DEVELOPMENT AND BENCHMARKING OF CODES FOR SIMULATION OF BEAM-BEAM EFFECTS AT THE LHC*

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Abstract

Beam dynamics at modern hadron colliders is strongly affected by effects of beam-beam interactions, which are complex phenomena and generally require extensive numerical simulations for accurate evaluation. In this note we describe the development of codes used for simulations of beam-beam effects at the LHC. We benchmark the codes on several test cases and report the results of this comparison.

1 DESCRIPTION OF CODES

1.1 Lifetrac

The code is described in more detail in Proceedings of PAC05 [1]. Below a brief summary of the code features is given. Lifetrac is a weak-strong beam-beam code that was originally created for simulating equilibrium distributions in electron-positron colliders (circa 1995). Eventually, the author added the functionality to simulate non-equilibrium distributions (2003-2004), making it a conventional macro particle tracking code. Lifetrac tracks a bunch of particles through the machine lattice and beam-beam interactions. The major design principles are the following

- Machine lattice is comprised of 6D linear maps and thin multipoles.
  - Linear maps are specified with the use of beta-functions (conventional or coupled), phase advances and M56. A PERL script will take MAD-X twiss output and convert it into a form readable by Lifetrac.

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  - Thin multipoles are read them from MAD-X with a PERL script
  - RF cavity (one or several per turn) is a sinusoidal kick.
  - Because the machine arcs are linear maps without chromatic dependence, there are two special methods for treatment of the lattice chromaticity. The linear (tune) chromaticity is implemented via an additional phase advance dependent on the particle's momentum, applied one or several times per turn. The second order (beta-function) chromaticity is added in the form of "chromatic drifts". They are added before and after the important IPs. For more details refer [1]. Both methods are symplectic.

- Beam-beam elements are 6D symplectic kicks (Hirata's formulae), their locations and parameters (beta-functions of the strong bunch, separation of the colliding bunches, crossing angle) are also read from MAD-X output. The main IPs are sliced longitudinally into arbitrary number of slices (12 was determined to work well for the Tevatron case of $\sigma_z/\beta^* \simeq 1$). Long-range IPs are not sliced (thin).

- An important feature for the Tevatron was the introduction of various noises in the form of random kicks applied once per turn. These kicks are specified using a correlation matrix.

- We typically track 10,000 particles for $10^6 - 10^7$ turns. Because the number of particles is not sufficient to describe beam emittances and lifetime with good precision, we enhance it by
  - Averaging the density distribution over the simulation step, usually 10,000 turns.
  - Using weighted distribution with more particles in the tails.

The code reads in all particles and then sends them to parallel nodes for tracking over the step. This is done via MPI. During a step nodes don't talk to each other. Then, at the end of each step the particle coordinates are collected by the head process to perform averaging and also for saving the simulation snapshot.

1.2 Sixtrack

The well-known SixTrack code [2] treats motion in the full 6d phase space [3] with the extended Hamiltonian, i.e. it should be used for large proton accelerators where the relative momentum deviation deltap is not excessively large. The 4d BB force is treated by the usual formalism [4] while the 6d case is treated à la Hirata [5] and updated by Ripken to include arbitrary crossing planes and coupling [6]. The code has been optimized for speed and an elaborate run environment has been set-up to allow for massive tracking studies [7].

1.3 SimTrack

SimTrack a simple C++ library designed for numeric particle tracking in the high energy accelerators. It adopts the 4^{th} order symplectic integrator [8, 9] for the optical transport in the magnetic elements. The 4-D and 6-D weak-strong beam-beam treatments [4, 5] are integrated in it for the beam-beam studies. The optical transfer through magnetic elements in SimTrack is benchmarked with Tracy-II and its 4-D and 6-D weak-strong beam-beam calculations are benchmarked with BBSIM [10].

SimTrack provides versatile functions to manage elements and lines. It supports a large range of types of elements. New type of element can be easily created in the library. SimTrack calculates Twiss, coupling and fits tunes, chromaticities and corrects closed orbits. AC dipole, AC multipole, and electron lens are all available in this library. SimTrack allows access and change of element parameters during tracking.

SimTrack has been extensively used for the nonlinear beam dynamics studies for RHIC Au ion and polarized proton operation and for the RHIC head-on beam-beam compensation. Functions to calculate tune footprint, tune and amplitude diffusion, and dynamic aperture are provided. It also analytically calculates nonlinear resonance driving and detuning terms from sextupoles and octupoles.

