

# BEAM STUDIES IN THE PS BOOSTER: HEAD-TAIL INSTABILITY

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Machine operation:

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R=25m

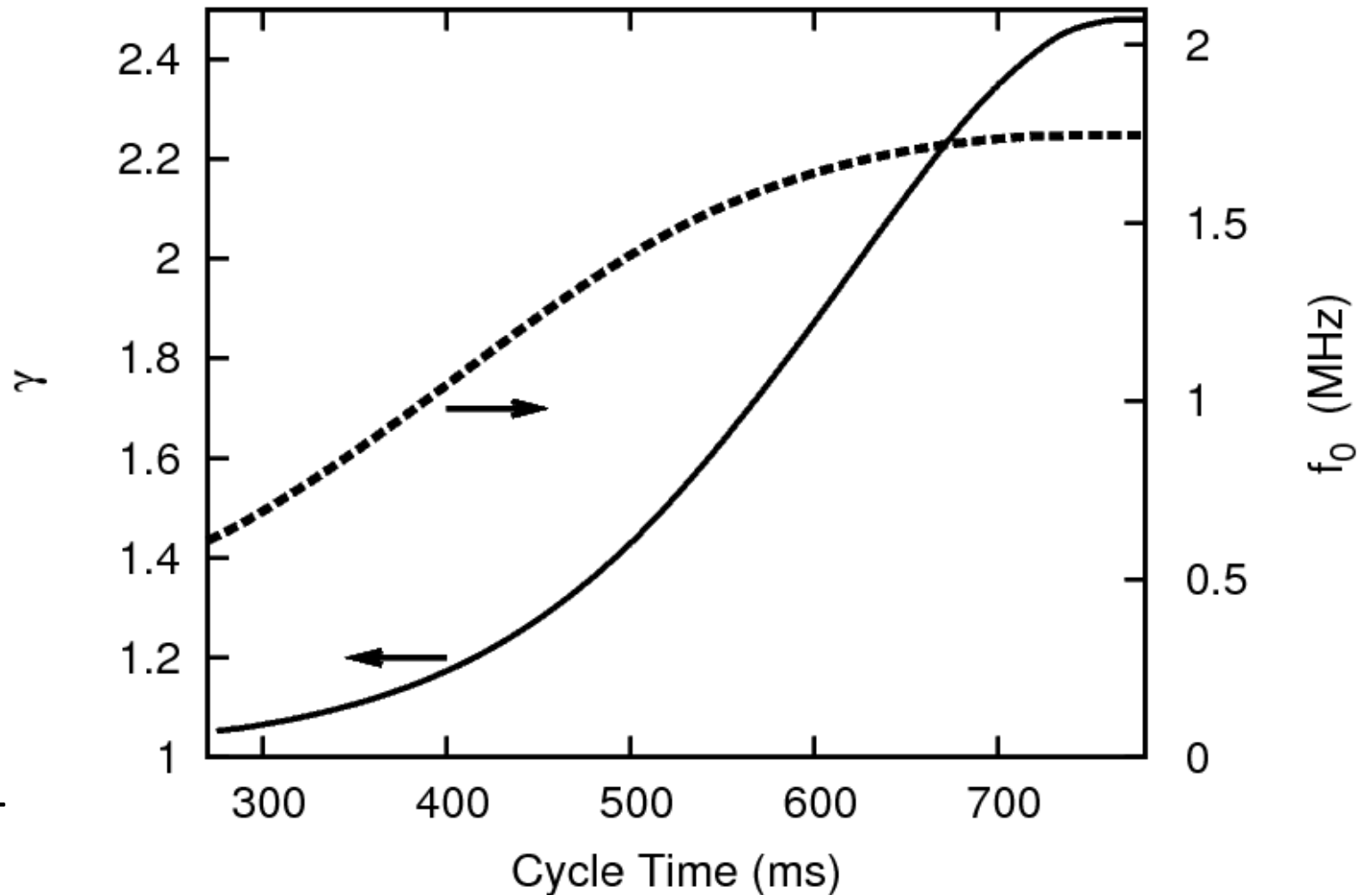
Kin. energy  
50 MeV  
1.4 GeV

Betatron tunes

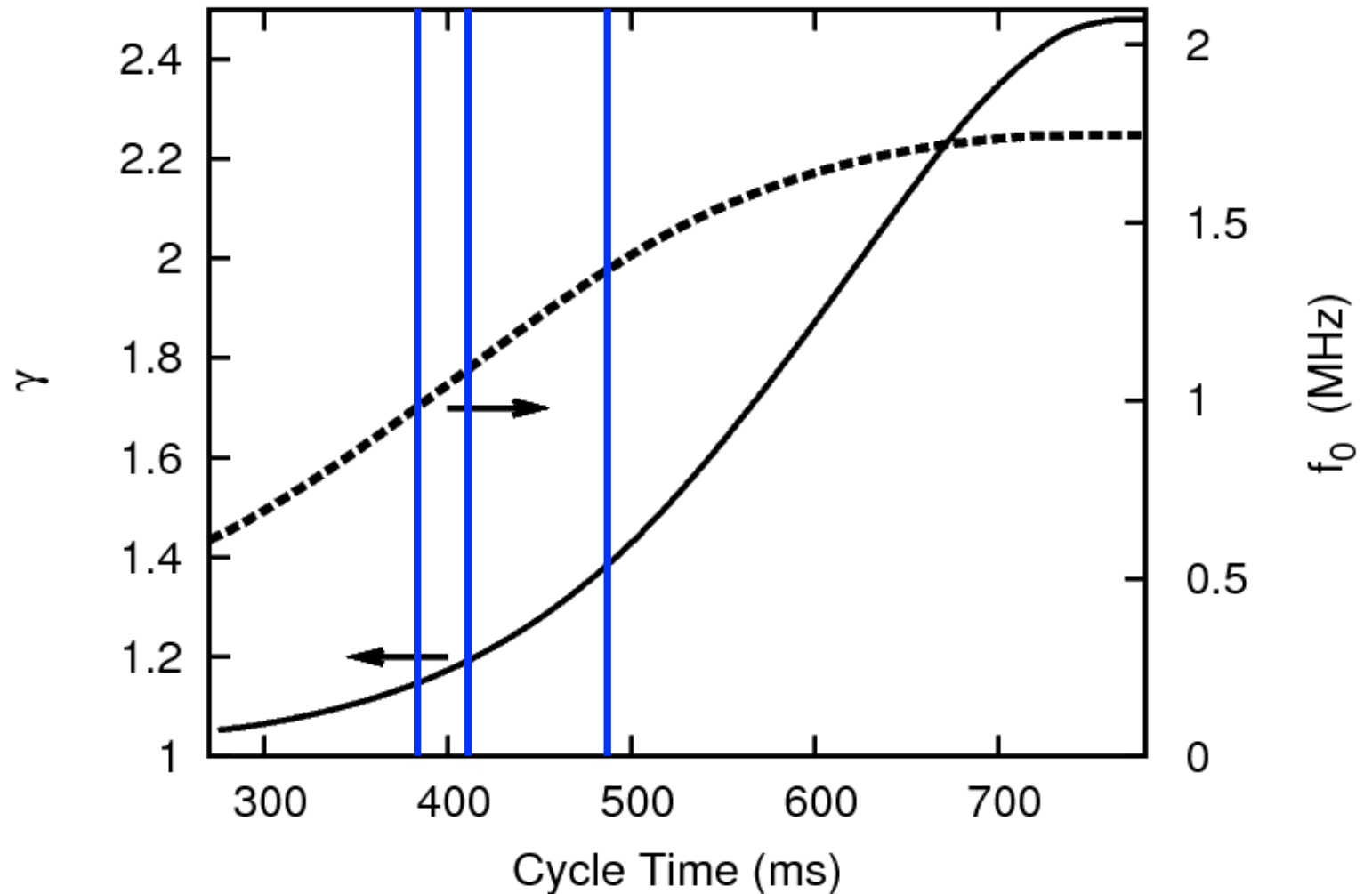
$Q_h=4.27-4.17$

$Q_v=4.65-4.20$

$\xi_h=-0.95$   $\xi_v=-2.1$



once the transverse feedback system is switched off:  
strong transverse oscillations and losses (within ms)



The instability:

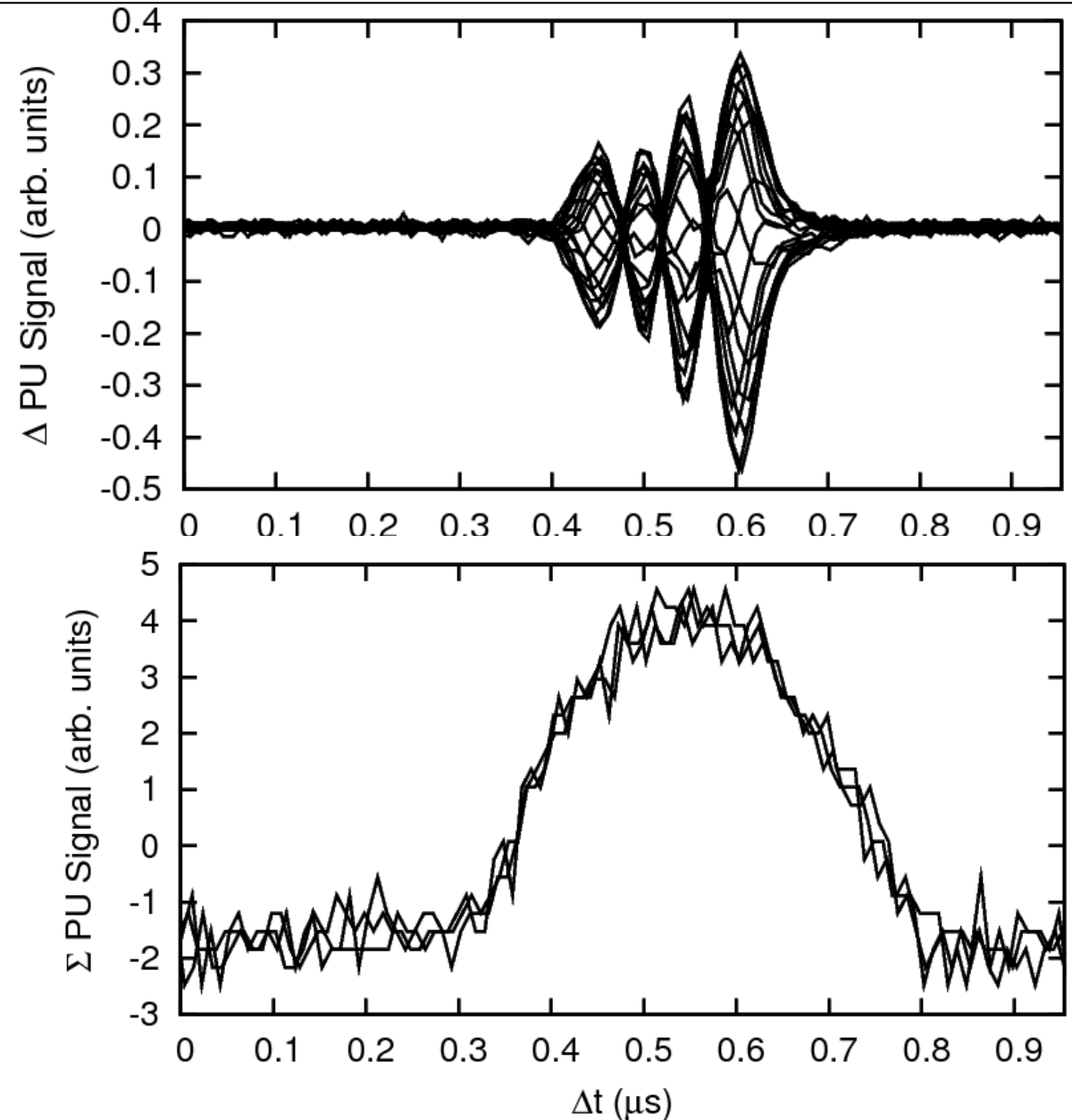
a standing-wave structure  
with the  $\xi$ -wiggles within;  
here 3 knots, i.e.  $k=3$  mode  
(rarely that nice)

nice exponential growth

slower than the  
synchrotron motion,  
 $\Delta Q/Q_s < 0.3$



an unstable head-tail mode



an example for the  
exponential growth

The instability at C392ms:

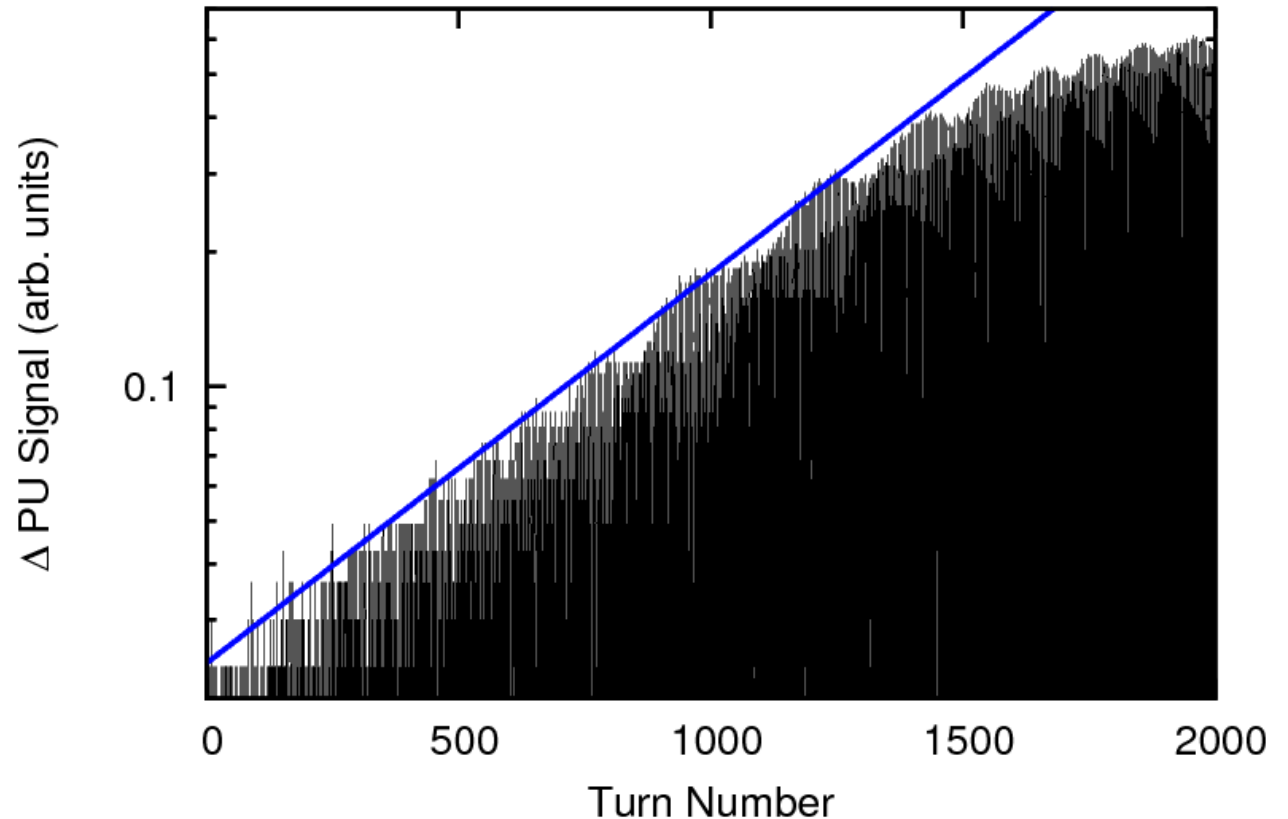
$$\Delta Q = 3.1 \times 10^{-4}$$

$$\tau = 0.53 \text{ ms}$$

$$\Delta Q / Q_s = 0.16$$

a general assumption so  
far has been:

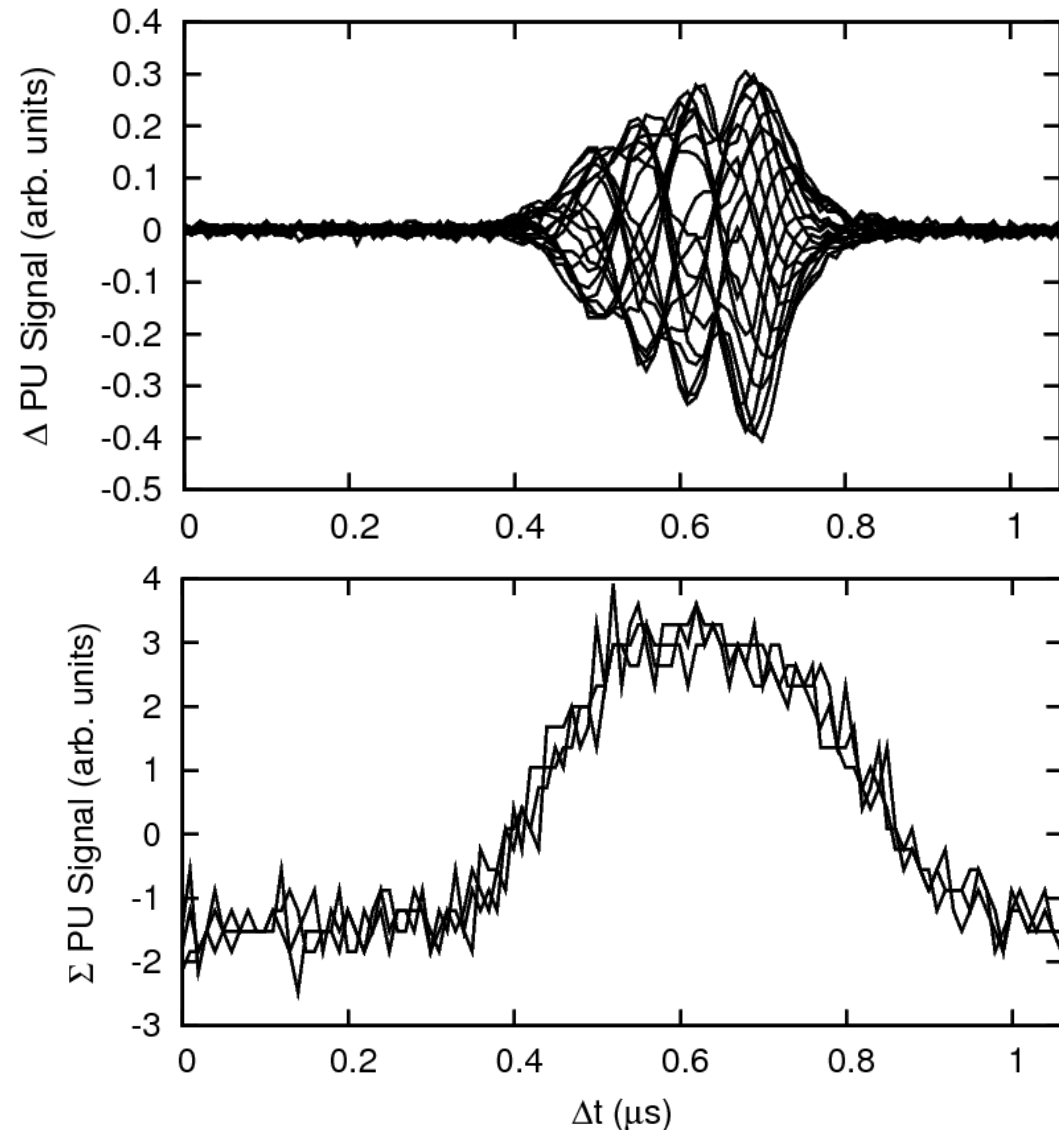
driven by the Resistive-  
Wall Impedance



