Transverse Oscillations for Coasting Beams with Strong Space Charge

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Möhl – Schönauer Equation

- In 1974, D. Möhl and H. Schönauer suggested to describe coasting beam oscillations by the following equation:

$$\frac{d^2 x_i}{dt^2} + \Omega_i{}^2 Q_i{}^2 x_i + 2\Omega_0{}^2 Q_0 \left[Q_c \bar{x} + Q_{sc} \left(x_i - \bar{x} \right) \right] = 0 \,.$$

lattice — wake — space charge

$$Q_{sc} = Q_{sc}(\mathbf{J}_i); \qquad Q_{sc}(0) = -\frac{N r_0}{4\pi \beta^2 \gamma^3 \varepsilon_{\mathrm{rms}}} \,.$$

action

Coherent tune shift, in a frequency domain:

$$Q_c = -i \frac{N r_0 \beta_x}{\gamma C} \frac{Z_x}{Z_0}\,; \qquad Z_0 = \frac{4\pi}{c} = 377\,\mathrm{Ohm}$$

Space charge term

$$\frac{d^2 x_i}{dt^2} + \Omega_i{}^2 Q_i{}^2 x_i + 2\Omega_0{}^2 Q_0 \left[Q_c \bar{x} + Q_{sc} \left(x_i - \bar{x} \right) \right] = 0 \,.$$

- The only non-trivial term in this equation is the space charge force:

Space charge term

$$\frac{d^2 x_i}{dt^2} + \Omega_i^{\,2} Q_i^{\,2} x_i + 2\Omega_0^{\,2} Q_0 \left[Q_c \bar{x} + Q_{sc} \left(x_i - \bar{x} \right) \right] = 0 \,.$$

- The only non-trivial term in this equation is the space charge force:

$$Q_{sc} \left(x_i - \bar{x} \right)$$

Space charge term

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- The only non-trivial term in this equation is the space charge force:

$$Q_{sc} \left(x_i - \bar{x} \right)$$

- Why non-linear space charge forces are described by a linear term? Does it mean that it is valid only for K-V (constant density profile) distribution?

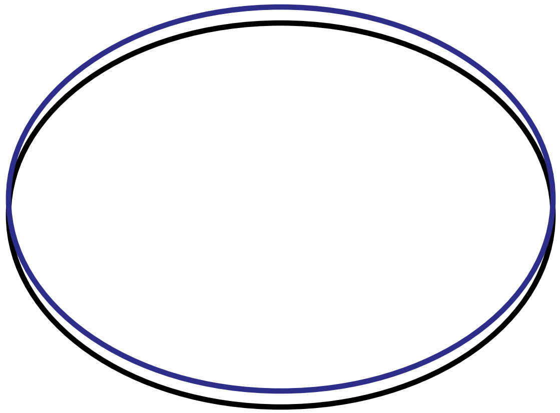
- No, it is valid for any beam profile. The reason is following:

Comments to Möhl-Schönauer Equation (MSE)

Space charge term: $Q_{sc}(\mathbf{J}_{\perp})\left(x_i - \bar{x}\right)$

- The single-particle motion consists of 2 parts:
  - free oscillations (typically with beam size amplitude, or $J_{\perp} \simeq \varepsilon_{\mathrm{rms}}$)
  - driven by the coherent offset oscillations (much smaller than the beam size).

- The equation describes the driven oscillations only, so it results from linearization of the original non-linear space charge term over infinitesimally small coherent motion, and averaging over the betatron phases of the free incoherent oscillations.



MSE as a unique choice

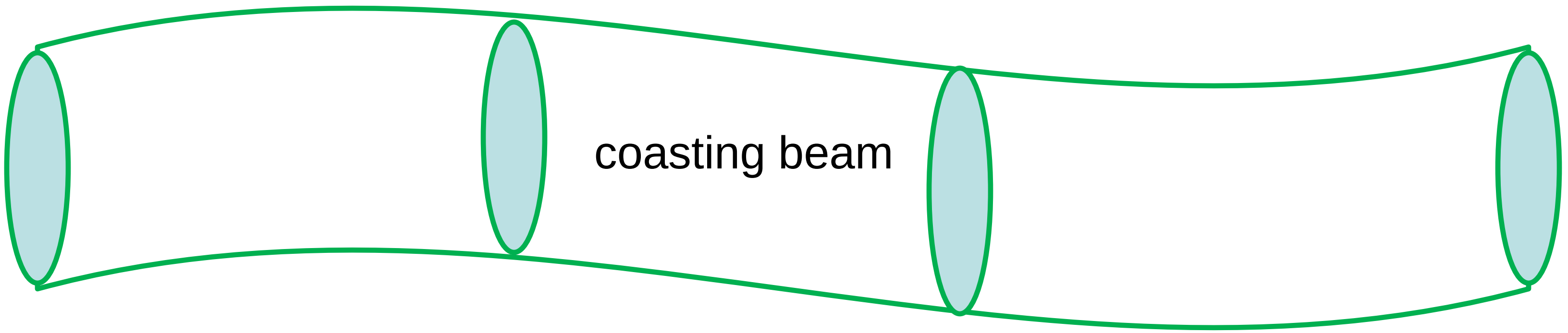
- This equation is a unique possibility for
  - A linear equation (driven oscillations are small!)
  - With time-independent coefficients (more on that below…)
  - With the given tunes and incoherent SC tune shifts $Q_{sc}(\mathbf{J}_\perp)$
  - Space charge force $\propto (x_i - \overline{x})$