To study the proton beam lifetime with head-on beam-beam compensation scheme in RHIC, SimTrack is used to calculates the proton particle loss and emittance growth with about 5,000 macro-particles up to 2×10^7 turns. To speed up the particle tracking, the integration steps of dipoles and quadrupoles are reduces. The tunes and chromaticities need to be re-matched before tracking.

To save the computing time involved in the lifetime calculation, we adopt a hollow Gaussian distribution for initial proton particles. The advantage of this method is that there are more macro-particles in the bunch tail and therefore there are more particles lost in the tracking. With this method the particle loss rate below 1%/hour or 0.007% particle loss in 2×10^6 turns can be detected with RHIC lattices.

For the emittance calculation, same as Lifetrac, we calculate $< x^2 >$, $< y^2 >$ and $< z^2 >$ with the coordinates of all macro-particles in all turns in each 10^4 turn tracking. By doing that, the emittance changes of 0.03% in 2×10^6 turns can be detected.

Since SimTrack is programable and flexible, it is also intensively used for RHIC general linear optics calculation and optimization. Many Mathematica tools like high resolution tune finding, SVD method and polynomial fitting are all available in this code.

2 BENCHMARKING WITHOUT BEAM-BEAM

Beam-beam off

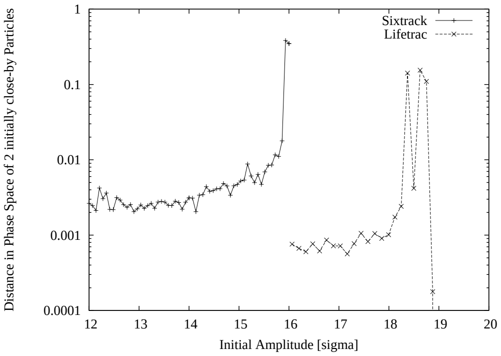

Figure 1: Distance in phase space of two initially close particles after 1E6 turns of tracking vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0 (4D case). Initial angle in Ax,Ay plane is 15 degrees.

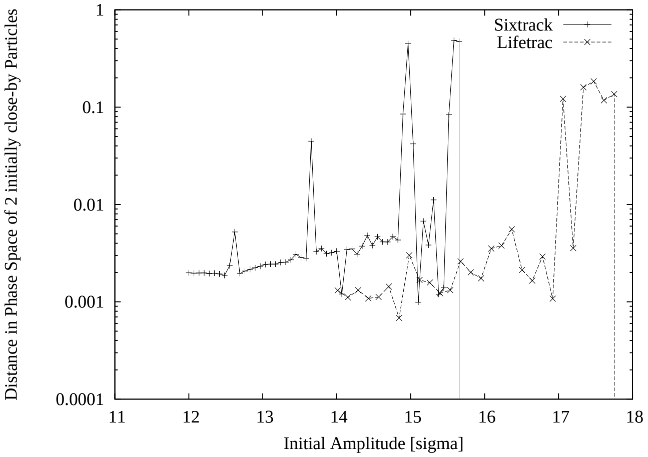

Figure 2: Distance in phase space of two initially close particles after 1E6 turns of tracking vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0 (4D case). Initial angle in Ax,Ay plane is 45 degrees.

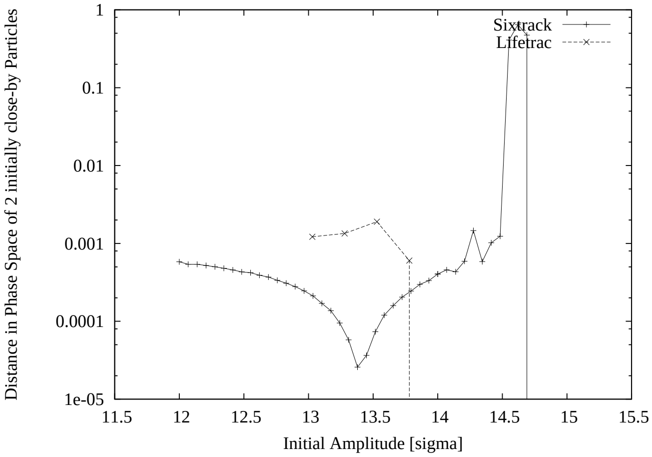

Figure 3: Distance in phase space of two initially close particles after 1E6 turns of tracking vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0 (4D case). Initial angle in Ax,Ay plane is 75 degrees.