**Instability at C386ms  
single rf**

$N_p = 370e10$   
 $\Delta Q = 2.3e-4$ ,  
 $\Delta Q/Q_s = 0.13$

**the mode  $k=3$   
the mode structure is not an  
ideal head-tail, it is modified  
by the impedance**



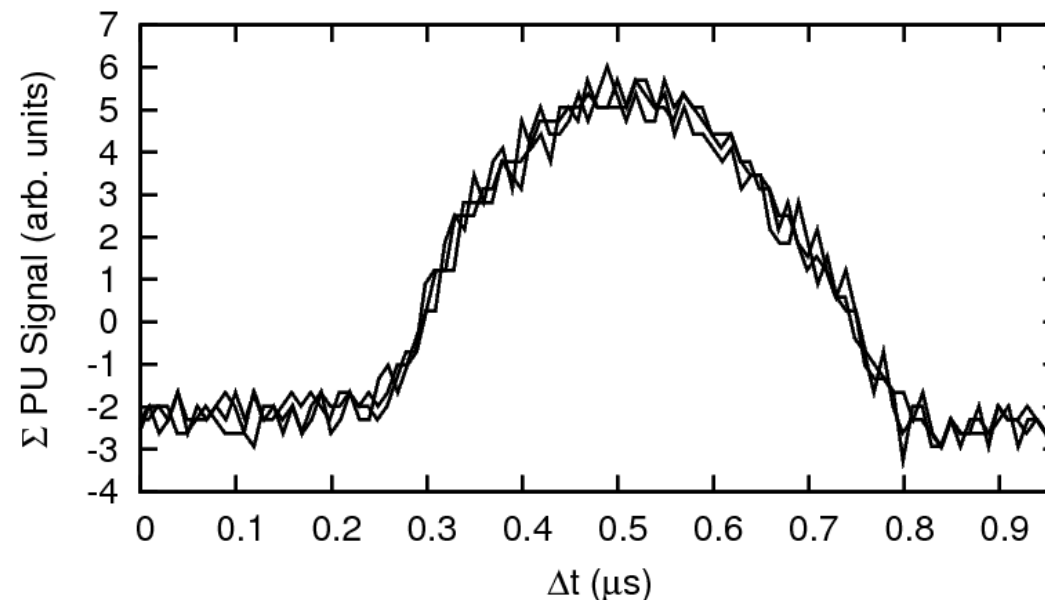
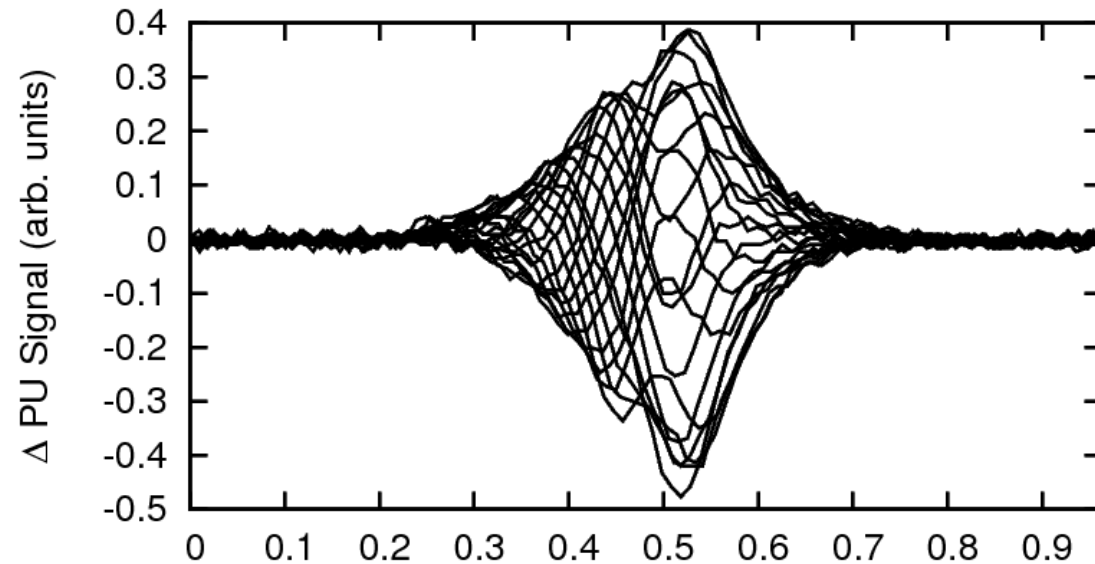
Instability at C386ms  
single rf,  
higher intensity

$$N_p = 600e10$$

$$\Delta Q = 4.4e-4,$$

$$\Delta Q/Q_s = 0.24$$

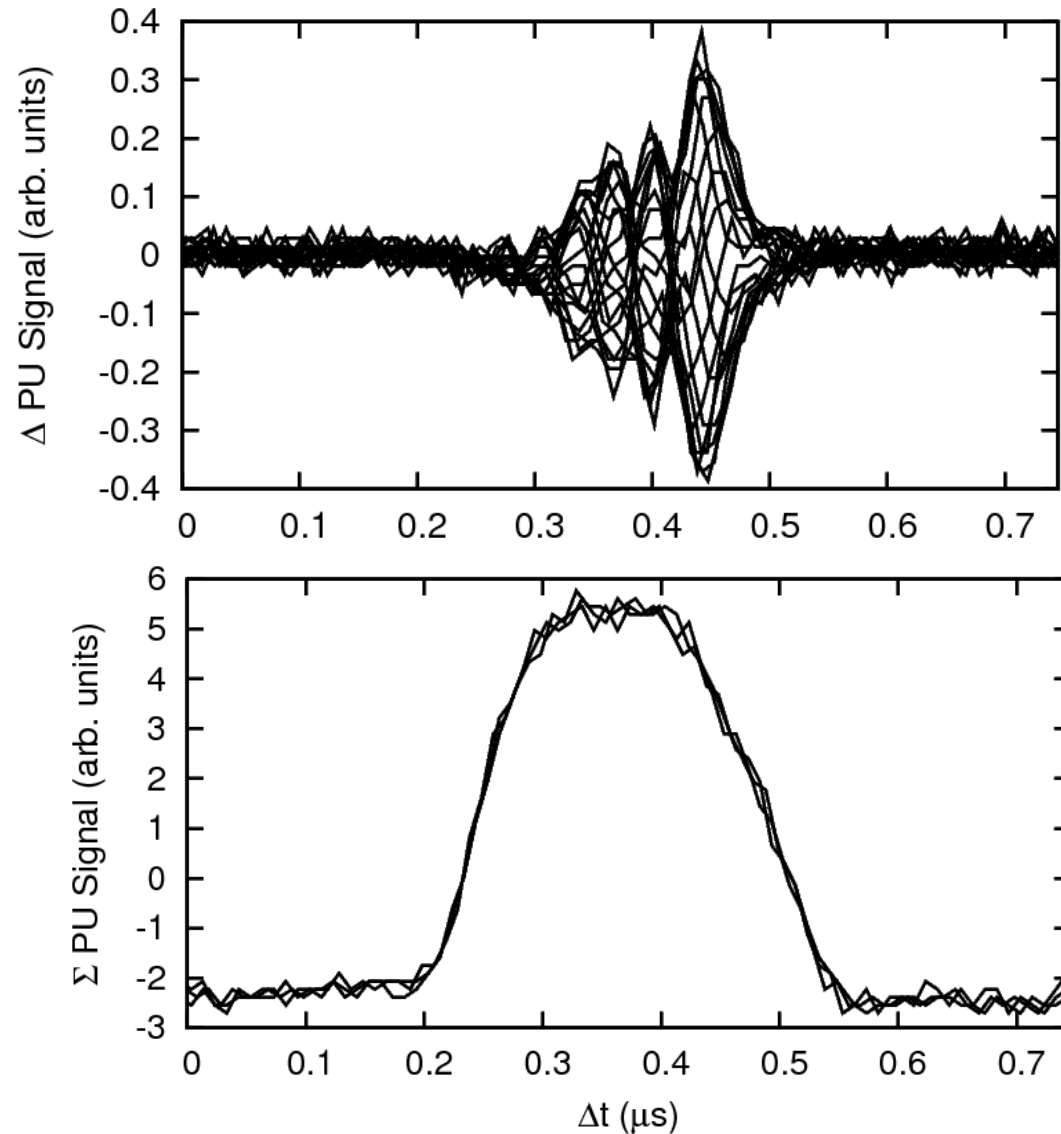
the mode structure  
is stronger deformed  
by the driving impedance



**Instability at C491ms  
single rf**

$N_p = 380e10$   
 $\Delta Q = 0.6e-4$ ,  
 $\Delta Q/Q_s = 0.064$

**the mode  $k=4$ ?  
 higher mode index  
 for later CTimes**

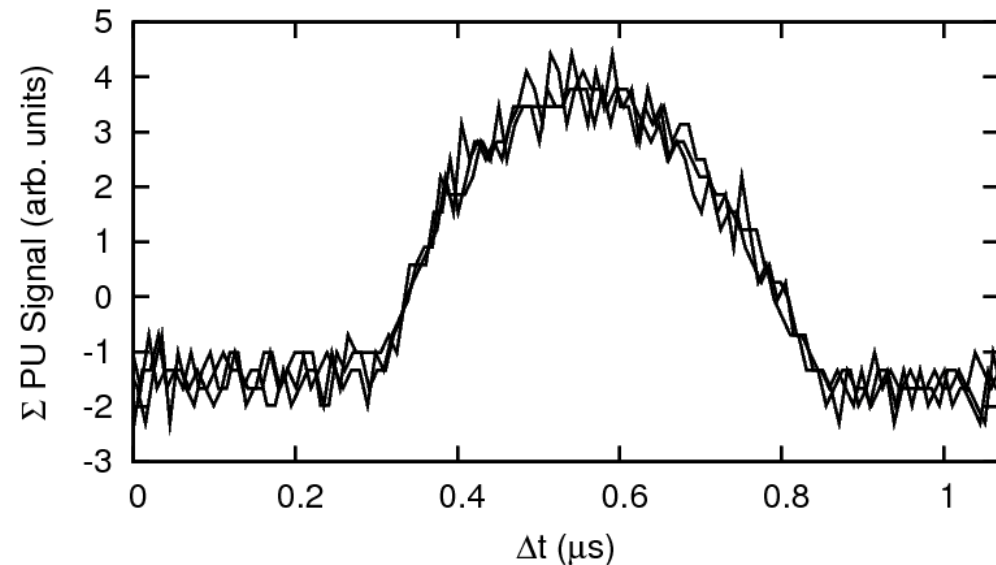
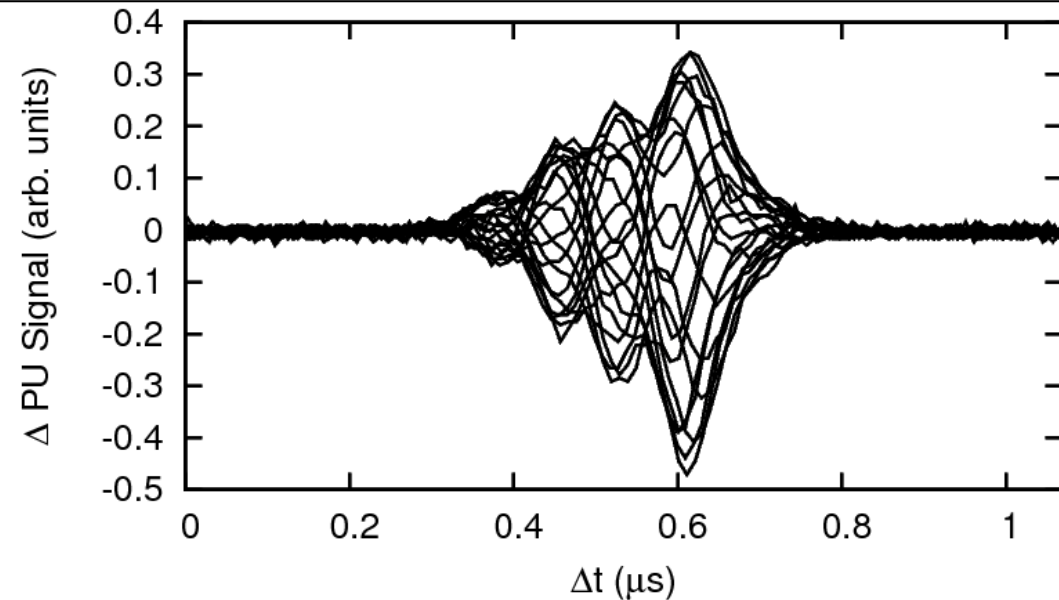




Instability at C383ms  
single rf,  
large transverse emittance

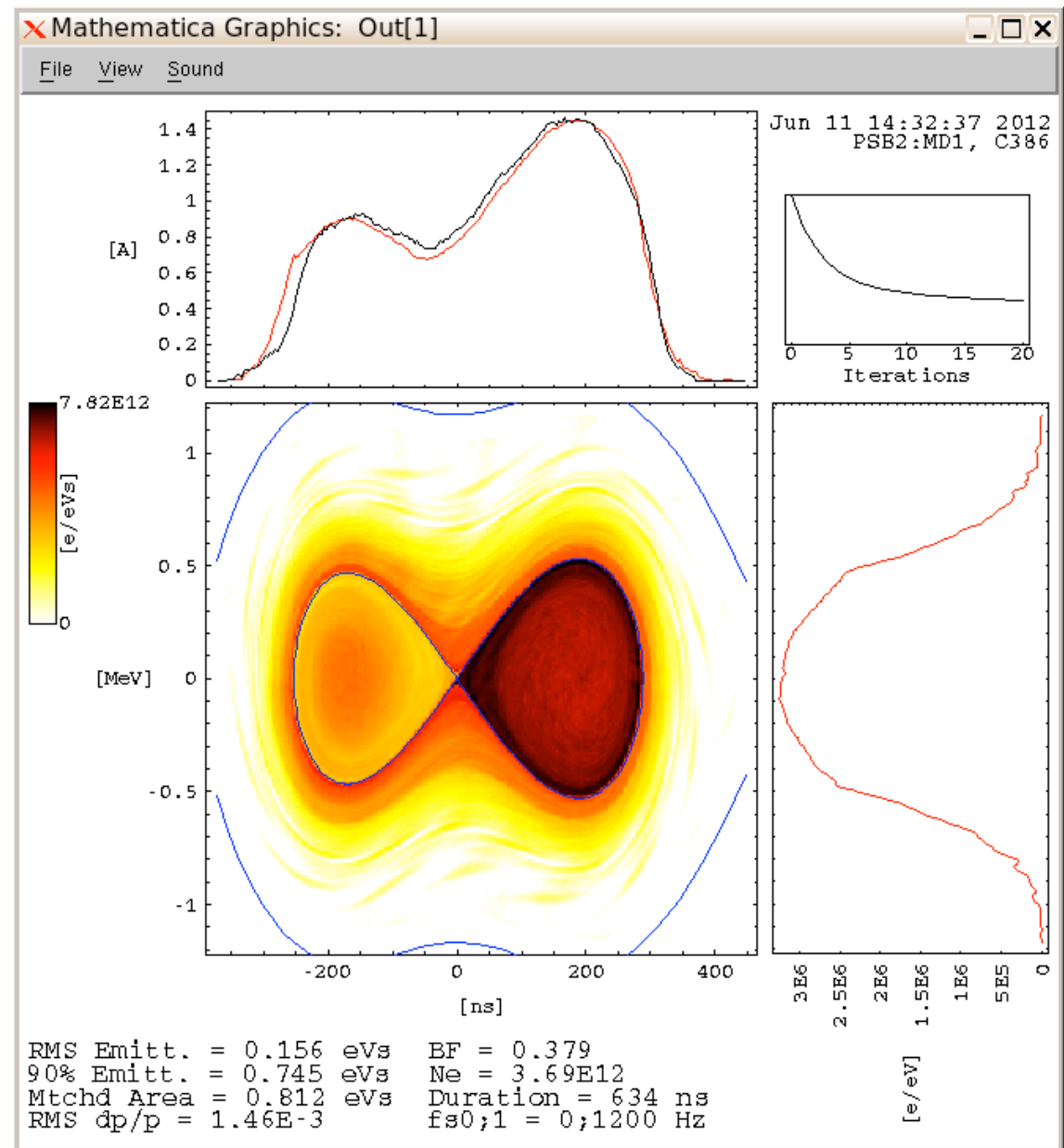
$N_p = 400e10$

the mode  $k=3$   
no clear effect of weaker  
space charge



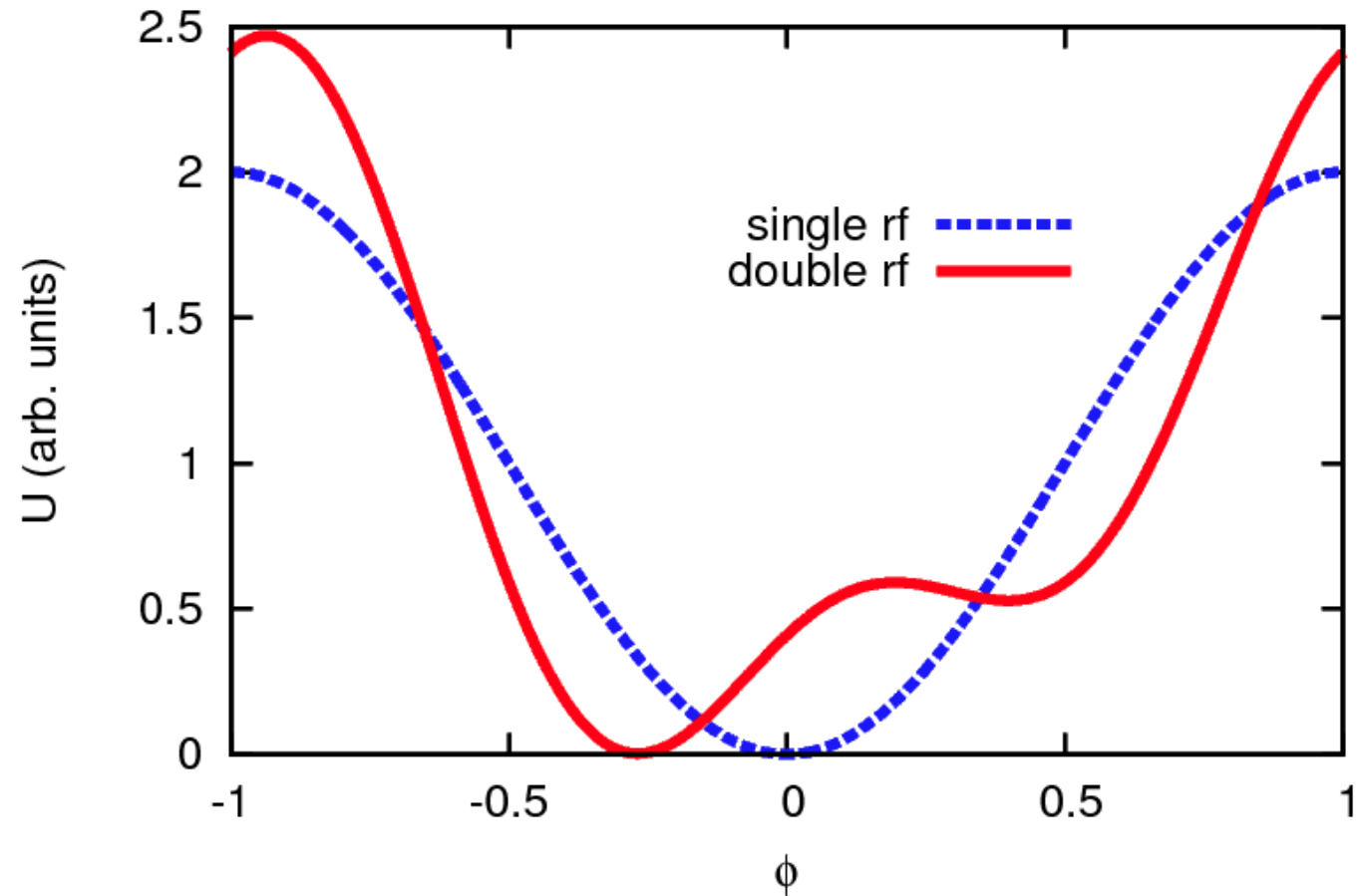
## double rf, standard at PSB

the h=2 cavity is shifted,  
the voltage of the h=2 cavity  
is the same (8kV).



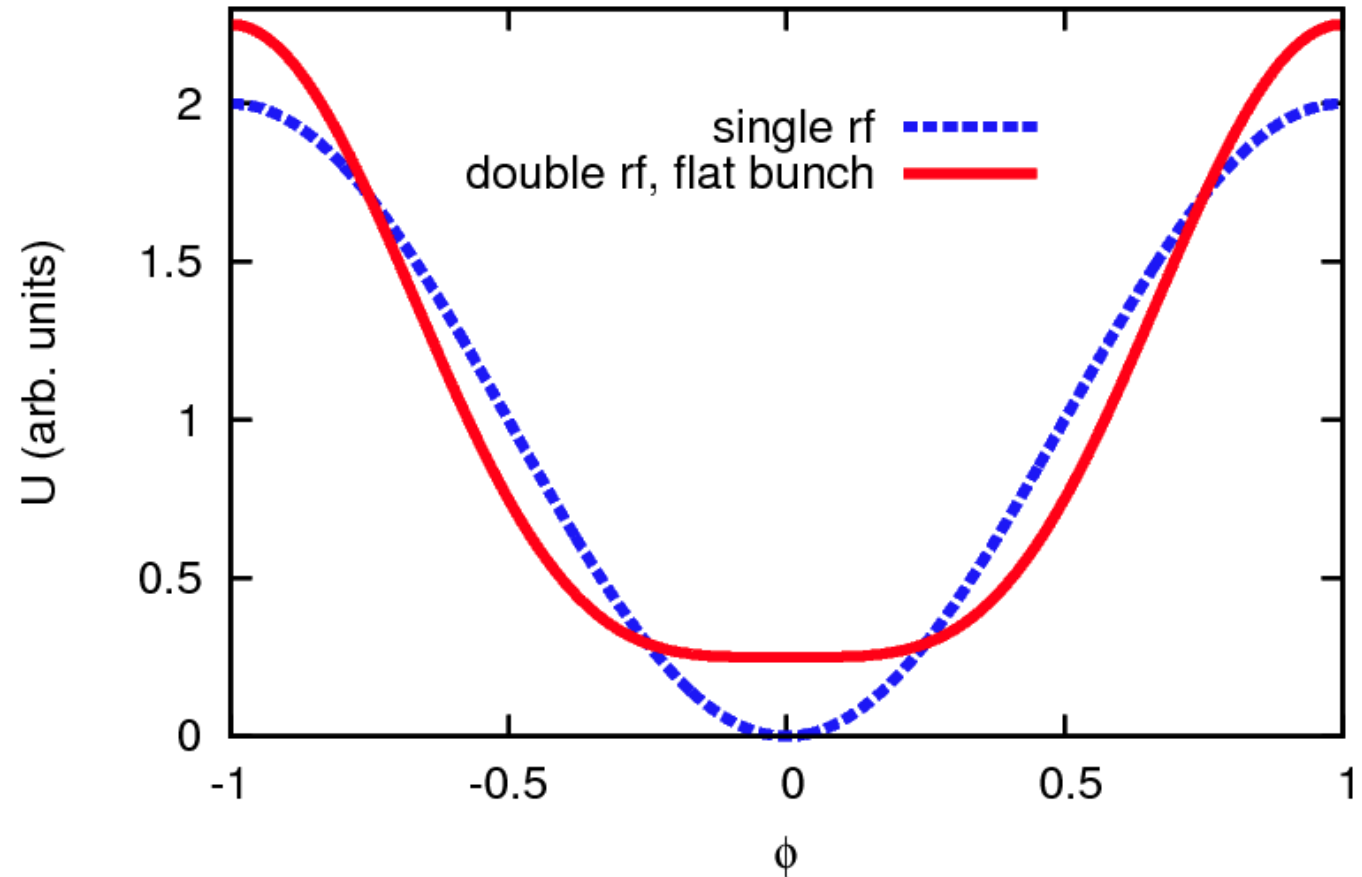
an example

related to the  $h=1$  cavity:  
 the  $h=2$  cavity is shifted by  $0.8\pi$ ,  
 the voltage of the  $h=2$  cavity is the same.



