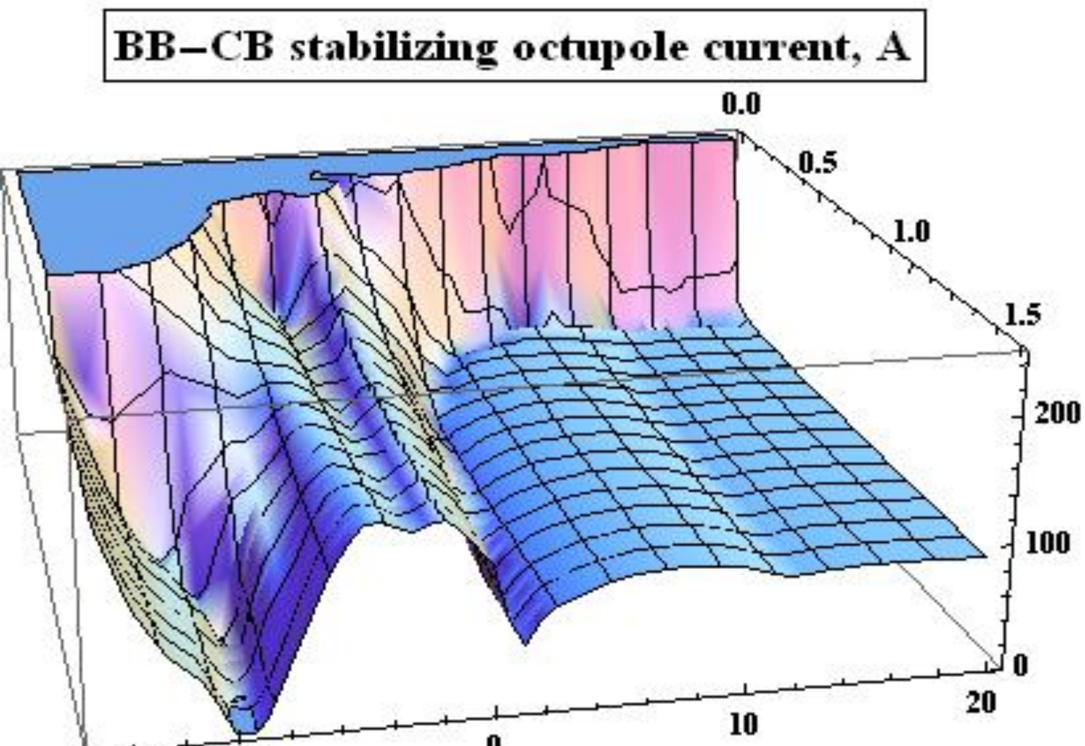
To be stable, the coherent tune shift has to be inside the SD.
For today, NHT uses simplified SDs, based on the transverse octupole nonlinearity only and Gaussian transverse distribution.



