Electron Cloud Instability

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Clouds and Clocks

- “My clouds are intended to represent physical systems which are highly irregular, disorderly, and more or less unpredictable. I shall assume that we have before us a schema or arrangement in which a very disturbed or disorderly cloud is placed on the left. On the other extreme of our arrangement, on its right, we may place a very reliable pendulum clock, a precision clock, intended to represent physical systems which are regular, orderly, and highly predictable in their behavior… There are lots of things, natural processes and natural phenomena, which we may place between these two extremes - the clouds on the left, and the clocks on the right.”

  Karl Popper, “Of Clouds and Clocks, an approach to the problem of rationality and the freedom of man”, to a memory of Arthur Compton.

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How to describe clouds?

- Electron clouds are irregular, poorly reproducible, very complicated phenomena. They are popperian clouds indeed. That is why it does not seem reasonable to make efforts for detailed depicting of their forms – the cloud changes faster than the artist is able to image its contour.

- We may still hope to catch roughly the main parameters of these objects, being able at least very approximately orient ourselves between them.

- The more complicated and irregular is the object, the simpler is its reasonable mathematics. Let’s try to stick with simple estimations, and do not loose main factors.

Main Factors

- E-cloud influences incoherent and coherent oscillations of beam particles in various aspects.
  - It works as a static lens, shifting up all coherent and incoherent tunes.
  - It gives a significant tune spread. With the size of the e-cloud similar to the proton beam size, the nonlinear tune spread is comparable to the tune shift. The tune spread is important for the Landau damping (LD) .
  - As a reactive medium, e-cloud works as a sort of low-Q impedance.
- Thus, e-cloud introduces both the impedance and LD.

Who of these antagonists is going to win?

Static focusing

- We assume the relevant e-cloud transverse size equal to the proton size. The incoherent tune shift follows:

$$\delta\nu_{pe} \simeq \frac{\pi n_e r_p R^2}{\gamma \nu_b}$$

- The rms spread of the tune shifts is assumed comparable to its average value.

- Tunes for all the beam coherent modes are going up as well due to this, by the same value.

Wake function

- Following [Burov & Dikansky,1997], e-cloud wake can be modeled as a low-Q resonator:

$$W(\tau) \simeq W_0 \sin(\omega_e \tau) \exp\left(\omega_e \tau / 2Q\right);$$

$$W_0 = \frac{4\pi n_e r_e c L}{a^2 \omega_e}, \quad Q \sim 2-3, \ \tau < 0$$

equivalent to a shunt impedance $\frac{R_s}{Q} \simeq Z_0 \frac{n_e r_e c L}{a^2 \omega_e}; \ Z_0 = \frac{4\pi}{c} = 377\,\mathrm{Ohm}$

Here n_e is the average e-cloud density inside the proton beam size of the radius a, L is the length of the e-cloud affected part of the machine (typically 50-100% of the entire circumference) and