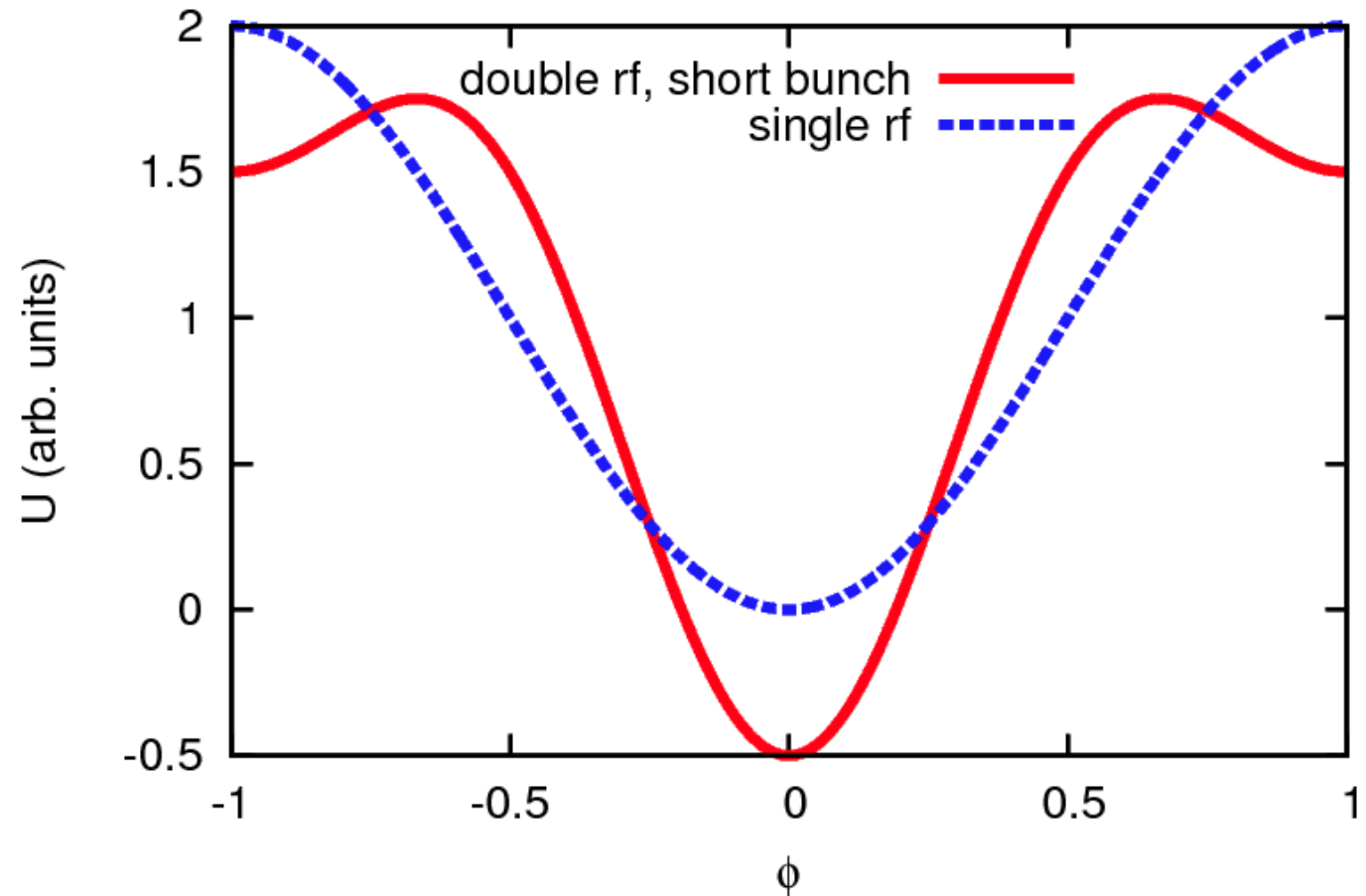
## double rf, flat bunch

related to the  $h=1$  cavity:  
 the  $h=2$  cavity is shifted by  $\pi$ ,  
 the voltage of the  $h=2$  cavity is a half (4kV).



## double rf, short bunch

related to the  $h=1$  cavity:  
 the  $h=2$  cavity shift is equal zero,  
 the voltage of the  $h=2$  cavity is the same.



**Instability at C384ms  
double rf, short bunch**

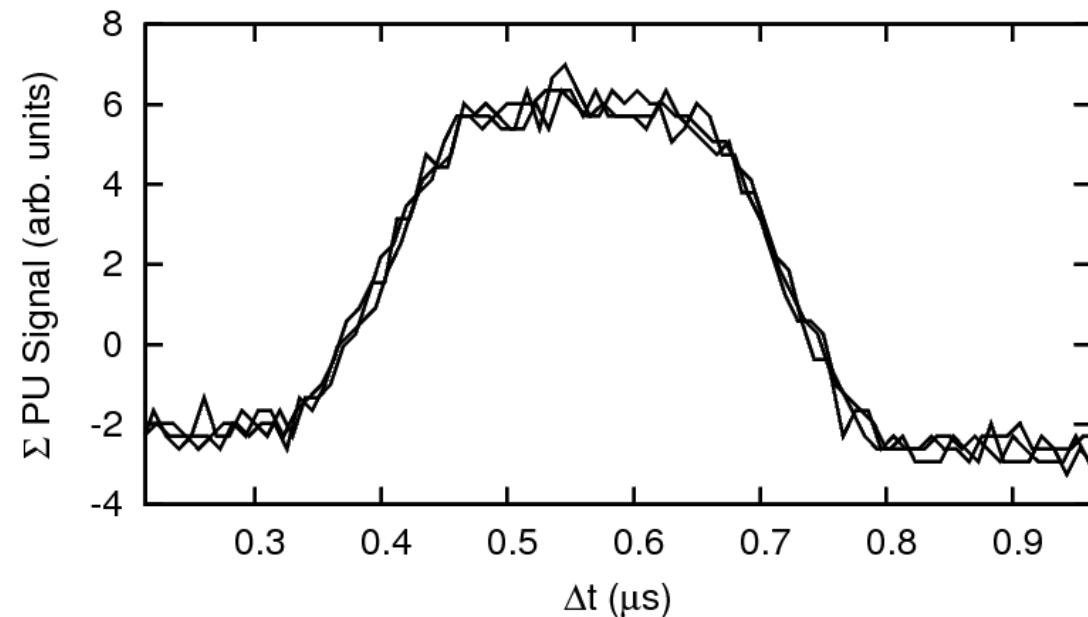
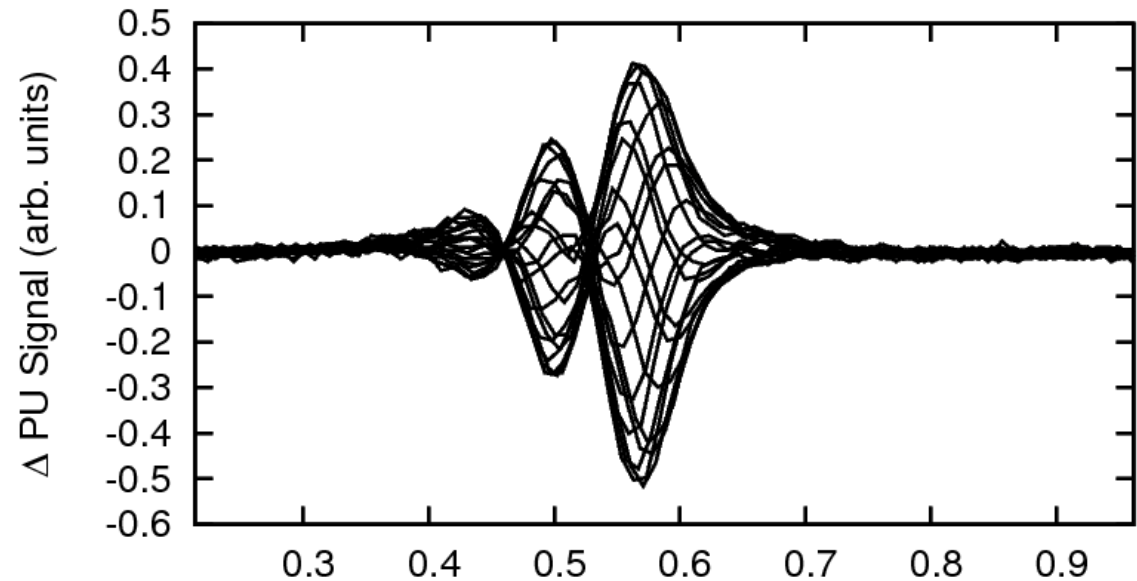
$$N_p = 610e10$$

$$\Delta Q = 3.8e-4,$$

$$Q_s = 3.17e-3$$

$$\Delta Q / Q_s = 0.12$$

**the mode  $k=2$ ,  
result of a shorter bunch**

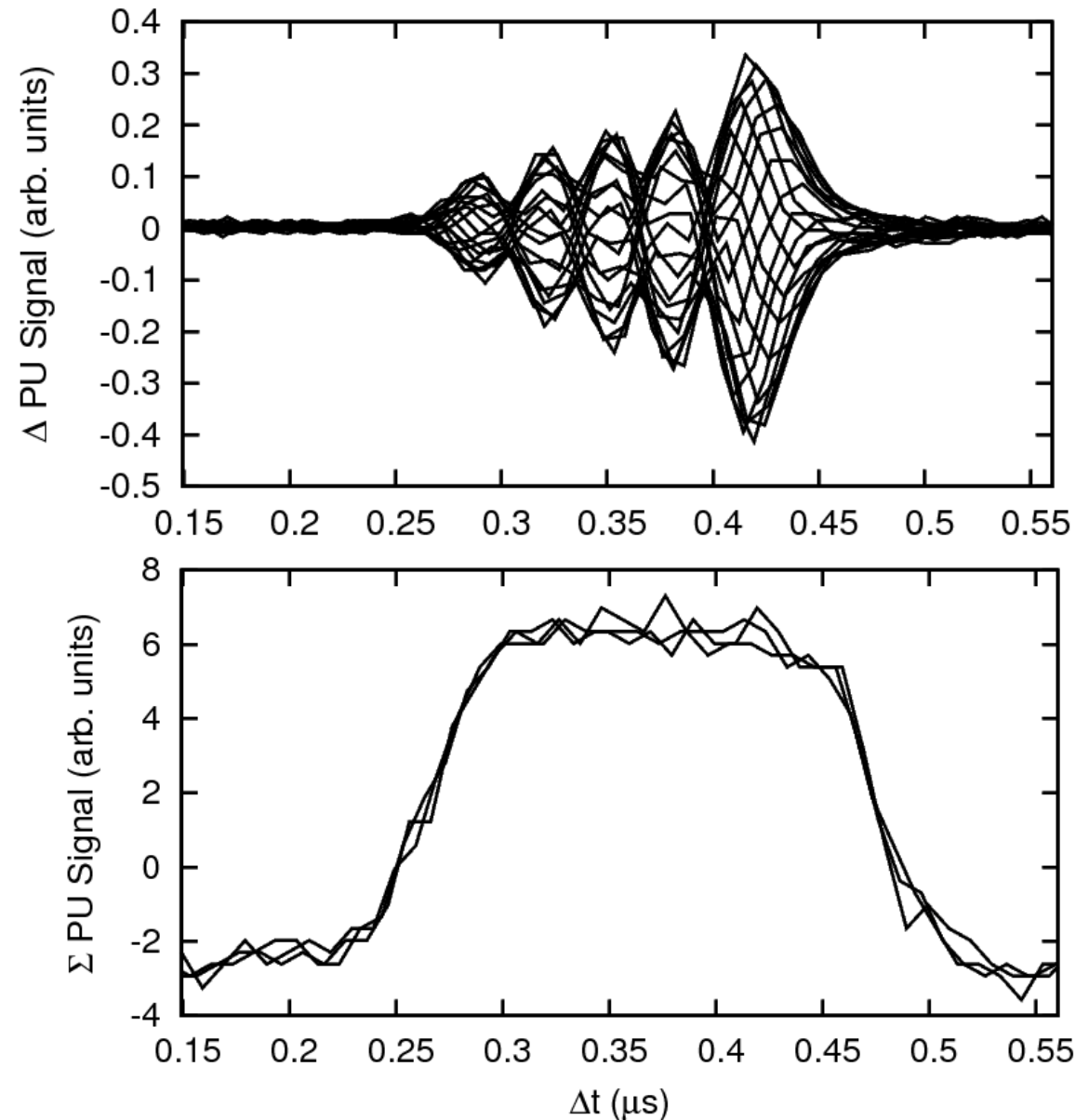


**Instability at C493ms  
double rf, short bunch**

$N_p = 500e10$

$Q_s = 1.62e-3$

**the mode  $k=4$   
higher mode index  
for later CTimes**



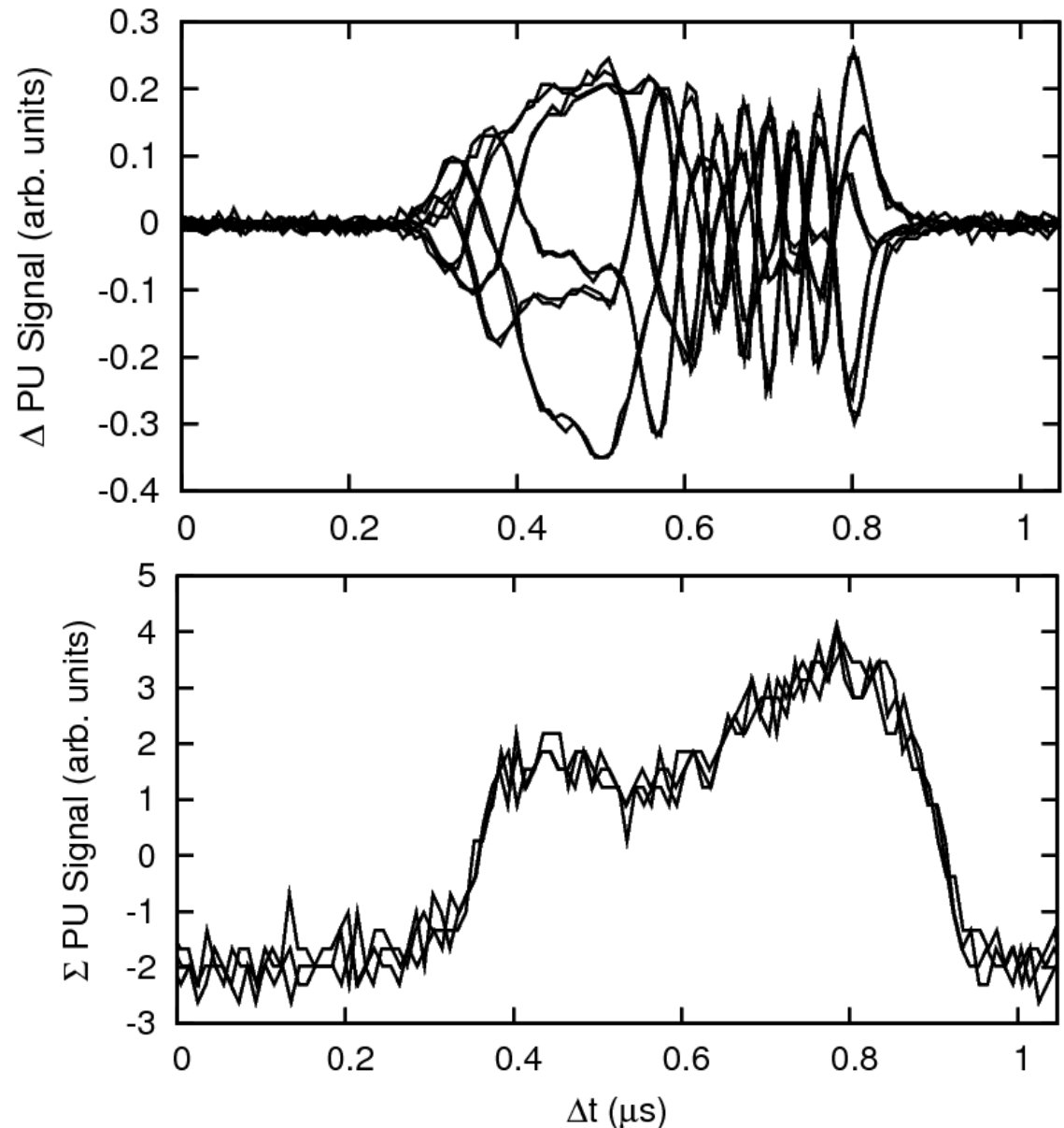
**Instability at C394ms  
double rf, PSB standard**

$$N_p = 500e10$$

$$\Delta Q = 1.3e-4$$

$$\Delta Q/Q_s = 0.071$$

**a complex mode structure  
in double rf**

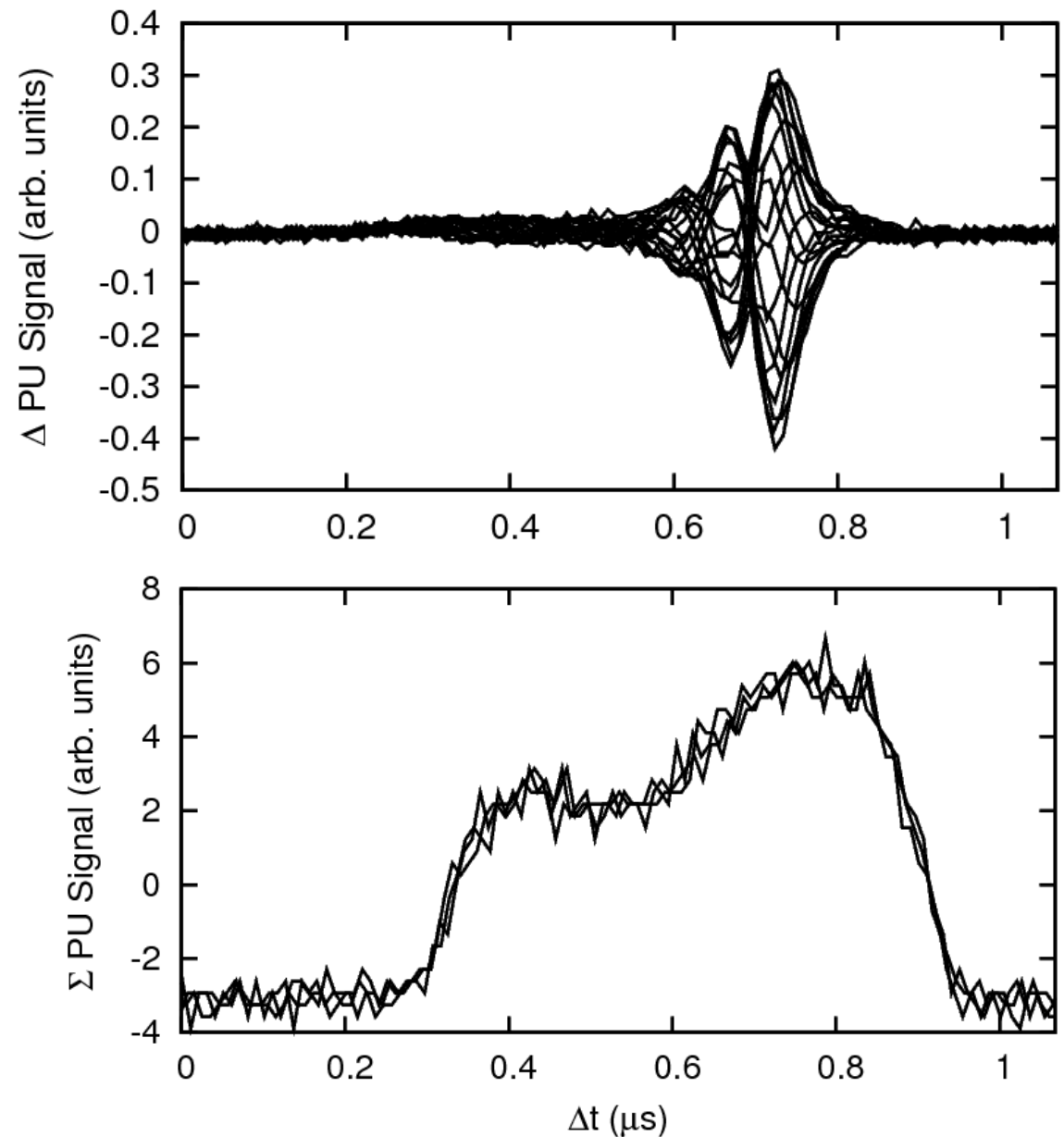




Instability at C394ms  
 double rf, PSB standard  
 higher intensity

$N_p = 950e10$

looks like the  $k=2$  mode in  
 the more dense, tail half of  
 the bunch



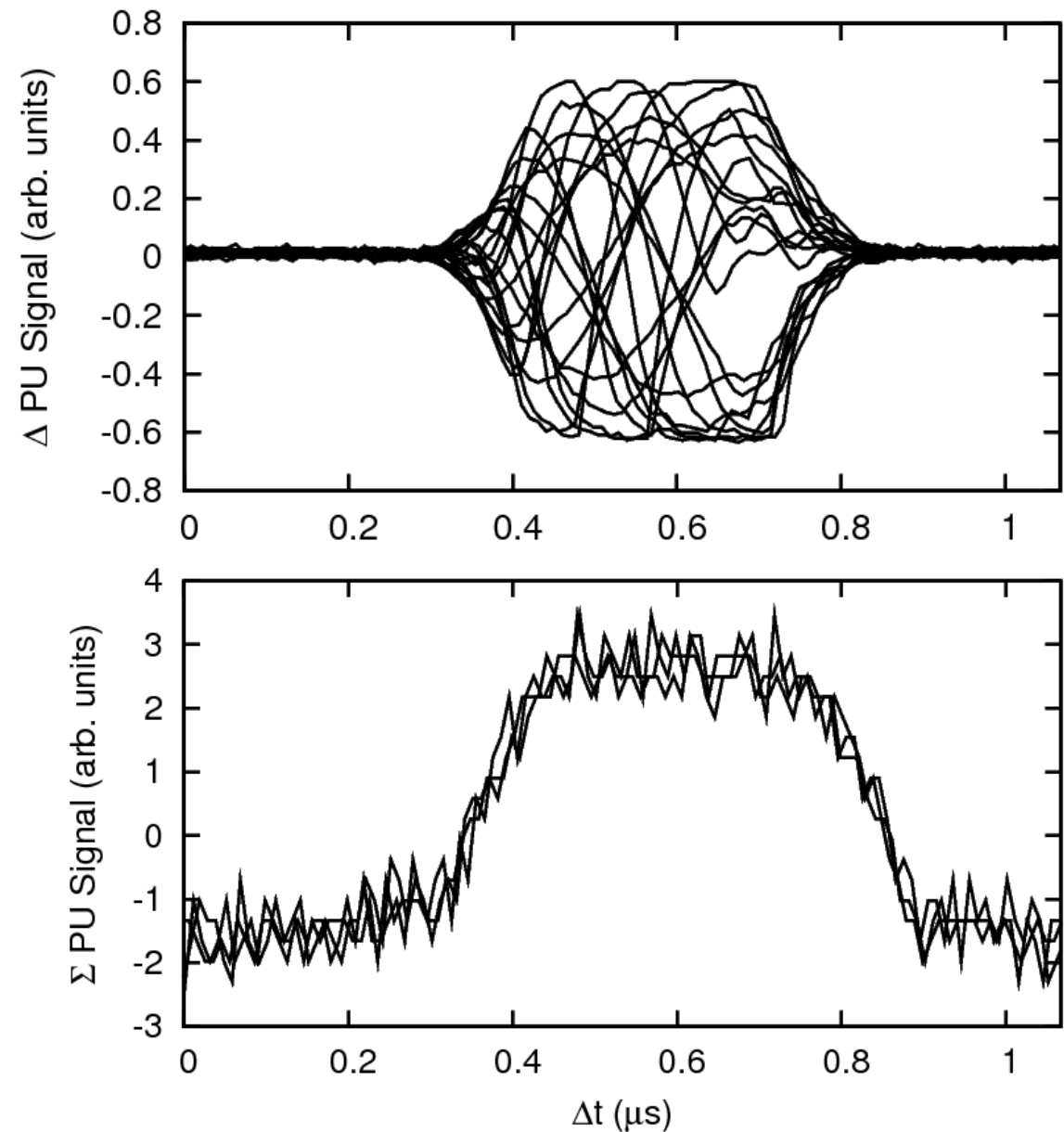
**Instability at C385ms  
double rf, flat bunch**

