Synchro-Betatron Motion in Circular Accelerators

Kevin Li

March 30, 2011

Outline of Part I

1. Motivation
   - Collective effects in the longitudinal plane
   - Numerical and computational tools in accelerator physics
2. Basic Model
   - Basic physics
   - Specialisation to classical electromagnetic theory
3. Basic Dynamics
   - The symplectic structure

Outline of Part II

4 The Transverse Hamiltonian
- Canonical transformations
- Coordinate and rescaling transformation
- Transverse dynamics

Outline of Part III

Part I

Motivation and Model Introduction

Outline

SPS ecloud effects

Frozen synchrotron motion:

Dynamics: 17.6e10 protons, 1e12 electrons

Tune footprint:

SPS ecloud effects

Linear synchrotron motion:

Dynamics: 17.6e10 protons, 1e12 electrons

Tune footprint:

SPS ecloud effects

Nonlinear synchrotron motion:

Dynamics: 17.6e10 protons, 1e12 electrons

Tune footprint:

Motivation 1

- Synchrotron motion does not preserve the longitudinal position over several turns
- The tune footprint is obtained over several turns
- The color dimension looses its meaning

$\Rightarrow$ We need to find a quantity that is preserved under synchrotron motion to refurnish the color dimension with a meaning

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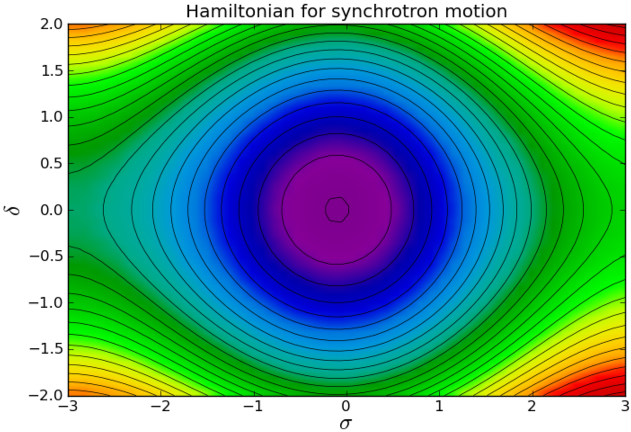
Motivation 1



Equations of longitudinal dynamics

Some times the bunch is made to sit in an **accelerating bucket** only to compensate for external losses

• in a lepton storage ring, to compensate for synchrotron radiation losses

• in general, a bunch in a stationary bucket can move to a synchronous phase different from 0 or π in order to compensate for impedance losses (see further)

$$\begin{cases} \dfrac{d\zeta}{dt} = -\eta\beta c\delta \\ \dfrac{d\delta}{dt} = \dfrac{eV_m}{p_0 C}\left[\sin\left(\dfrac{h\zeta}{R} + \Phi_s\right) - \sin\Phi_s\right] \end{cases}$$

$$H = -\frac{1}{2}\beta c\eta\delta^2 + \frac{eV_m}{2\pi h p_0}\cos\left(\frac{h\zeta}{R}\right) + \frac{eV_m \sin\Phi_s}{p_0 C}\zeta$$

[1]G. Rumolo

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But where does this derive from?

[1] G. Rumolo

Motivation 2

Numerical and Computational Tools in Accelerator Physics

An introduction

Werner Herr
CERN, BE Department

http://cern.ch/Werner.Herr/METHODS

Hamiltonian of particle in EM fields

For the Hamiltonian of a (relativistic) particle in a electro-magnetic field we have:

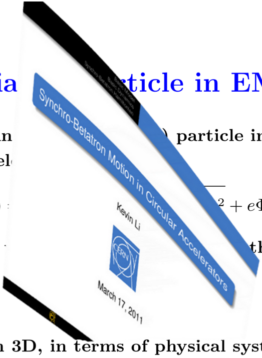
$$\mathcal{H}(\vec{x},\vec{p},t) = c\sqrt{(\vec{p} - e\vec{A}(\vec{x},t))^2 + m_0^2c^2} + e\Phi(\vec{x},t)$$

where $\vec{A}(\vec{x},t)$ is the vector potential and $\Phi(\vec{x},t)$ the scalar potential