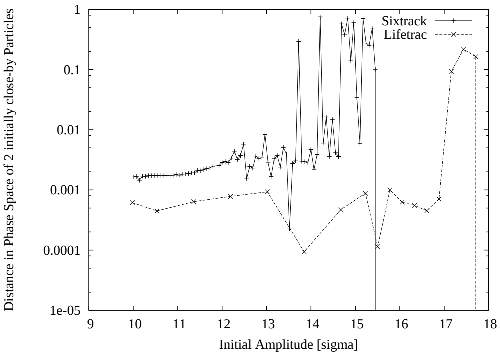

Figure 4: Distance in phase space of two initially close particles after 1E6 turns of tracking vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0.00027 (6D case). Initial angle in Ax,Ay plane is 15 degrees.

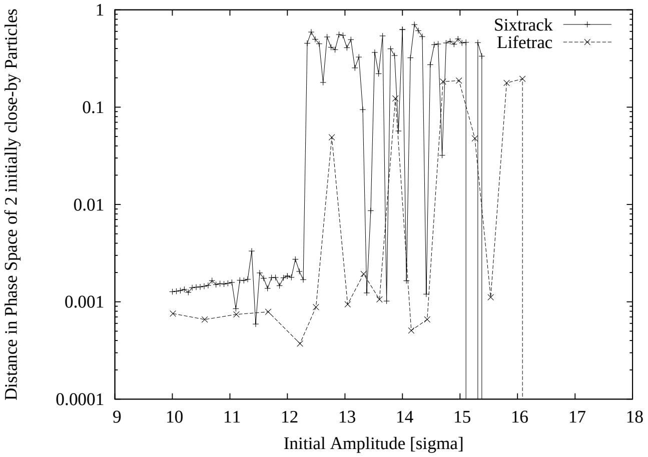

Figure 5: Distance in phase space of two initially close particles after 1E6 turns of tracking vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0.00027 (6D case). Initial angle in Ax,Ay plane is 45 degrees.

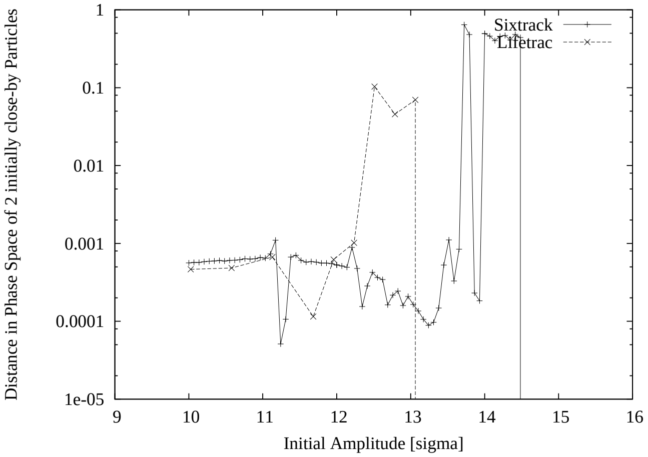

Figure 6: Distance in phase space of two initially close particles after 1E6 turns of tracking vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0.00027 (6D case). Initial angle in Ax,Ay plane is 75 degrees.

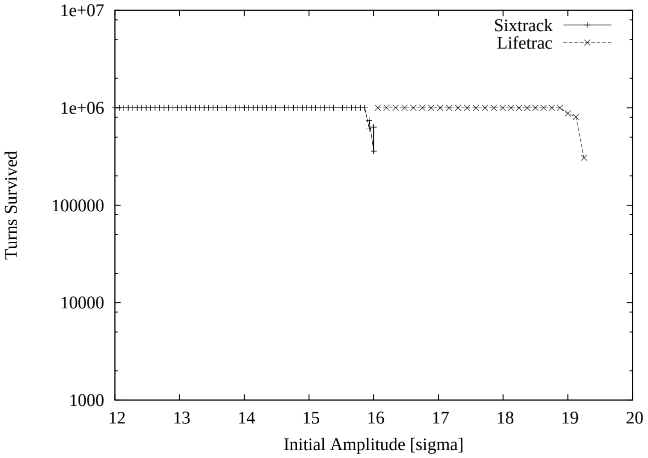

Figure 7: Particle survival time in turns vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0 (4D case). Initial angle in Ax,Ay plane is 15 degrees.

