Nested HT Method: Impedance, Damper, Radial Modes and Coupled Bunches

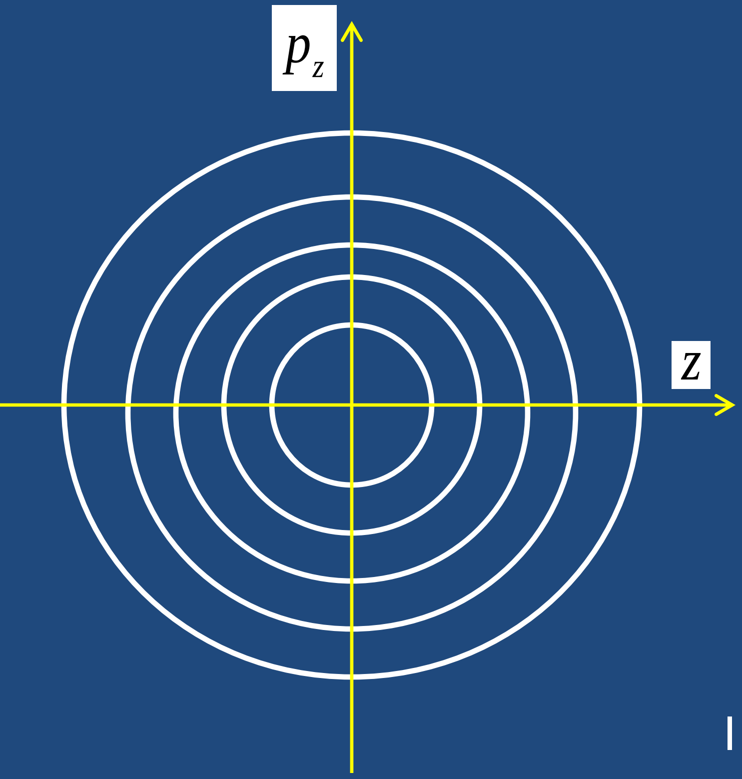
A. Burov

Fermilab-LARP

special thanks - to Elias, Nicolas, Simon, Xavier, Benoit, Gianluigi, and Tatiana P.

CERN, LMC Aug 29, 2012

Nested Head-Tail Basis



$$\psi_{l\alpha} \propto \exp(il\phi + i\chi_\alpha \cos\phi - i\Omega_l t)\ ;$$

$$\chi_\alpha = \frac{Q'\omega_0 r_\alpha}{c\eta}\ ;$$

$$\Omega_l = \omega_b + l\omega_s\ .$$

I am using n_r equally populated rings which radii r_α are chosen to reflect the phase space density.

Main Equation, single bunch

- In the water-bag single bunch approximation, beam equations of motion can be presented as in Ref [A. Chao, Eq. 6.183]:

$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X$$

where X is a vector of the HT mode amplitudes,

$$(\hat{S} + \hat{Z})_{lm\alpha\beta} = -il\delta_{lm}\delta_{\alpha\beta} - i^{l-m}\frac{\kappa}{n_r}\int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau_\alpha - \chi_\alpha) J_m(\omega\tau_\beta - \chi_\beta)$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l}\frac{d}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta)$$

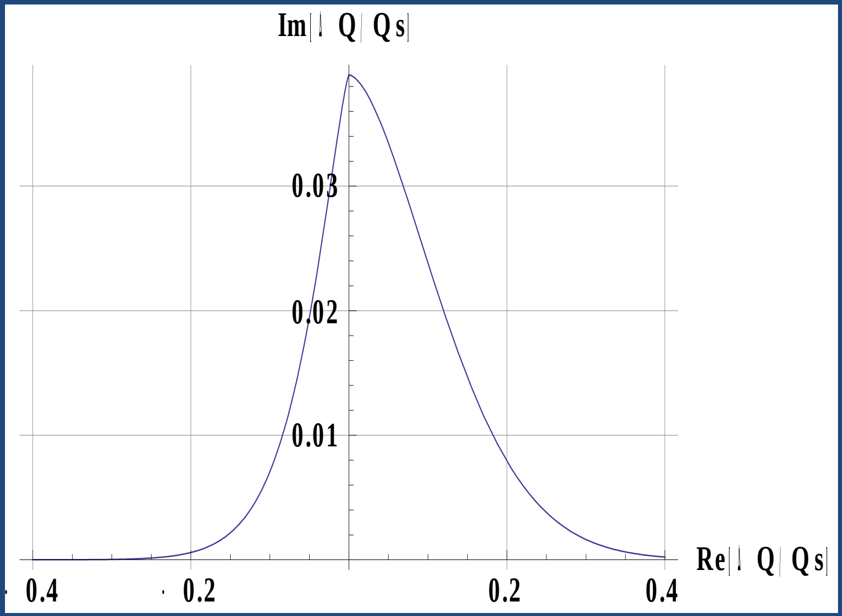
d is the damper gain in units of the damping rate, $\kappa = \dfrac{N_b r_0 R_0}{8\pi^2 \gamma Q_b Q_s}$

time is in units of the angular synchrotron frequency.

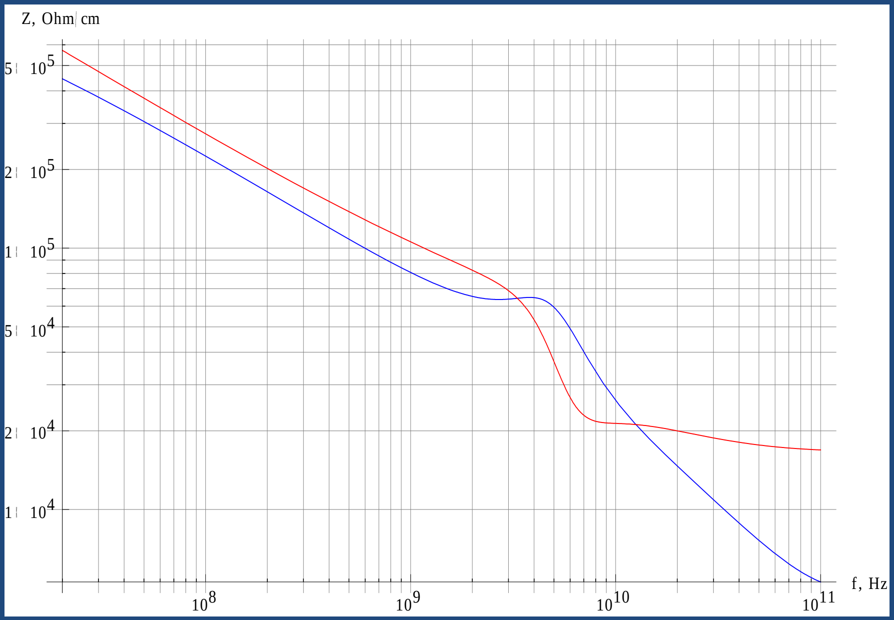
Analysis of solutions

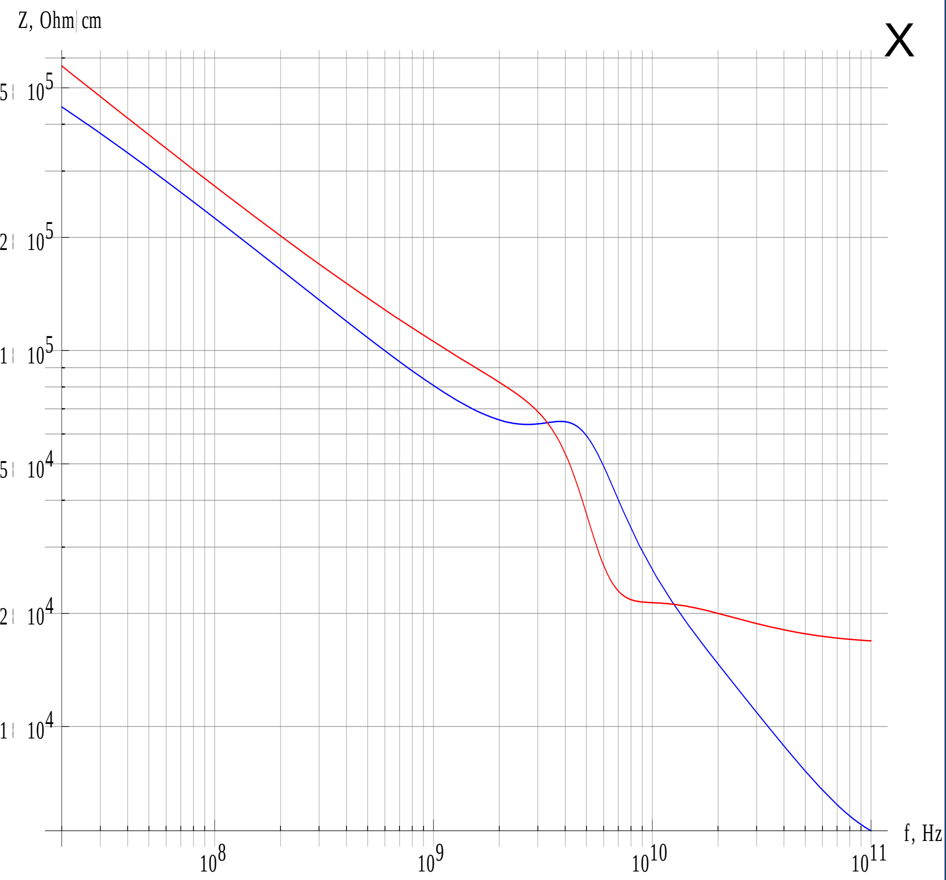
1. For every given gain and chromaticity, the eigensystem is found for the LHC impedance table (Nicolas M.).
2. The complex tune shifts are found from the eigenvalues $\Delta\Omega_{l\alpha} = \Omega_{l\alpha} - l$
3. The stabilizing octupole current is found from the stability diagram (Xavier B.) for every mode, then max is taken.

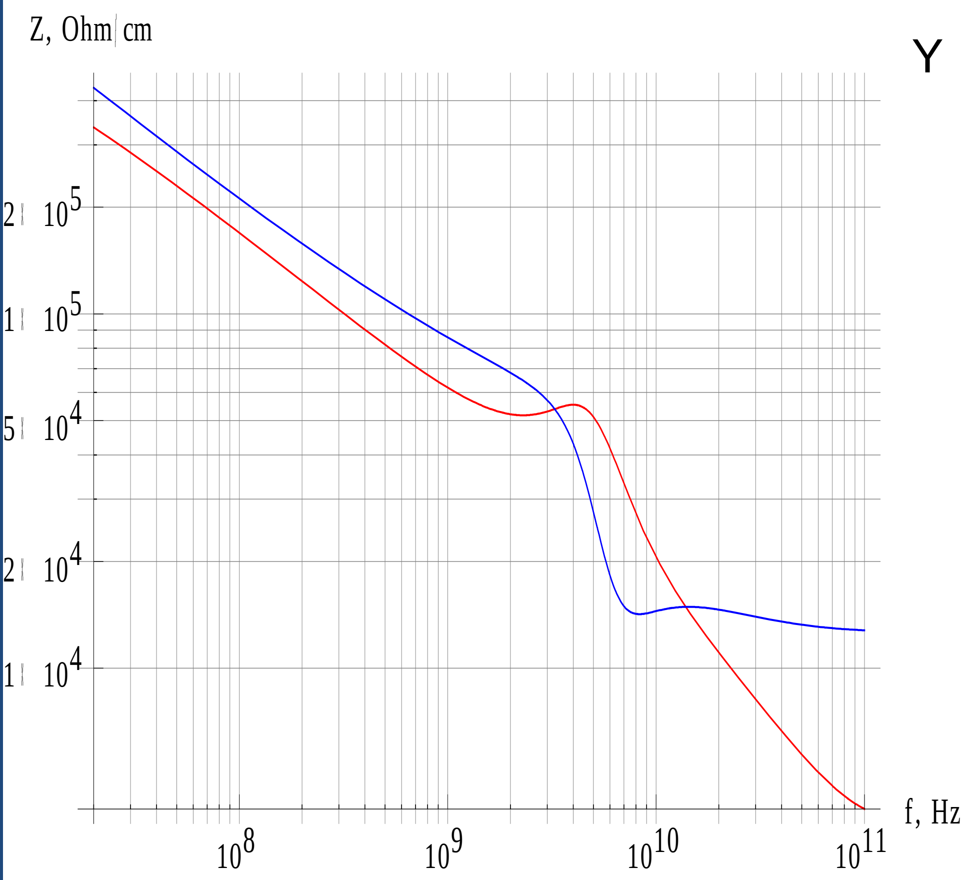
Stability diagram at +200 A of octupoles



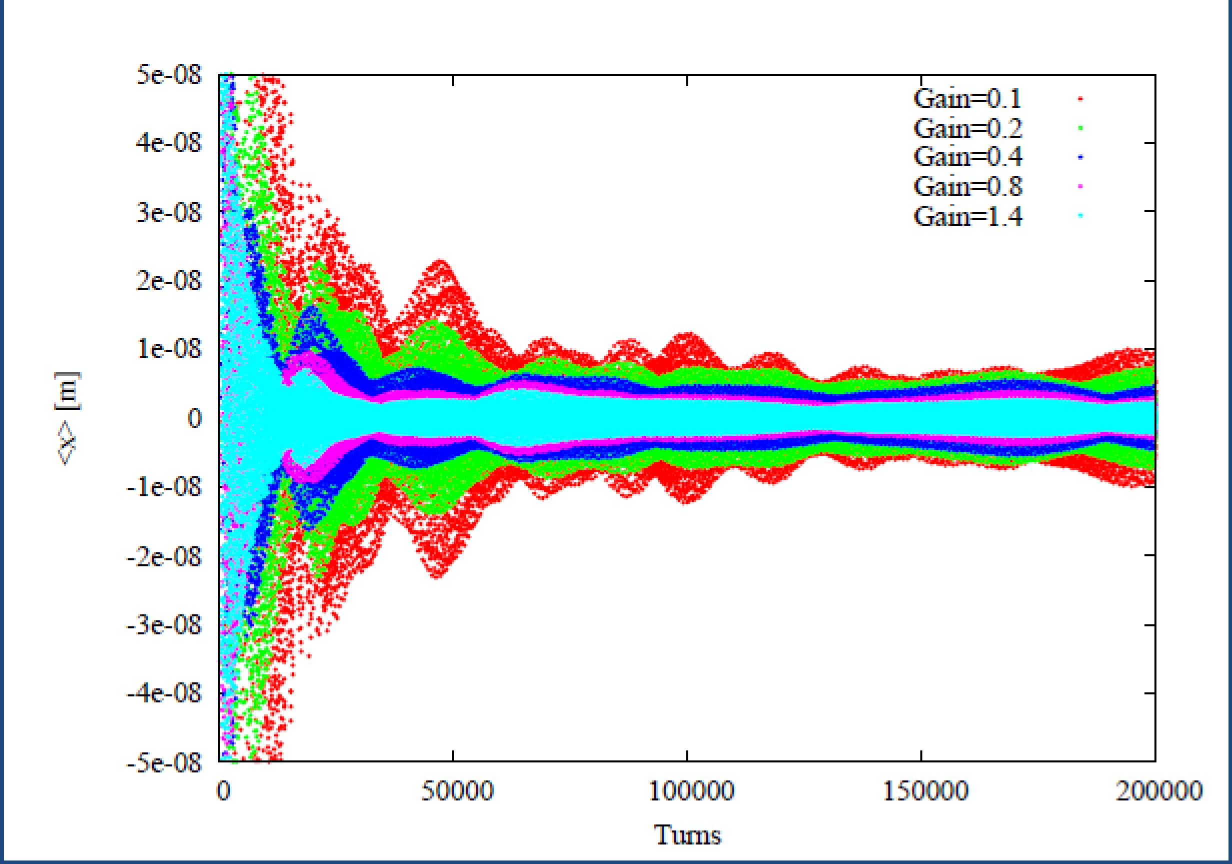
Impedances (Nicolas M.)




Z, Ohm|cm
X
5| 10^5
2| 10^5
1| 10^5
5| 10^4
2| 10^4
1| 10^4
10^8
10^9
10^10
10^11
f, Hz


Z, Ohm|cm
Y
2| 10^5
1| 10^5
5| 10^4
2| 10^4
1| 10^4
10^8
10^9
10^10
10^11
f, Hz

Single Bunch Tracking (S. White)



Simon 's gain scan with his (so far) single bunch tracking code for chroma=-6. Gain=0.1 (700 turns of the damping time) appears to be a threshold in agreement with the above slide.

Coupled Equidistant Bunches

Main idea:

For LHC, the bunches are well-separated, so the wake function of the neighbor bunch can be taken as flat within the bunch length.

The only difference between the bunches is CB mode phase advance, otherwise they are all identical.

Thus, the CB kick felt by any bunch is proportional to its own offset, so the CB matrix $\hat{C}$ has the same structure as the damper matrix $\hat{D}$:

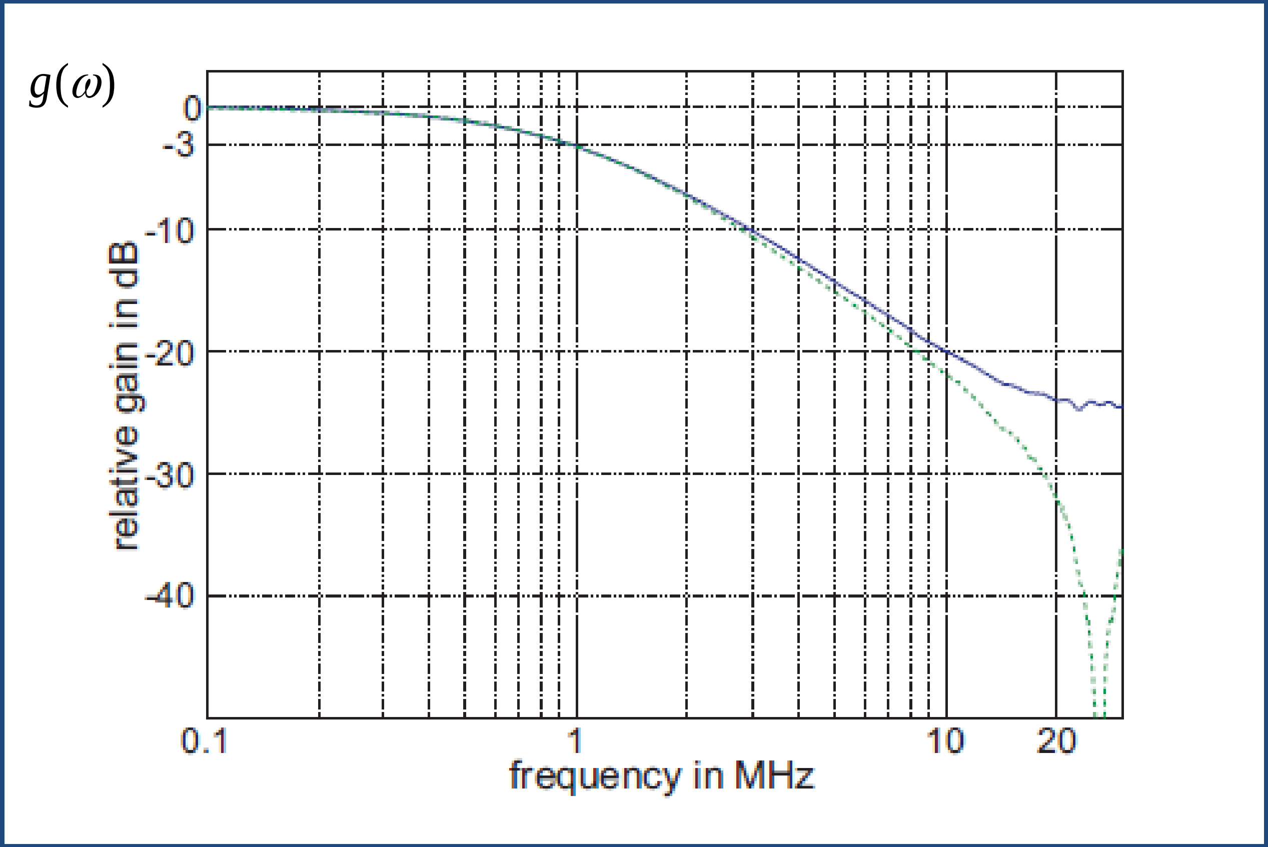
$$\dot{X} = \hat{M}_0 \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X + \hat{C} \cdot X;$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l} \frac{d_\mu}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta); \qquad \hat{C} = 2\pi i \kappa W(\varphi_\mu) \hat{D} / d_\mu;$$

$$W(\varphi_\mu) = \sum_{k=1}^{\infty} W(-ks_0) \exp(-ik\varphi_\mu); \;\; \varphi_\mu = (1 - \{Q_x\}) + \frac{2\pi\mu}{M_b}; \;\; |\mu| \le \frac{M_b}{2}$$

Wake and impedance are determined according to A. Chao book.