In another form (in 3D, in terms of physical systems):

$$\mathcal{H} = \frac{p_x^2 + p_y^2}{2(1 - \frac{2p_t}{\beta} + p_t^2)} - \frac{xp_t}{\beta\rho} + \frac{x^2}{2\rho^2} + \frac{(1-\beta^2)p_t^2}{2\beta^2} + k_1\frac{x^2 - y^2}{2} + V(x,y)$$

Hamiltonia … rticle in EM fields



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Goal

What this presentation should be about:

- not a presentation of new results
- by no means any claim for mathematical rigor
- rather an attempt to gather different ressources to summarize the known theory in a more or less complete and comprehensible manner
- rather with an appeal to physical intuition

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Literature

Physics:

- [Goldstein: Classical Mechanics]
- [Jackson: Classical Electrodynamics]
- [Huang: Statistical Mechanics]
- [Peskin/Schroeder: Quantum Field Theory]

Applied Hamiltonian dynamics:

- [T. Suzuki: 1985]
- [K. Symon: 1997]
- [S. Tzenov: 2001]

Outline

We create our universe: two manifolds and one map

Initialisation:

Basic objects

- Parameter manifold: $\mathcal{M} \cong \mathbb{R}^m$
- Configuration manifold: $\mathcal{N} \cong \mathbb{R}^n$
- Map: $\Phi : \mathcal{M} \to \mathcal{N}$

Derived objects

- World bubble: $\Theta = \Phi(U \subset \mathcal{M}) \cong \mathbb{R}^m$
- Phase space: $\Omega = T^*\mathcal{N} \cong \mathbb{R}^{2n}$
- Jacobian: $\mathcal{J} = D\Phi \in \mathbb{M}(m \times n, \mathbb{R})$

All physics is in finding Φ

We create our universe: two manifolds and one map

Initialisation:

We create our universe by declaring two manifolds and connecting them with a map

- The parameter manifold $\mathcal{M}$ is our world
- The configuration manifold $\mathcal{N}$ is some quantity we are interested in
- The map Φ is an embedding of our world into the target space and as such describes the evolution of the target space quantities

We create our universe: two manifolds and one map

Why do we need manifolds and all that stuff?

- In our intuition we are (always) using them
- We (always) start with a collection of points which are, a priori, completely unstructured (i.e. a mesh of an accelerator structure (without connectivity information) or the time steps in a particle tracking code (with no ordering))
- We want to be able to talk about neighbourhoods, derivatives, tangent spaces, metrics in order obtain a predictable evolution (a function of the parameter manifold) for any quantity that lives in our world. Our collection of points must thus be endowed with a smooth connectivity which is done formally via a differentiable structure (equivalence class of atlases where an atlas is a family of compatible charts on an open cover of the parameter manifold[1]). Then, locally, our collection of points becomes isomorph to the Euclidean space; it locally obtains the structure of a linear vector space within which we are fully equipped with all our well-known tools of calculus

[1] s. Abraham, Marsden pp. 31

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Basic physics: the principle of least action

In this case we can define:

Action

The action S is defined as the volume of the world bubble:

$$S = \int_{\Theta} d\Phi$$

The principle of least action

Given a fixed subspace U, a map Φ is physical if and only if the action S is stationary

$$\delta S = 0$$

The Lagrangian

To introduce a useful formalism it is expedient to write the action as

Action

$$S = \int_{\Phi(U)} d\Phi = \int_U d^m x \sqrt{\det(\mathcal{J}\mathcal{J}^T)} = \int_U d^m x \, \mathcal{L}$$