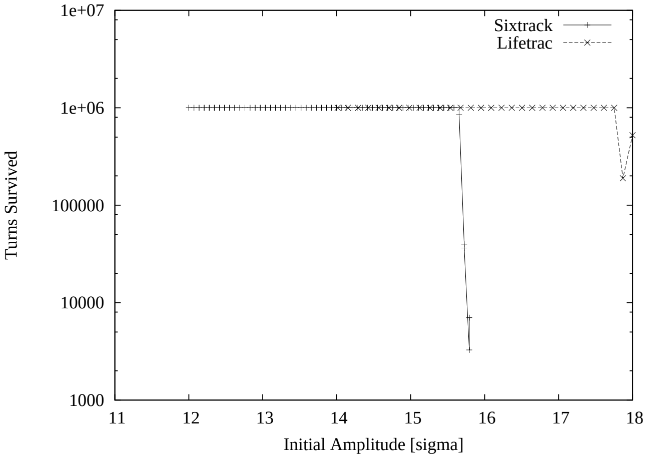

Figure 8: Particle survival time in turns vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0 (4D case). Initial angle in Ax,Ay plane is 45 degrees.

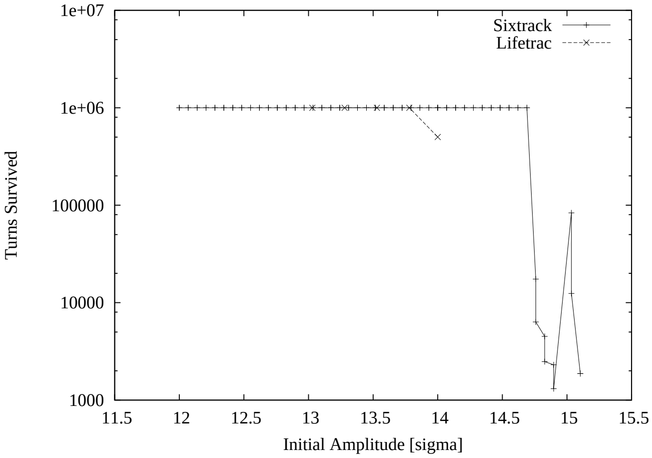

Figure 9: Particle survival time in turns vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0 (4D case). Initial angle in Ax,Ay plane is 75 degrees.

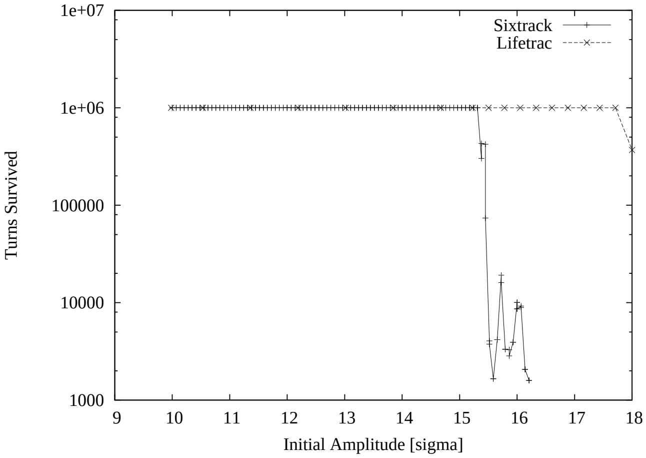

Figure 10: Particle survival time in turns vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0.00027 (6D case). Initial angle in Ax,Ay plane is 15 degrees.