Time-independent $Q_{sc}(\mathbf{J})$ = rigid-slice approximation

$$\frac{d^2 x_i}{dt^2} + \Omega_i^{\,2} Q_i^{\,2} x_i + 2\Omega_0^{\,2} Q_0 \left[Q_c \bar{x} + Q_{sc} \left(x_i - \bar{x} \right) \right] = 0 \,.$$

lattice — wake — space charge

- The assumption for the rigid slice approximation is that a core of any beam slice moves as a whole, so there is no inner motion in it.



- This is a good approximation, when the space charge is strong enough:

$$| Q_{sc} | >> \Delta Q_i \,.$$

- Otherwise, the beam shape oscillates as well, and MSE is not necessarily valid.

Coasting beam: Landau damping (Burov, Lebedev, 2008):

$$\Lambda = -\pi \left\langle \Delta Q_{sep} \right\rangle \int \Delta Q_{sep} f_x J_x \ \delta(\Delta Q_l + Q_{sc} - \mathrm{Re}\, \nu_c) d\Gamma, \qquad f_x \equiv \partial f / \partial J_x \ ;$$

$$\Delta Q_{sep} \equiv \mathrm{Re}\, \Delta Q_c - Q_{sc}(J_x, J_y)\,, \qquad \left\langle \Delta Q_{sep} \right\rangle \equiv -\left(\int \frac{f_x J_x \ d\Gamma}{\Delta Q_{sep}} \right)^{-1} ;$$

$$\mathrm{Re}\, \nu_c = \mathrm{Re}\, \Delta Q_c + \delta Q^{(1)} + \delta Q^{(2)}\,,$$

$$\mathrm{Im}\, \nu_c = \mathrm{Im}\, \Delta Q_c - \Lambda$$

$$\delta Q^{(1)} = -\left\langle \Delta Q_{sep} \right\rangle \int \frac{\Delta Q_l f_x J_x \ d\Gamma}{\Delta Q_{sep}}\,,$$

$$\delta Q^{(2)} = -\left\langle \Delta Q_{sep} \right\rangle \int \frac{\Delta Q_l^2 f_x J_x \ d\Gamma}{\Delta Q_{sep}^2}\,.$$

Landau Damping

- Landau damping is proportional to the phase space density of the resonance particle:

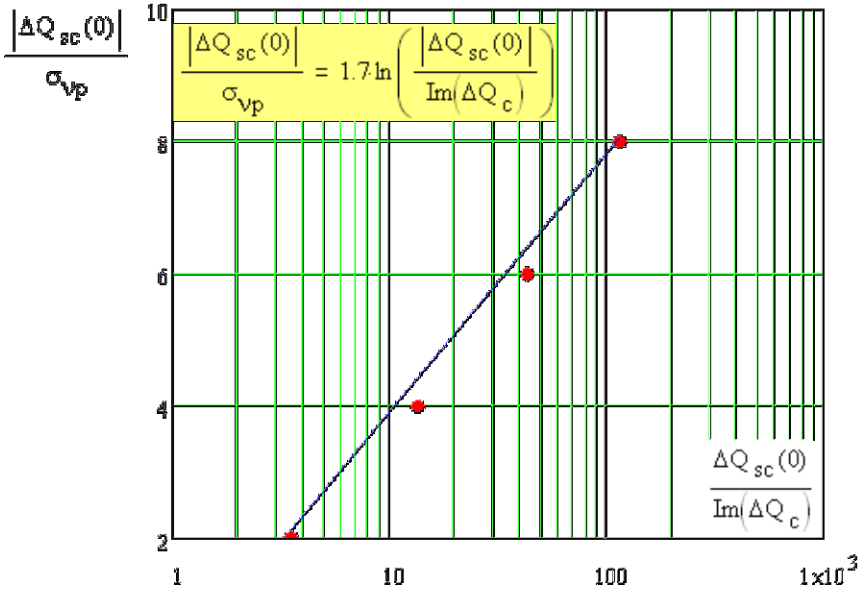
$$\Lambda = -\pi \left\langle \Delta Q_{sep} \right\rangle \int \Delta Q_{sep} f_x J_x \ \delta(\Delta Q_l + Q_{sc} - \mathrm{Re}\,\nu_c) d\Gamma$$