Thus, we have introduced the Lagrangian

Lagrangian

$$\mathcal{L} = \sqrt{\det(\mathcal{J}\mathcal{J}^T)}$$

Classical limit and electromagnetic theory

- Introduce classical limit

$$\delta = n\lambda^3 \ll 1$$

δ: characteristic dimensionless density parameter of a quantum gas
$\lambda = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$: thermal de Broglie wavelength
The action becomes the length of the world-line:
$S = \int dl \, , \; dl^2 = -c^2 dt^2 + d\vec{x}^2$

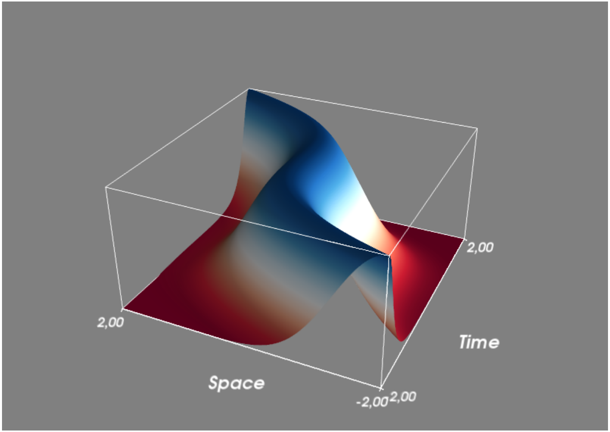
- Introduce electromagnetic theory via U(1)-gauge coupling by moving from the standard to the covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu + igA_\mu$$

A: gauge fields
The action becomes the covariant length of the world-line:
$S = \int D_\mu l = \int L \, dt$

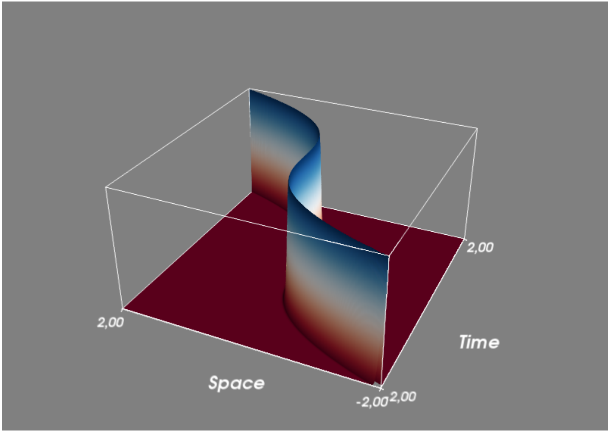
Intuition of the classical limit

A small number of particles with a wavefunction that is represented as an evolving Gaussian wavepackage: $\psi(x,t) \rightarrow \exp\left(-\frac{x^2}{\sigma^2}\right)$



Intuition of the classical limit

For sufficiently many particles at low densty constructive superposition of wavefunctions establishes a correlation between space and time coordinates via delta-functions: $\psi(x,t) \to \delta(x(t) - x') \to x(t)$



Intuition of electromagnetic theory

- Gauge invariance of the Dirac field

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

- Directional derivative

$$\vec{\nabla}_{\vec{e}}\,\vec{\psi}(\vec{x}) = \lim_{h\to 0}\frac{\vec{\psi}(\vec{x}+h\,\vec{e})-\vec{\psi}(\vec{x})}{h}$$

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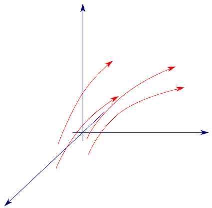