$$N_p = 450e10$$

$$\Delta Q = 5.1e-4$$

$$\Delta Q/Q_s = 0.28$$

**a complex mode structure  
in double rf**



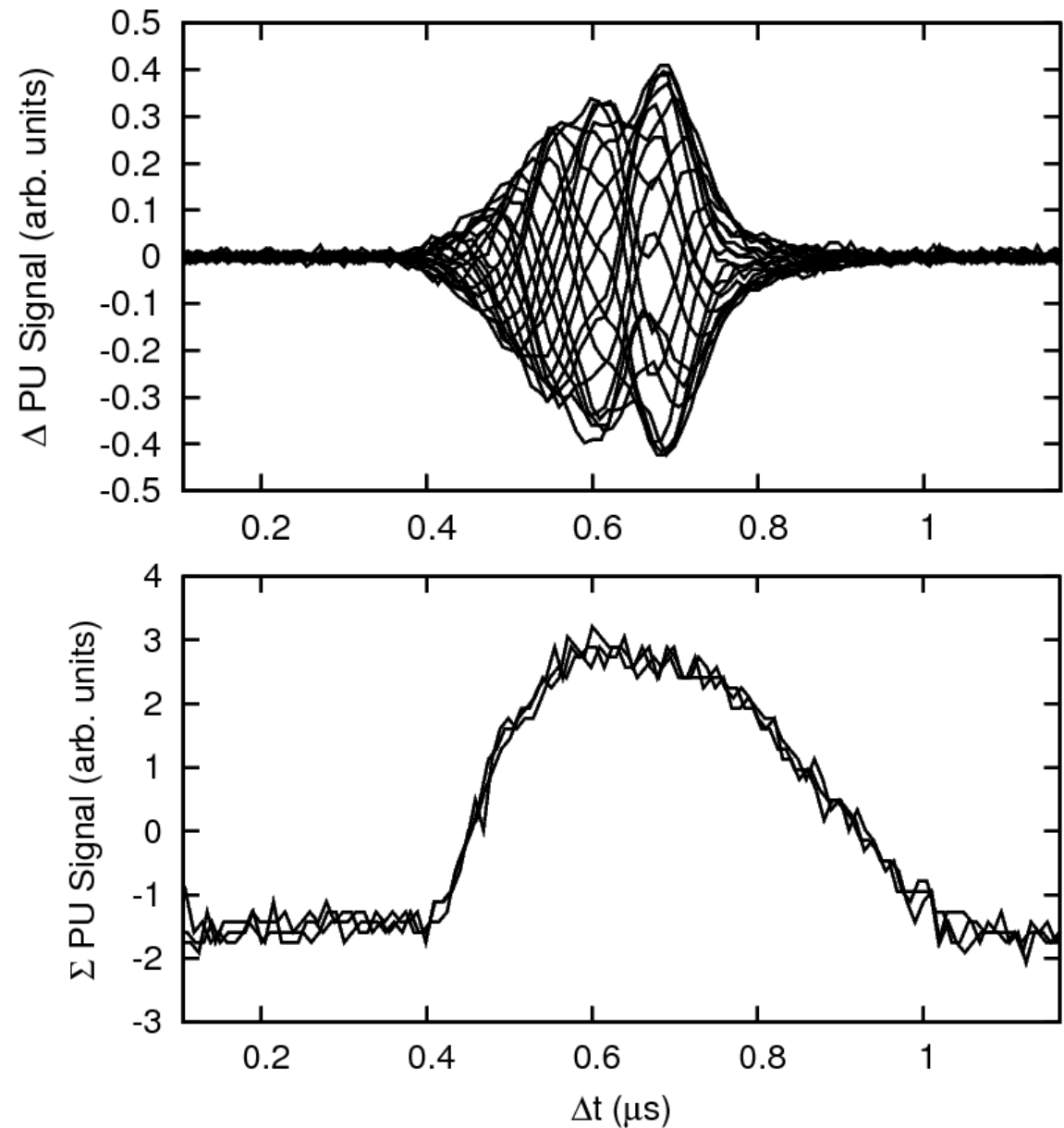
Instability at C385ms  
 single rf,  $V_0=4\text{kV}$   
 (until now 8kV)

$N_p=400e10$  (near  
 the threshold)

$\Delta Q=2.2e-4$

$Q_s=0.66e-3$

$\Delta Q/Q_s=0.33$

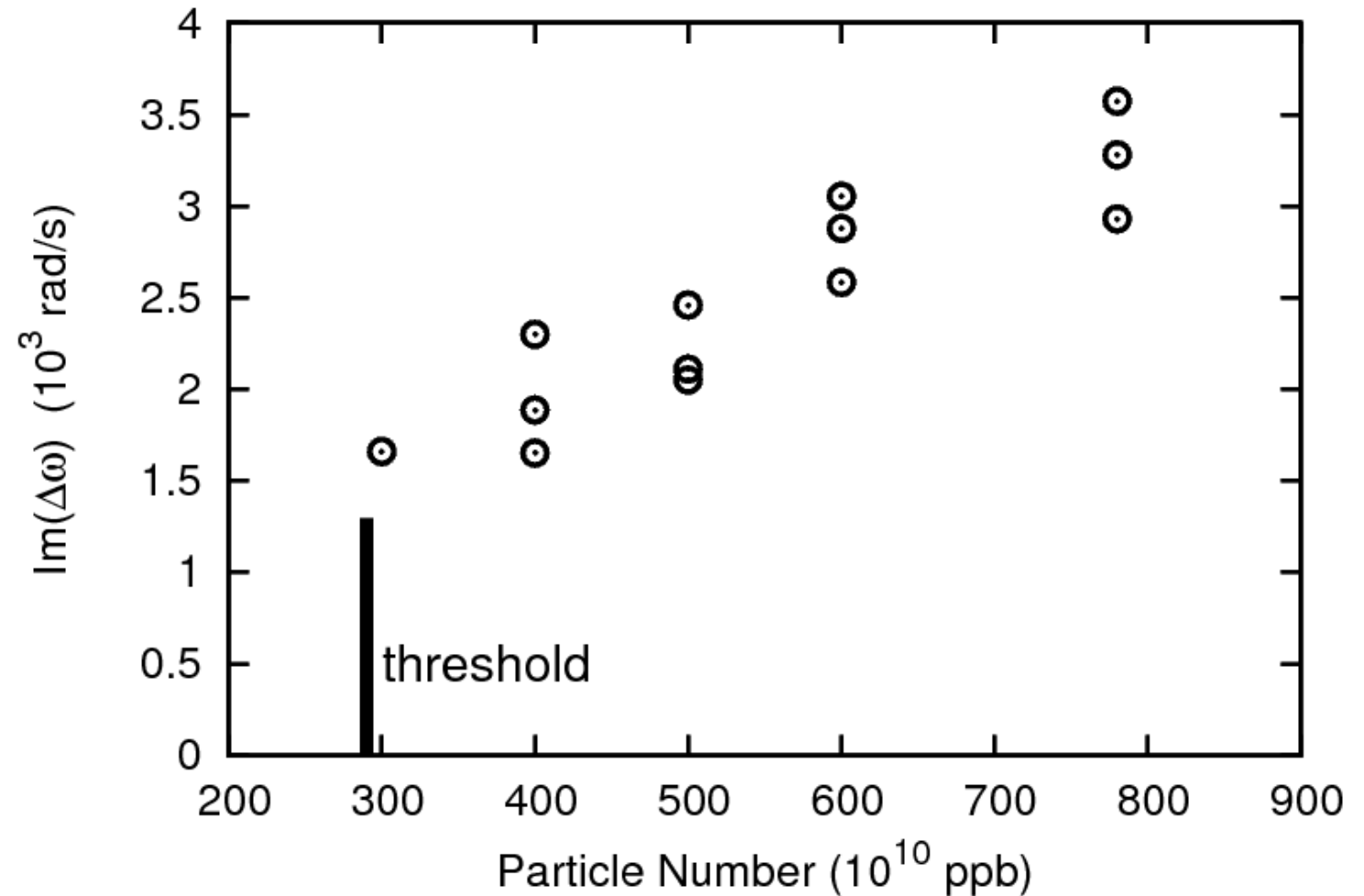


## questions appear

<b>At PSB we observe</b>	<b>Generally we know</b>
the instability occurs at reproducible cycle times	triggering of a collective instability is irregular (example: at PS flat-bottom)
the instability is always and only in the horizontal plane	the vertical Resistive-Wall impedance at PSB is larger than the horizontal
the instability has a clear intensity threshold, depending on settings	the head-tail instability has no intensity threshold
higher mode index $k$ at later Ctimes; lower mode index $k$ for shorter bunches; mode structure deformed by the $Z_{\perp}$ ; no clear effect of weaker space-charge	

As the imaginary part of the frequency, or the instability increment

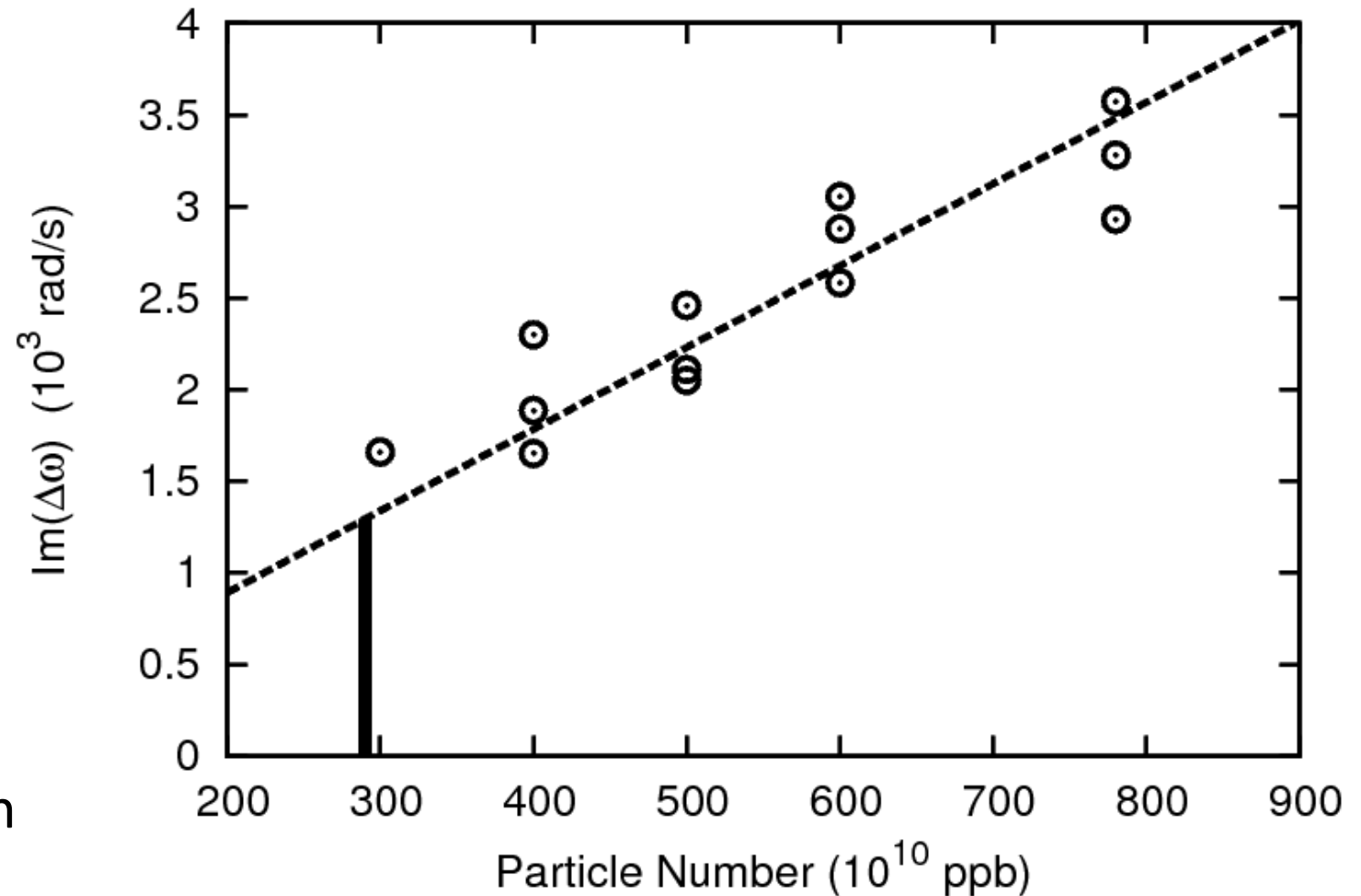
single rf  
 $V_0=8\text{kV}$



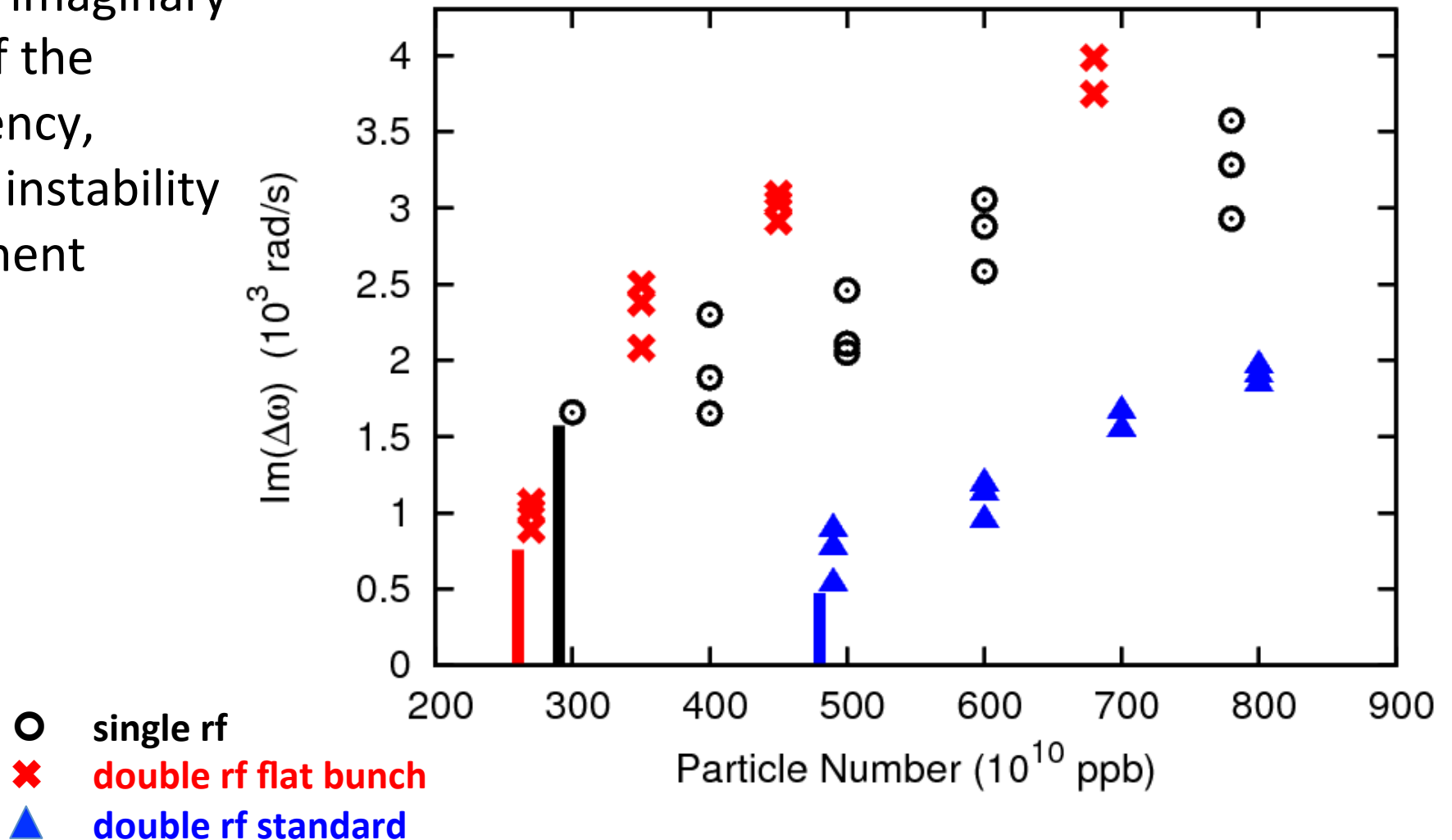
As the imaginary part of the frequency, or the instability increment