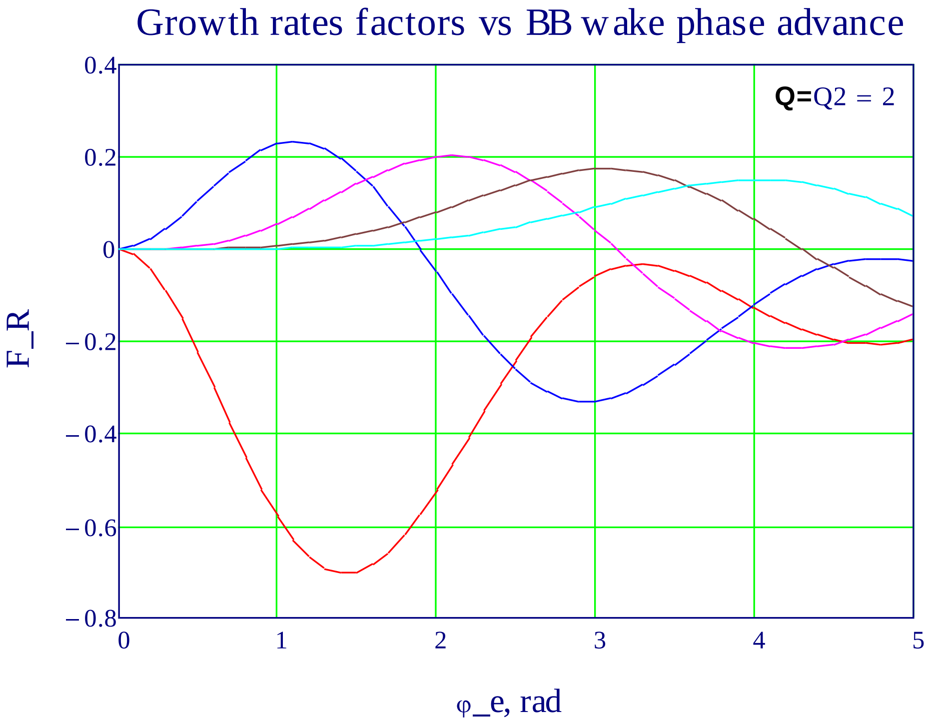
$$\omega_e = (c/a)\sqrt{N_b r_e / \sqrt{2\pi}\sigma_z}$$

is the frequency of electron oscillations in the space charge field of the bunch with rms length σ_z.

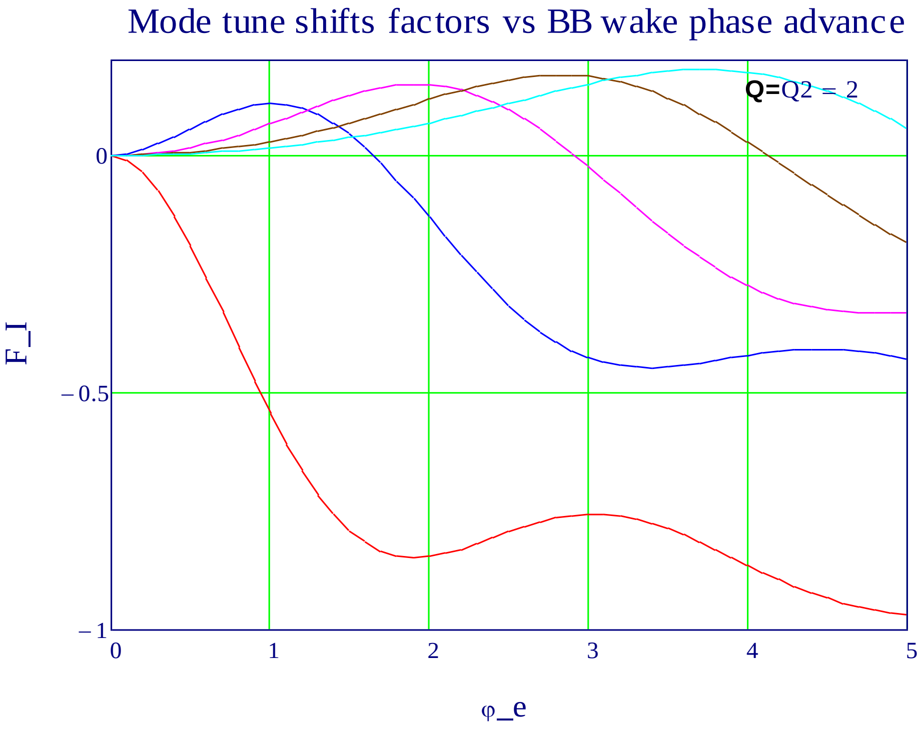
Weak Head-Tail (WHT)

- Application this wake function to the WHT tune shift and growth rate (A. Chao, Eq. 6.213, air-bag) results in (HT phase $\chi \le 1$) :

$$\mathrm{Im}[\Delta \nu_m] = \chi \delta \nu_{pe} F_R(m,\phi_e); \quad \mathrm{Re}[\Delta \nu_m] = \delta \nu_{pe} F_I(m,\phi_e); \quad \phi_e = \omega_e \sigma_z / c$$