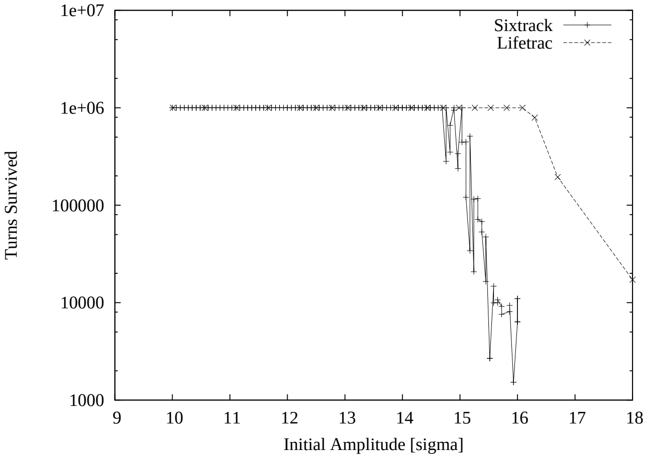

Figure 11: Particle survival time in turns vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0.00027 (6D case). Initial angle in Ax,Ay plane is 45 degrees.

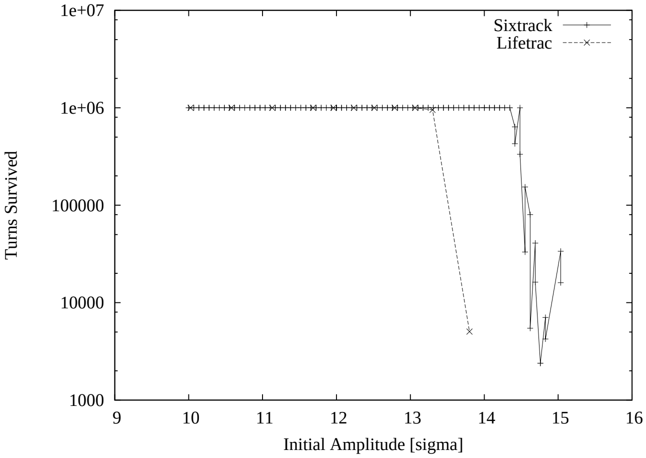

Figure 12: Particle survival time in turns vs. the initial amplitude. Beam-beam interaction is off, particles' momentum deviation is 0.00027 (6D case). Initial angle in Ax,Ay plane is 75 degrees.

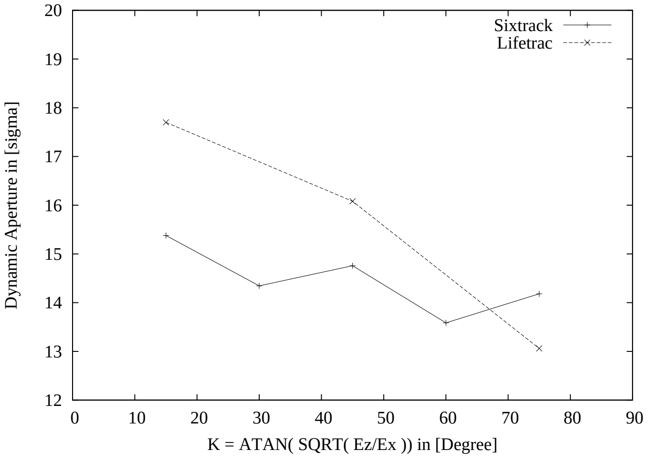

Figure 13: Dynamic aperture in beam sigma vs. initial angle in Ax,Ay plane. Beam-beam interaction is off, particles' momentum deviation is 0.00027 (6D case).

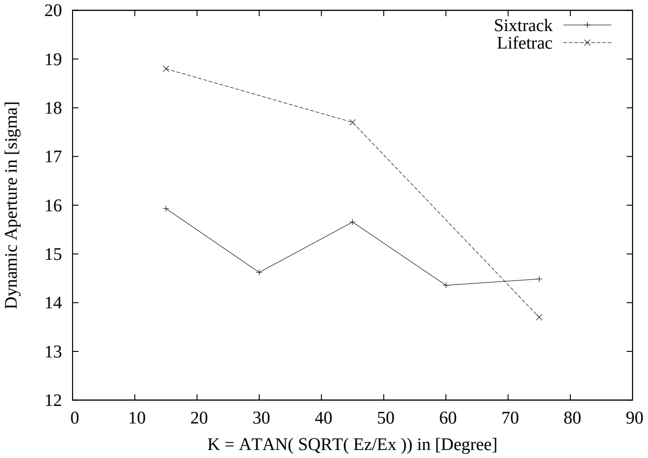

Figure 14: Dynamic aperture in beam sigma vs. initial angle in Ax,Ay plane. Beam-beam interaction is off, particles' momentum deviation is 0 (4D case).