single rf  
 $V_0=8\text{kV}$

the dashed line crosses the origin

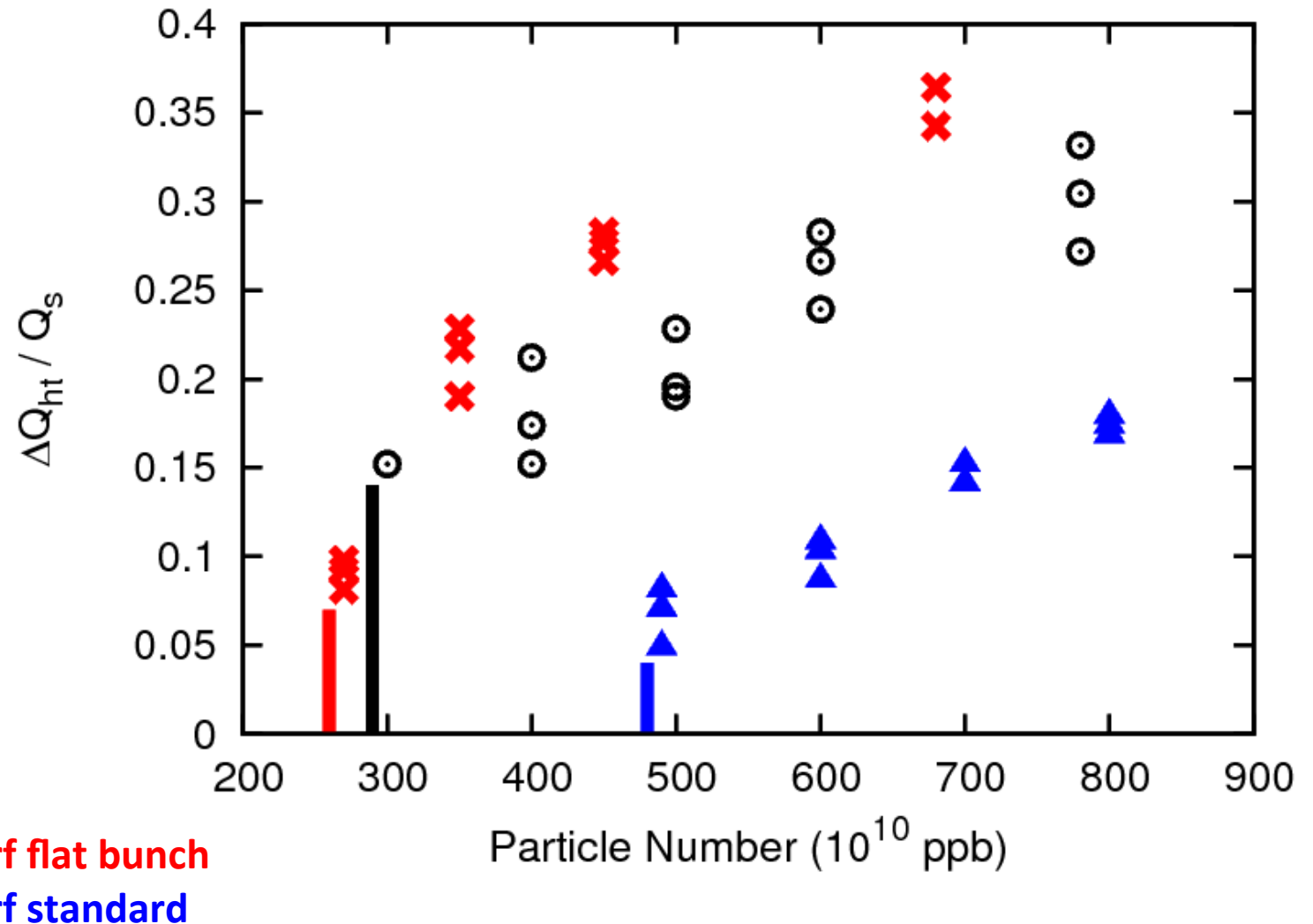


As the imaginary part of the frequency, or the instability increment



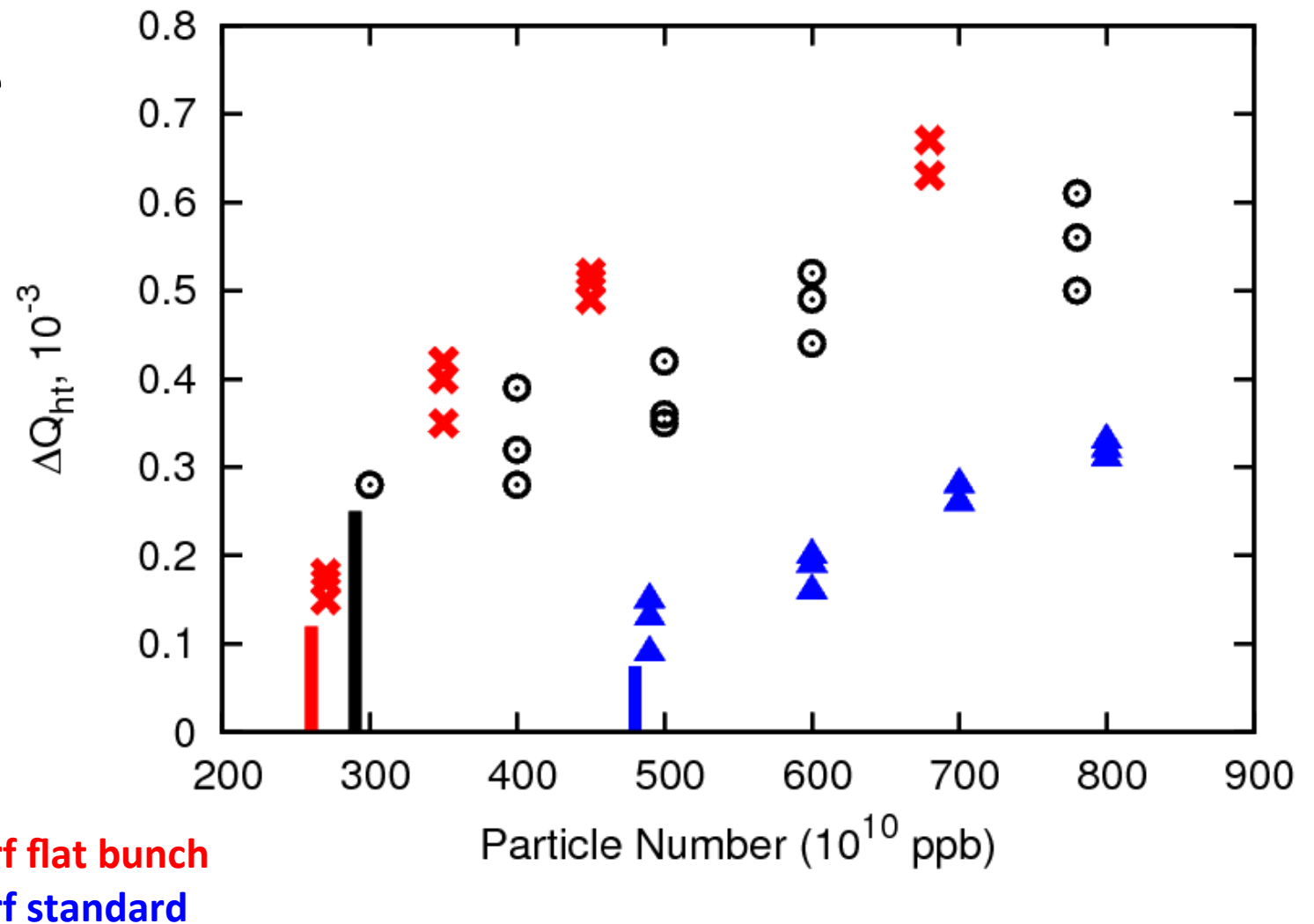
Normalized by  
the synchrotron  
frequency:

this is always  
large,  
thus the mode  
structures are  
strongly  
modified

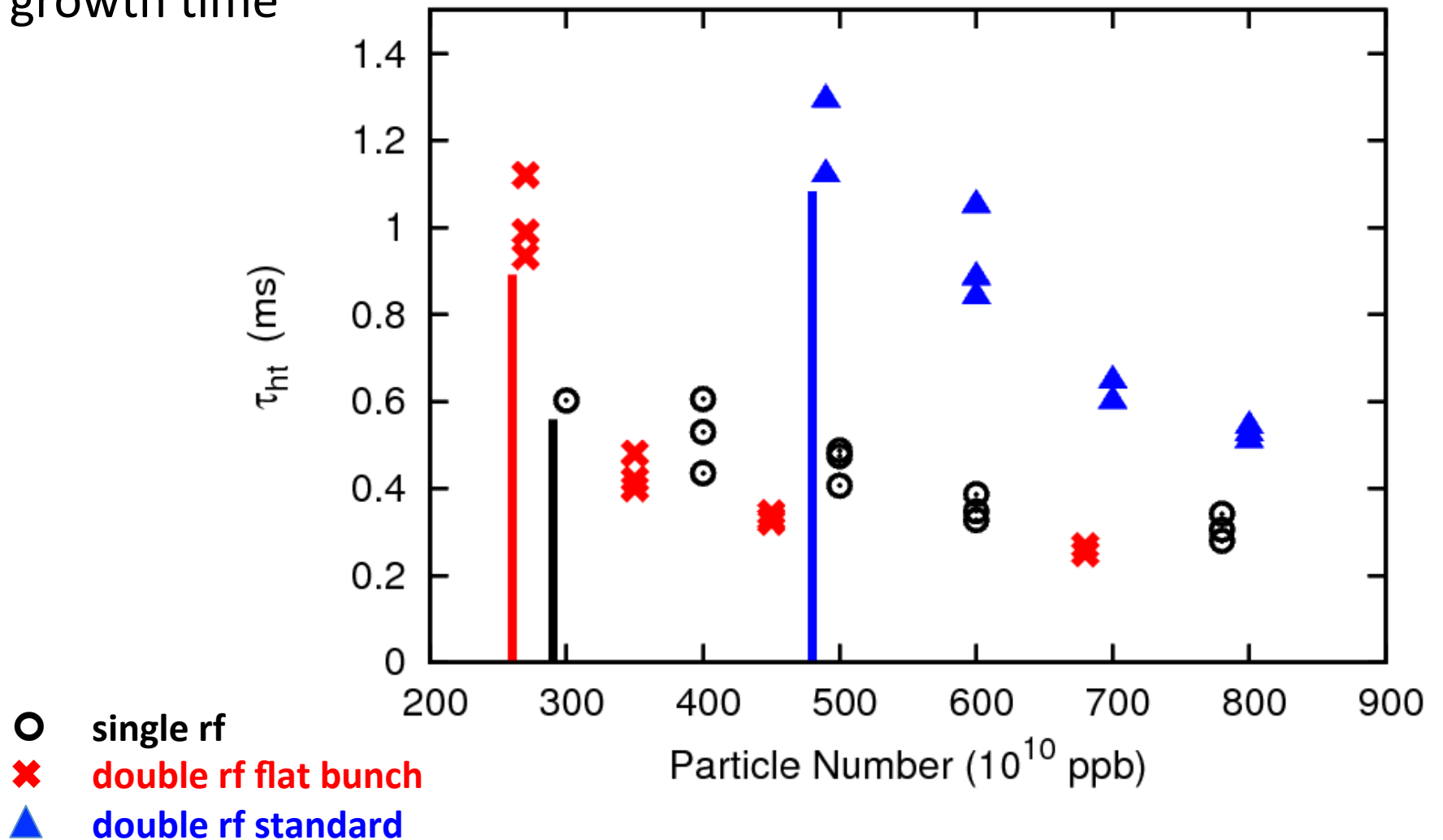




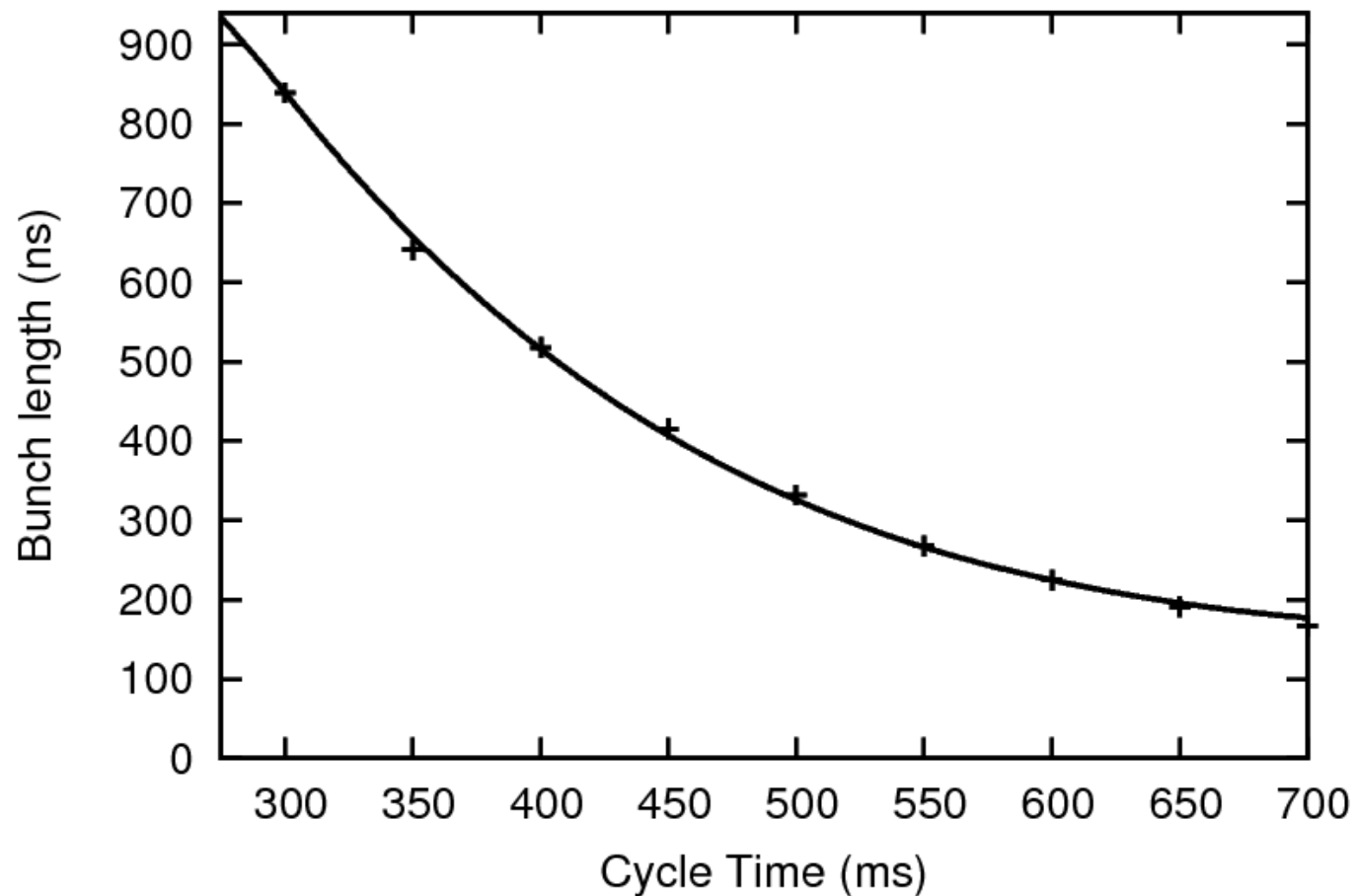
As the absolute value of the tune shift



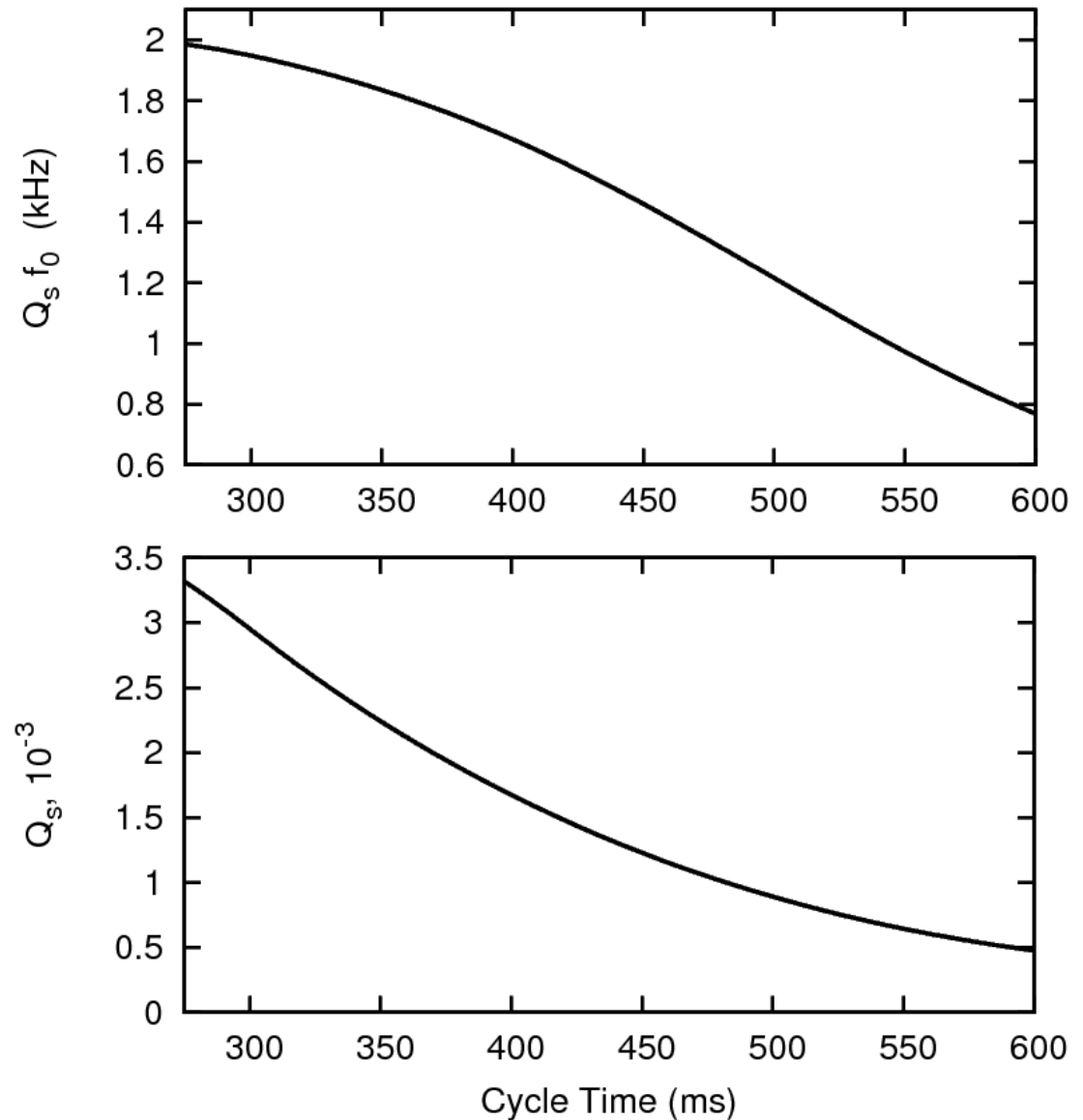
As the growth time



bunch length for the  $\varepsilon_z = \text{const}$ ,  
and measurements for single rf,  $V_0 = 8\text{kV}$



synchrotron frequency  
and  
the synchrotron tune  
for single rf,  $V_0=8\text{kV}$



the space-charge tune shift  
(rms-equiv. K-V beam)

$$\Delta Q_{sc} = \frac{\lambda_0 r_p R}{\gamma^3 \beta^2 \epsilon_{\perp}}$$

the space-charge parameter

$$q = \frac{\Delta Q_{sc}}{Q_s}$$

Elliptic cross-section:  
( $\epsilon_x, \epsilon_y$  rms emittances,  
 $\epsilon_{\perp}$  total for the rms-equivalent K-V)

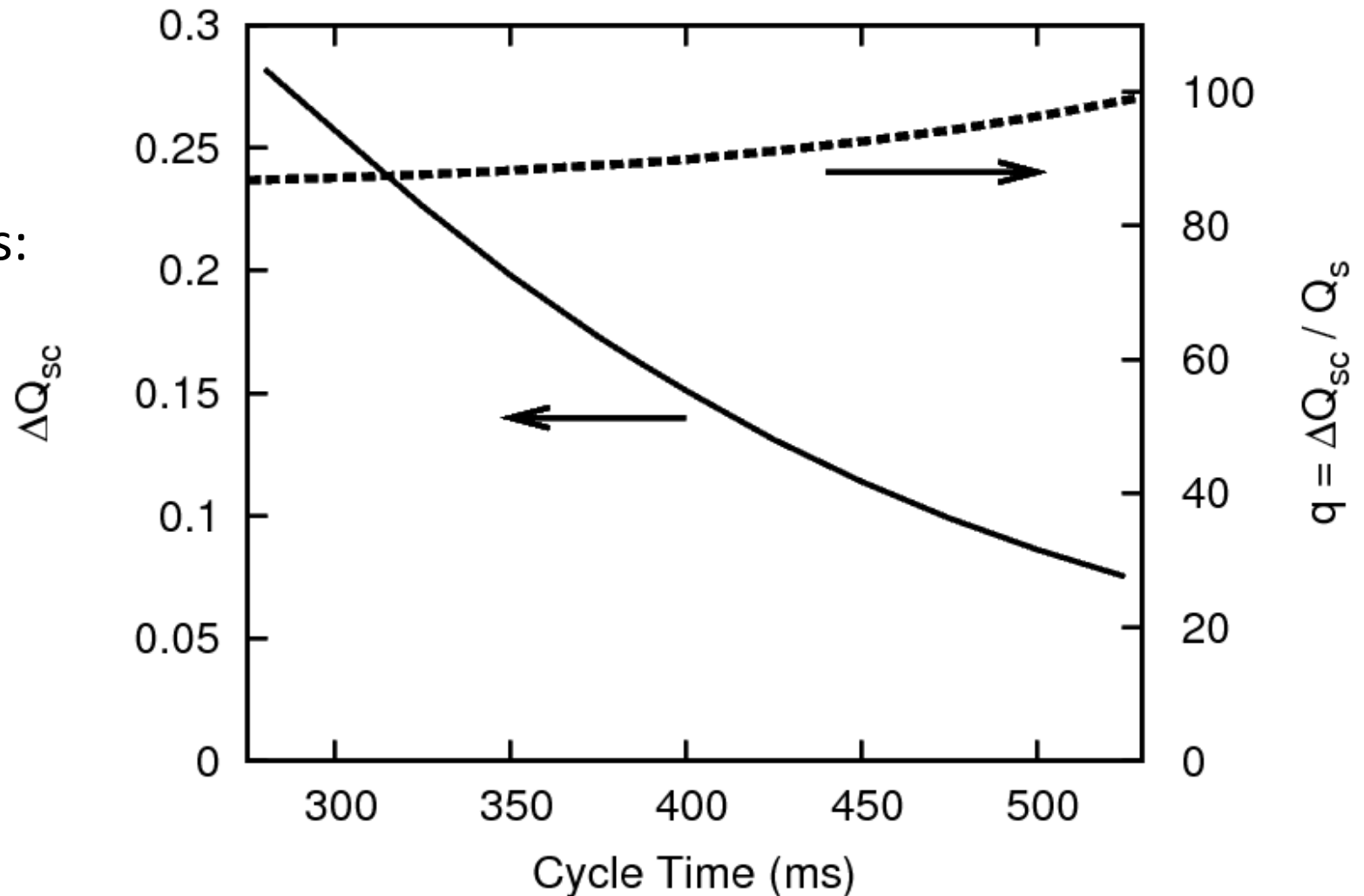
$$\epsilon_{\perp} = 2 \left( \epsilon_x + \sqrt{\epsilon_x \epsilon_y \frac{Q_{0x}}{Q_{0y}}} \right)$$

Gaussian profile:  $\Delta Q_{sc}^{\max} = 2\Delta Q_{sc}$

Space-charge tune spread:

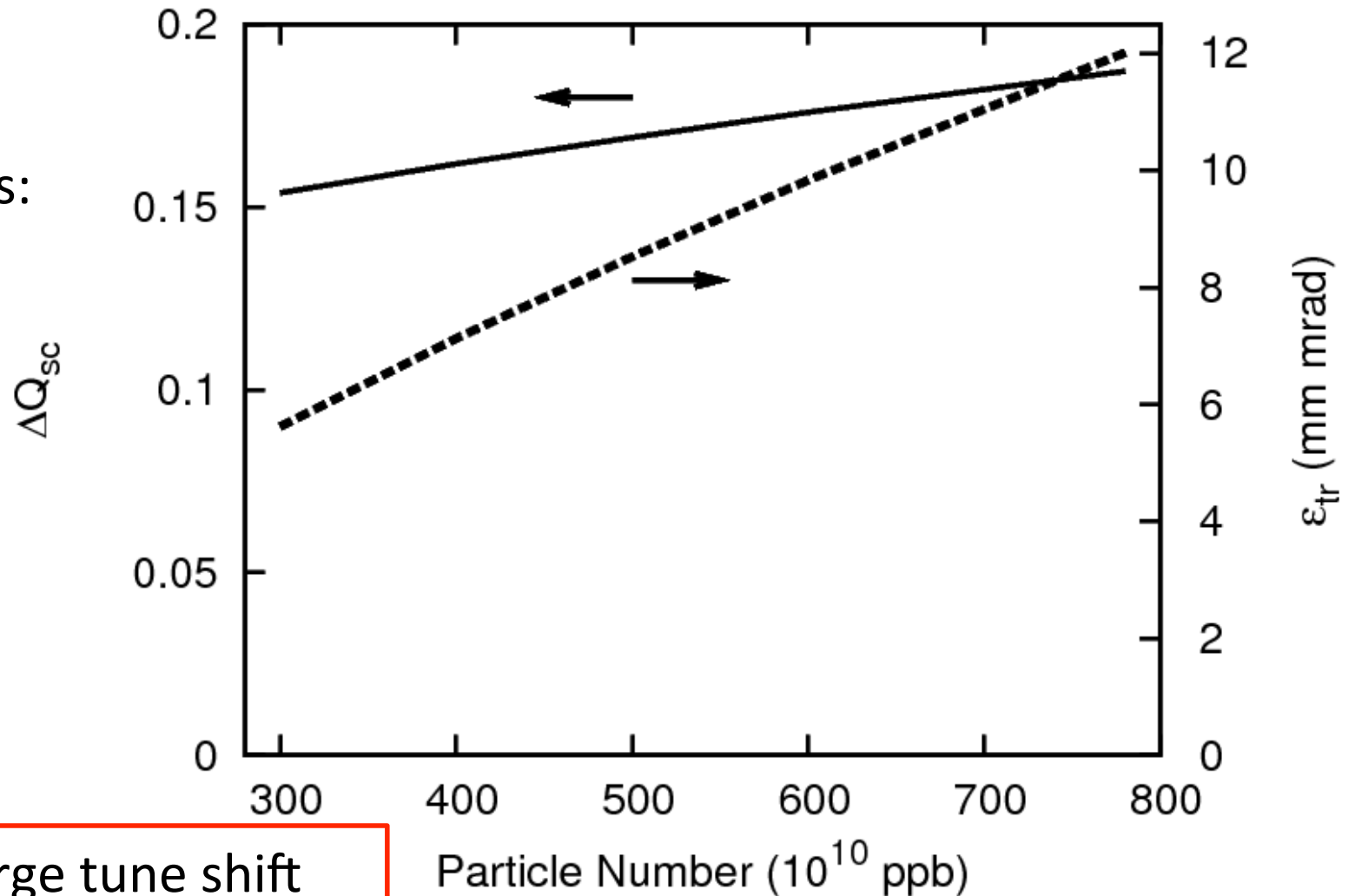
- different transverse amplitudes
- density variation along the bunch

from the measurements:  
 space-charge  
 tune shift for  
 single rf,  
 $V_0=8\text{kV}$ ,  
 $300\text{e}10$  ppb,  
 horizontal



**the space-charge parameter  $q$  is a flat function along the intensity;  
 very strong space charge regime => a minor change not crucial;  
 no Landau damping for the relevant ( $k < 6$ ) head-tail modes**

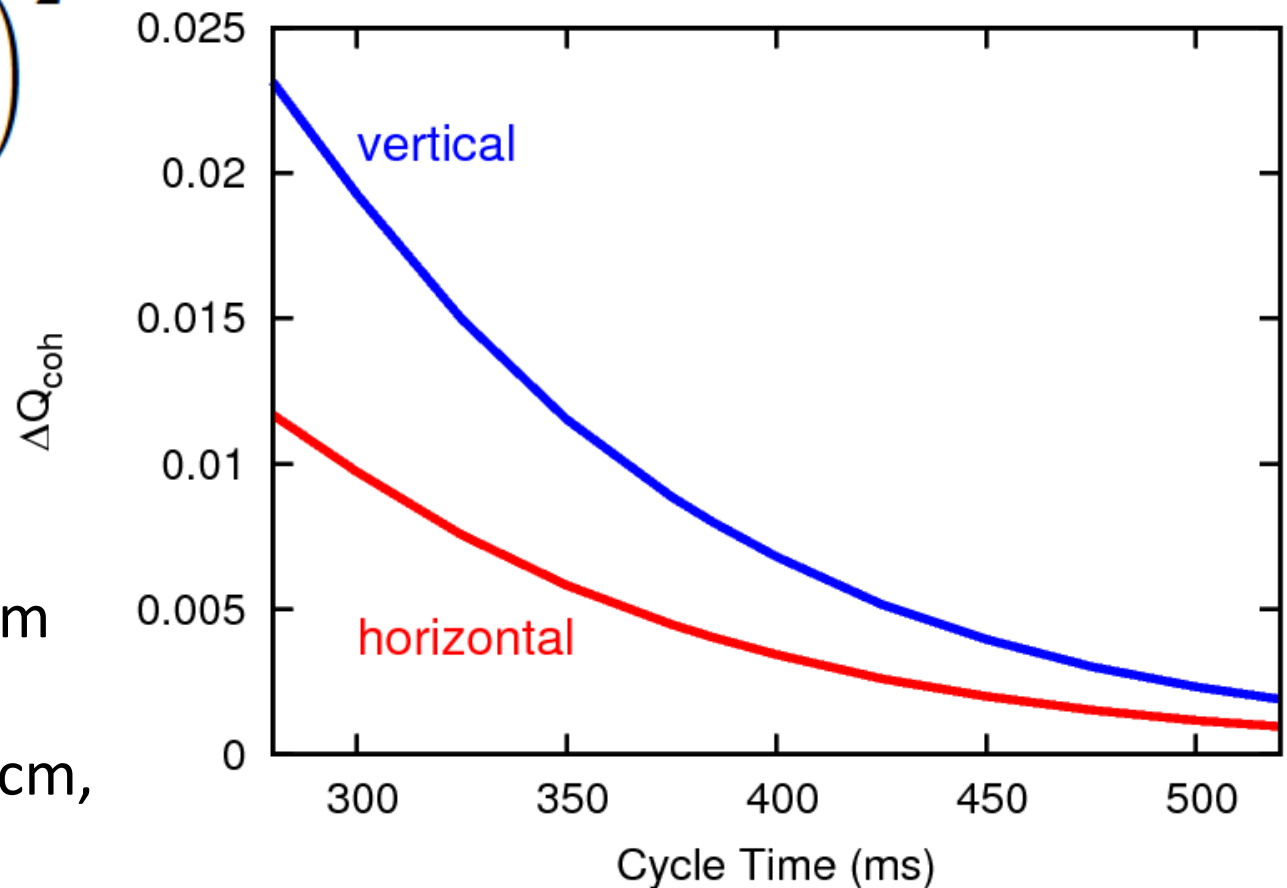
from the measurements:  
space-charge  
tune shift  
at C385,  
for single rf,  
 $V_0=8\text{kV}$ ,  
horizontal



the space-charge tune shift  
is a flat function along the  
intensity

$$\frac{\Delta Q_{\text{coh}}}{\Delta Q_{\text{sc}}} = \left( \frac{a_{\text{beam}}}{b_{\text{pipe}}} \right)^2$$

here for the PSB vacuum pipe assumed:  
 two-thirds circular  $b=8\text{cm}$ ,  
 one-third elliptic,  
 horizontal  $h=8\text{cm}$ ,  
 vertical  $w=3.5\text{cm}$





the eigenfrequencies of the bunch head-tail modes  
for the airbag bunch model  
with arbitrary space-charge and coherent force:

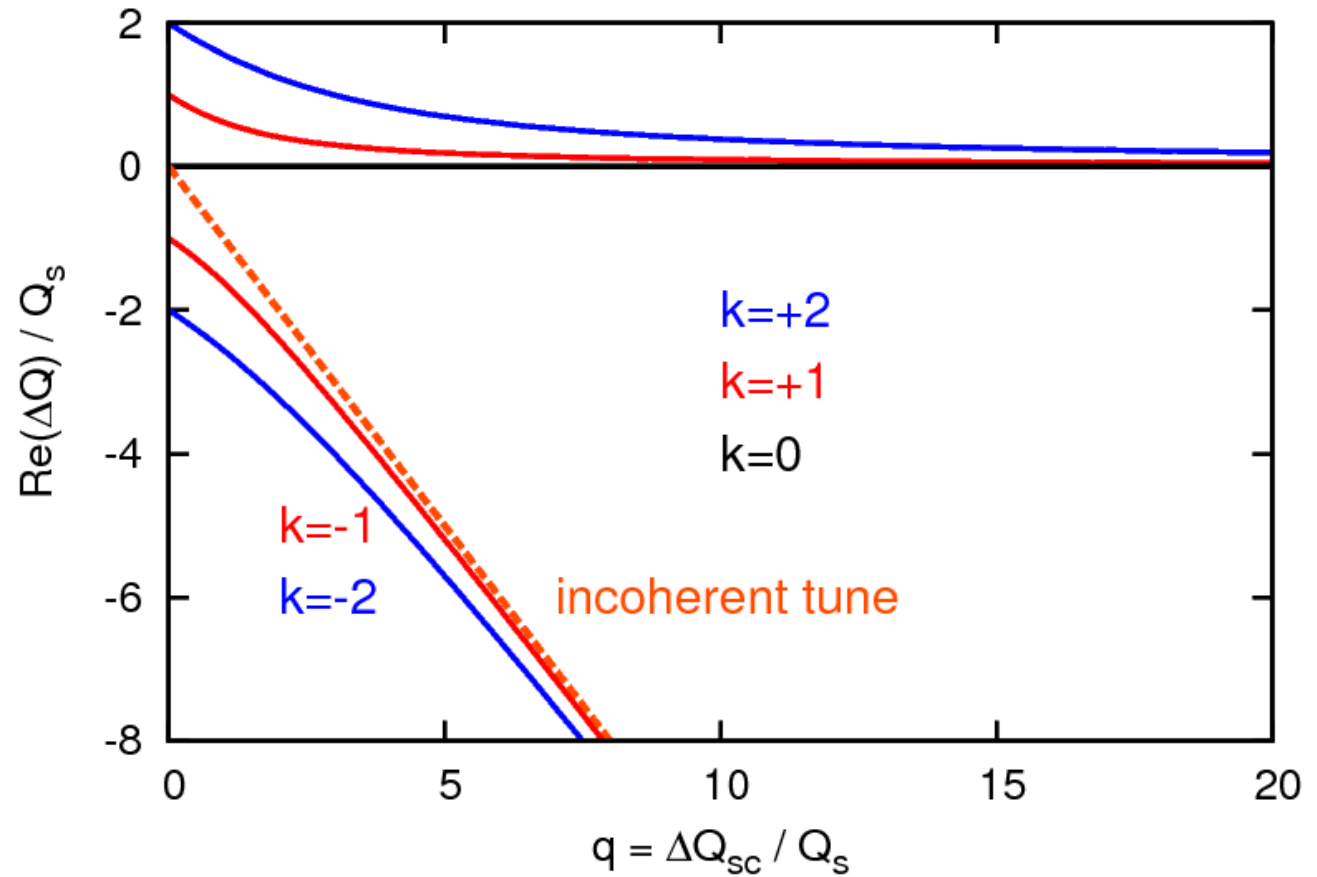
$$\Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_{coh}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc} - \Delta Q_{coh}}{2}\right)^2 + k^2 Q_s^2}$$

$$\frac{\Delta Q_k}{Q_s} = -\frac{q}{2} \left(1 + \frac{\Delta Q_{coh}}{\Delta Q_{sc}}\right) \pm \sqrt{\frac{q^2}{4} \left(1 - \frac{\Delta Q_{coh}}{\Delta Q_{sc}}\right)^2 + k^2}$$

O.Boine-Frankenheim, V.Kornilov, PRSTAB **12**, 114201 (2009)  
V.Kornilov, O.Boine-Frankenheim, PRSTAB **13**, 114201 (2010)  
M.Blaskiewicz, PRSTAB **1**, 044202 (1998)

with space charge  
only,  $\Delta Q_{\text{coh}} = 0$

the  $k=0$  mode is not  
affected;  
the positive modes  
close to  $Q_0$ ;  
the negative modes  
close to the incoherent  
tune and are strongly  
damped



with a coherent tune shift,

$$\Delta Q_{\text{coh}} / \Delta Q_{\text{sc}} = 0.1$$

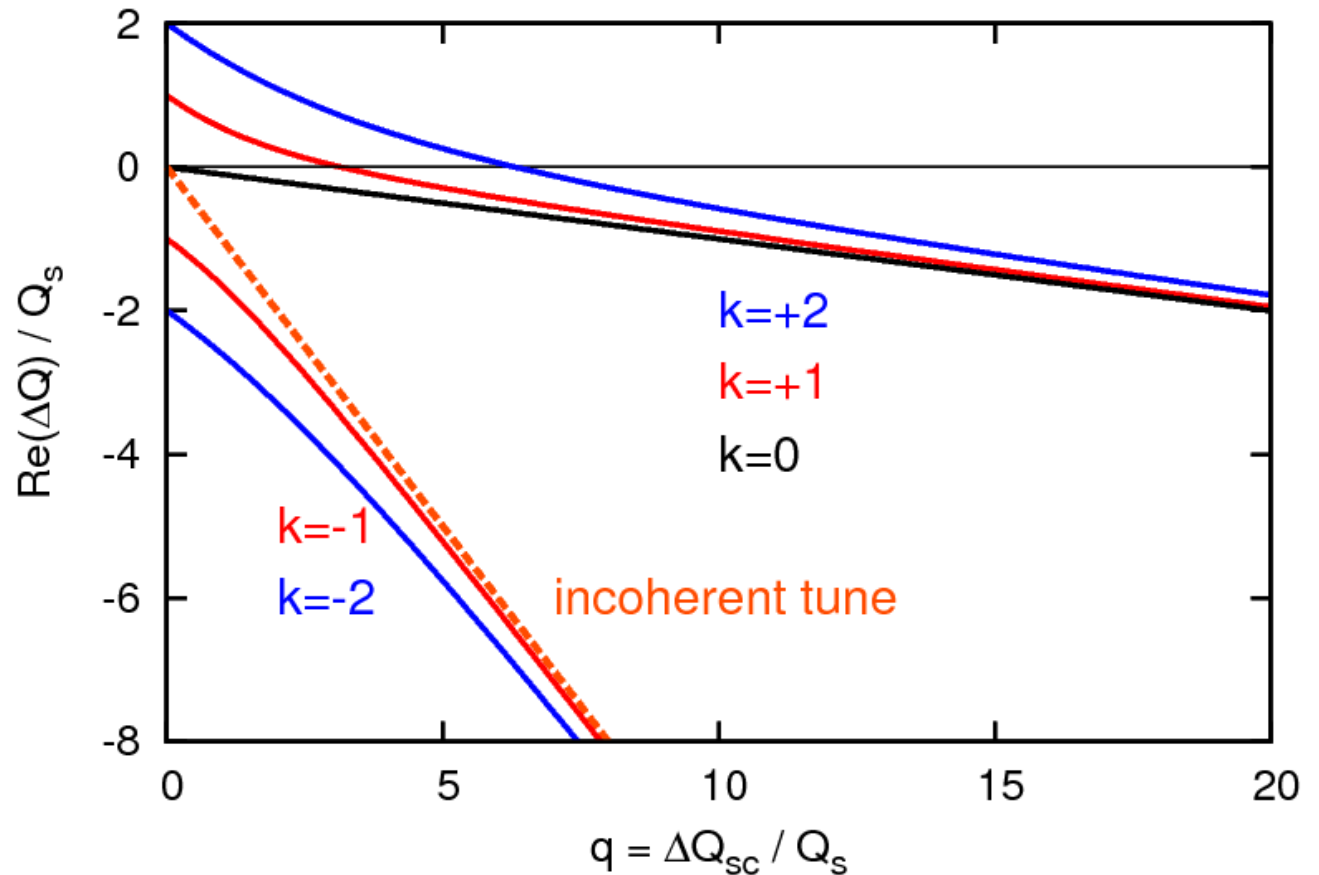
the  $k=0$  mode:

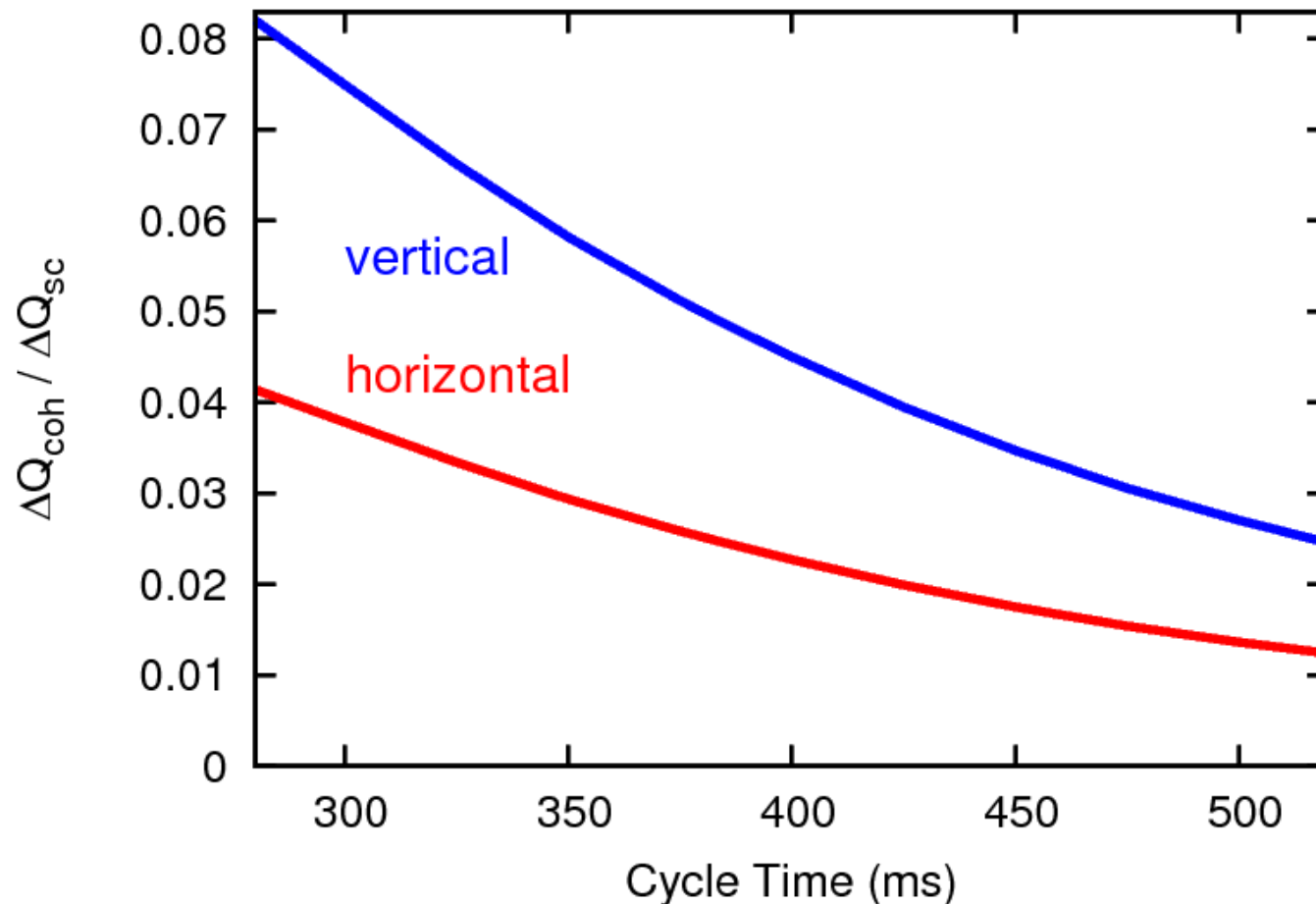
$$\Delta Q = -\Delta Q_{\text{coh}}$$

the  $k>0$  modes enter the incoherent spectrum

$$-2\Delta Q_{\text{sc}} < \Delta Q < 0$$

=> Landau damping

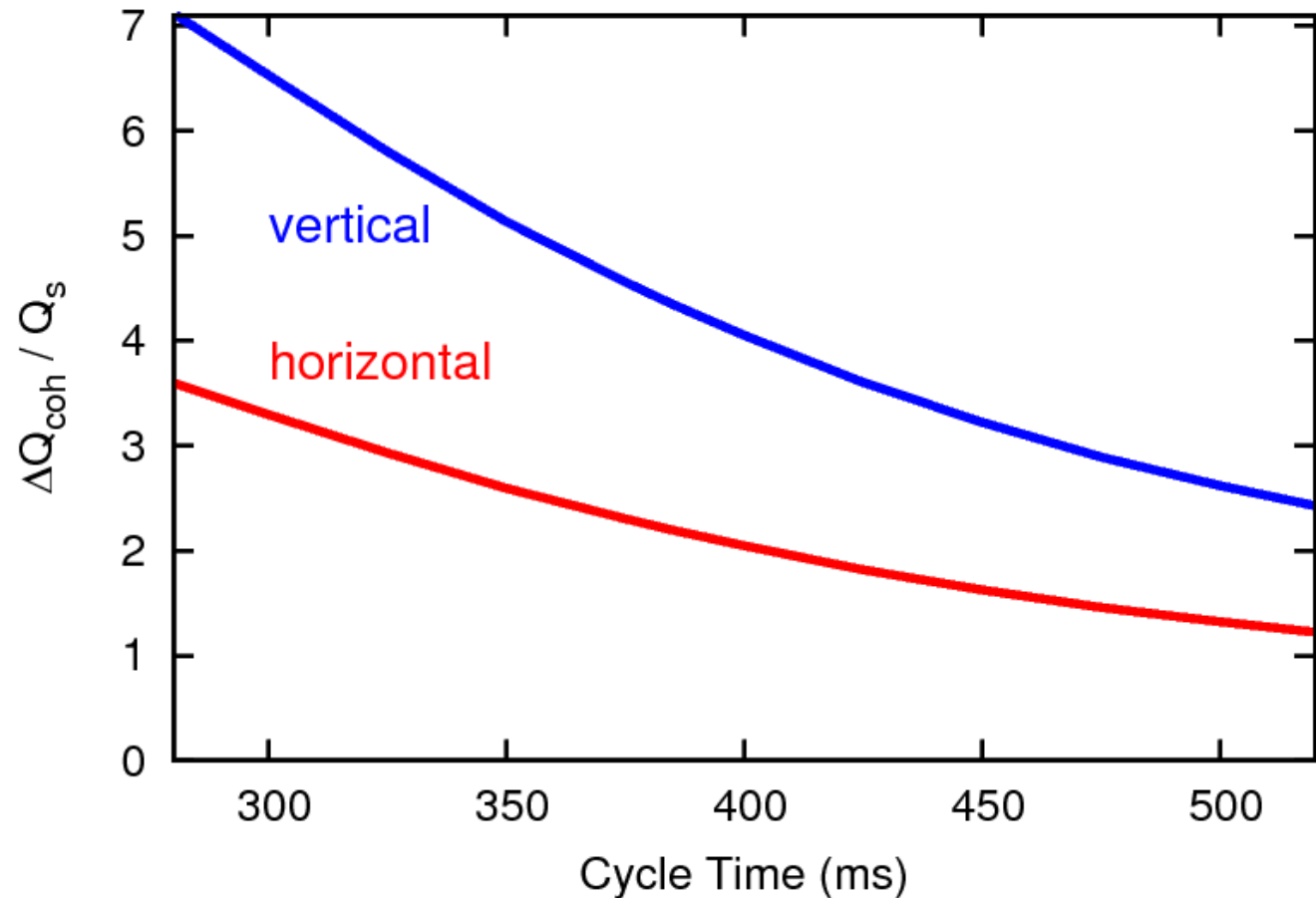




Landau damping is stronger in the vertical plane; the damping contribution decreases along the cycle. **This may contribute to the horizontal exclusiveness and to the later occurrence in CTime**

The real  $\Delta Q_{\text{coh}}$  is larger than the synchrotron tune.