Growth rates factors vs BB wake phase advance



Mode tune shifts factors vs BB wake phase advance

$$F_R(m,Q,\phi) = 3\int_0^{\infty} \frac{J_m(x\phi)J'_m(x\phi)}{1+Q^2(x-1/x)^2}\frac{dx}{x}; \quad F_I(m,Q,\phi) = \frac{3}{2}\int_0^{\infty} \frac{Q(x-1/x)J_m^2(x\phi)}{1+Q^2(x-1/x)^2}\frac{dx}{x}$$

WHT (2)

- These rates and phase advances have to be compared with the LD.
- Without space charge, LD is driven by the e-cloud rms tune spread

$$\delta_2 \nu_{pe} \simeq (0.3-0.5)\delta \nu_{pe}$$

- Thus, it can be concluded:
  - HT mode 0 can be unstable: its tune shift can exceed the spread, and its rate can be high. But for the proper sign/value of the chromaticity it is damped. Also it can be damped by a damper.
  - Higher HT modes are damped by LD – at least at $\chi < 1$, since $\delta_2 \nu_{pe} > \mathrm{Im}[\Delta \nu_m]$
  - Thus, all the HT modes can be damped (most likely are L-damped at any $\chi > 0$).
  - If there is SC so high that $\delta \nu_{sc} > \delta \nu_{pe}$, it kills LD, and WHT becomes possible.

TMCI

- For the TMCI, comparing the tune shifts of 0th and -1st modes (see the previous plot), we see that it happens at

$$\delta\nu_{pe} \simeq \nu_s / |F_I(0,\phi_e) - F_I(-1,\phi_e)| \simeq (2-3)\nu_s$$

also showing that the LD is going to dominate over the wake.

- SC does not matter if it is weak, $\delta\nu_{sc} < \nu_s$.

- Otherwise, it prohibits TMCI up to $m \simeq +\delta\nu_{sc} / \nu_s$. For those modes though LD catches up:

$$m\delta\nu_{pe} / \delta\nu_{sc} \simeq \delta\nu_{pe} / \nu_s \geq 1$$

- Thus, TMCI appears to be impossible with SC either.

Beam Breakup (BBU)

- For high enough e-cloud density the instability can be fast compared to the synchrotron time, so it is going to be BBU.

- The question is – can BBU win over the nonlinearity?

- Applying the coasting beam formula (A. Chao, Eq. 6.262), assuming the coasting beam line density equal to the beam peak density, we get for the most unstable wave number $k \simeq \omega_e / c$:

$$\mathrm{Im}[\Delta\nu_{\mathrm{bbu}}] \simeq 4\delta\nu_{pe}$$

or order of magnitude higher than the estimated LD contribution!

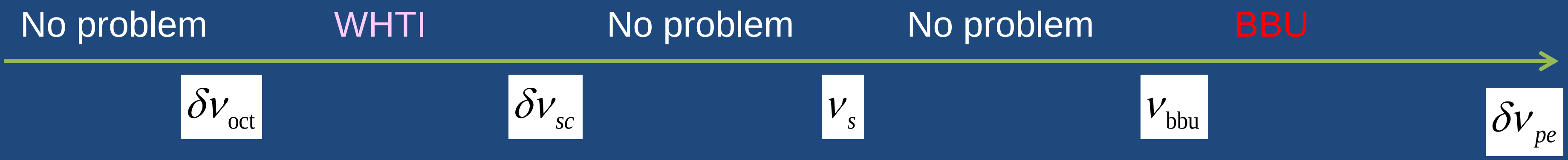
- Thus, BBU is going to kill the beam as soon as $\delta\nu_{pe} \gg \nu_s$.
- It can be stabilized by high chromaticity