3 BENCHMARKING WITHOUT BEAM-BEAM

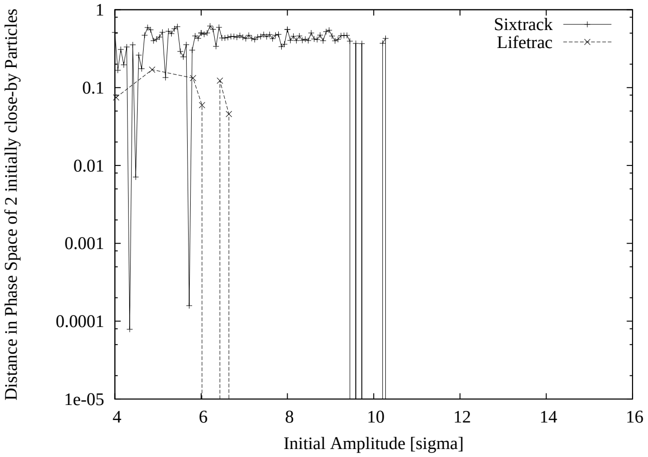

Figure 15: Distance in phase space of two initially close particles after 1E6 turns of tracking vs. the initial amplitude. Beam-beam interaction is on, particles' momentum deviation is 0 (4D case). Initial angle in Ax,Ay plane is 45 degrees.

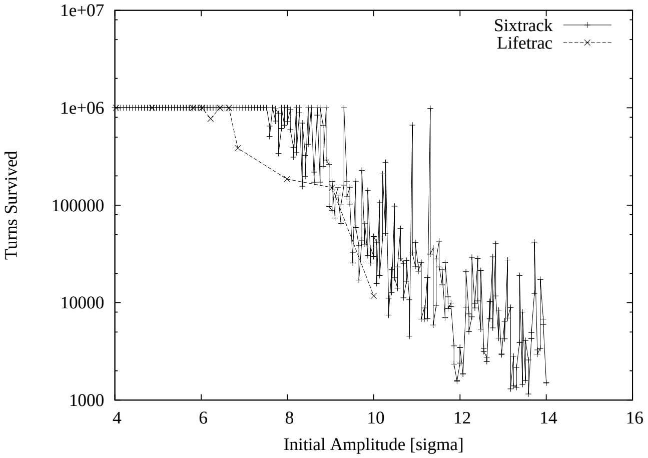

Figure 16: Particle survival time in turns vs. the initial amplitude. Beam-beam interaction is on, particles' momentum deviation is 0 (4D case). Initial angle in Ax,Ay plane is 45 degrees.

4 LHC tune foot-prints

The three plots in this sections show the LHC head-on BB without and with the long range BB. The captions describes the details.

The nominal BB tune shift is calculated as:

$$\Delta Q = \frac{r_p N}{4\pi\epsilon} \tag{1}$$

A good approximation for alternating crossing Ref. [11] is:

$$\Delta Q \approx \frac{r_p N \beta^*}{2\pi\gamma\sigma^*\sqrt{\sigma^{*2}+\theta^2\sigma_z^2/4}} \qquad (2)$$

