<u>Bunch Shape and Luminosity Profile</u>

J. Tückmantel, CERN, BE-RF, 20.07.10

It was remarked that the spatial luminosity profiles L(z) as measured by the experiments are looking very much Gaussian. From this fact it might be concluded that bunches in the machine *must* also have Gaussian profile. We will show that this conclusion is not correct for realistic data and that – even under the assumption of perfectly identical bunches – a de-convolution of L(z) will only give very imprecise bunch profiles.
It is well known that for two identical Gaussian bunches L(z) will be precisely Gaussian, then with half the σ^2 of the bunches. However, we will now calculate L(z) for a $\cos^2$ bunch profile that is often used to describe long bunches in proton machines[1] and that significantly deviates from a Gaussian shape with the same 'nominal bunch length', especially in the tails, Fig. 1.
The $\cos^2$ profile can be described by the parameter x with $-0.5 \le x \le 0.5$ relative to the full bunch length

$$\rho(x) \propto \begin{cases} \cos^2(x\cdot\pi) & [-\frac{1}{2} \le x \le +\frac{1}{2}] \\ 0 & [else] \end{cases} = \begin{cases} \big(1 - \cos(x\cdot 2\pi)\big)/2 \\ 0 \end{cases}$$

For simplification we have not normalized ρ(x), not important in the present context.

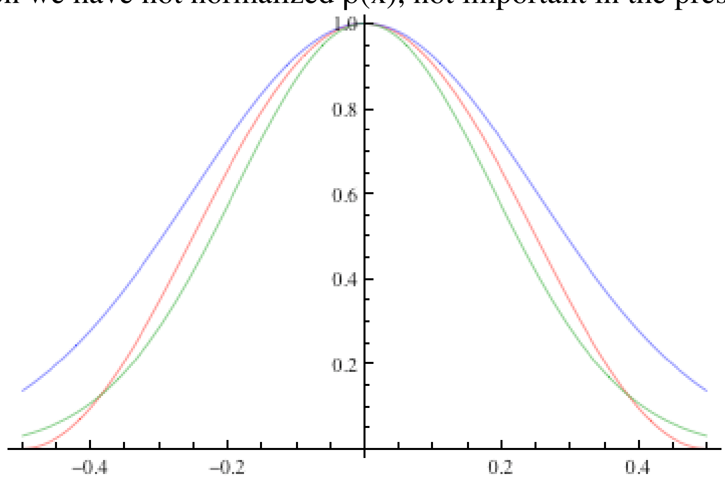

Fig.1: $\cos^2$ bunch profile (red), Gaussian profile with the same nominal "4σ bunch length" (blue) and Gaussian profile reproducing best the $\cos^2$ luminosity profile (green), see later.

The luminous profile is the density product of both bunches at a given z integrated over time. We assume two identical bunches both moving with v≈c in opposite direction. Without loss of generality the variables t and z are defined such that bunch centers meet at z=t=0 and the absolute bunch length is normalized to 1, i.e.

$$L(z) \propto \int_{-\infty}^{+\infty} \rho(z - c\cdot t)\cdot\rho(z + c\cdot t)\ dt$$

Taking the above $\cos^2$ profiles and considering that ρ is zero outside the given x-range, L(z) becomes here proportional to

[1] in contrast to the short Gaussian bunches in synchrotron radiation dominated electron machines

$$L(z) \;\propto \int_{(-1/2+|z|)/c}^{(1/2-|z|)/c} (1+\cos(2\pi(z-c\cdot t)))\cdot(1+\cos(2\pi(z+c\cdot t)))\; dt$$

which is mathematically equal to

$$L(z) \;\propto\; \frac{(2\cdot|z|-1)(2+\cos(4\pi|z|))+3\sin(4\pi|z|))}{4\pi}$$

We did not do a mathematical fit for the best matching Gauss curve for this L(z) but simply played with numbers and plotting the results. The closest match was obtained for $\exp(-28\cdot z^2)$, see Fig. 2, corresponding to $\sigma_L=1/\sqrt{56}\approx 0.134$. One sees (Fig. 2) that both curves are nearly indistinguishable, hence it is nearly impossible from real data to de-convolute the true bunch profile from L(z). This Gaussian luminosity profile would be created perfectly by Gaussian bunches of $\sigma_B=1/\sqrt{28}\approx 0.19$, i.e. bunches would have a "4σ bunch length" of 0.75 compared to a bunch length of 1 for the $\cos^2$ bunches.