$$\xi\delta p / p \simeq (1-3)\delta\nu_{pe}$$

Conclusions

1) $\delta\nu_{sc} < \nu_s$:

$$\nu_{\text{bbu}} \simeq 10\max\left(\xi\delta p/p, \nu_s\right)$$



No problem

$\delta\nu_{\text{oct}}$

WHTI

$\delta\nu_{sc}$

No problem

ν_{s}

No problem

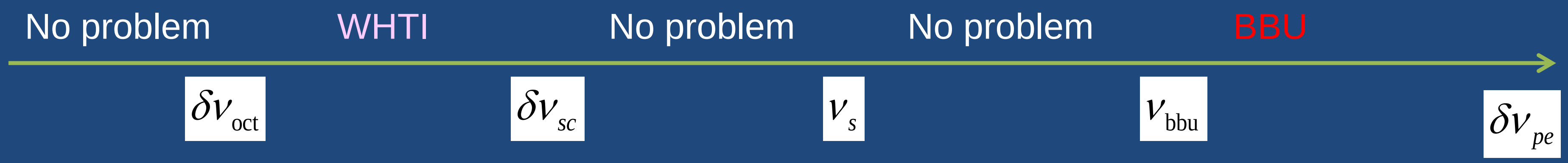
ν_{bbu}

BBU

$\delta\nu_{pe}$

Conclusions

1) $\delta\nu_{sc} < \nu_s$:



No problem WHTI No problem No problem BBU

$\delta\nu_{\text{oct}}$ $\delta\nu_{sc}$ ν_s ν_{bbu} $\delta\nu_{pe}$

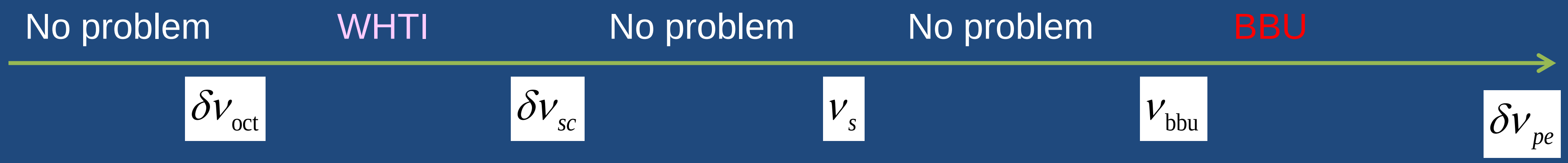
2) $\nu_s < \delta\nu_{sc} < \nu_{\text{bbu}}$:

No problem WHTI WHTI No problem BBU

$\delta\nu_{\text{oct}}$ ν_s $\delta\nu_{sc}$ ν_{bbu} $\delta\nu_{pe}$

Conclusions

$1)\quad \delta v_{sc} < v_s :$



No problem

WHTI

No problem

No problem

BBU

δv_{oct} δv_{sc} v_s v_{bbu} δv_{pe}

$2)\quad v_s < \delta v_{sc} < v_{\mathrm{bbu}} :$

No problem

WHTI

WHTI

No problem

BBU

δv_{oct} v_s δv_{sc} v_{bbu} δv_{pe}

$3)\quad \delta v_{sc} > v_{\mathrm{bbu}} :$

No problem

WHTI

WHTI

BBU

δv_{oct} v_s v_{bbu} δv_{pe}

Conclusions

- Since strong nonlinearity of the e-cloud yields high tune spreads

  $$\delta_2 \nu_{pe} \sim (0.3-0.5)\delta \nu_{pe} ,$$

  neither weak nor strong head-tail instabilities (TMCI) on the e-cloud only appear to be possible.

- With SC $\delta \nu_{sc} > \delta \nu_{pe}$, WHTI should be expected.

- TMCI (zero-chroma instability) does not look possible at any SC.

- BBU is not stopped by e-cloud nonlinearity. It can be stopped by the high chromaticity.

Many thanks for everyone of you!