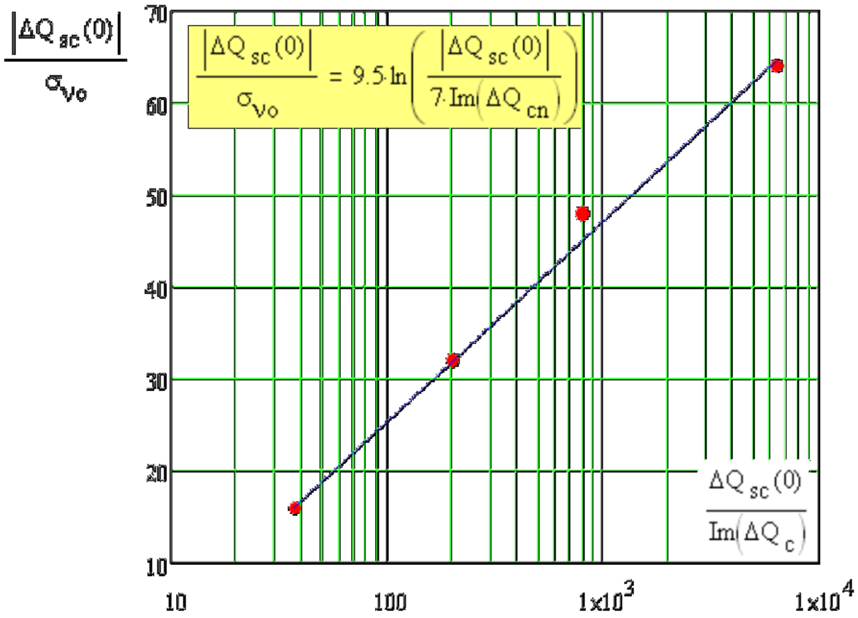
- Thus, with strong space charge, it is determined by the distribution tails.

- Typically, the far tails are neither predictable nor measurable, and not even reproducible in many cases. That is why, at strong space charge, the stability thresholds are predictable and reproducible with a moderate accuracy. In the RR it was 20-30% in the intensity.

Coasting Beam Thresholds (Burov, Lebedev, 2008)

- Thresholds are determined by $\Lambda = \operatorname{Im}\Delta Q_c$. For a round Gaussian beam:





Chromatic threshold

$$\sigma_{vp} \equiv |\xi - n\eta| \left(\frac{\delta p}{p}\right)_{\rm rms}$$

$$\Delta Q_l = (\xi - n\eta)\hat{p} \equiv \sigma_{vp}\hat{p}/\sigma_p$$

Octupole threshold

$$\Delta Q_i = \sigma_{vo}(J_x + J_y)/2\varepsilon \ > 0\,;$$

$$\langle J_x \rangle = \langle J_y \rangle = \varepsilon$$

Direct application to bunched beams

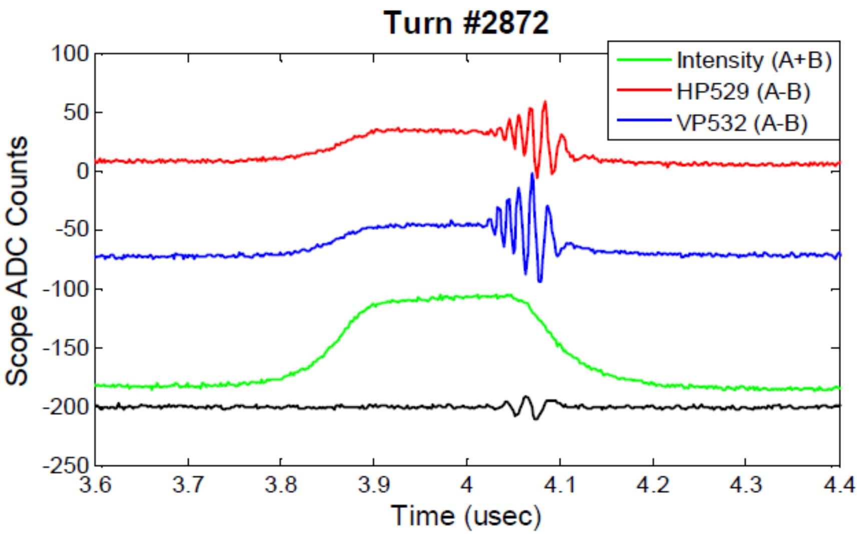
- Coasting beam results sometimes can be applied to bunched beams.

  – For short-wavelength and fast oscillations (much shorter than the bunch length, much faster than the synchrotron frequency) – the so-called microwave instability;

  – For multi-bunched beam, at zero chromaticity and long wavelength (much longer than the bunch spacing)

Antiproton instabilities at Fermilab Recycler Ring

Typical instability snapshot (L. Prost et al, 2011,
http://lss.fnal.gov/archive/test-tm/2000/fermilab-tm-2498-ad.pdf):



- Instability was always at the tail
- Frequency 70 MHz – by the damper
- Threshold prediction : ±30%