During the cycle, other transverse impedance must cause mode coupling and a fast TMCI. Space-charge tune shifts prevent it.



**This might be an experimental proof of the mode coupling suppression by space charge**

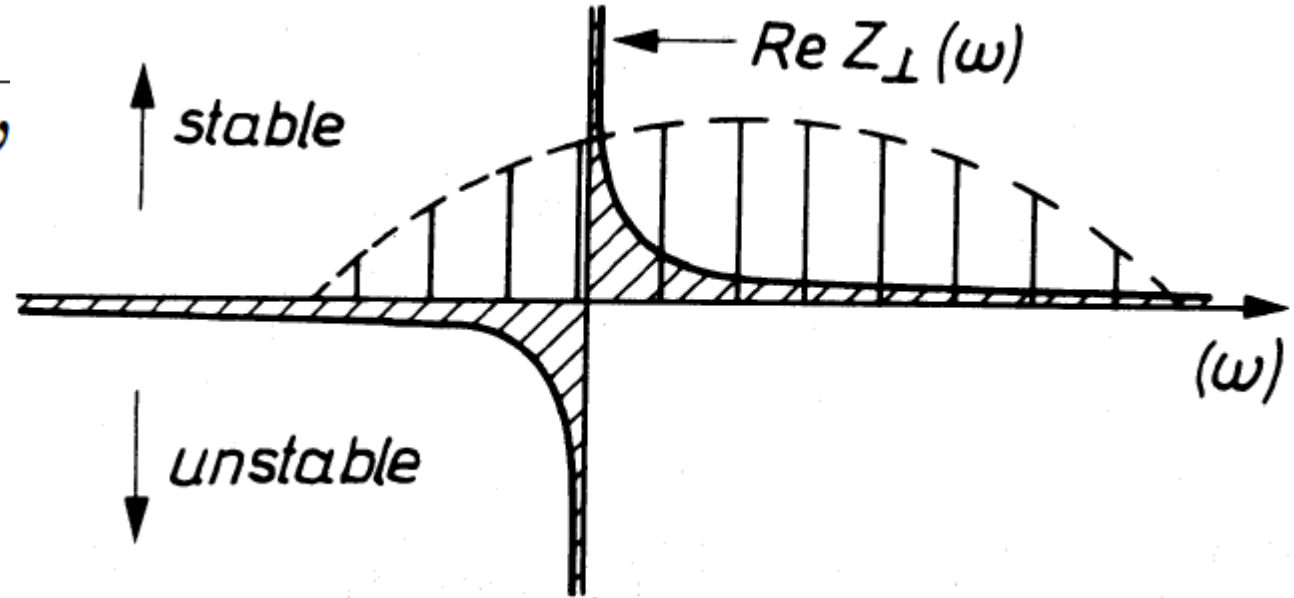
Theory predictions: Blaskiewicz prstab 1998; Burov prstab 2009

F.Sacherer 1974

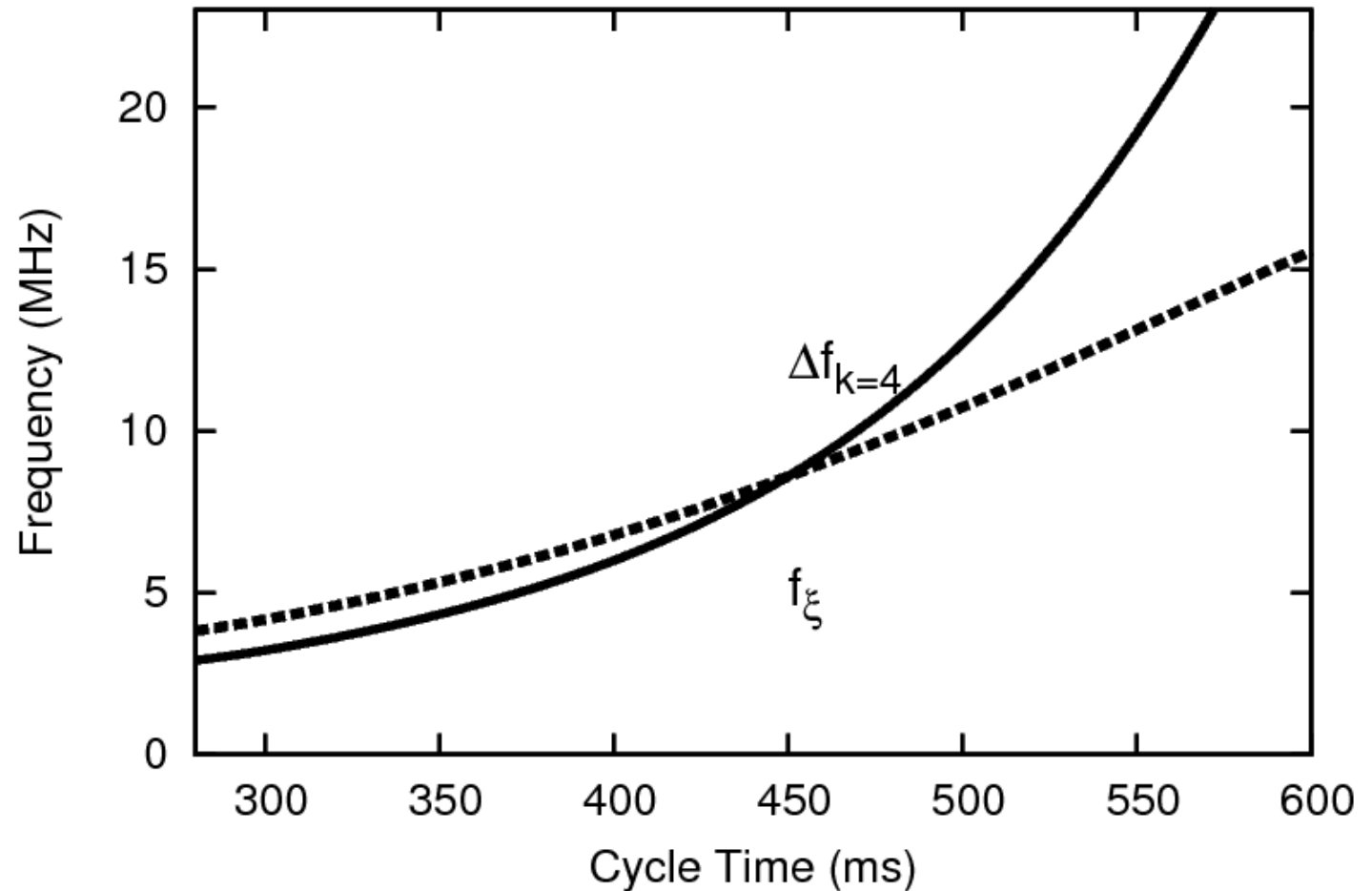
$$\Delta Q_k = \frac{\Upsilon}{1+k} \frac{\sum (-i) Z_{\perp}(\omega_p) h_k(\omega_p - \omega_{\xi})}{\sum h_k(\omega_p - \omega_{\xi})}$$

$$\omega_p = (p + Q_0)\omega_0 + k\omega_s$$

$$\Upsilon = \frac{I_0 q_{ion}}{4\pi \gamma m c Q_0 \omega}$$



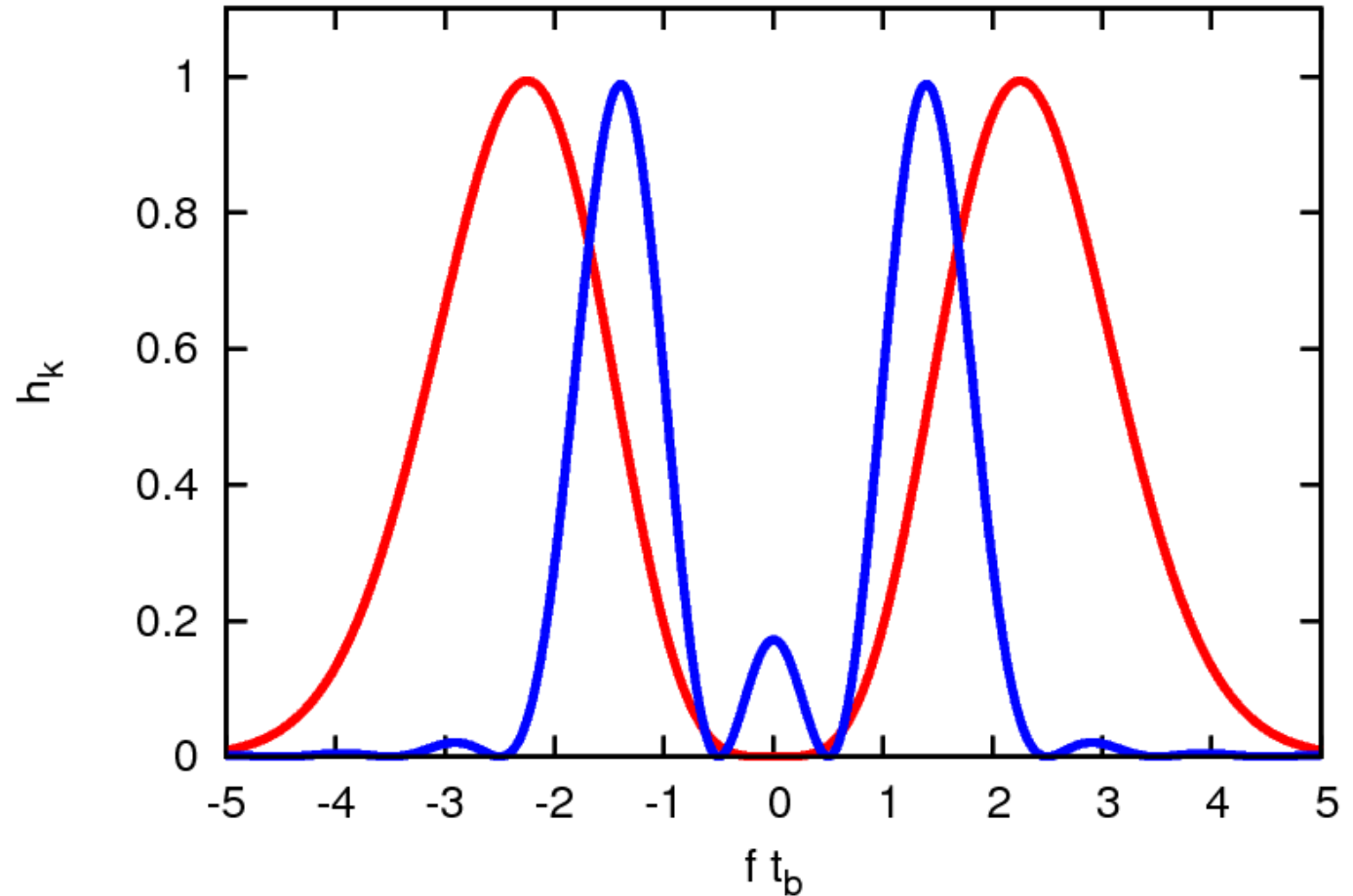
Evolution along the cycle of the chromaticity frequency shift (the  $k=0$  mode) and the spectrum position of the  $k=4$  mode.



Beam spectrum of the  $k=2$  mode for different bunch profiles:  
the sinusoidal bunch,  
the Gaussian bunch.

A realistic bunch is more complicated

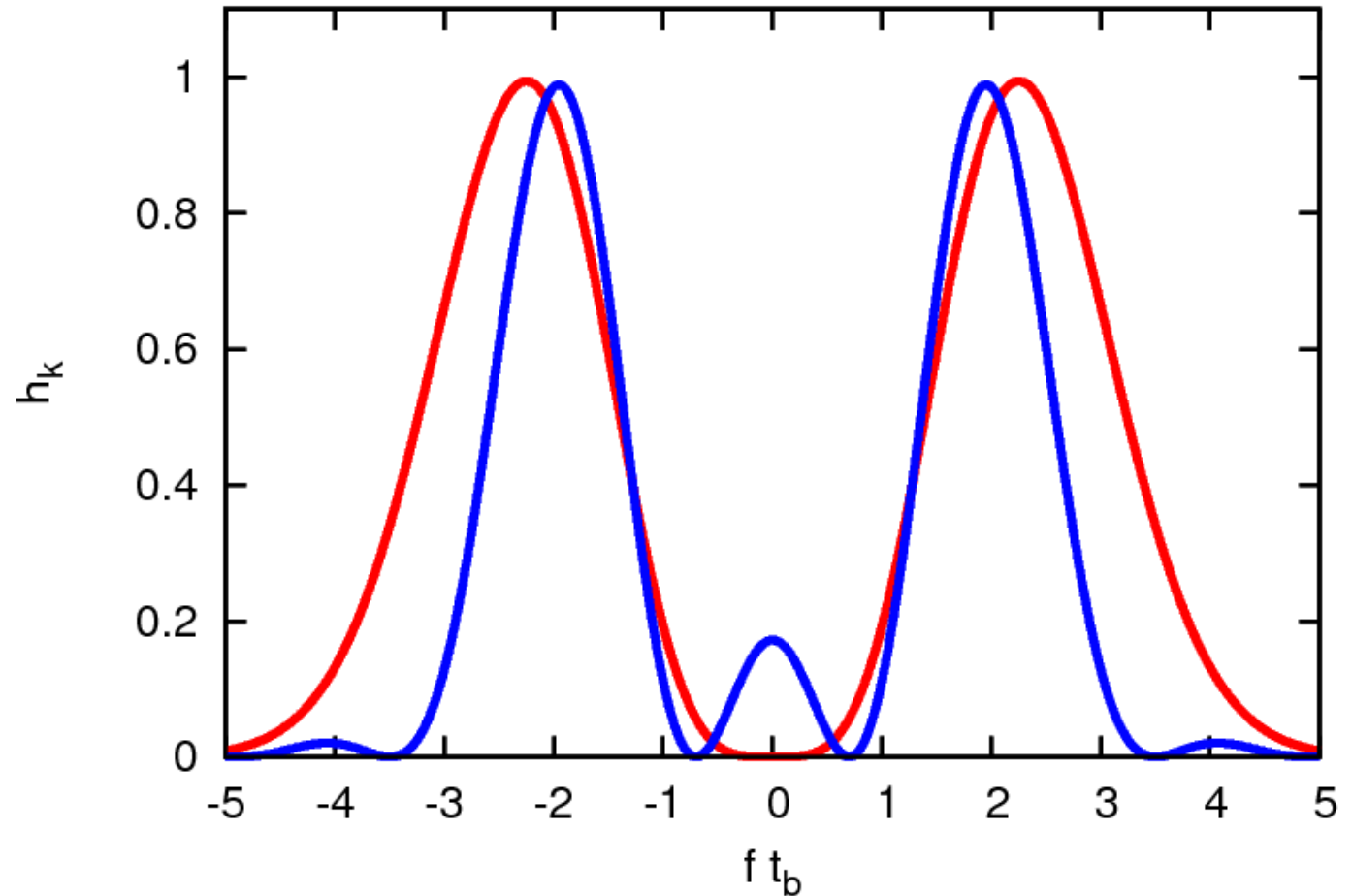
This causes some uncertainty for the impedance the bunch couples to.





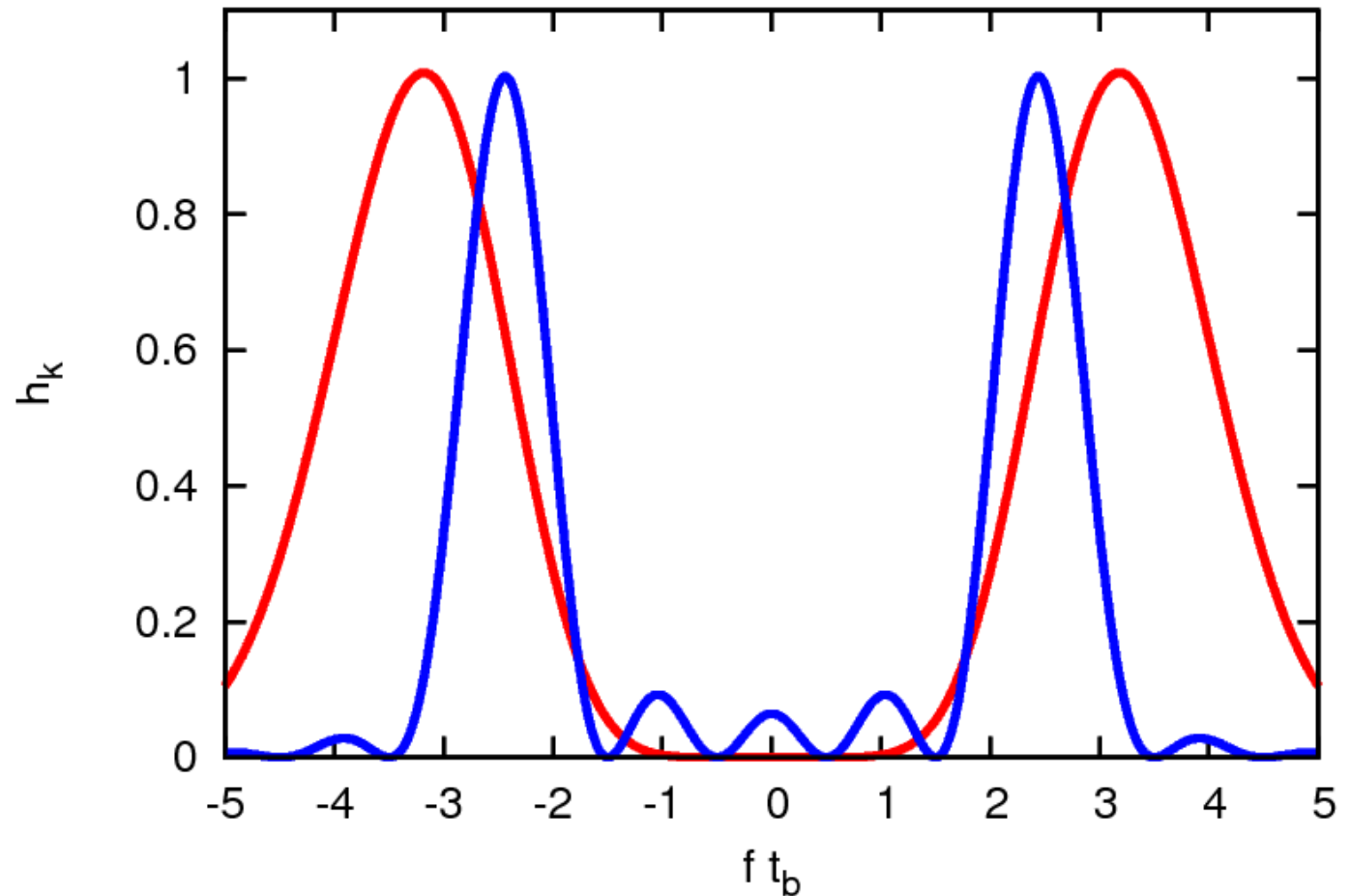
To cope with this difference, the spectrum of a sinusoidal bunch can be stretched, here by a factor 1.4.

the sinusoidal bunch  
the Gaussian bunch



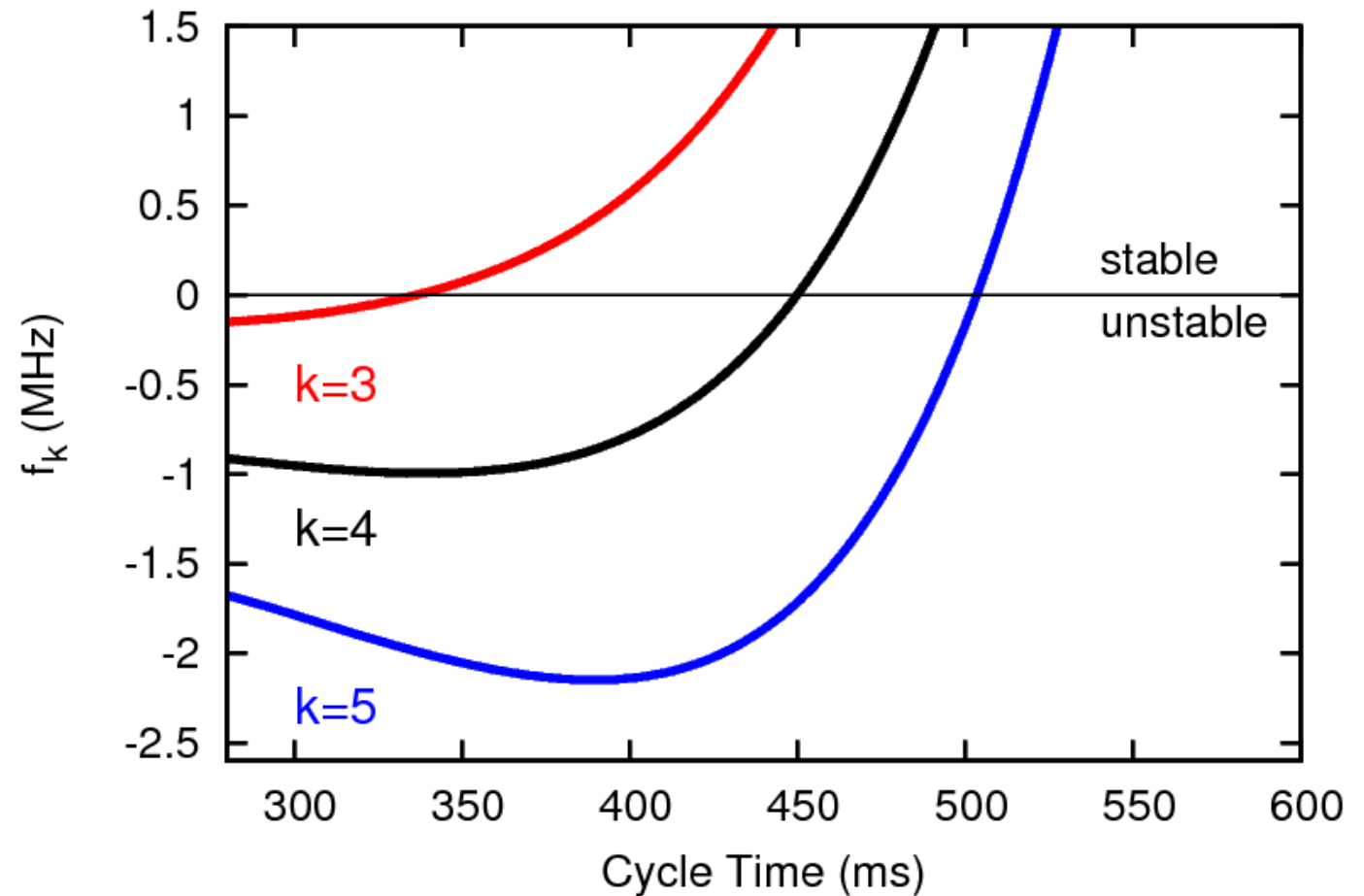
The spectrum width is also an uncertainty factor.