Damper response function (impedance)



Current ADT response (W. Hofle, D. Valuch) .
At 10 MHz it drops 10 times.

How CB damping rate is related to this function?
Below gain is measured in omega_s units, gain=1.4 is equivalent to 50 turns of the damping time.

CB Mode Damping Rate

With $g(\omega)$ as the frequency response function of the previous plot, the time-domain damper's "wake" is

$$G(\tau) = \int_0^\infty g(\omega)\cos(\omega\tau)\,d\omega/\pi\,,$$

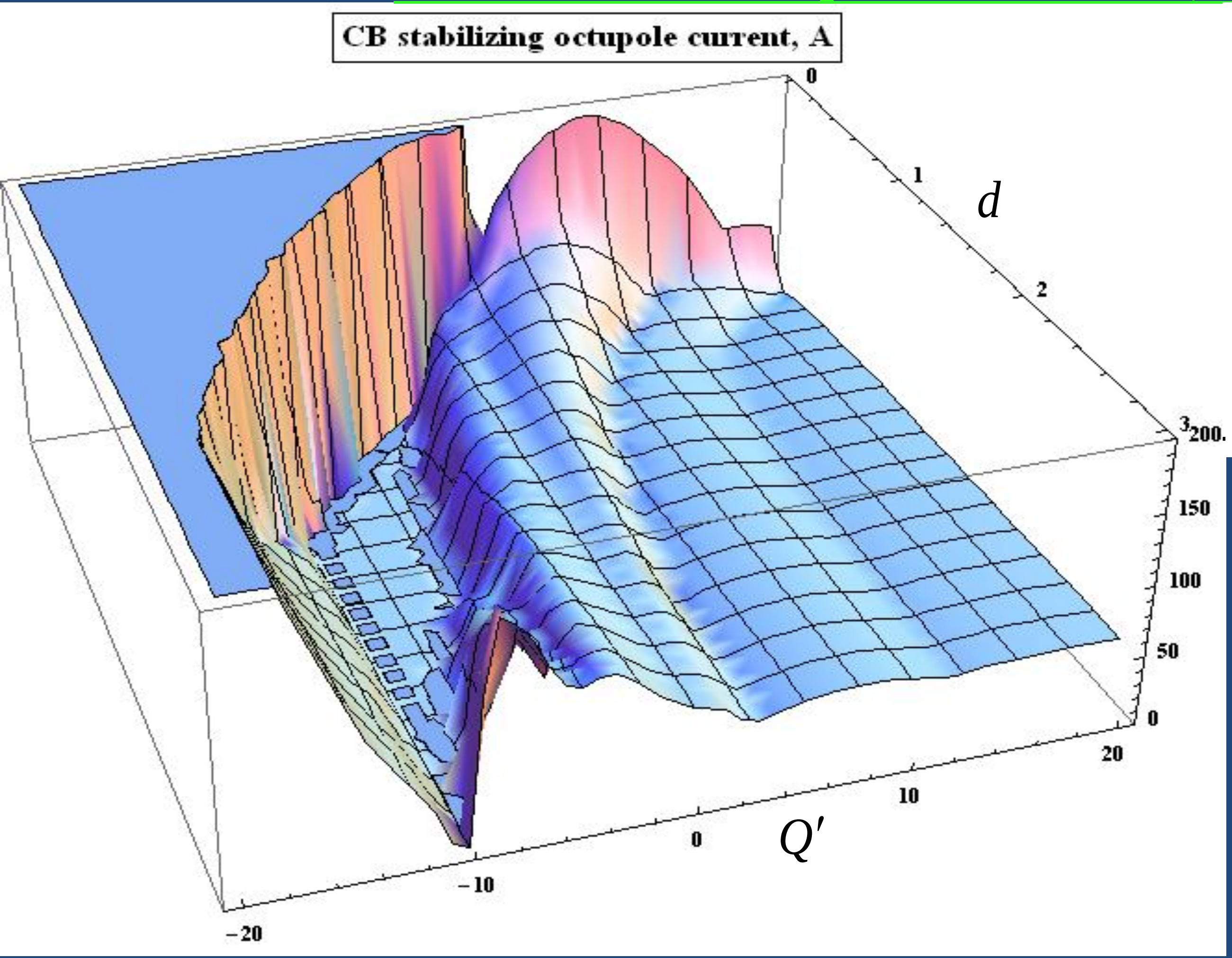
assuming this response to be even function of time (no causality for the damper!).

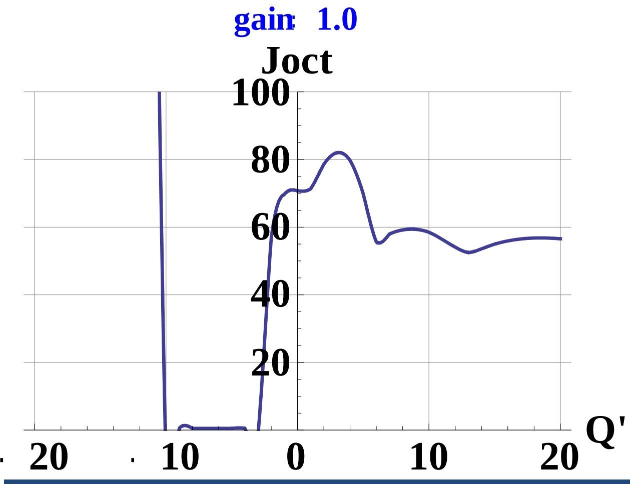
From here:

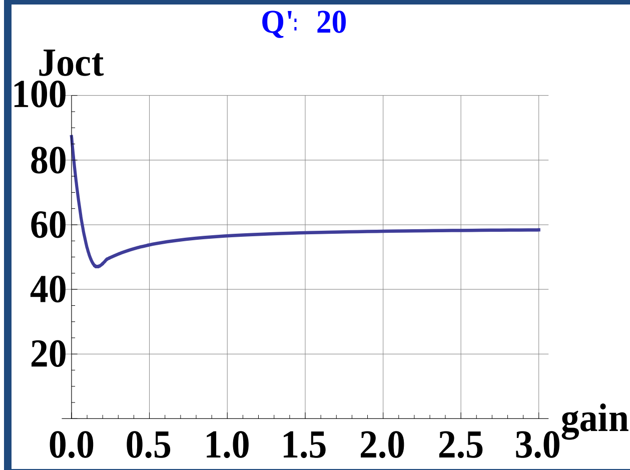
$$d_\mu = d\,\frac{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)\cos(k\varphi_\mu)}{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)};$$

where d is the rate provided for low-frequency CB zero-head-tail modes at zero chromaticity.

Threshold octupole current, Ioct<0





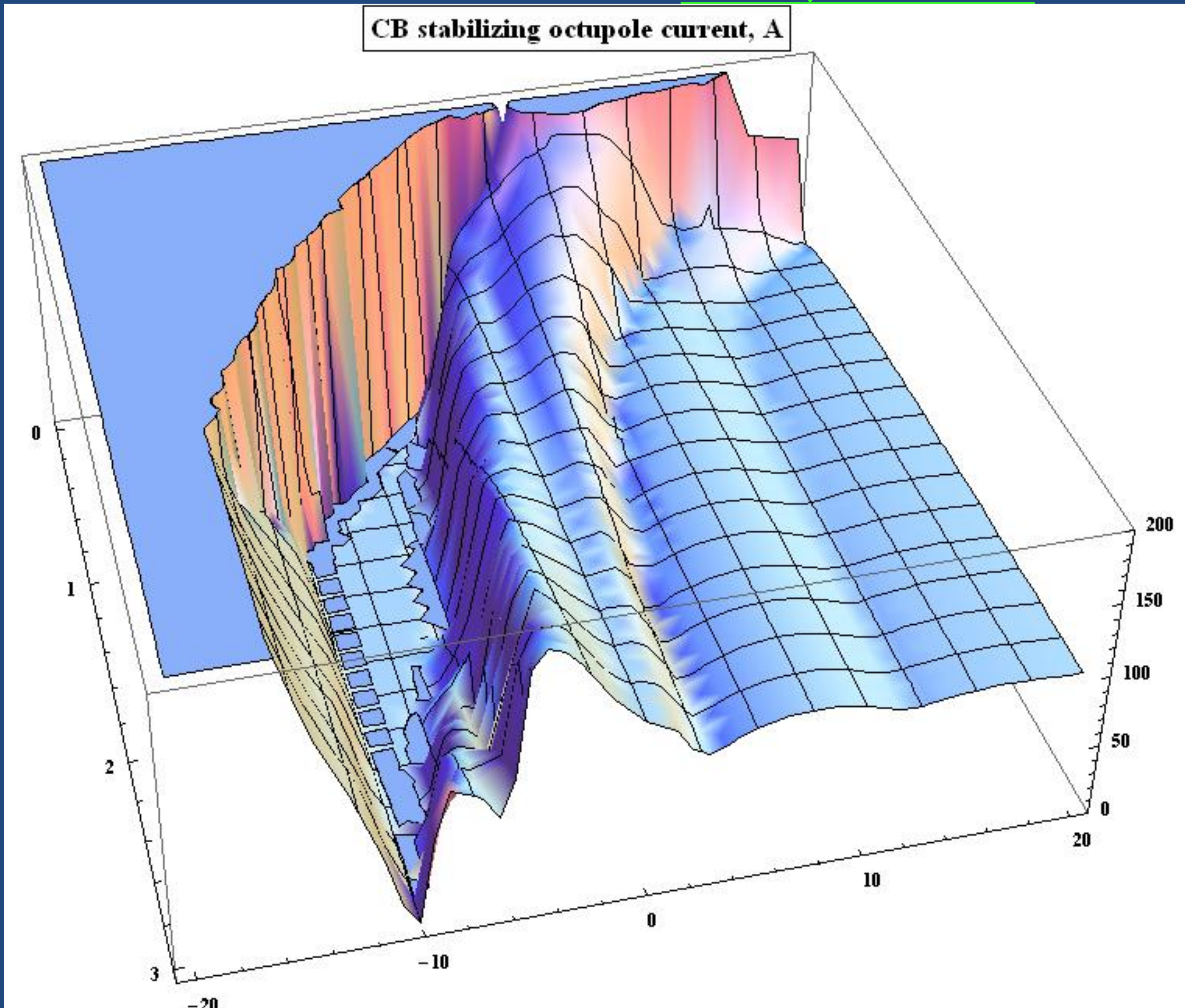


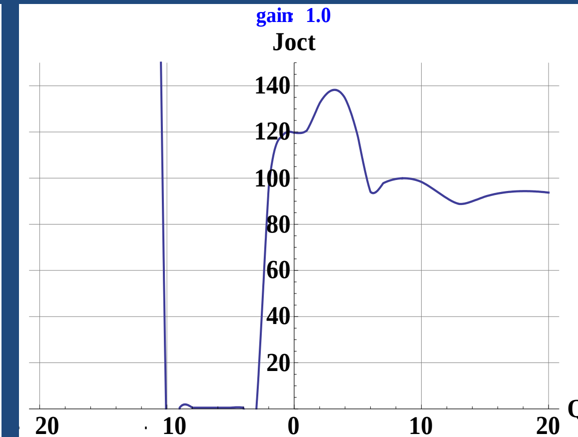
Note area of zero-octupole stability.

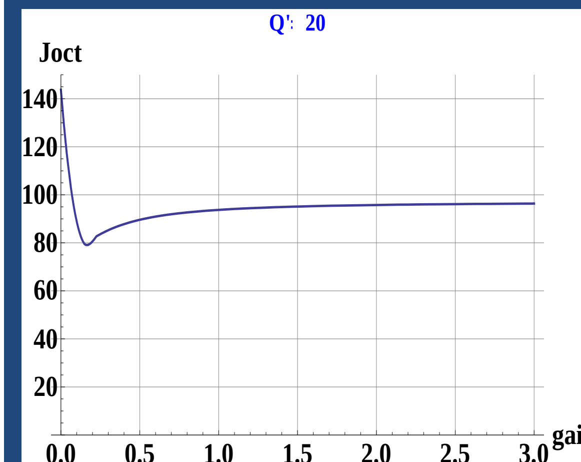
$N_{\mathrm{b}} = 1.5 \cdot 10^{11};\ \ \tau_{\mathrm{bb}} = 50\mathrm{ns};\ \ \varepsilon = 2\mu\mathrm{m};\ \ E = 4\mathrm{TeV}.$

It is not a strip, as for a single-bunch, but a triangle.
Note the steepness. Note damper is not needed at chroma>15.

Same, Ioct>0

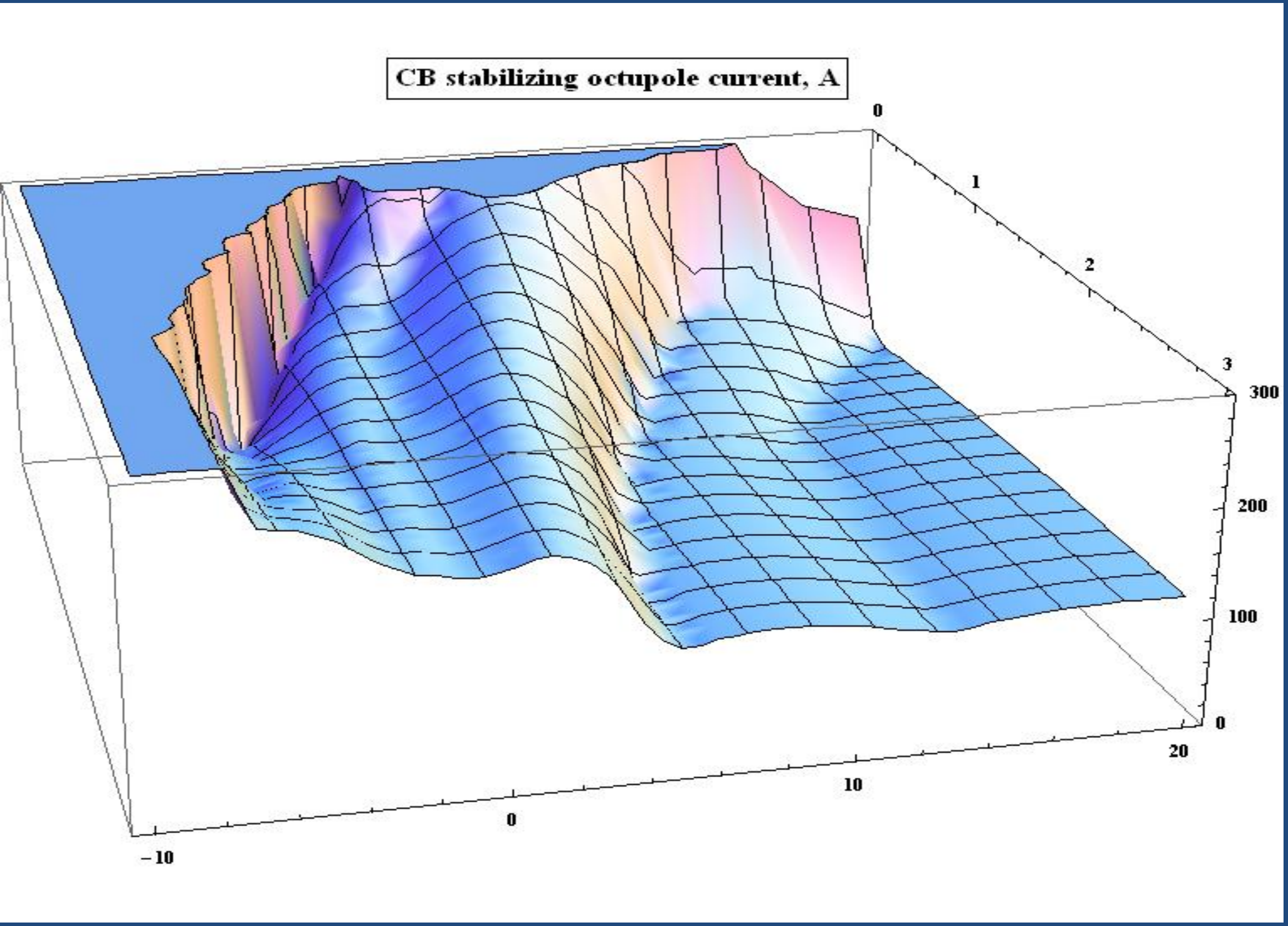


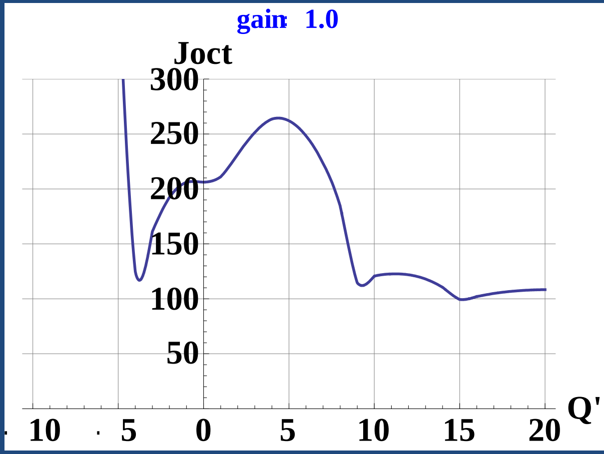


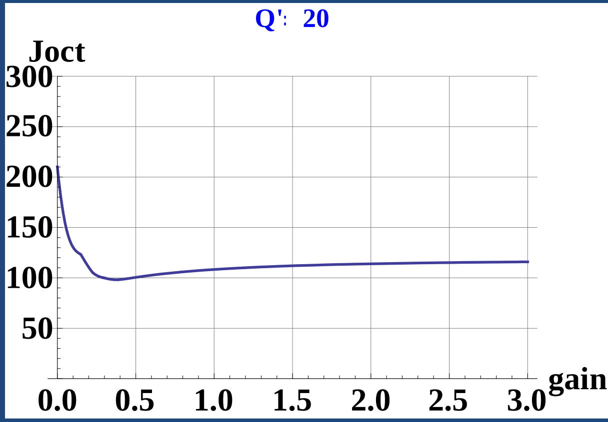


Essentially the same plot, rescaled with 1.6 times higher current

2 times nominal impedance, Ioct<0:





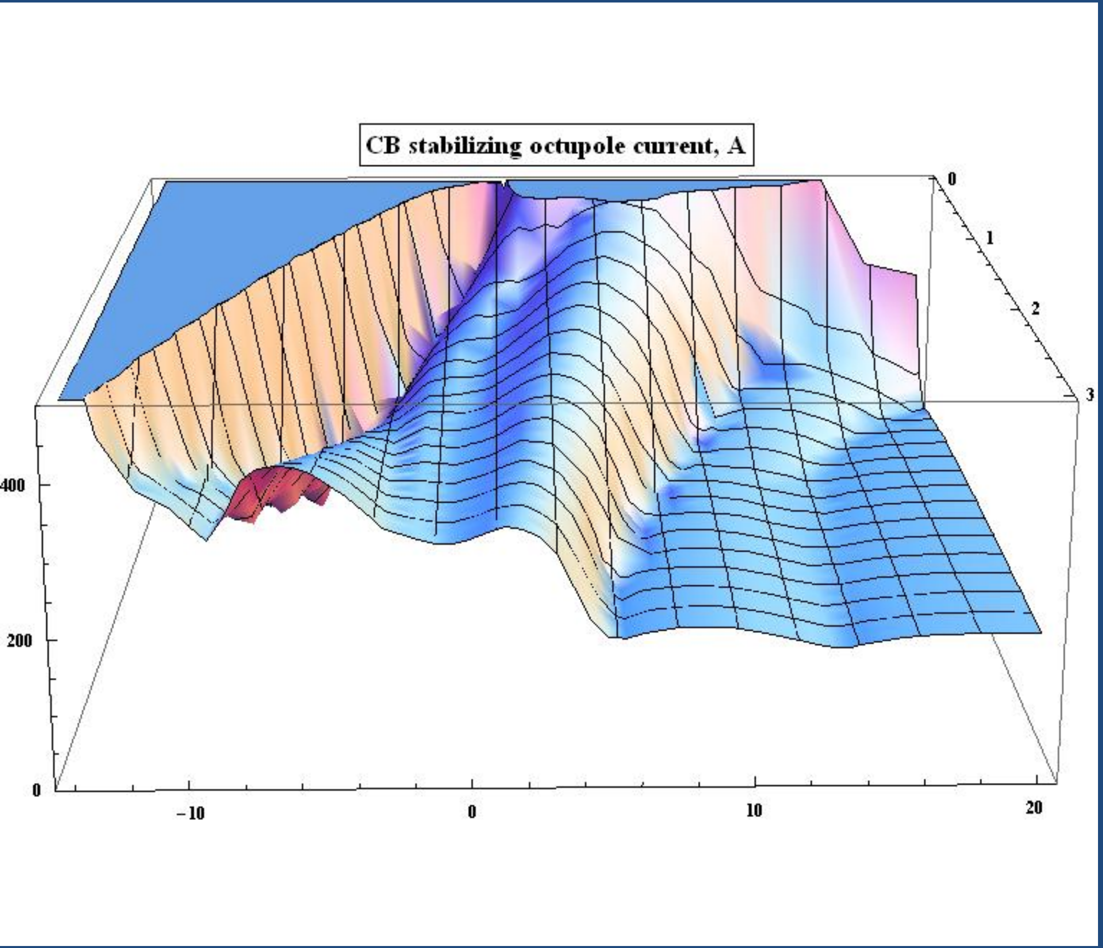


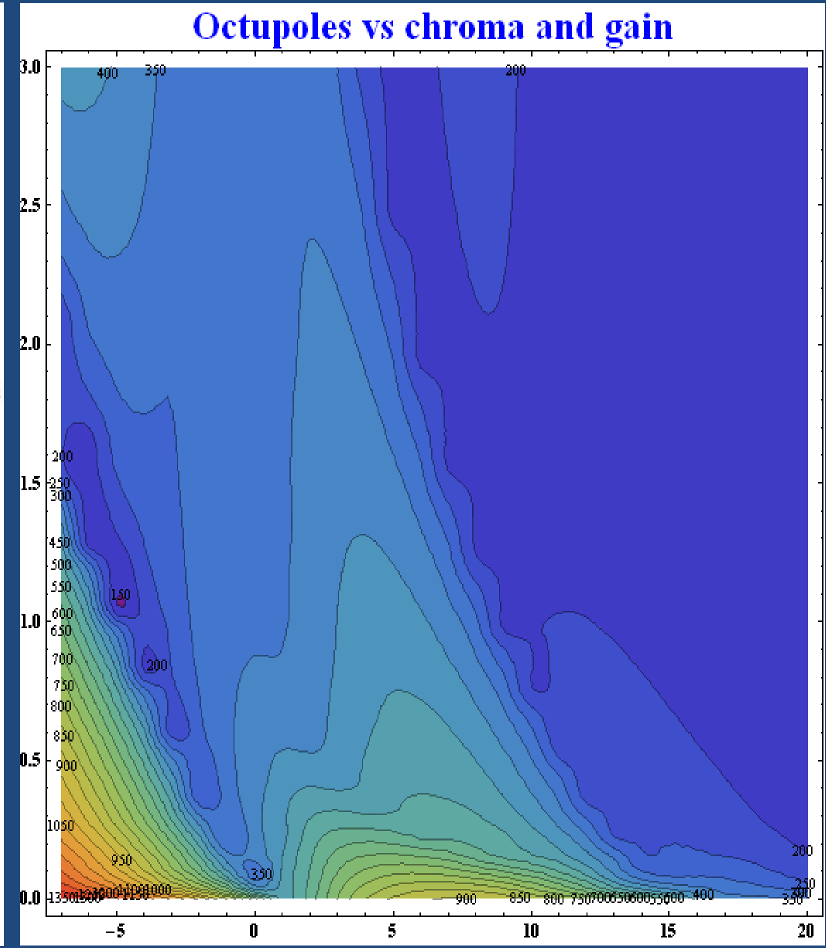
No-octupole stability area disappeared.

If the real impedance is doubled, then we are staying in the right area of chromaticity>10.

In this area the octupole current doubles with the impedance doubling.

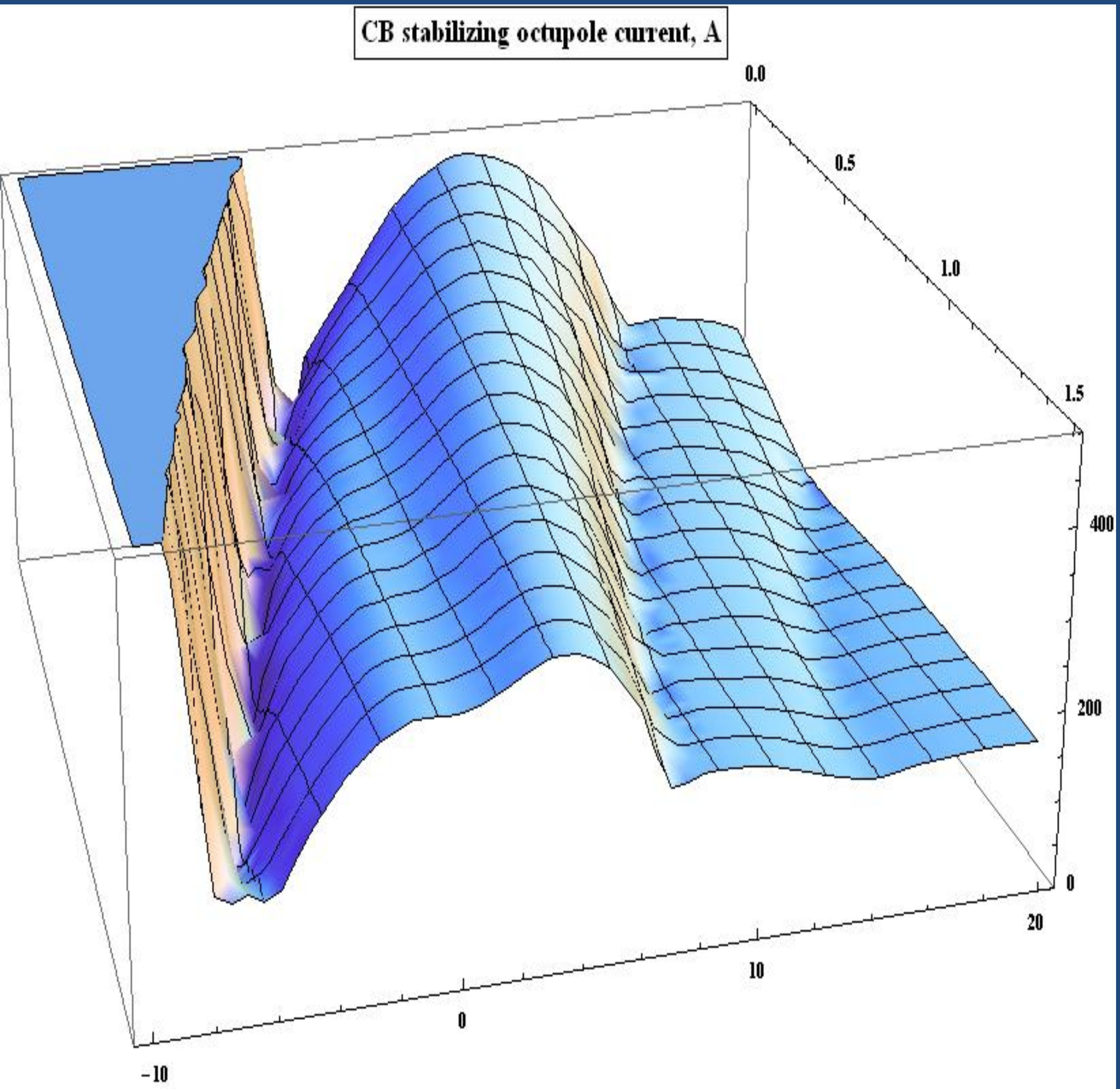
2 times nominal impedance, Ioct>0

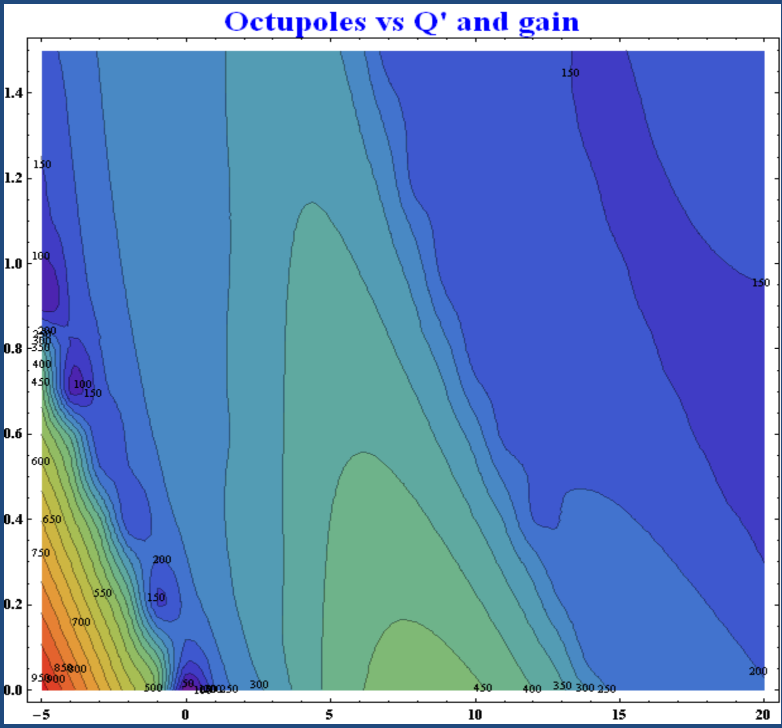


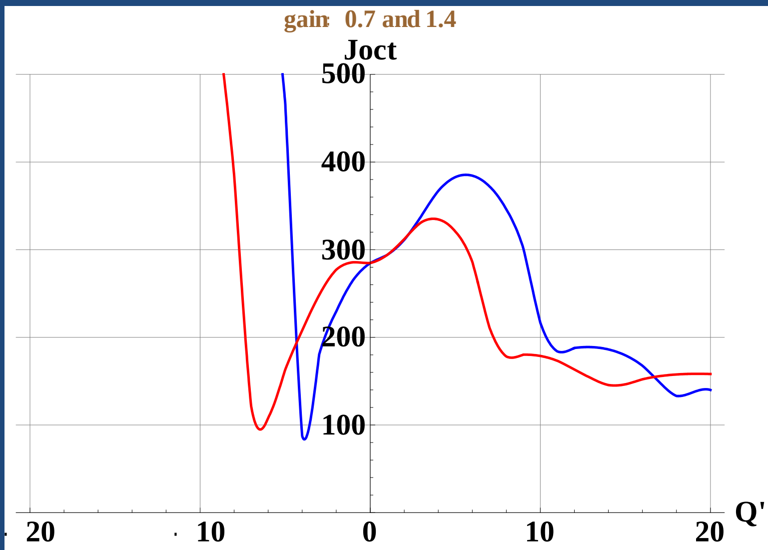


Nothing new – the same scaling factor ~ 1.6 compared with the negative octupoles.