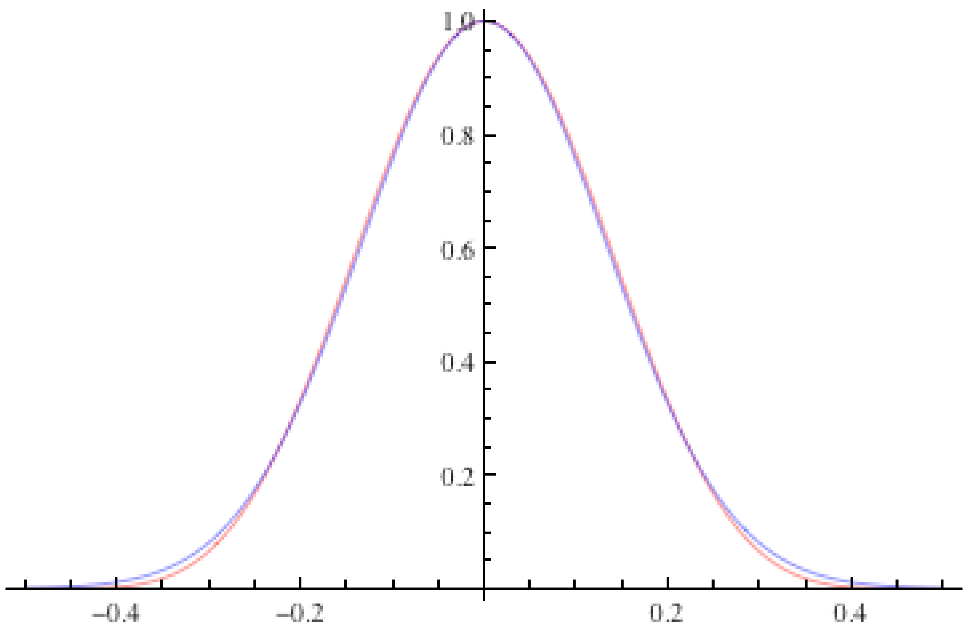

Fig.2: L(z) for $\cos^2$-bunches (red, 4σ=1) and Gaussian bunches (blue). The Gaussian profile drawn is $\exp(-28\cdot z^2)$.

On top of these facts, established for two absolutely identical bunches, one should consider that there is a certain bunch-to-bunch scatter in shape which in general makes things look 'even more Gaussian'. This is true for any two interacting bunches, if only one bunch-pair would be gated out, and even more for the integral L(z) measured over the whole set of bunches (full beam).

Conclusion: It is practically impossible to de-convolute the true bunch-profile from the longitudinal luminosity profile even if measured very precisely by the experiments. This statement even holds when the measurement would be done in gating only a single bunch pair and assuming that both bunches have precisely the same longitudinal density distribution.

Comparison with LHC measurements:
Form Ph. B.: The experiments measure – integrated over all bunch crossings – a Gaussian luminous region with σ_L=60 mm (± a few mm for the different experiments, but very stable along the coast for the bunches now with 'calibrated length' by blow-up during the ramp), i.e. σ_L=180 ps in 'time domain'. From this one would conclude that bunches – if assumed Gaussian – would have $\sigma_B=180\cdot\sqrt{2}$ ps = 255 ps to create such a luminous region. On the other hand the RF claims from the 'Juliameter' a 4σ bunch length[2] of 1350 ps or σ=338 ps. The ratio of these is 0.75, exactly the ratio of the $\cos^2$ full bunch length and the best-fitting (luminosity wise) Gaussian type bunches as obtained above.

This rises (again) the discussion about what is "the" bunch length (and, maybe, if a single number is sufficient to grasp the essential quantities of a bunch)

[2] which is in fact measured as full-width-at-half-maximum and then rescaled to the nominal 4σ bunch length