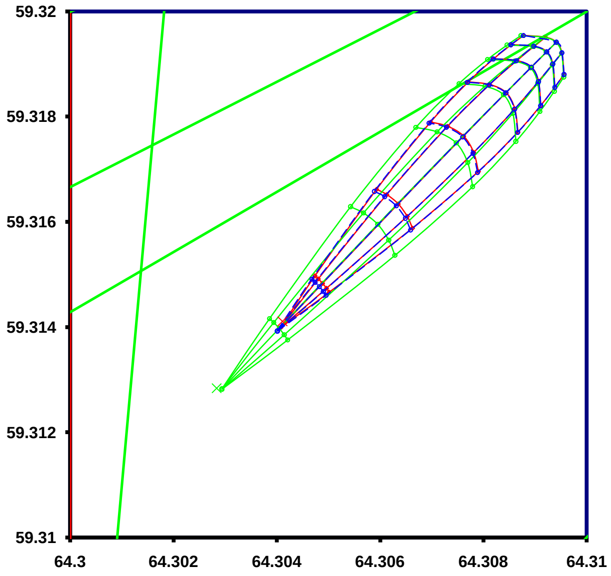

Figure 17: LHC head-on BB in IP1 & IP5 with nominal crossing angle. Green is the 4D case of a single head-on BB per IP while in red the 6D case (13 slices) is shown. The large green and red “X” are the theoretical tuneshifts for the nominal and crossing angle case (see Eq. 20 in Ref. [11]). They are in excellent agreement within the precision of the settings of the parameters. In blue is overlaid the 4d BB tune footprint where the head-on is manually distributed over 5 slices properly placed longitudinally. The close agreement with the 6d case justifies why for the LHC 4D BB kicks, when distributed over several kicks, seems to be sufficient at least for the nominal LHC. This issue has to be reviewed for the LHC upgrade since the crossing angle will be larger. Lines depict all resonances up to 10^{th} order.

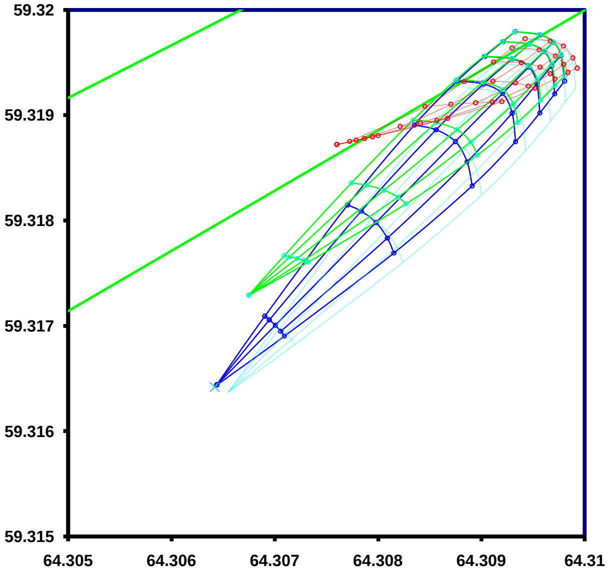

Figure 18: LHC head-on BB in IP1 only with nominal crossing angle. Dark blue shows the 4D BB with deltap = 0, when large deltap = 0.00027 is used the weak coupling between transverse and longitudinal phase space leads to the slightly shifted light blue tune footprint. For a single head-on BB there is no longer a compensation due to the crossing angle in one plane only: in green is shown the effect at deltap = 0, while for deltap = 0.00027 the shift and distortion is significant (shown in red). The bluish "X" symbol shows the nominal BB tune-shift for one BB interaction. Lines depict all resonances up to 10^{th} order.

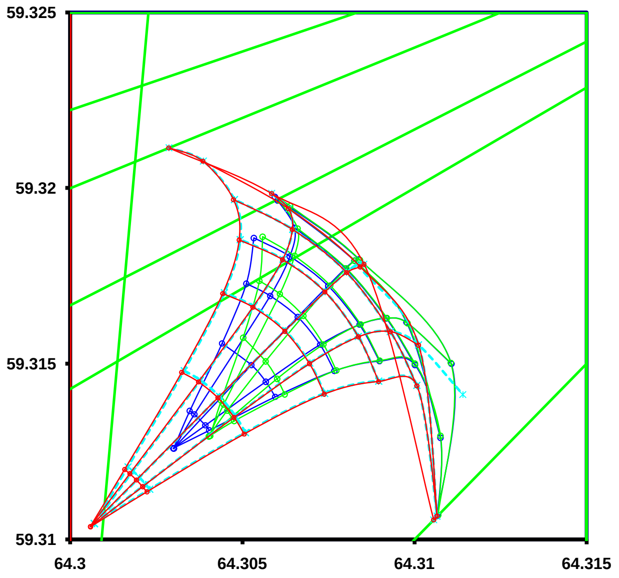

Figure 19: LHC head-on and parasitic BB kicks. The tune footprint for the 4d BB case (5 slices) and the the 6d BB (13 slices) are practically the same (red and dotted cyan footprint respectively). However for large deltap = 0.00027 the tune footprint shrinks when in IP1 & IP5 the 4d head-on BB kicks are replaced by 6d head-on BB kicks (blue footprint) and it shrinks further when all 4 IPs have 6d head-on BB kicks (green footprint). All parasitic BB kicks are treated as 4d BB kicks. Lines depict all resonances up to 10^{th} order.