2⊗SB Impedance, 0⊗CB, oct>0

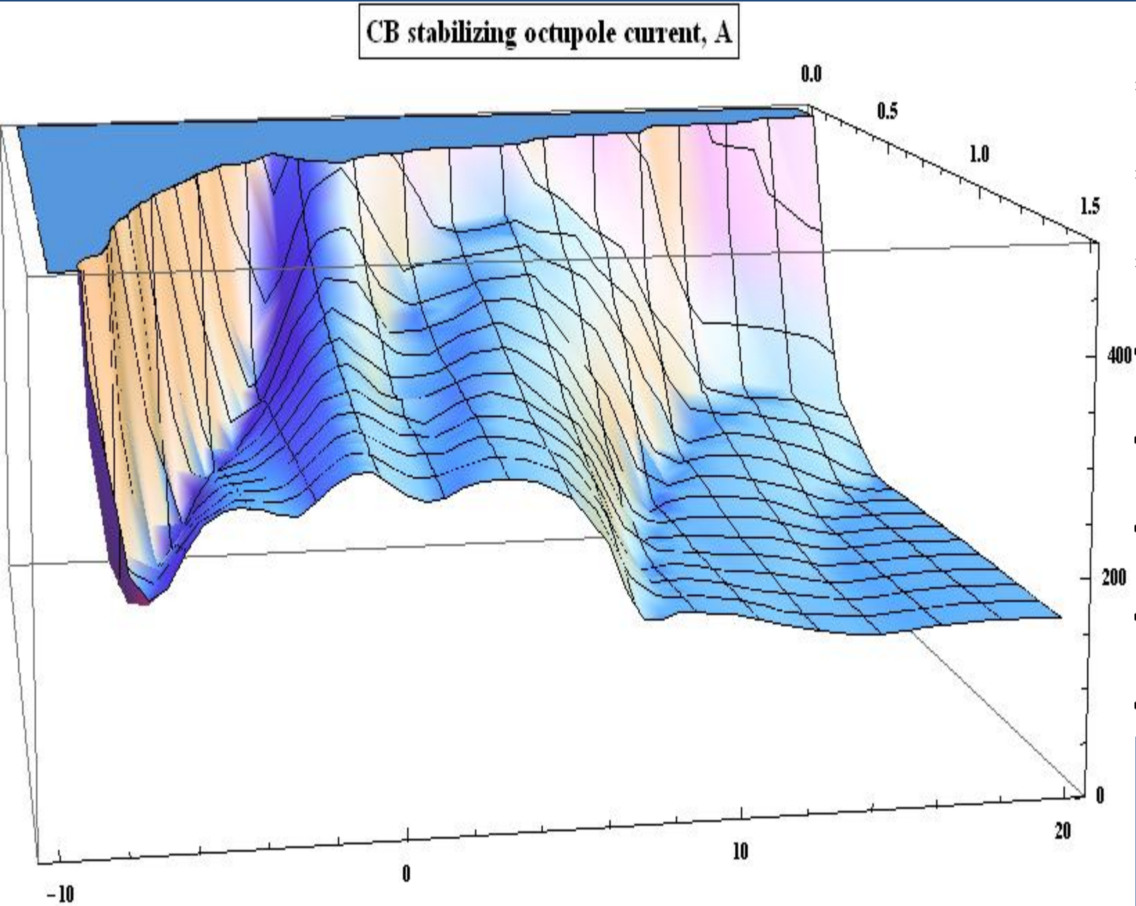


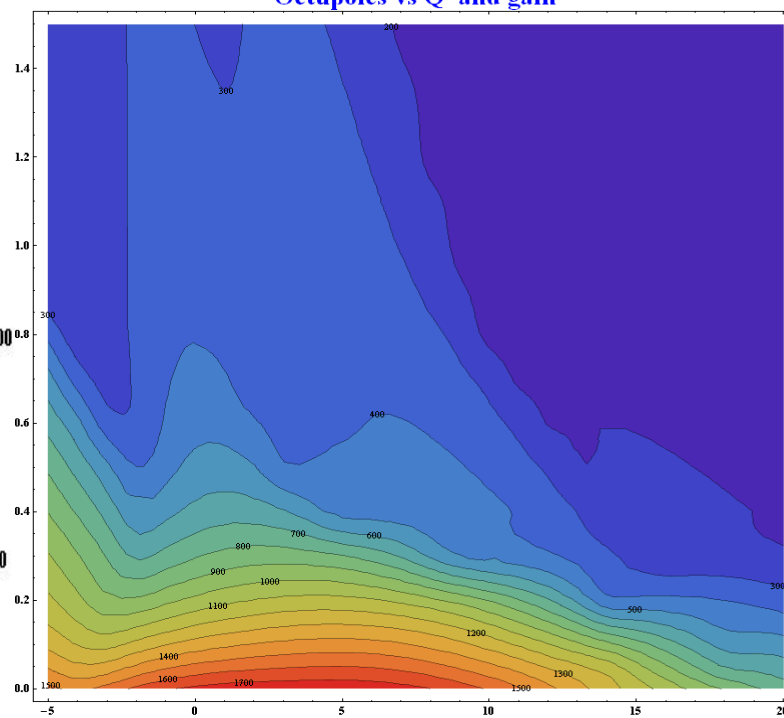




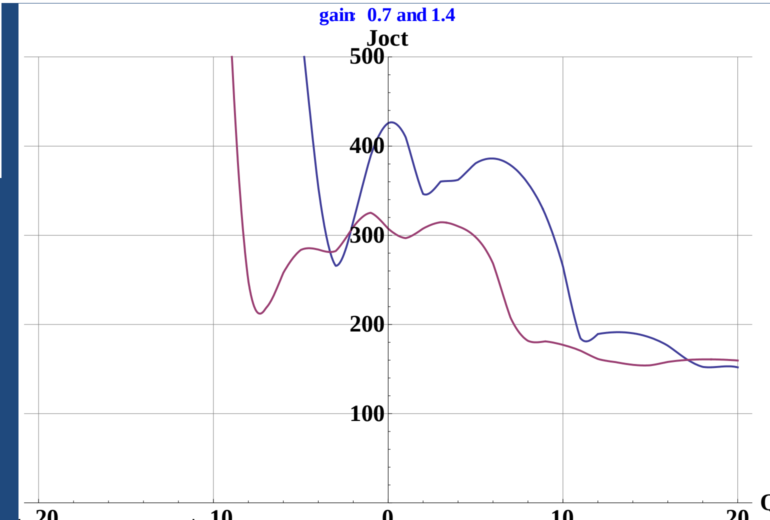
TMCI onset

2×Single bunch impedance, 6×CB Impedance, oct>0



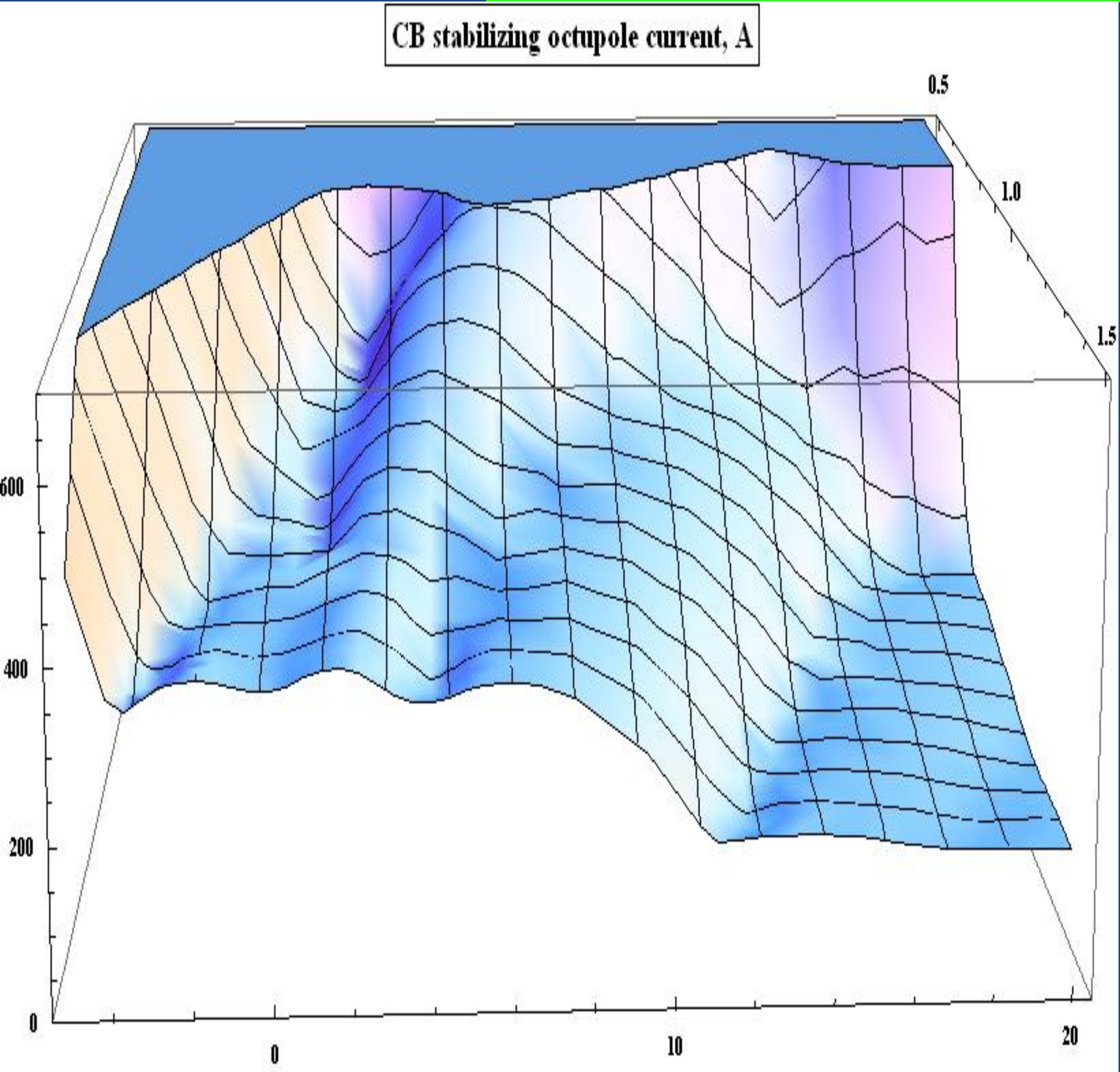


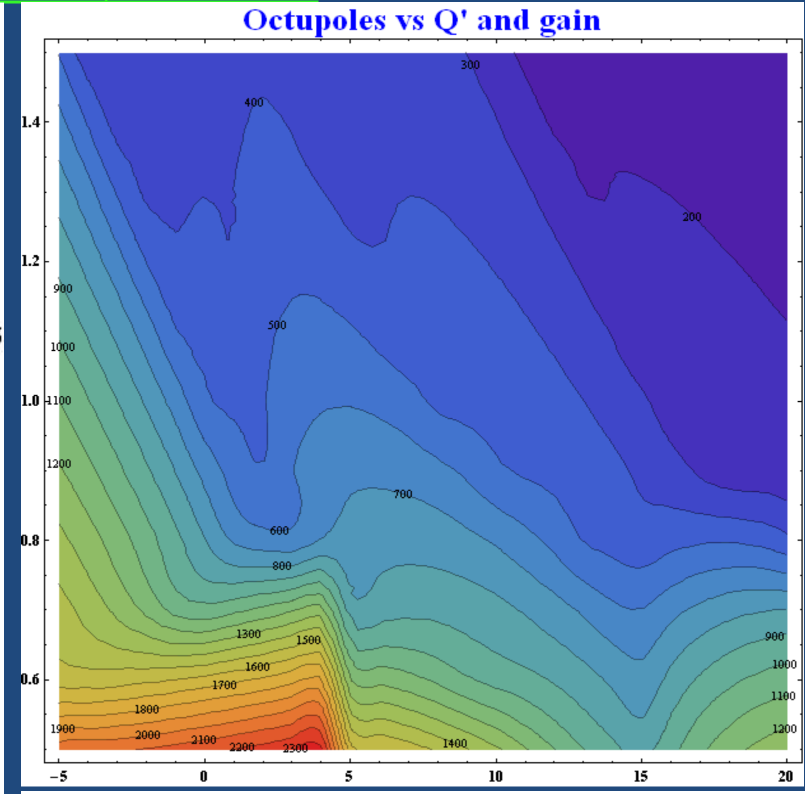
gain 0.7 and 1.4



At Q’>0 – insensitive to CB.

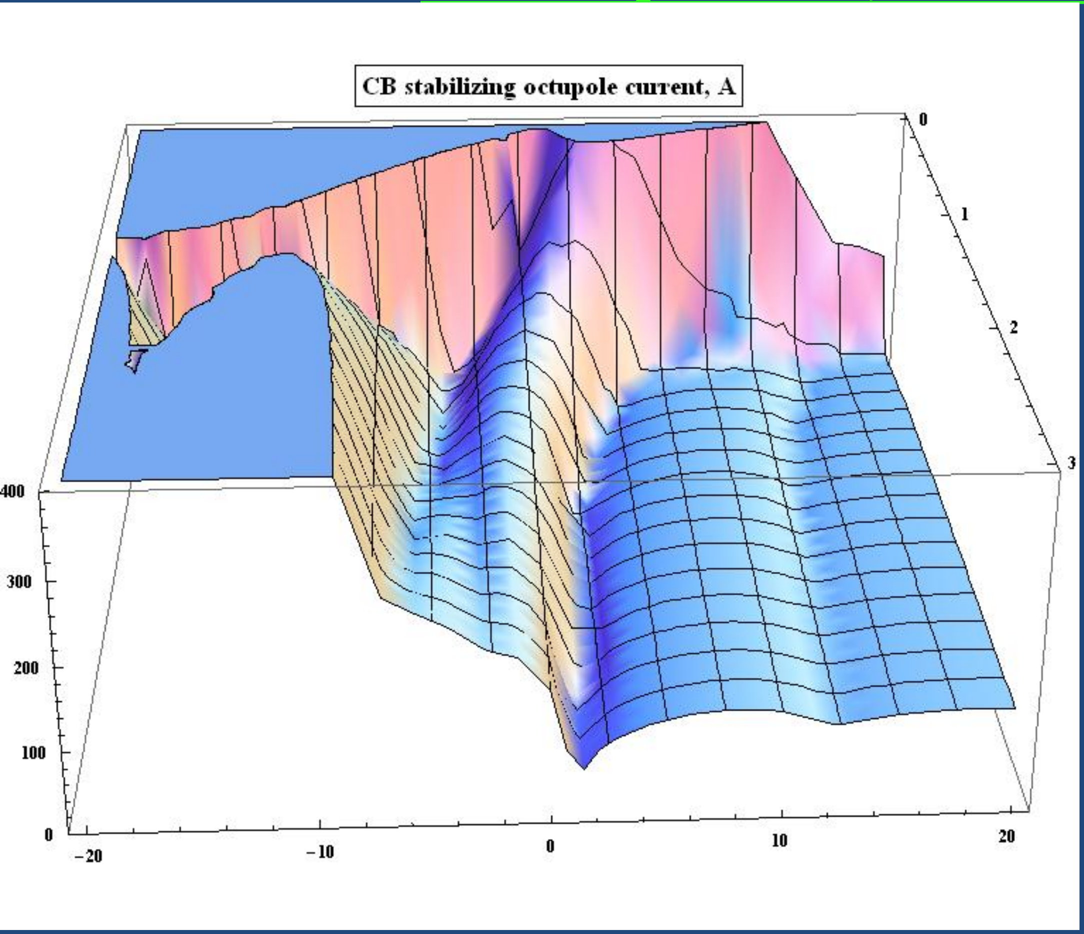
2×SB, 20×CB Impedances, oct>0

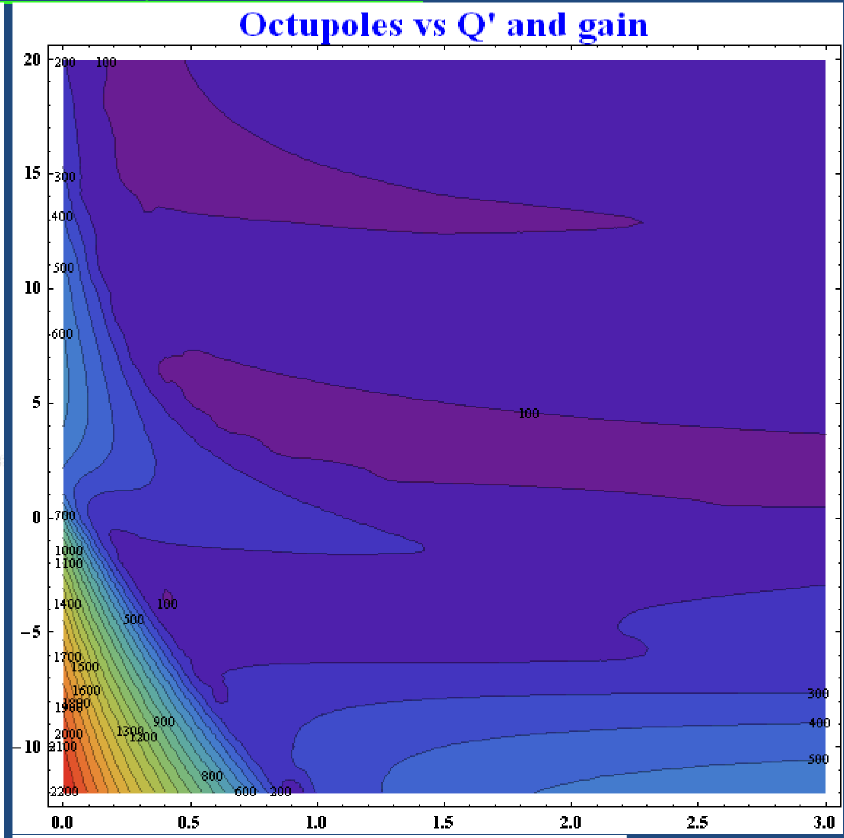


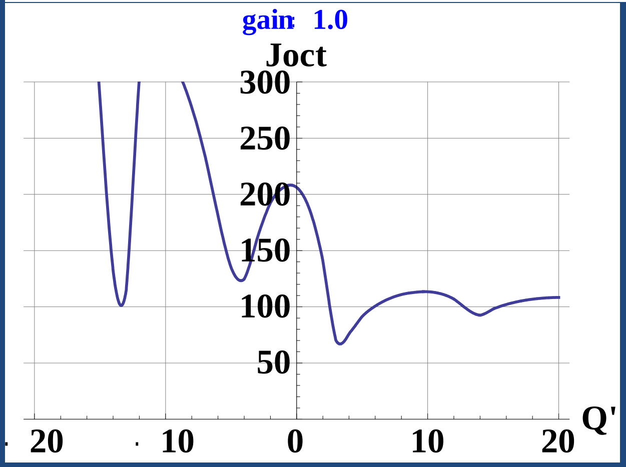


At least here the plateau starts later...

2 ⊗ Impedance, Flat Gain, Ioct<0:

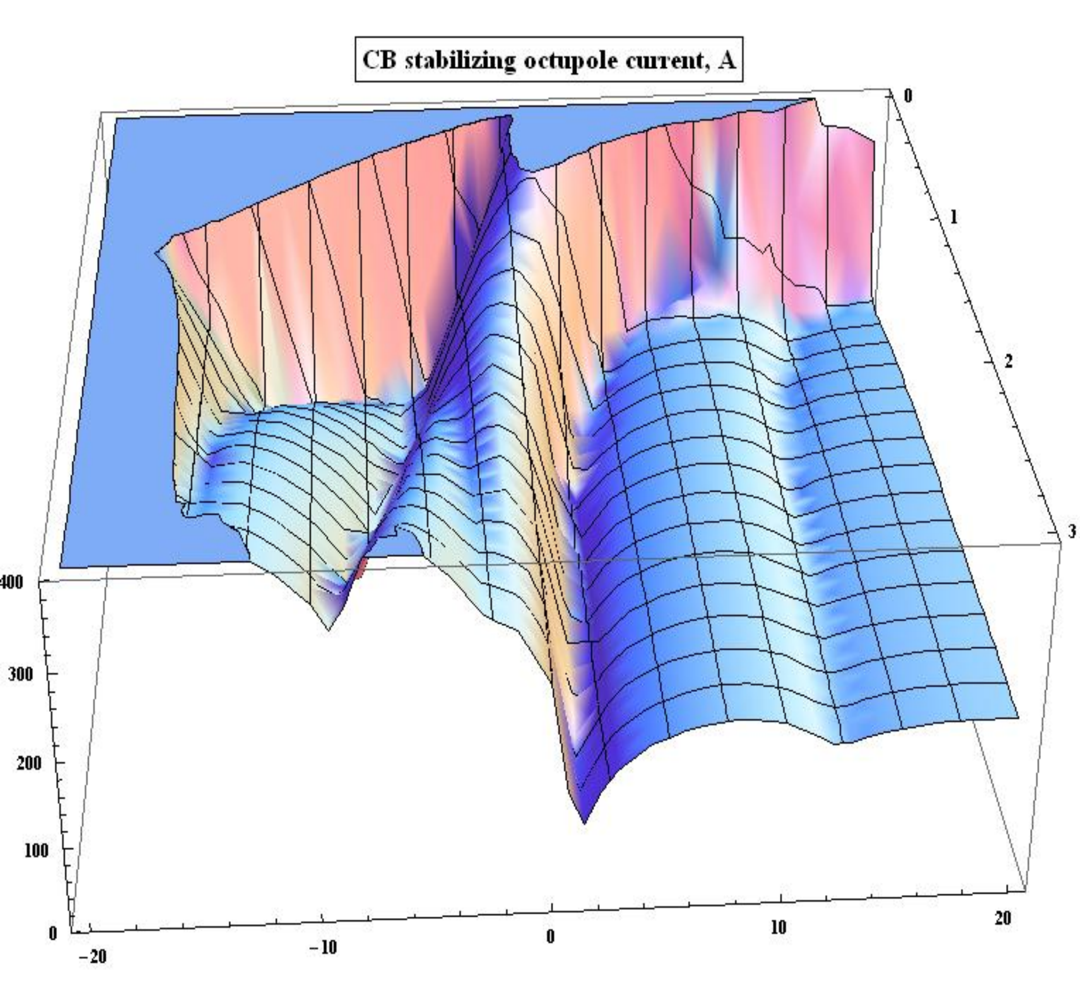




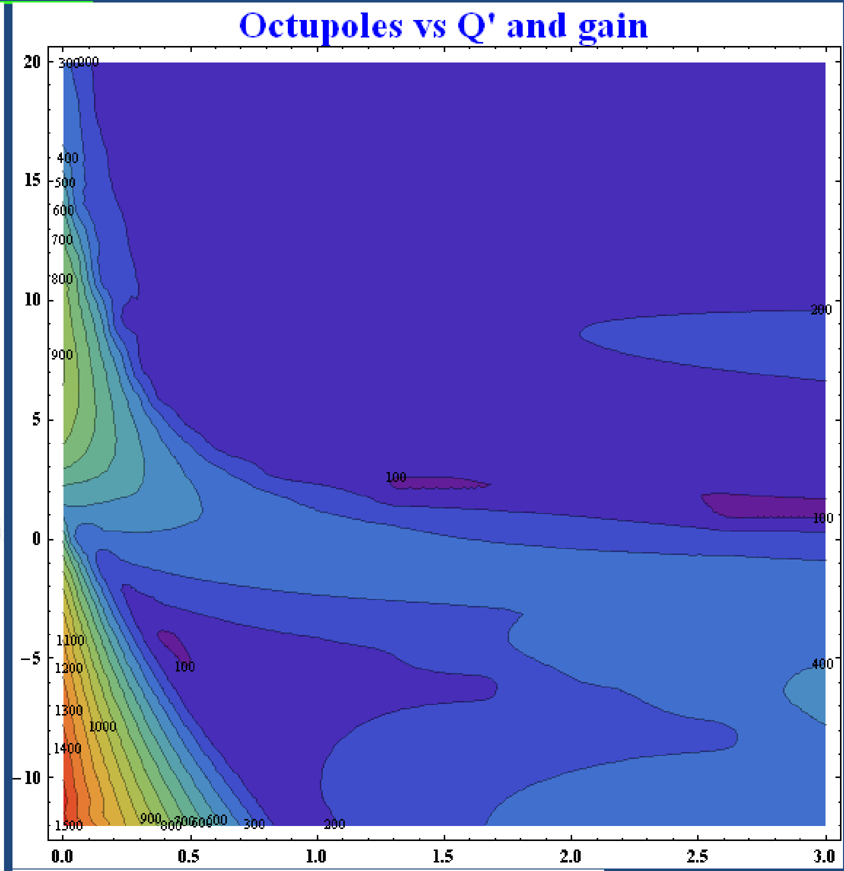


No difference with the current ADT for $Q' \geq 10$.

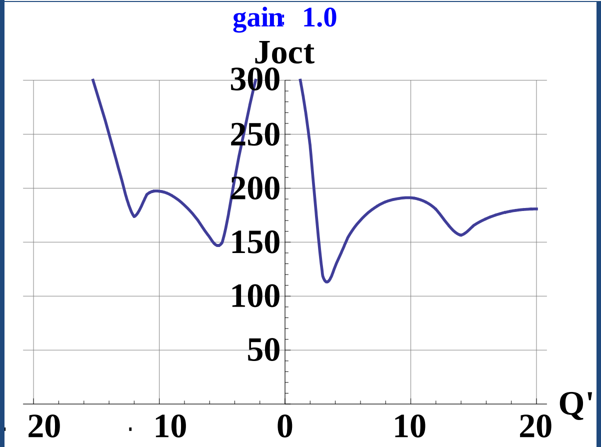
Same, Ioct>0:



CB stabilizing octupole current, A

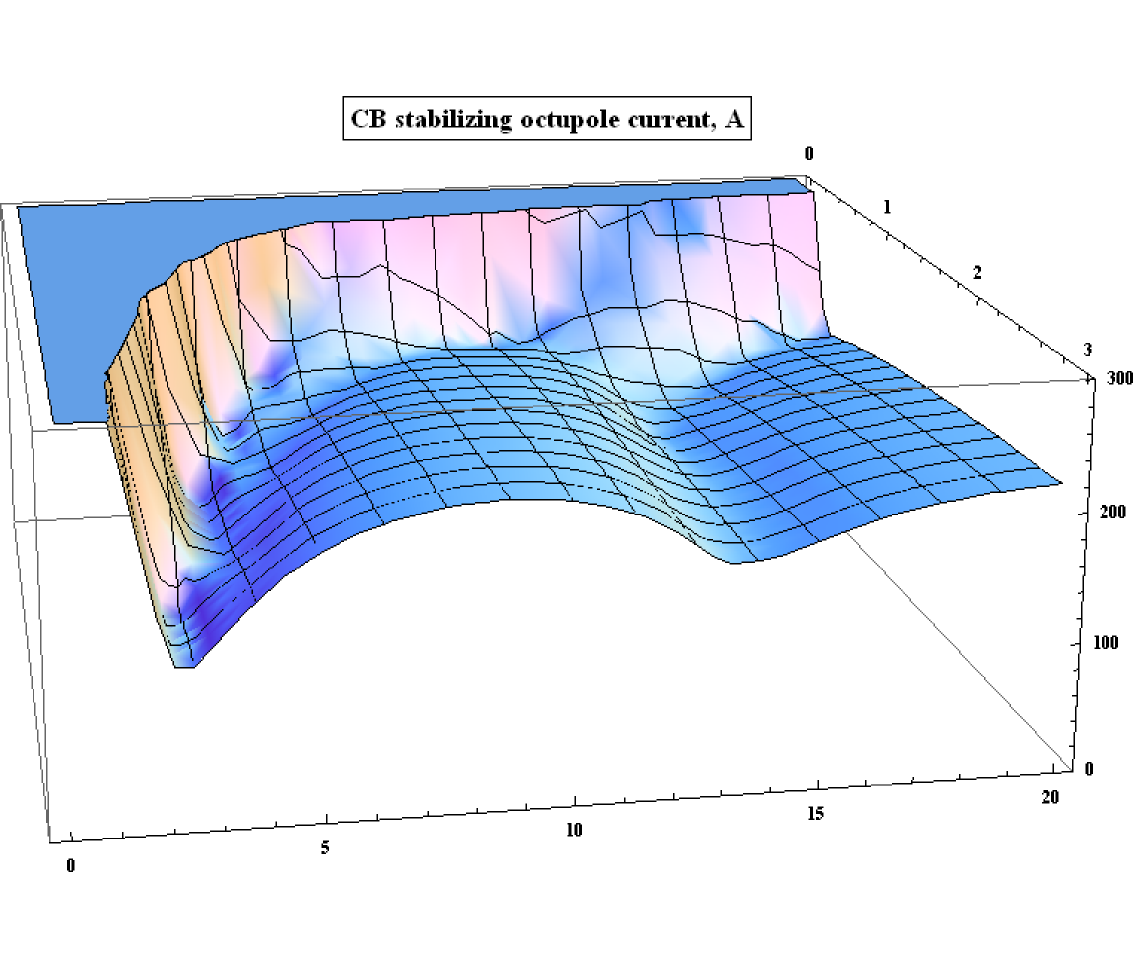


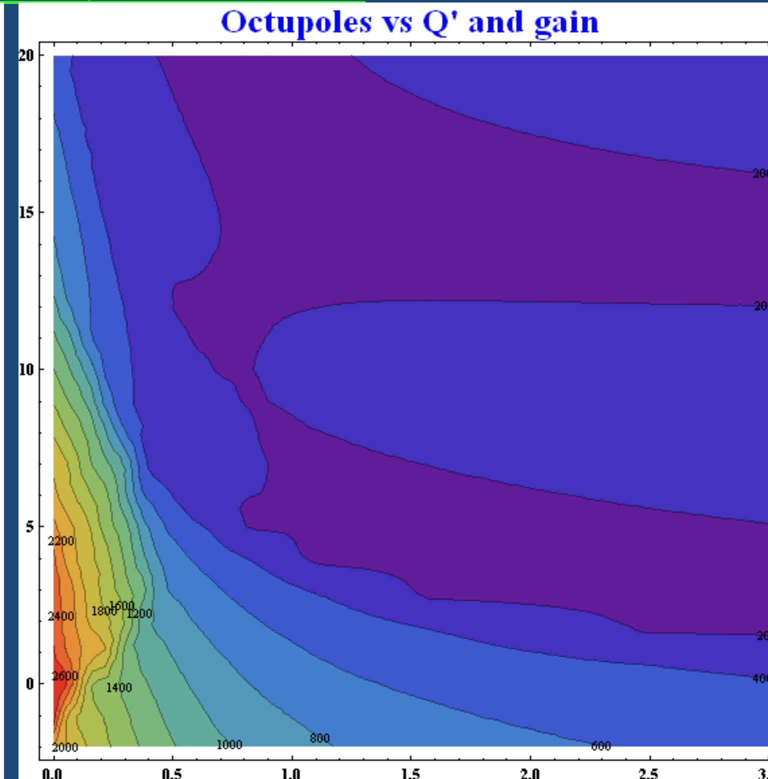
Octupoles vs Q' and gain



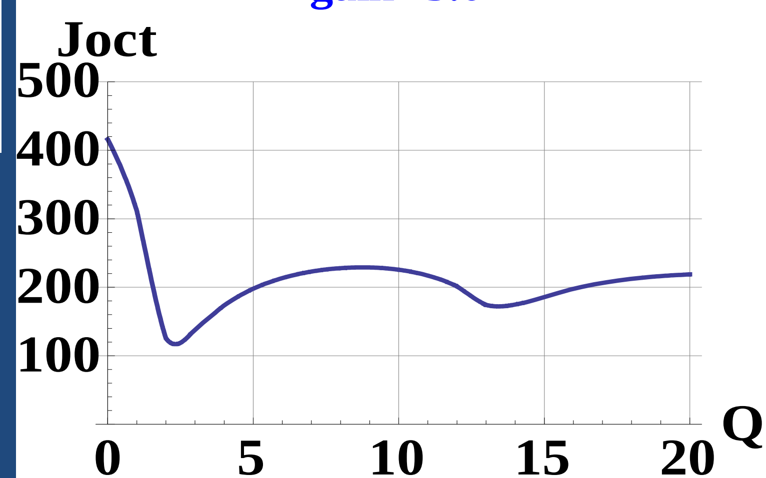
gain 1.0
Joct
Q'

4 ⊗ Impedance, Flat Gain, Ioct<0:

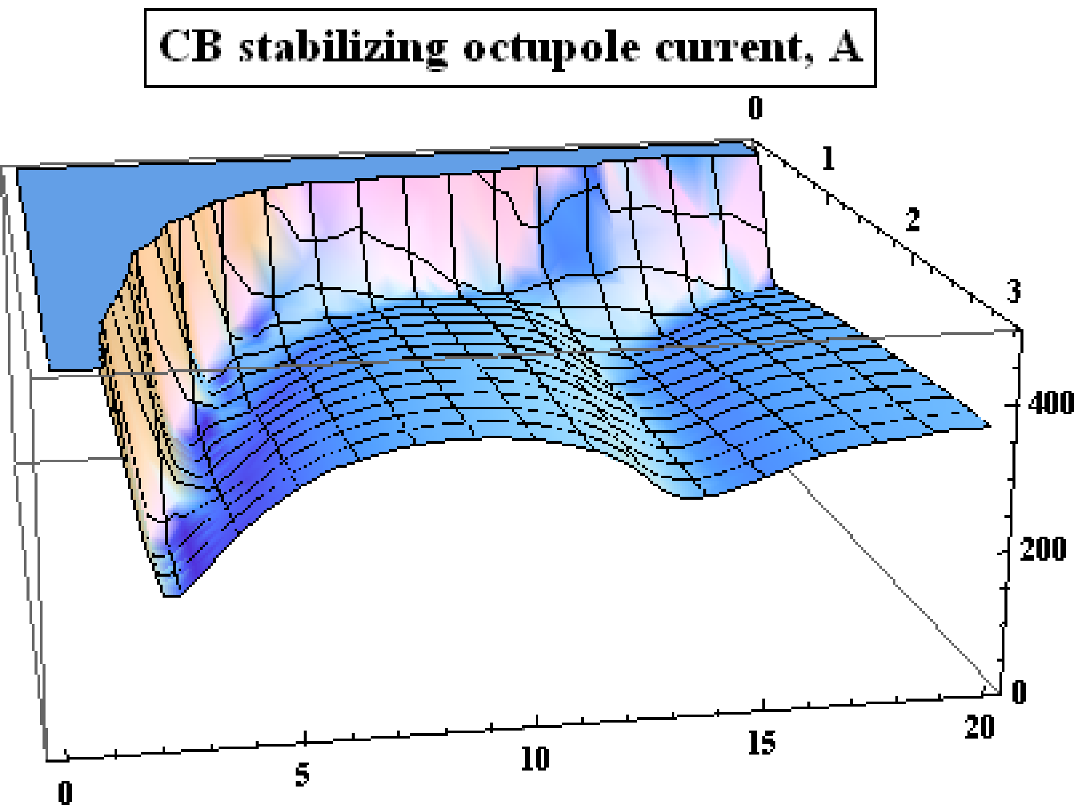


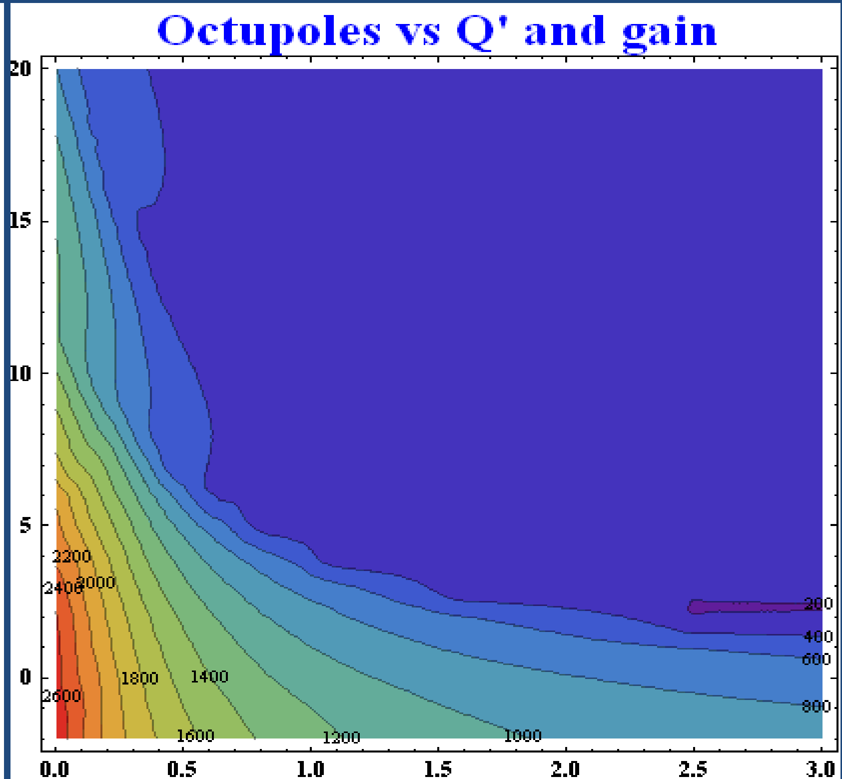


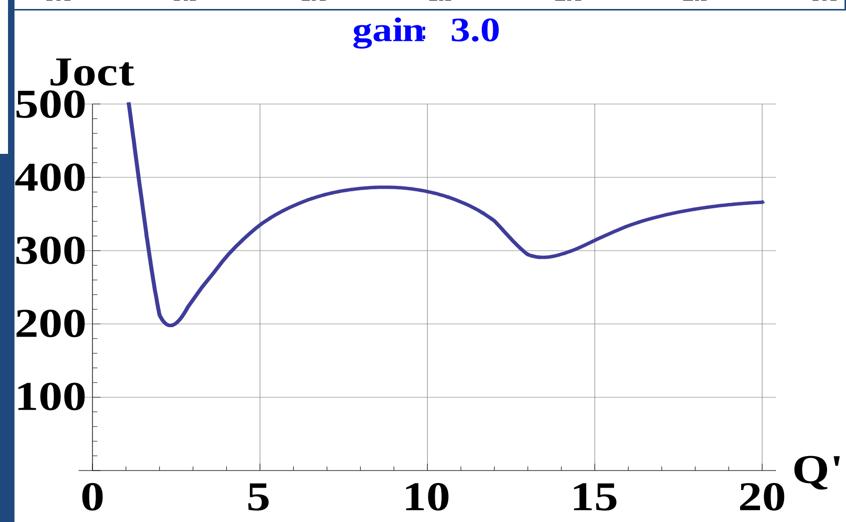
gain 3.0



4 ⊗ Impedance, Flat Gain, Ioct>0:

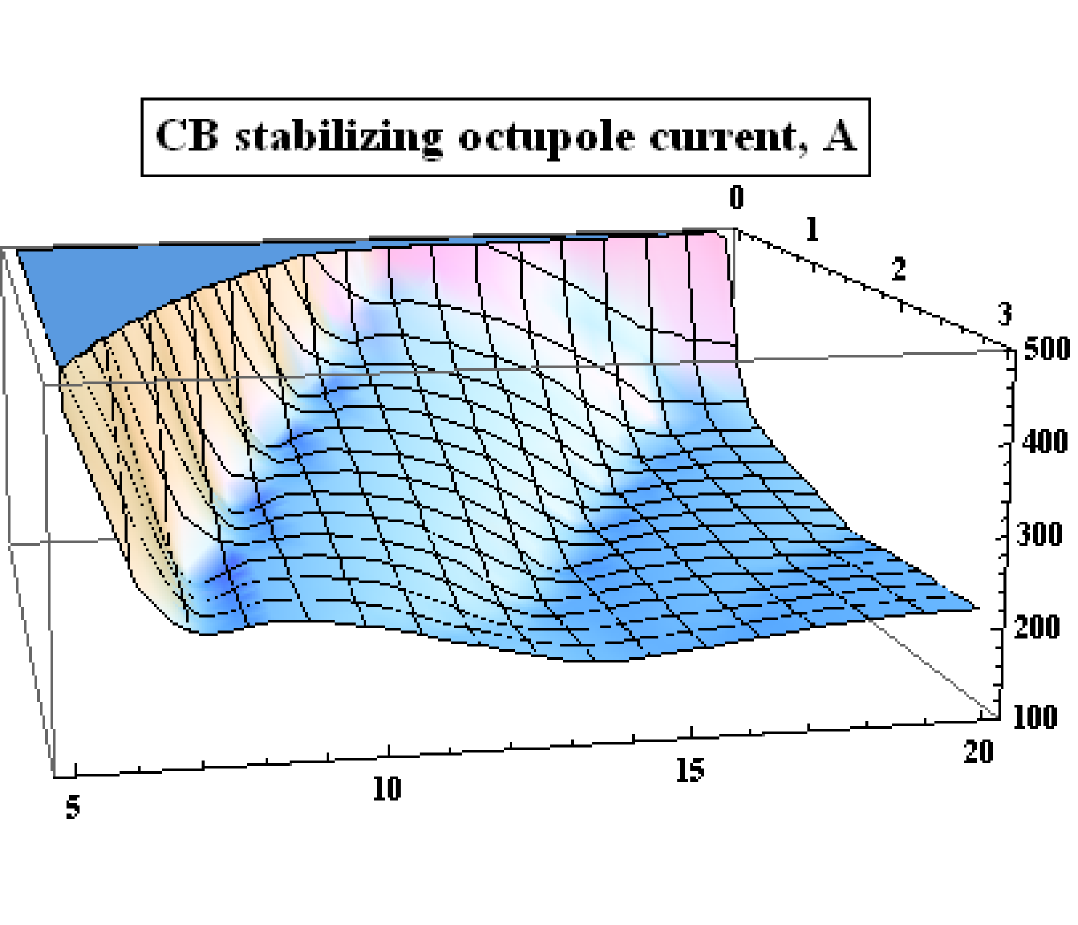


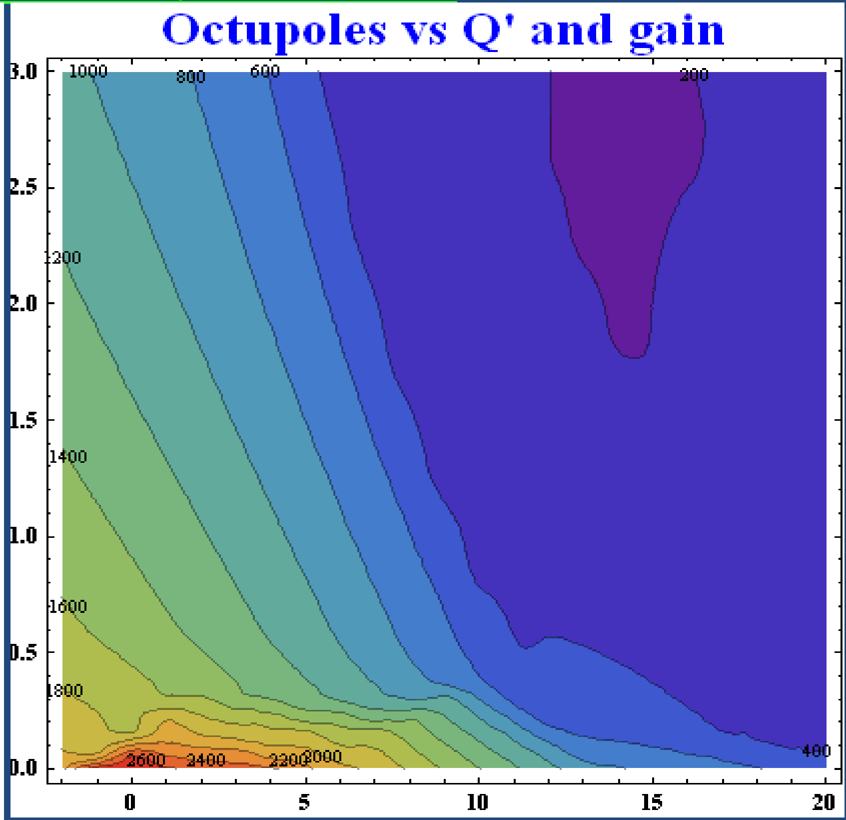


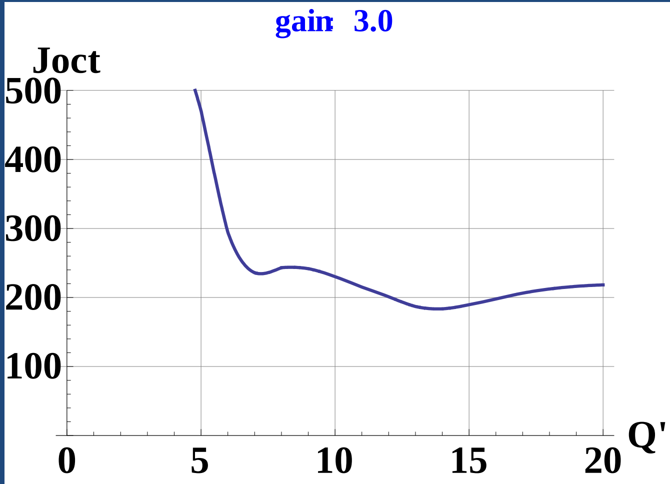


Same polarity factor ~ 1.6

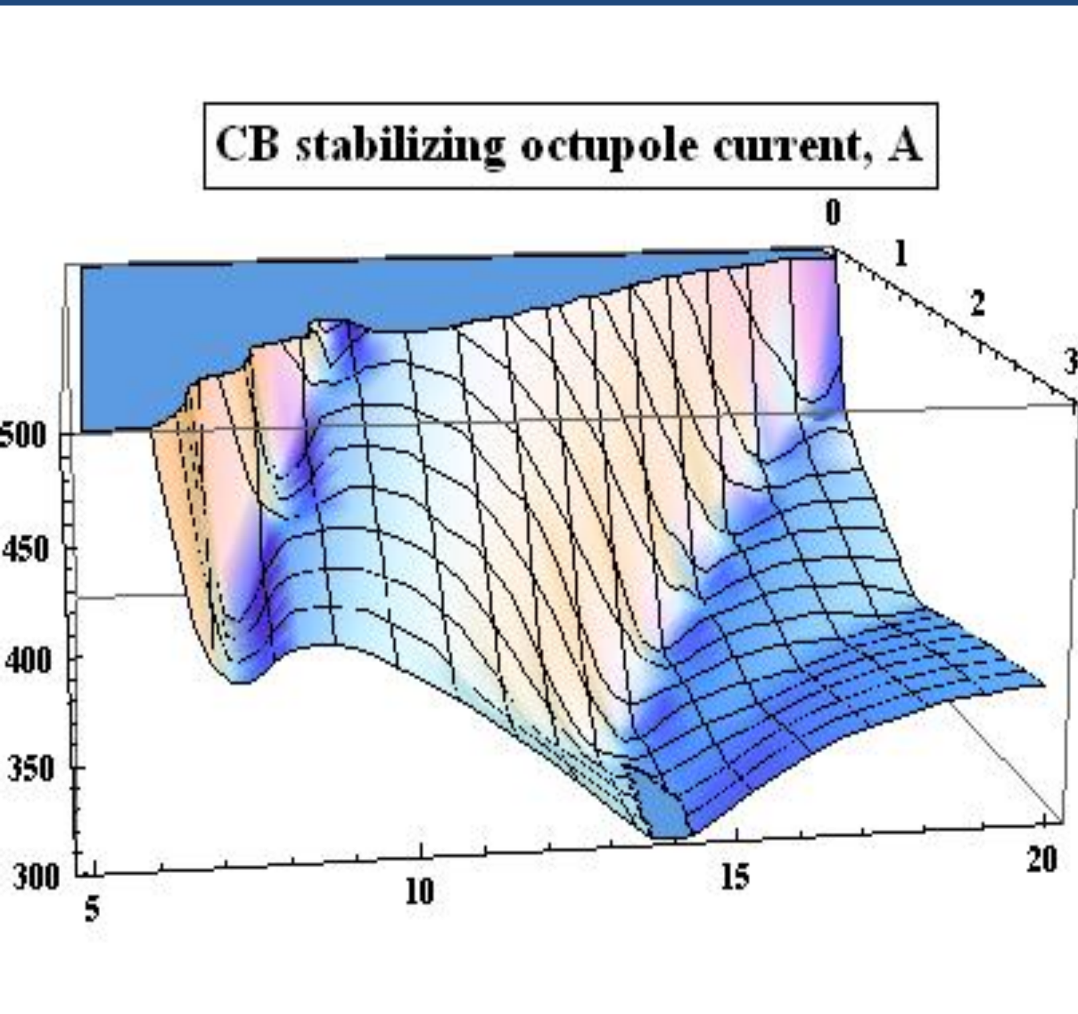
4 ⊗ Impedance, Current ADT, Ioct<0:

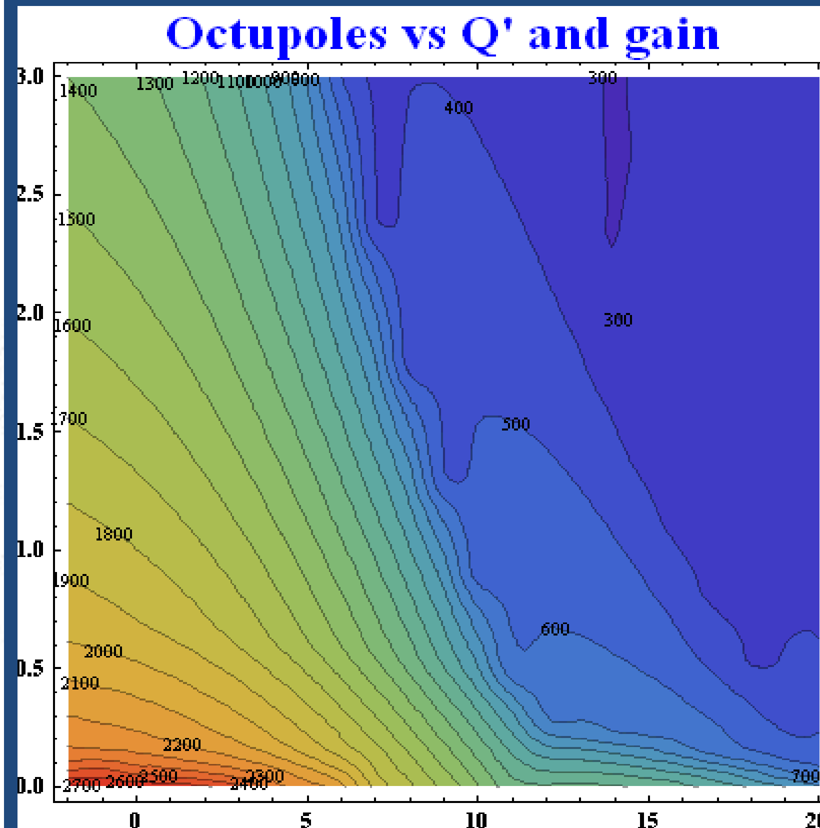


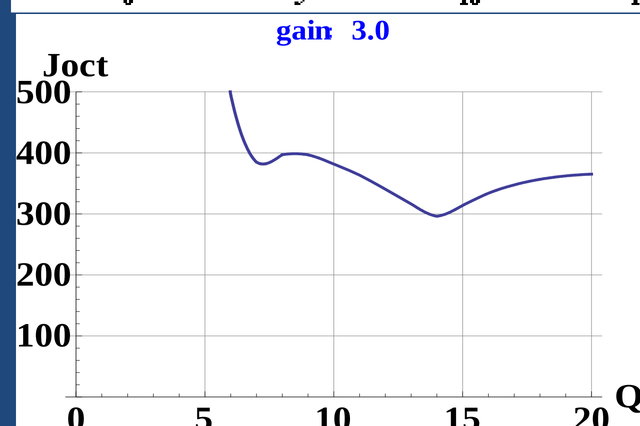




4 ⊗ Impedance, Current ADT, Ioct>0:







Summary: power of the model

- Method of nested head-tail modes (NHT) is implemented on a base of Mathematica. It allows to find coherent tunes for all the modes, solving the eigenproblem at its 3D set (azimuthal $\otimes$ radial $\otimes$ coupled-bunch).

- The external tables: impedance/wake, ADT frequency profile, stability diagram.

- Based on that, the threshold octupole current as a function of the gain amplitude and chromaticity is calculated.

- To test any new beam, impedance or gain profile, with 5 radial, 21 azimuthal and 11 representative CB modes it takes only 25 min on my 3 years old laptop. CPU time is quadratic with the number of modes and is independent of the number of bunches.

Summary: checking of the model

- A good agreement (20% or better) with a single-bunch tracking code of Simon is seen. Cross-check is ongoing with the tracking code upgrading. Cross-check with the Head-Tail code is planned.

- Some recently found disagreements with previously started Nicolas-Alexey (NA) computation (multiple bunches, no radial modes) are resolved, mistakes are corrected (showing an importance of parallel approaches!)

- A recommendation of NA to increase chromaticity and keep the max gain is confirmed by LHC operations. NHT leads to the same recommendation.

Summary: Good Gain Valley

- The damper is effective only at the valley of negative chromaticity, namely $-10 \le Q' \le 0.$ Outside this valley its effect on the threshold octupole current is vanishing. However, inside the valley, sufficiently high gain provides stability even without octupoles at all!

- To provide this valley, the gain has to (almost) flat, i.e. the damper has to be bunch-by-bunch. Otherwise its effective gain is determined by the minimal gain value at the CB band (10 MHz for 50 ns beam).

- Assuming 2 times higher impedance as nominal, the reliable area of zero-octupole stability is excluded at current ADT: the canyon is too narrow.

- For optimal flat ADT frequency profile the Good Gain Valley can be reached!

Summary: Low Octupole Plateau

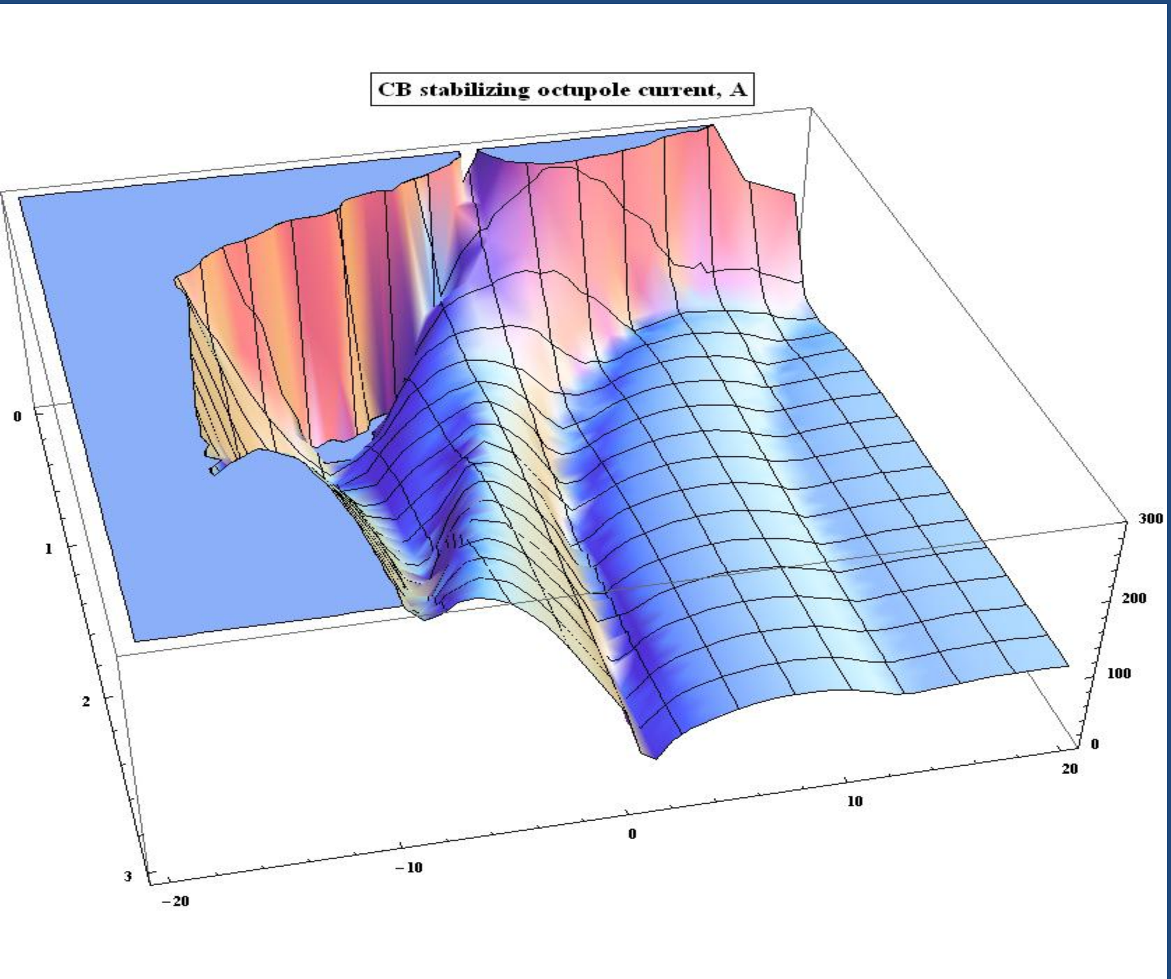
- Outside the GG valley, the damper is almost useless, stability is determined by the octupoles only.
- Lowest octupoles are at high chroma plateau $Q' \geq 12-15$.
- For focusing (positive) polarity, the required octupole current is ~ 1.6 times higher than for the defocusing (negative).
- Assuming 2 times nominal impedance, it yields +200 A for 50 ns beam with 1.5E11 p/b at the LO plateau.

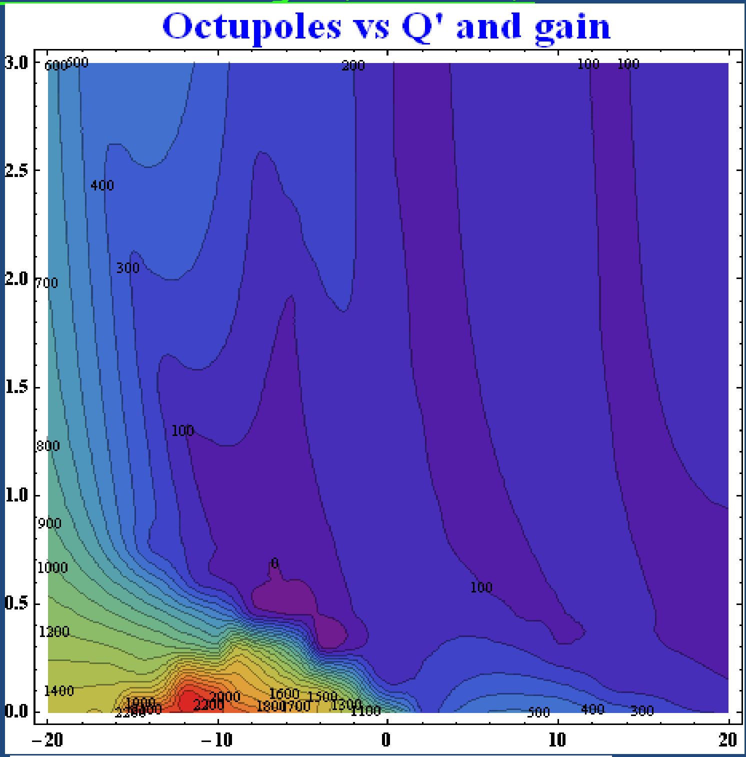
Next steps

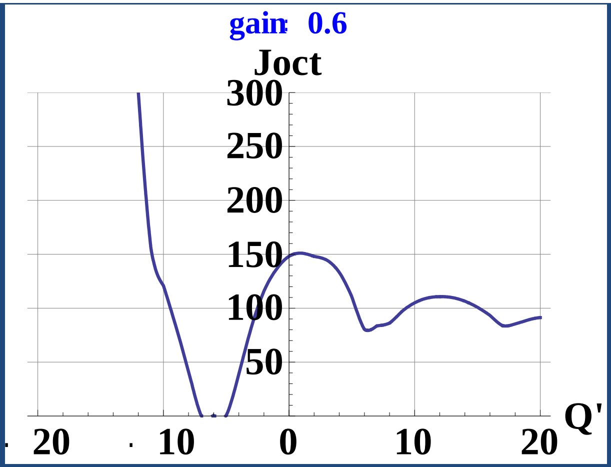
- To include train structure;
- To include quadrupolar wakes/impedances;
- To include coupled bunches in tracking code of Simon White using the same idea as for this code; to include real batches and trains there.
- At every stage to reach agreement with the parallel NA computations and Simon W. / Head-Tail tracking.
- To apply this powerful tool for PSB, PS, SPS…

Many thanks for your attention!

2*impedance, Ioct<0, gain dphase=45 deg (+foc)