Here the bunch spectrum of the  $k=4$  mode for  
the sinusoidal bunch,  
the Gaussian bunch.



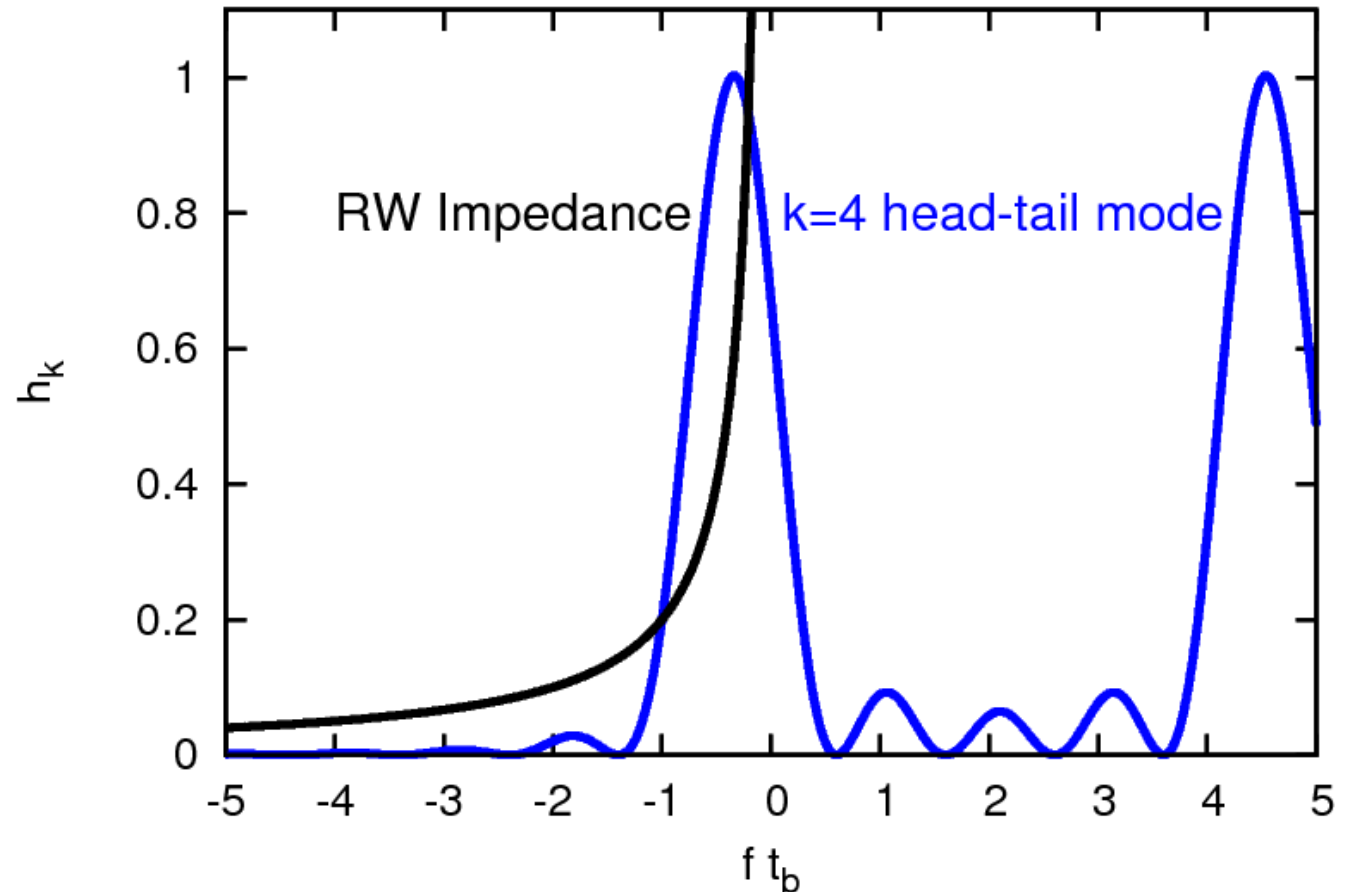
Evolution of the lower (unstable) bunch spectrum part along the PSB cycle .

Higher-order modes cross the Resistive-Wall Impedance later in Ctime.



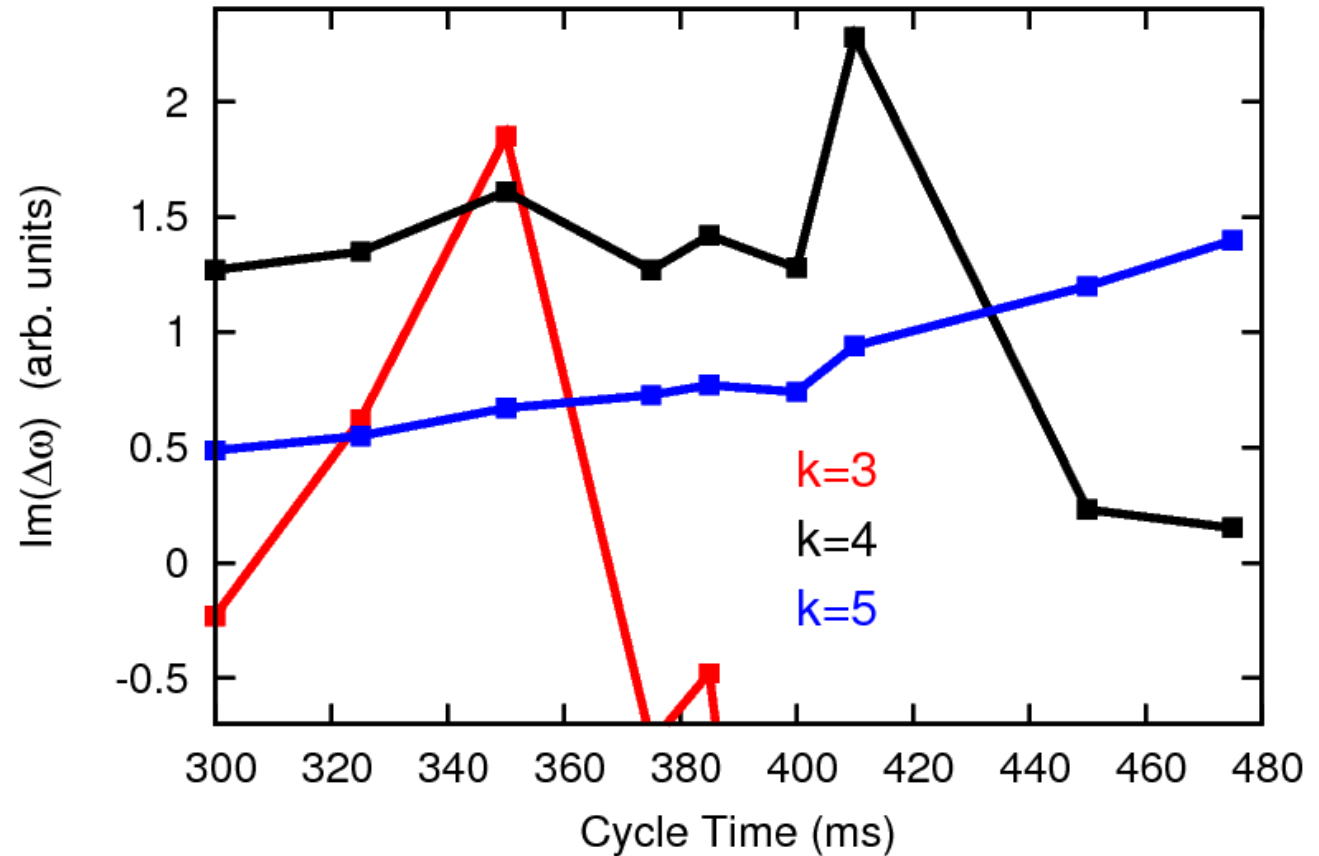
As the bunch spectrum migrates during the cycle, it can become unstable due to coupling to the Resistive-Wall impedance, here a “narrowband” impedance.

Another low-frequency (MHz) narrowband impedance can not be ruled out.



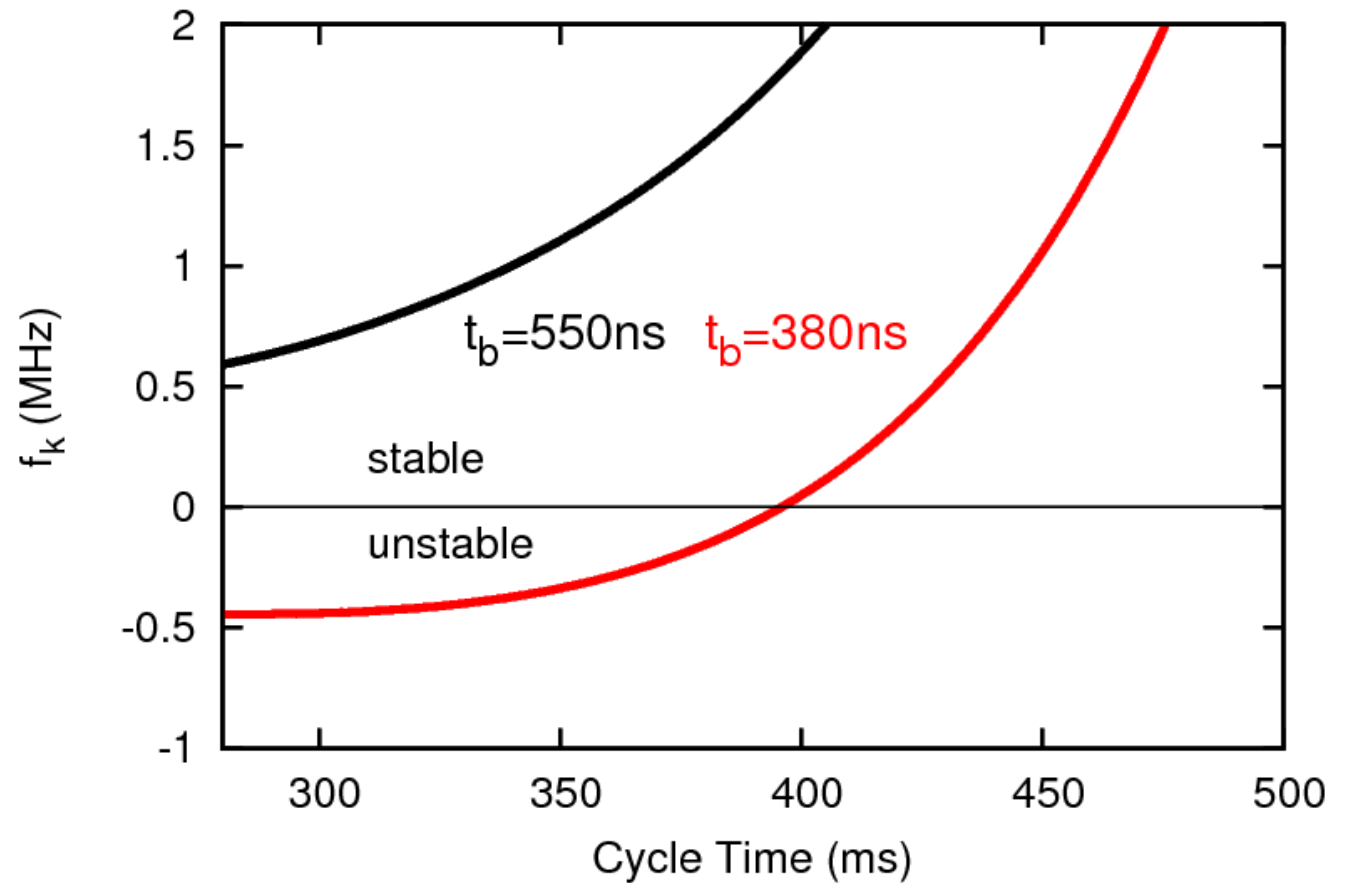
Growth rates of the head-tail modes along the PSB cycle, as given by the Sacherer model, for the Resistive-Wall impedance

the main uncertainty for the quantitative estimations is due to the bunch spectrum (here a preliminary example for  $\xi=-0.8$ , detailed analysis in progress)



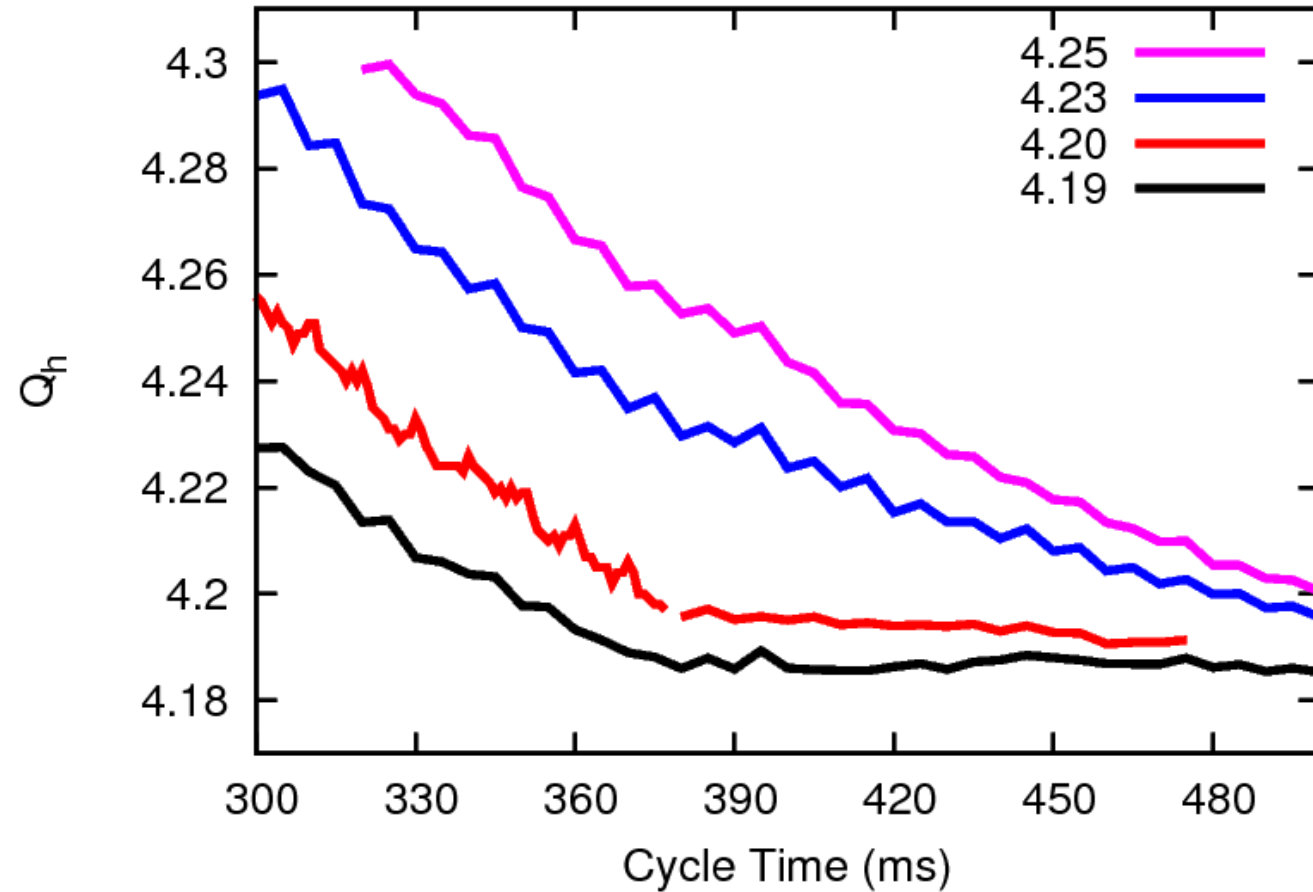
Lower-order head-tail modes for shorter bunches.

Evolution of the  $k=2$  lower spectrum peak along the PSB cycle.

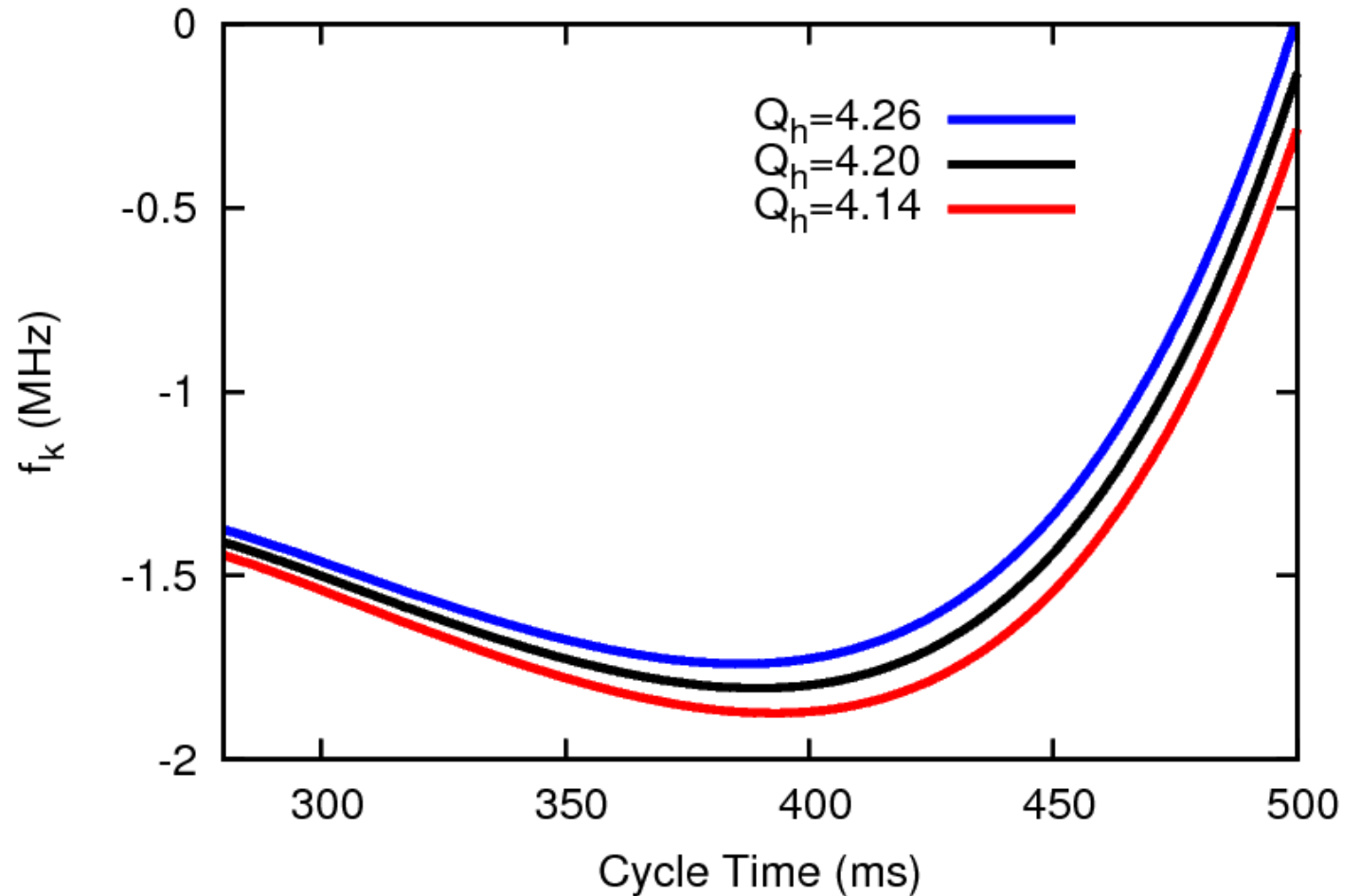


By changing the lattice tunes a (small) systematic shift of the instability Ctime has been observed.

- $Q_h=4.19$ : around C384
- $Q_h=4.20$ : around C386
- $Q_h=4.23$ : around C389
- $Q_h=4.25$ : around C392



Effect of the lattice betatron tune on the evolution of the frequency position for the  $k=4$  mode.





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The unstable head-tail modes observed during the PSB ramp are normally strongly deformed by the impedance

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Intensity thresholds, growth rates and the mode structure are compared for single rf bucket and for double rf types: PSB standard, flat-bunch, short-bunch

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The PSB bunches are in the strong space-charge regime

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The Landau damping due to image charges with the direct space charge, if strong enough for PSB parameters, may contribute to the horizontal instability and to the later Ctimes of the instability

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Analysis of the time evolution of the head-tail modes according to the Sacherer theory can explain higher-order modes for later Ctimes, lower-order modes for shorter bunches, and the Resistive-Wall impedance (or a low-frequency narrowband) as the driving force.

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