5 LHC Dynamic Aperture for 4D & 6D BB

Fig. 20 shows the comparison between the 4D & 6D dynamic aperture (DA) over 10'000, 100'000 and 1'000'000 turns respectively.

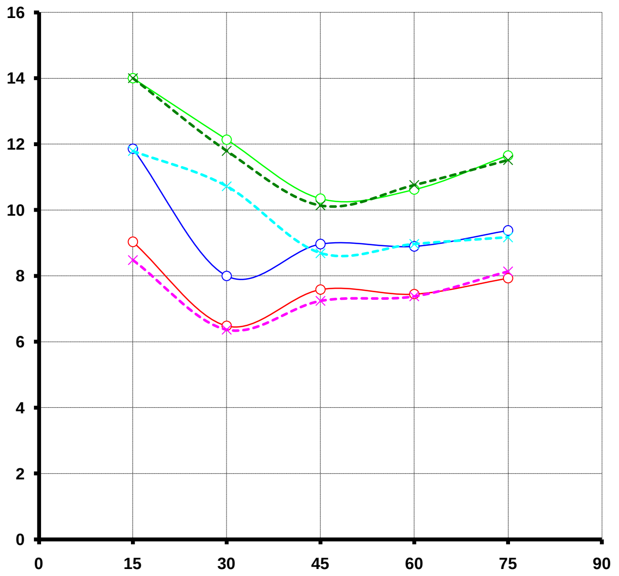

Figure 20: LHC dynamic aperture for the 4D and the 6D BB (13 slices) treatment. The curves in shades of green are DA for 10'000, bluish for 100'000 and reddish for 1'000'000 turns respectively. The solid lines with circles are from the 4D BB while the dashed curves with an "X" character are the results from 6D BB tracking.

It is interesting to note that the long-term DA is similar despite the fact the tune foot-prints (see Fig. 19) are very different for the two types of BB treatments. This can only mean that the amplitude (in sigma) where the tune foot-prints fold over is more relevant for the particle stability than how far the tune foot-print extends for the low amplitude particles.

6 RHIC tune foot-prints

We compared the tune footprints without and with beam-beam interactions with the Blue ring lattice for the RHIC 2011 250 GeV polarized proton run. We set the tunes in absence of BB to (28.695, 29.685) and linear chromaticities to (1,1). The nonlinear multipole field errors from interaction regions are included. The β^* at interaction points IP6 and IP8 are 0.7 m. In the simulation study, we used one RF cavity whose voltage is set to 300 kV. The proton bunch intensity is 1.5×10^{11}. The bunch length is 0.4545m. In the weak-strong beam-beam simulation, we split the strong beam into 13 slices in both codes. The caption describes the details of the plot.

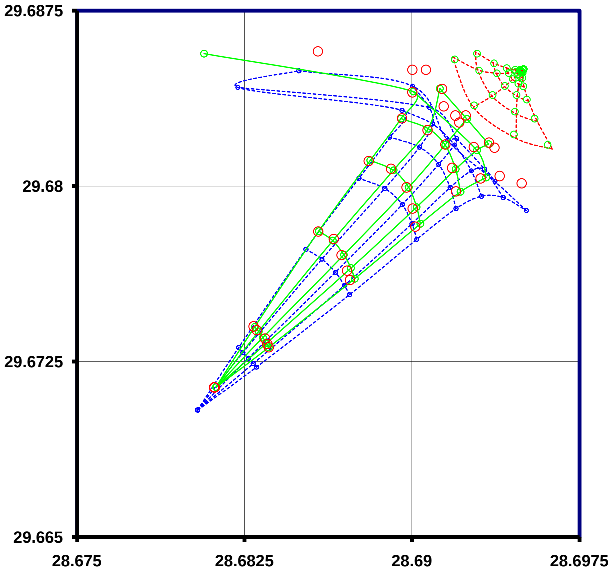

Figure 21: In broken red lines the SixTrack 4d case is shown without BB with data from SimTrack shown on top as green circles. The green solid lines show SixTrack calculations in 6d with 6d BB (13 slices). Also in this case the SimTrack results (red circles) are very close by. This tracking was done with deltap = 0.001. As a comparison (broken blue line) the same case is shown for deltap = 0. The deltap = 0.001 tune foot-print is shifted due to relatively large Q” but apparently the 6d BB treatment does not lead to an additional shrinking of the tune foot-print.

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