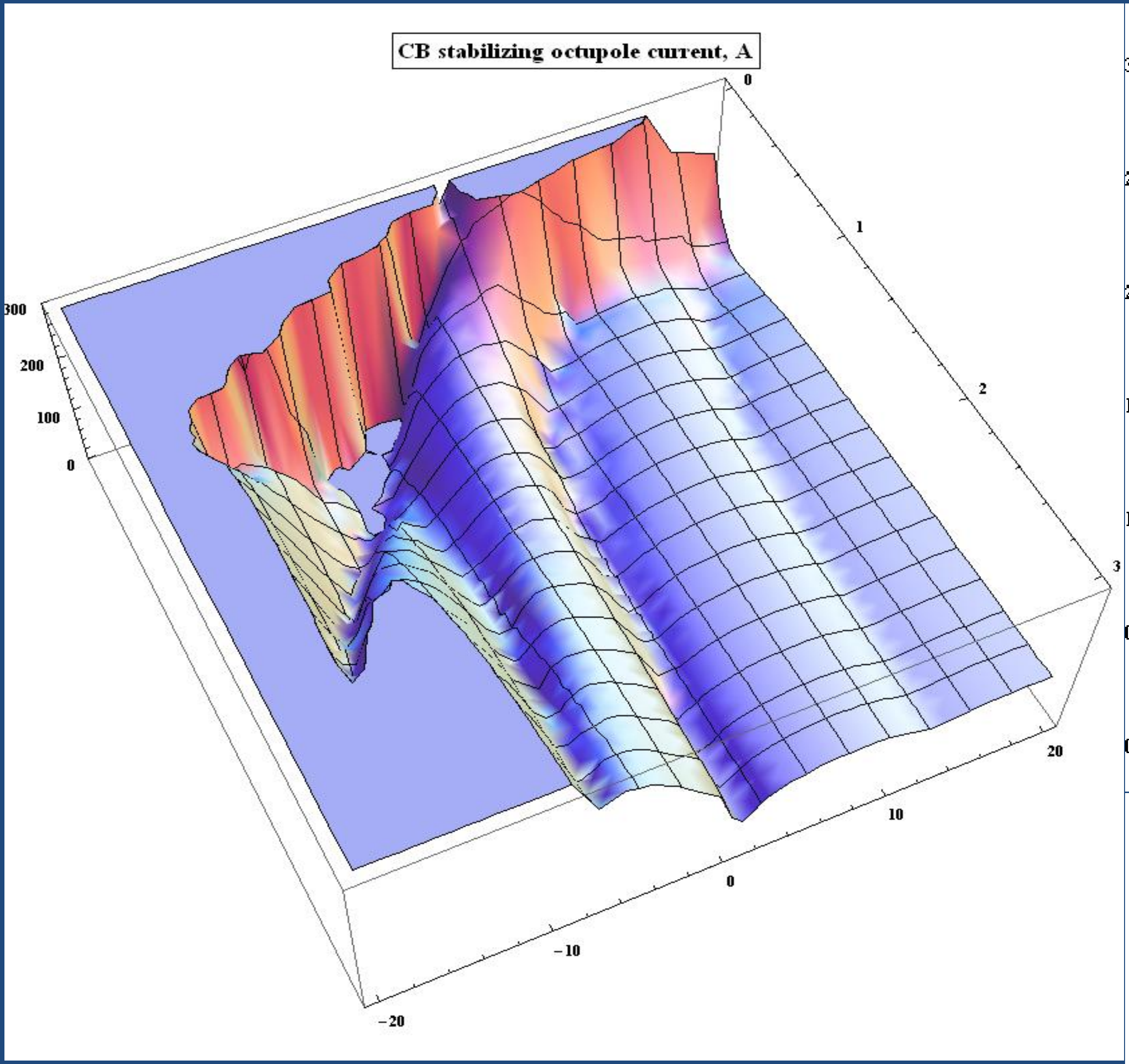


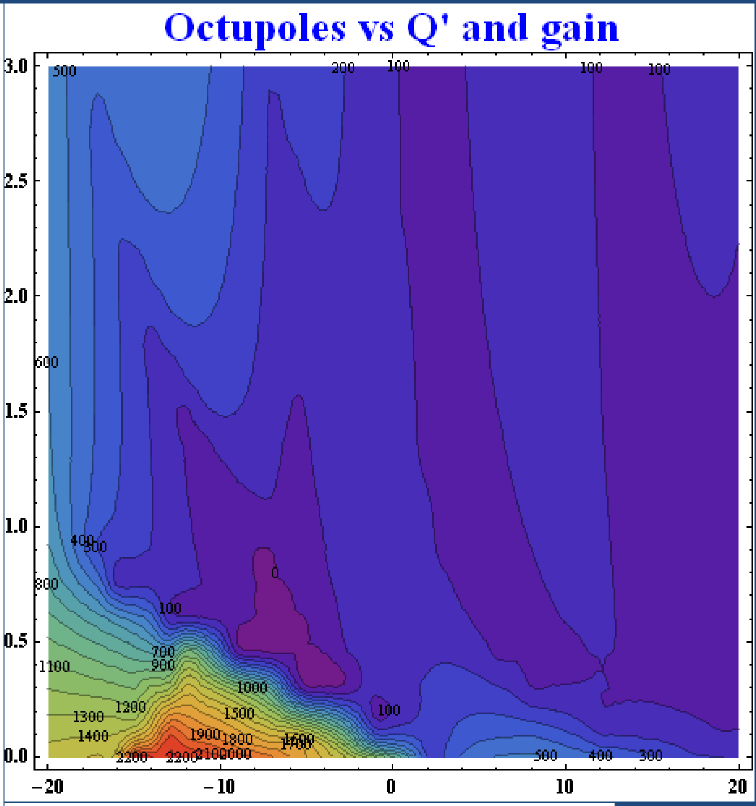


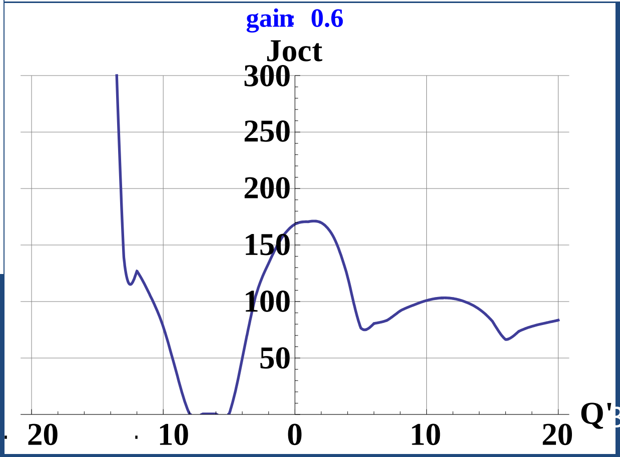


No-octupole stability area is still here! – at its threshold

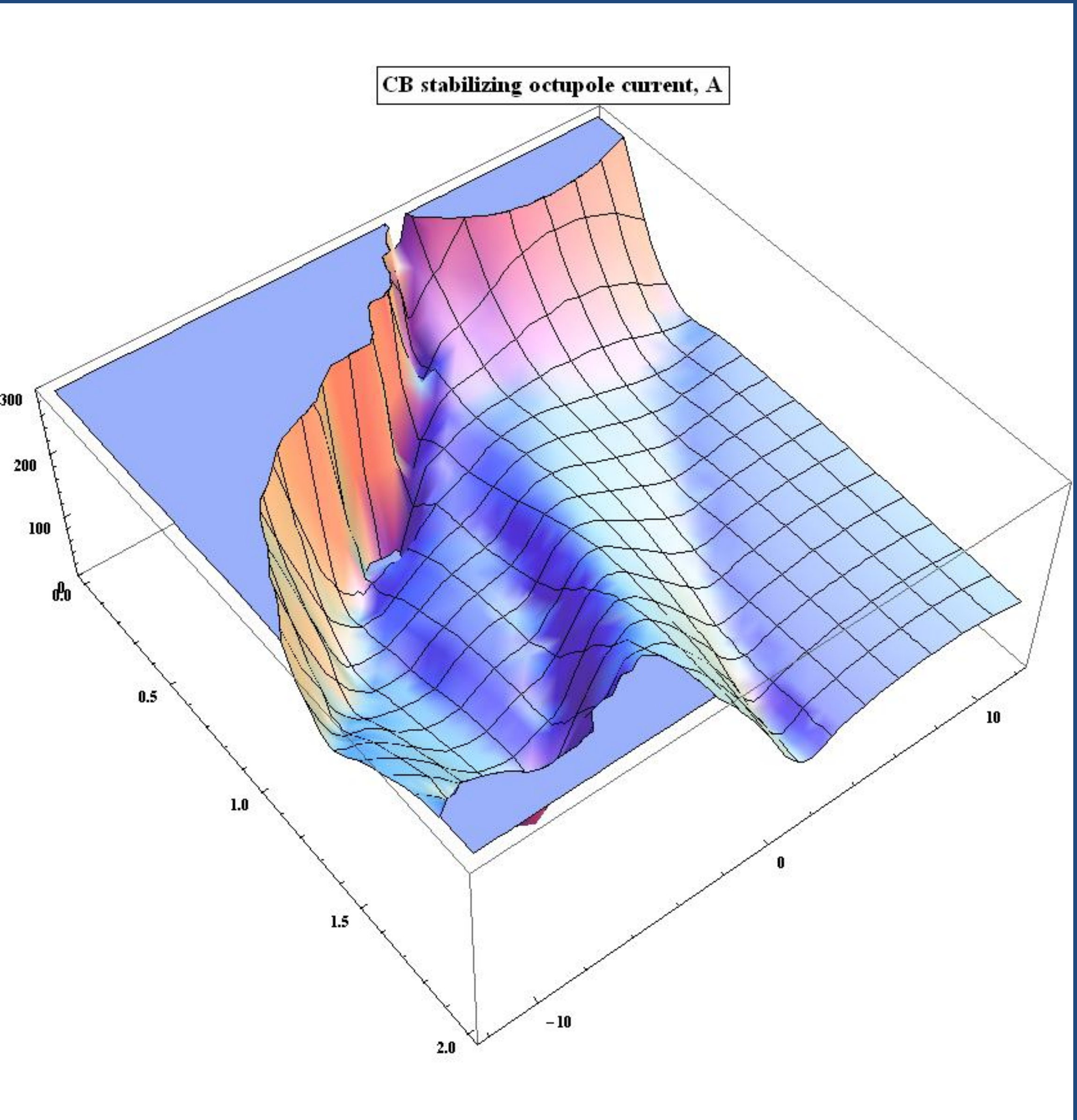
2*impedance, Ioct<0, gain dphase=+22.5 deg to foc

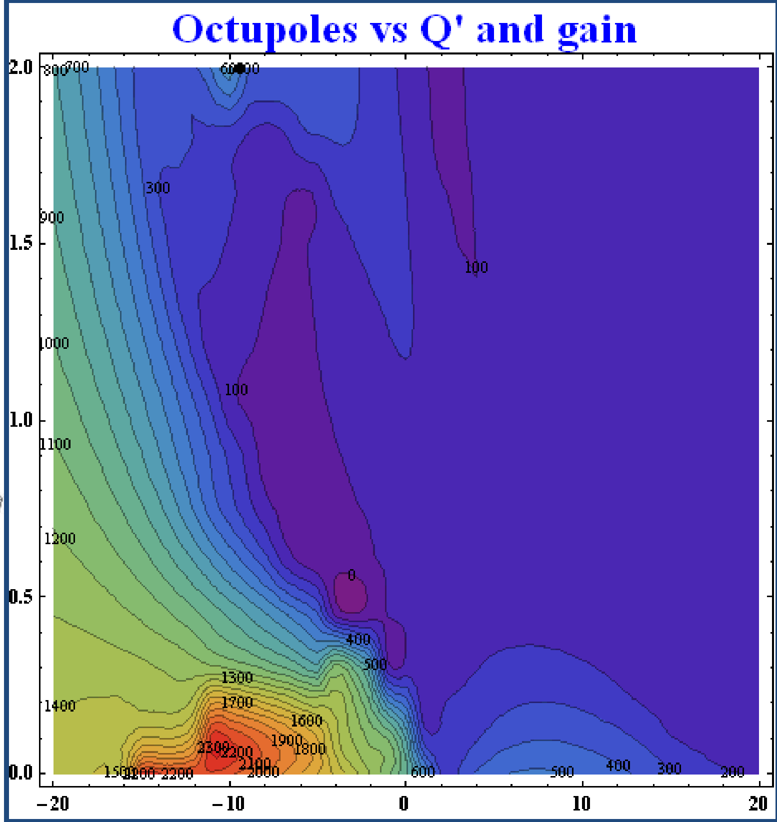




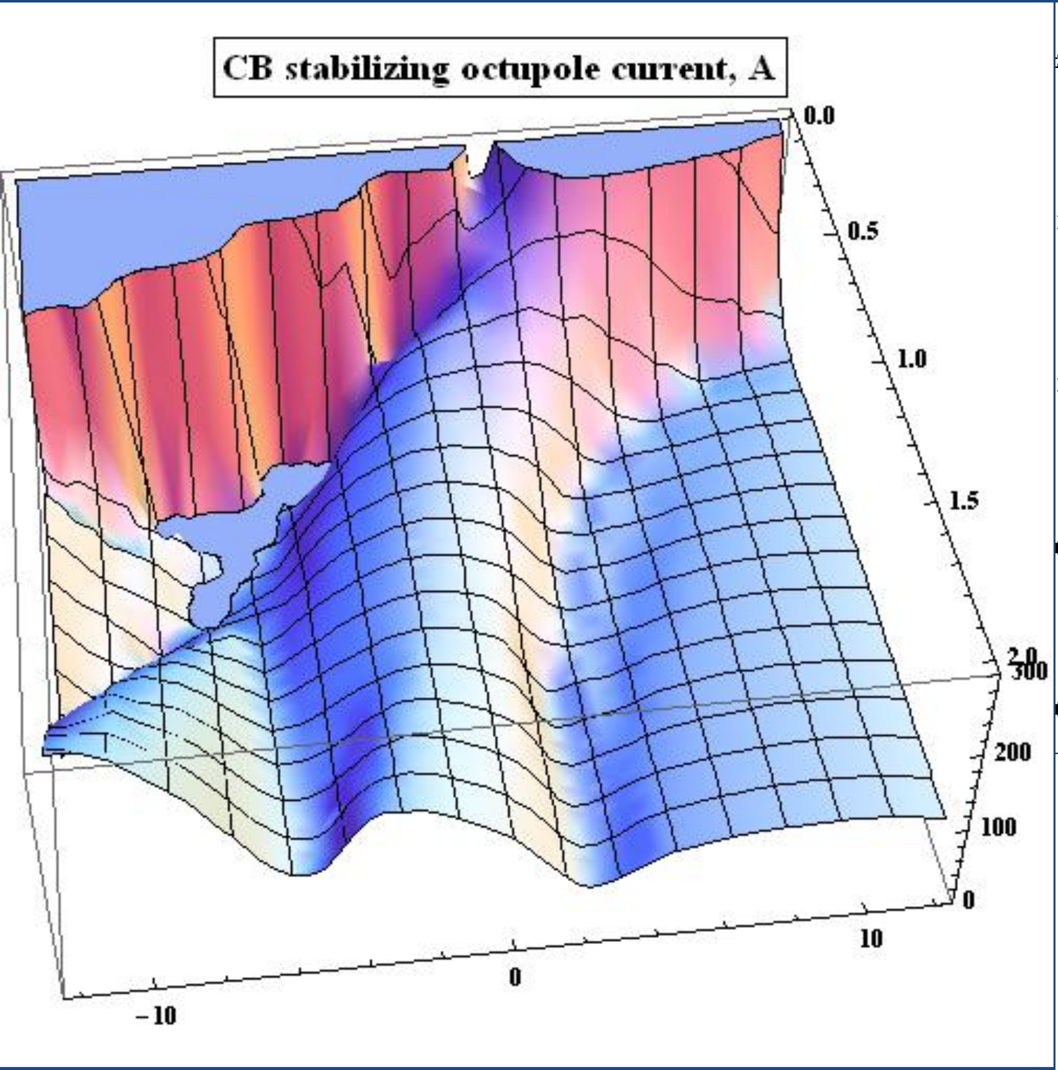


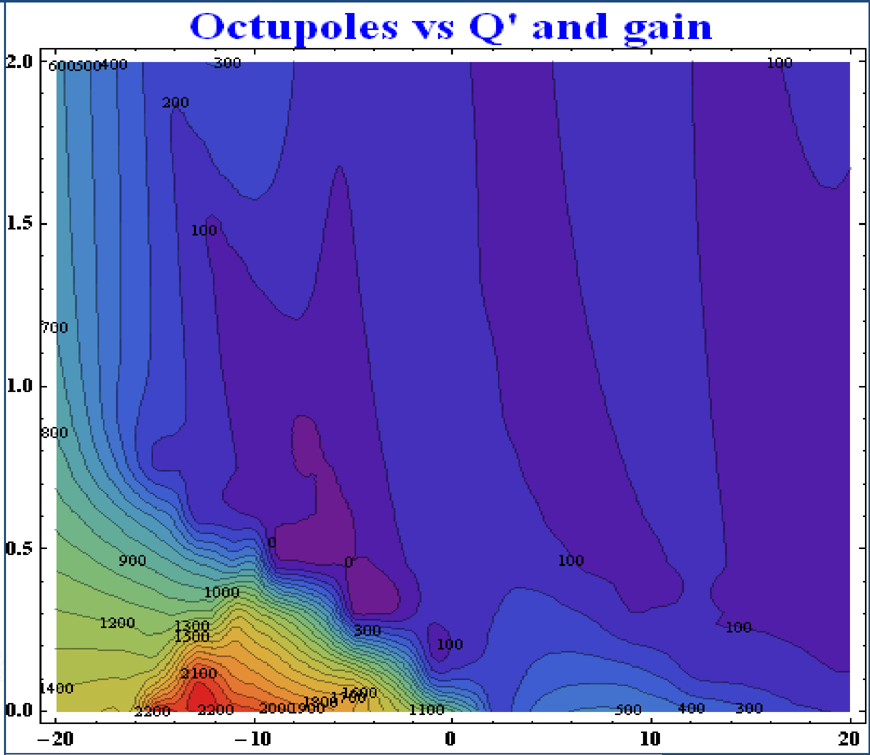
Same, gain dphase=+67.5 deg to foc

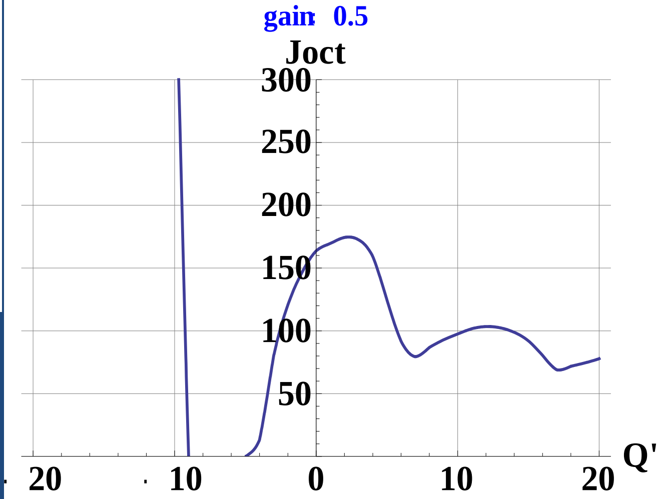




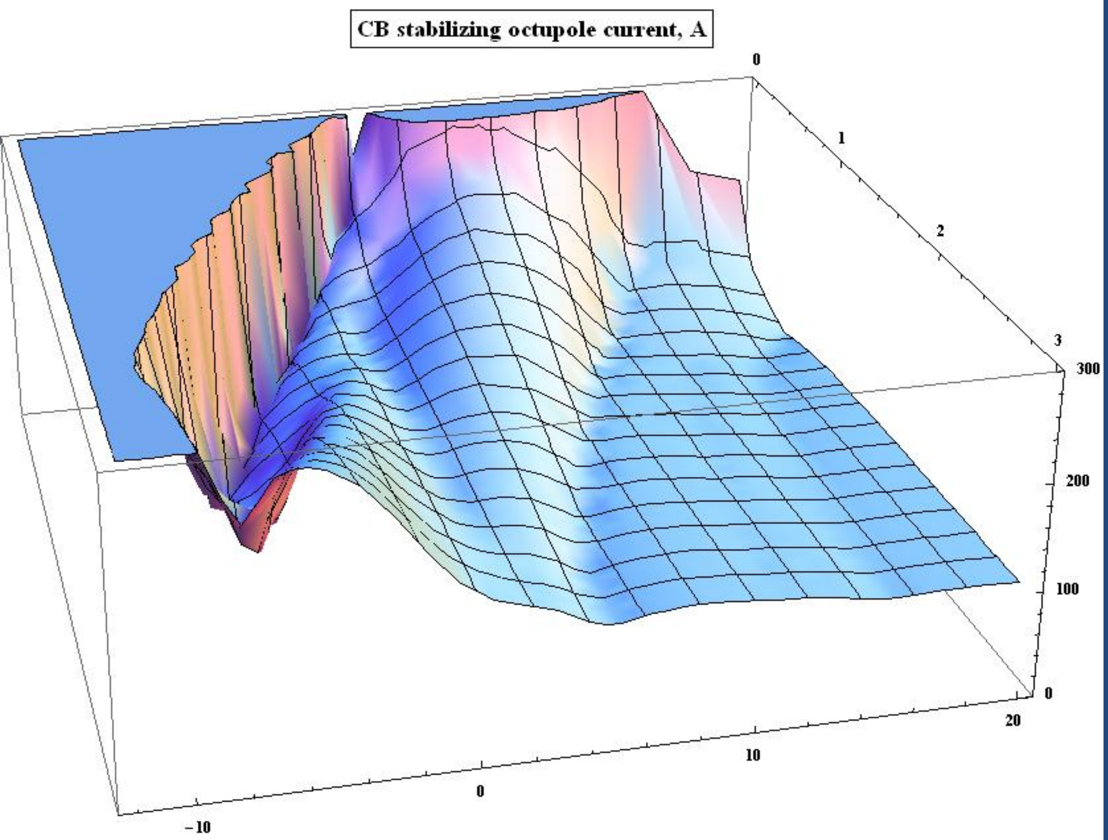
2*Impedance, gain dphase=+30 deg to foc, flat, Ioct<0