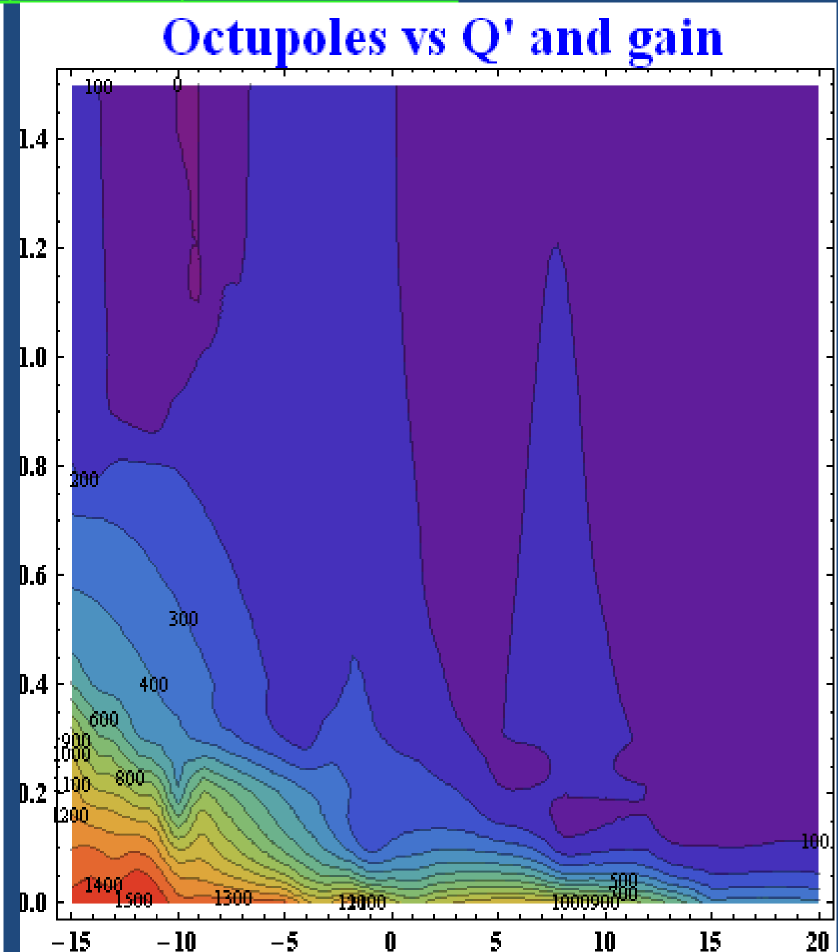
<= Stability diagram for +200 A of octupoles and Gaussian distribution.

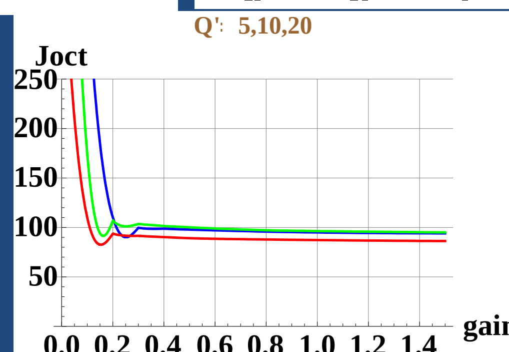
In the near future plans – to take longitudinal non-linearity into account

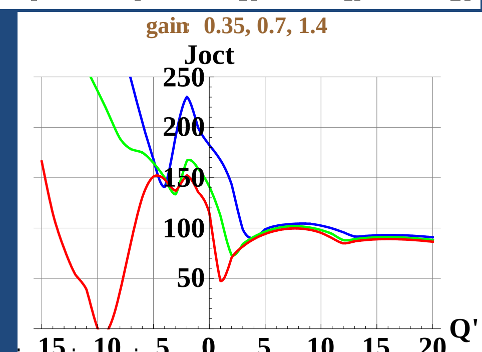
1⊗Imp, CB, CBB $\psi = \pi/2$, MO+, bbb ADT



incoherent $dQ_{bb} / Q_s \big|_{IR1,5} = \pm 1.1; \quad \psi = \pi / 2$







An example of NHT threshold results for:

nominal H impedance,
50ns beam,
2 IRs with specified phase difference,
new bbb ADT,
focusing octupole polarity.

AB

Summary: power of the model

- Method of nested head-tail modes (NHT) is implemented on a base of Mathematica. It allows to find coherent tunes for all the modes, solving the eigenproblem at its 4D set:

  azimuthal ⊗ radial ⊗ coupled-bunch ⊗ beam-beam.

- The external tables: impedance/wake, ADT frequency profile, distribution functions and nonlinearities.

- Based on that, all the coherent modes with all the details are computed.

- To test any new beam, impedance or gain profile, with 5 radial, 21 azimuthal and 15 representative CB modes for 40 chromaticity points and 20 gain points takes only 25 min on my 3 years old laptop.

- With all the new features it is not going to take much longer.

Next steps

- To include longitudinal plane into SD.
- To include train structure.
- To include detuning wakes/impedances.
- To make all that user friendly and public.

Many thanks for your attention!