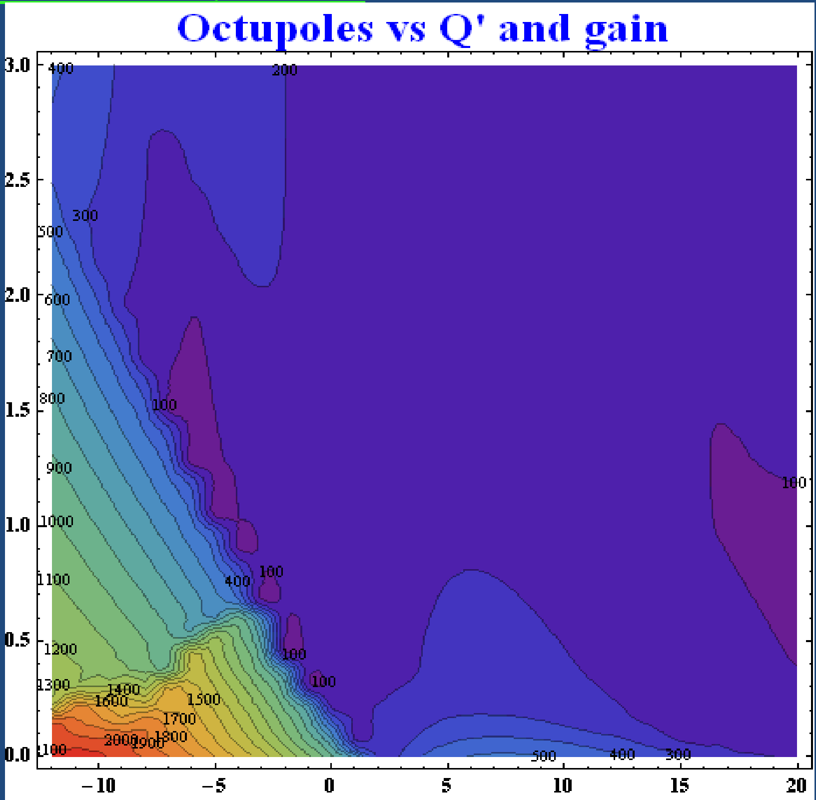


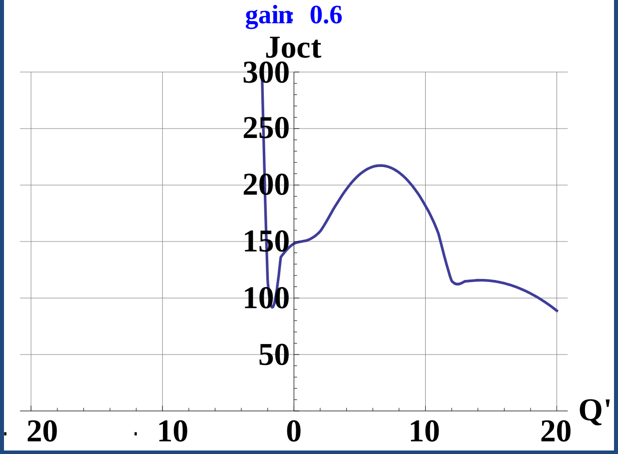




gain dphase=+30 deg to foc, ADT

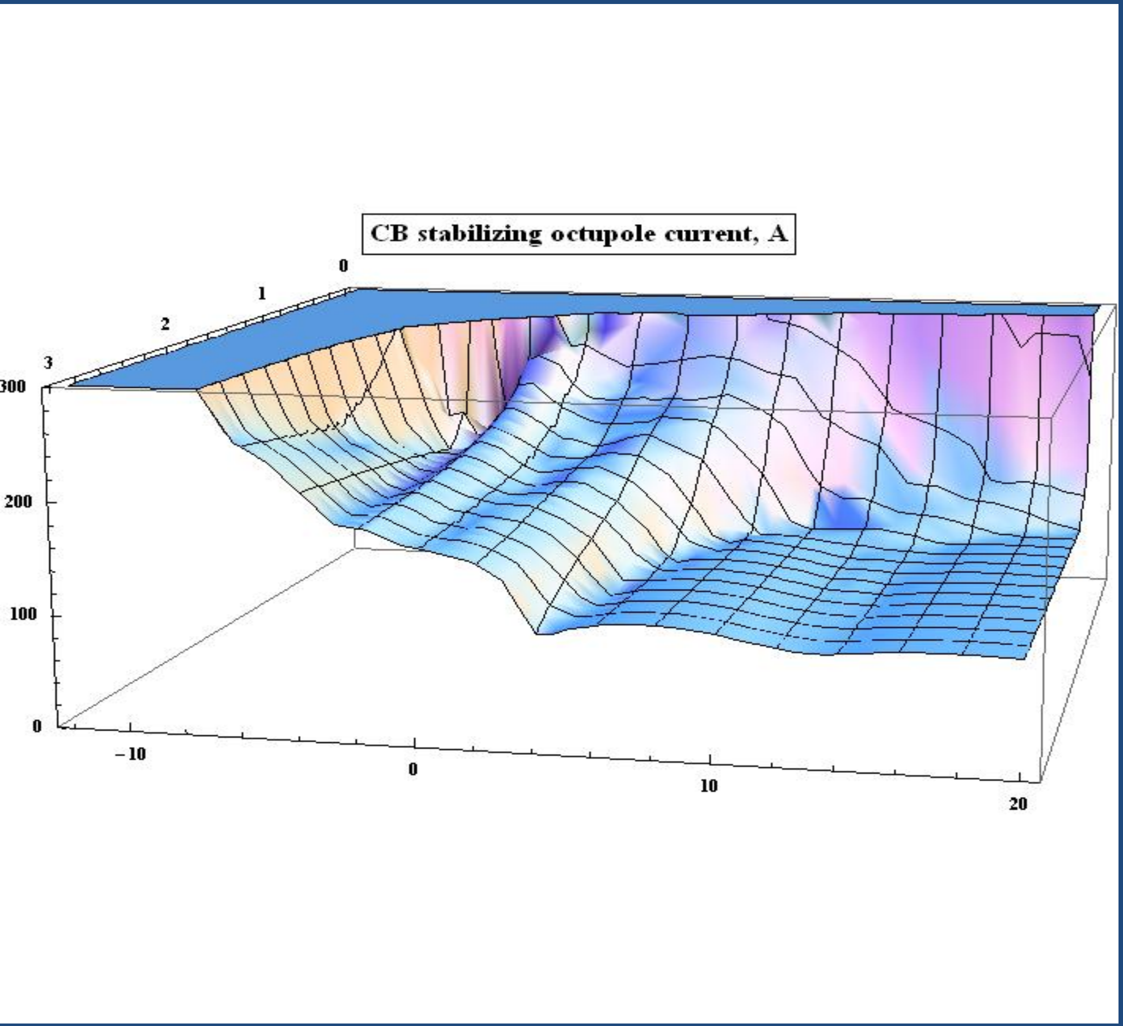


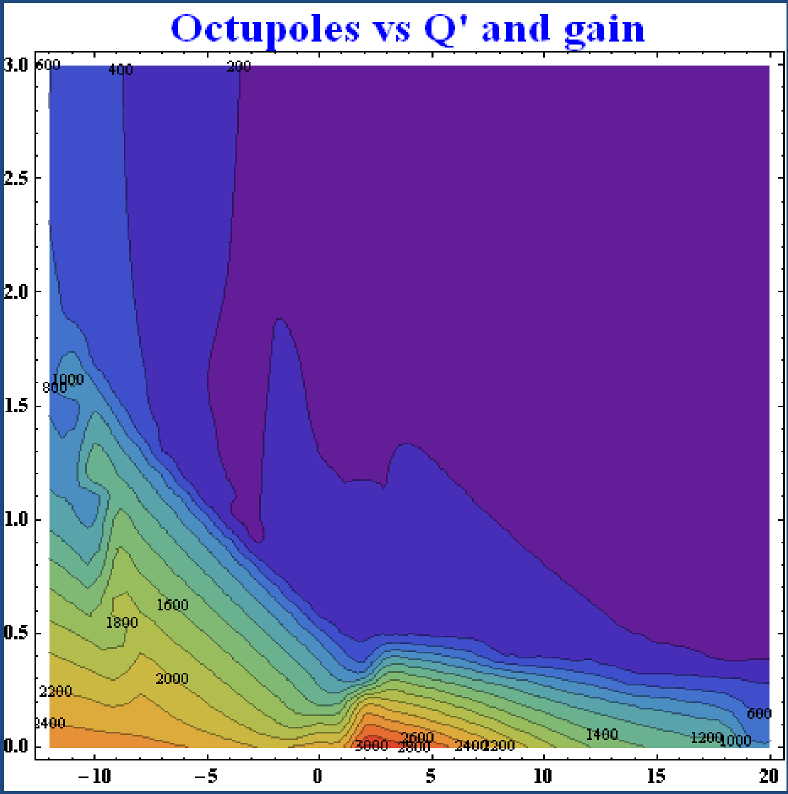


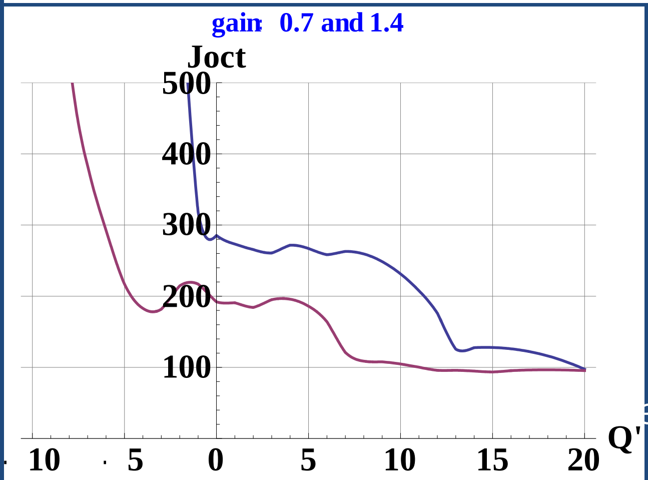


No more the golden valley.

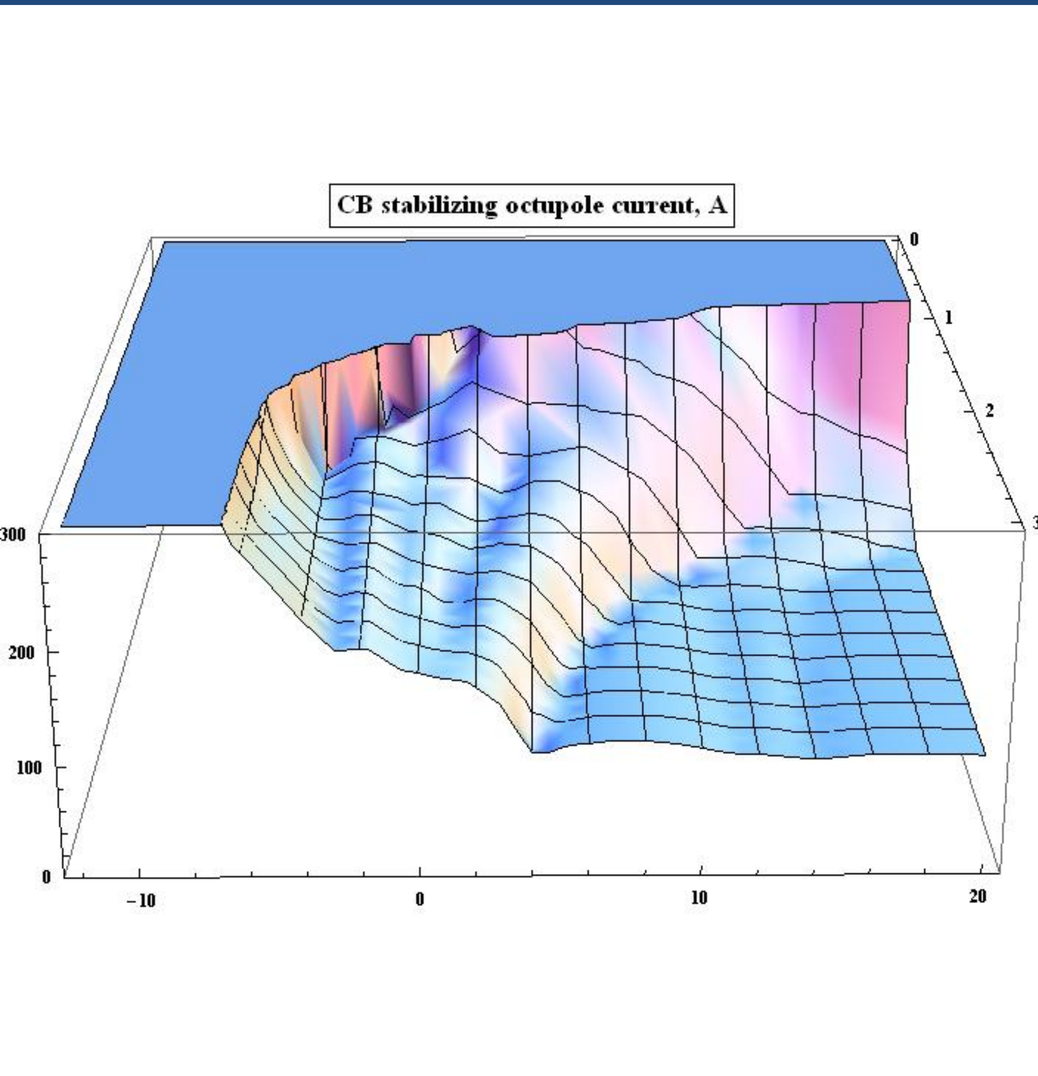
ADT, CBFactor=5, ImpFactor=2, Joct<0

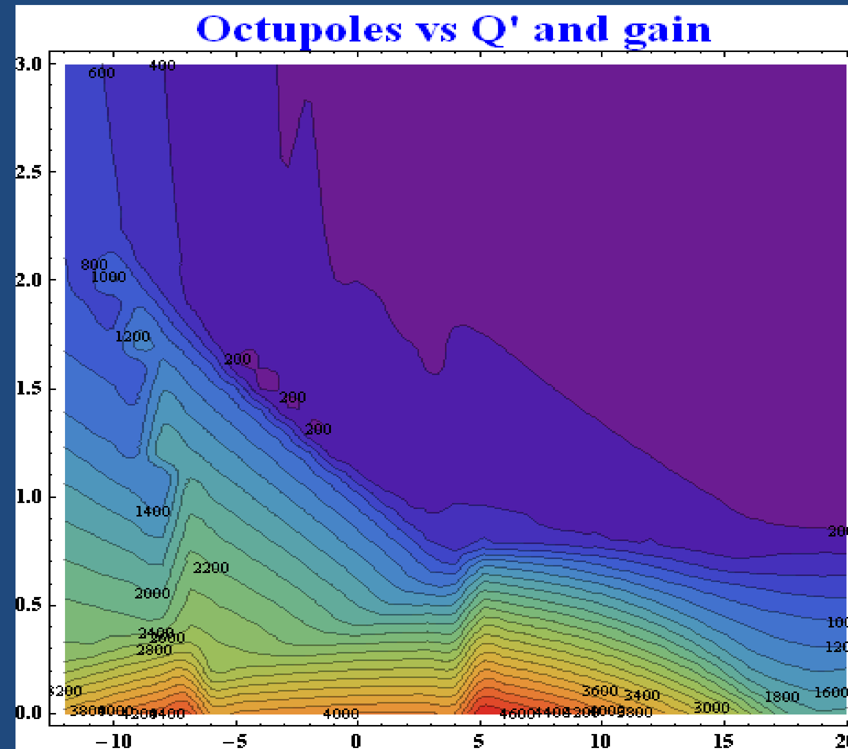


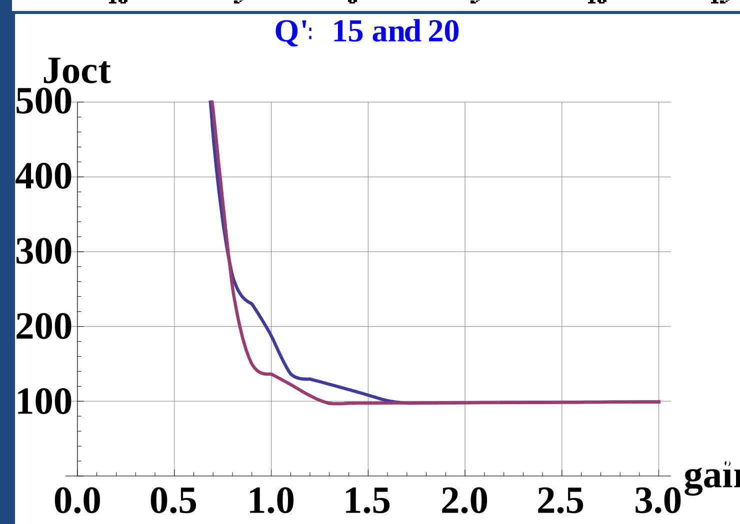




ADT, CBFactor=10, ImpFactor=2, Joct<0







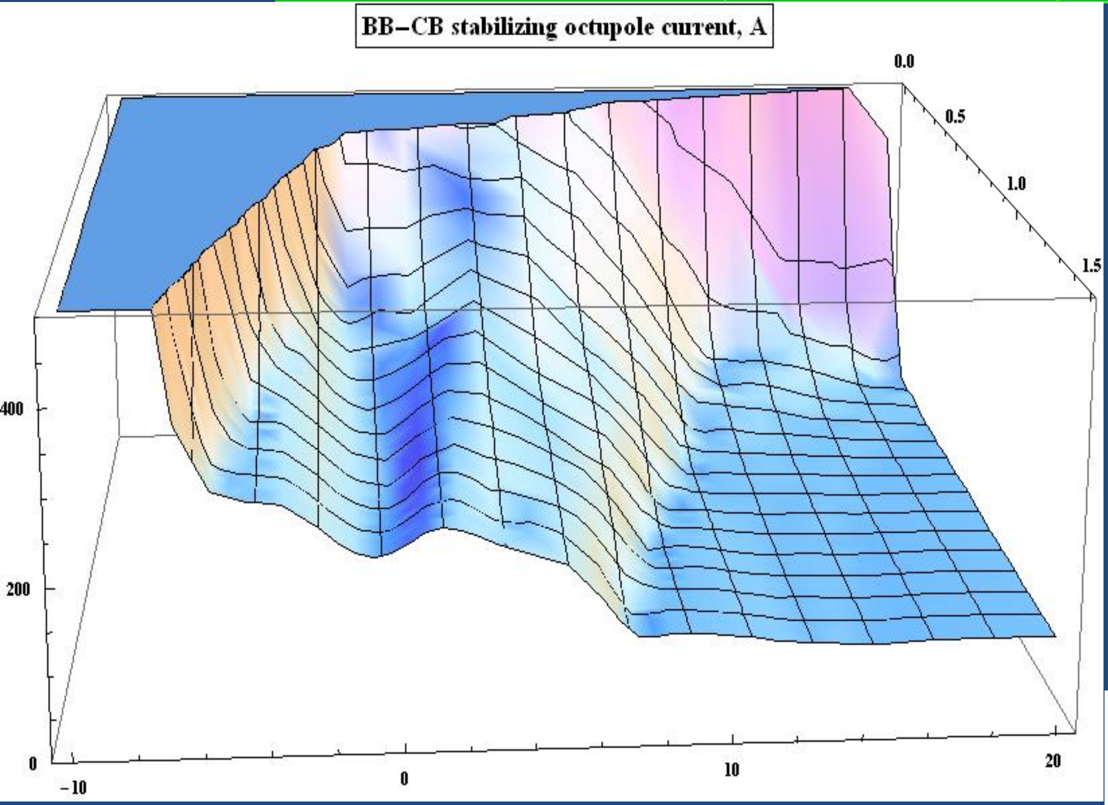
MD data analysis

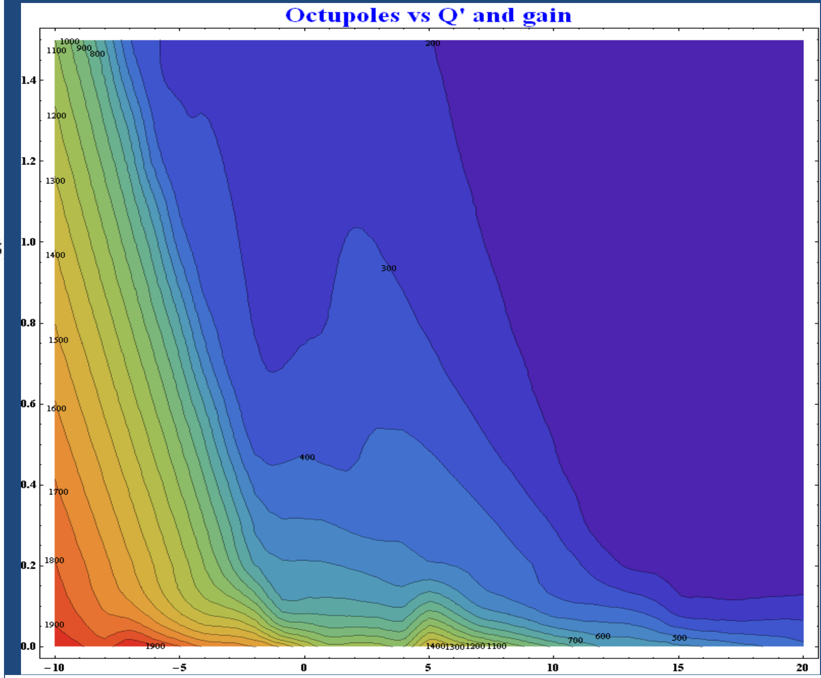
Fill Number	stage	focusing octupole current [A]	Q'	ADT damping time [turns]	hidden nonlinearity as additional octupole current, [A]	NHT computed impedance factor, Z/Z_nom
2744 (MD)	before squeeze	-100	4.1 +/- 0.3	200	-10 or -60	1.5 or 2
2771 (MD)	after squeeze	-20	4 +/- 0.5	100	-60	1.5
2771 (MD)	after squeeze	-240	-5 +/- 0.5	100	-60	2
2771 (MD)	after squeeze	-60	1 +/- 0.5	100	-60	2
2771 (MD)	after squeeze	-400	2.4 +/- 0.6	inf	-60	1.5

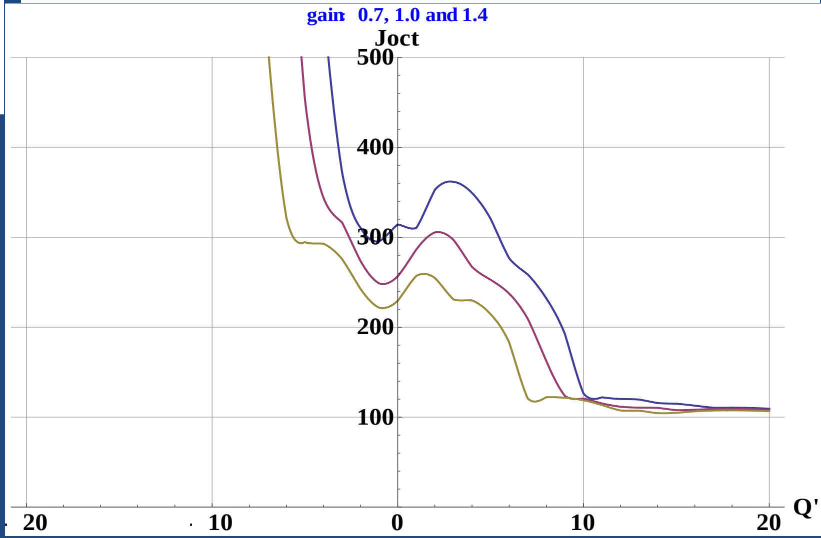
MD data on the threshold octupole currents were analyzed by NHT assuming hidden machine nonlinearity and impedance factor in comparison to the nominal of Nicolas. The best fit gives $Z / Z_{\mathrm{nom}} = 1.5 - 2.0$, and the nonlinearity $\Delta I \cong -60\mathrm{A}$ after and $\Delta I \cong -10\mathrm{A} - 60\mathrm{A}$ before the squeeze.

The data were taken for 1.4-1.5E11 p/b, 2pi mm*mrad, Beam 2 only with 1380 50ns bunches, the instability was vertical for all the cases.

2⊗(SB and CB), dQbb1=Qs, J<0, dφ15=π, Imp







Now beam-beam is included,
but it does not play a role.