$\vec{\psi}(\vec{x})$

$\partial_\mu \vec{\psi}(\vec{x})$

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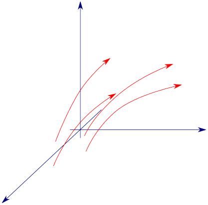
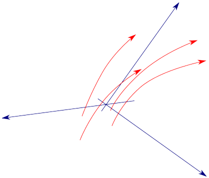


$$\xrightarrow{e^{i\alpha(x)}}$$



$$\vec{\psi}(\vec{x}) \quad \longrightarrow \quad \vec{\psi}(\vec{x}+h\,\vec{e})$$

$$\partial_\mu\vec{\psi}(\vec{x}) \quad \longrightarrow \quad D_\mu = (\partial_\mu + ieA_\mu)\,\vec{\psi}(\vec{x})$$

One-parameter action and electromagnetic Lagrangian

One-parameter action and electromagnetic Lagrangian

$$S = \int \mathcal{L} \, d^4x \qquad \Bigg\| \qquad S = \int L \, dt$$

$$\mathcal{L} = -\rho_m c \sqrt{\dot{x}_\mu \dot{x}^\mu} + j_\mu A^\mu \qquad \Bigg\| \qquad L = -mc\sqrt{1 - \frac{\vec{v}^2}{c^2}} - qV + q\vec{v} \cdot \vec{A}$$

Electromagnetic Hamiltonian

Hamiltonian

A Legendre transform of the Lagrangian

$$H = P\,\dot{q} - L \quad \text{with} \quad P = \frac{\partial L}{\partial \dot{q}}$$

yields the Hamiltonian

$$H(q, P, t) = \sqrt{(\vec{P} - q\vec{A})^2 c^2 + m^2 c^4} + qV \tag{1}$$

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Least action and Hamilton equations of motion

$$\begin{aligned}
\delta S &= \delta \int P\,dq - H\,dt \\
&= \int \delta P\,dq + P\,\delta(dq) - \delta H\,dt - H\,\delta(dt) \\
&= \int_{\text{P.I.}} dq\,\delta P - dP\,\delta q - \frac{\partial H}{\partial q}\,dt\,\delta q - \frac{\partial H}{\partial P}\,dt\,\delta P - \frac{\partial H}{\partial t}\,dt\,\delta t + dH\,\delta t \\
&= \int \left(dq - \frac{\partial H}{\partial P}\,dt\right)\delta P - \left(dP + \frac{\partial H}{\partial q}\,dt\right)\,\delta q + \left(dH - \frac{\partial H}{\partial t}\,dt\right)\,\delta t \\
&= 0
\end{aligned}$$

Equations of motion

$$\dot{q} = \frac{\partial H}{\partial P}\,, \quad \dot{P} = -\frac{\partial H}{\partial q}\,, \quad \dot{H} = \frac{\partial H}{\partial t}$$

Symplectic structure

The Legendre transform makes the independent variable time and together with the principle of least action/equations of motion unleashes the full symplectic structure of the theory yielding:

- the symplectic manifold (Ω, ω^0) (Phase space, Poisson bracket)

$$\omega_0 : T_q\mathcal{N} \times T_q\mathcal{N} \to \mathbb{R}\,, \quad (u, v) \mapsto \omega_0(u, v)$$

- because ω_0 is nondegenerate, it defines a 1-form

$$\omega_1 : T_q\mathcal{N} \to T_q^*\mathcal{N}\,, \quad u \mapsto \omega_0(u, \cdot) \quad (\langle u|)$$

- let's use this 1-form to implicitly define a very special vector field

$$\omega_0(X_H, Y) = -dH(Y) \Leftrightarrow (JX_H, Y) = -(\vec{\nabla} H, Y)$$

$q \in \mathcal{N}$, $J = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$ symplectic structure matrix

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Hamiltonian vector field

We have thus defined the Hamiltonian vector field

$$X_H = J \cdot \vec{\nabla} H =: H :$$

What is so special about this Hamiltonian vector field?

- Infinitesimal time evolution

$$X_H = J \cdot \vec{\nabla} H = \sum_{\alpha=1}^{f} \frac{\partial H}{\partial q^\alpha} \frac{\partial}{\partial p_\alpha} - \frac{\partial H}{\partial p_\alpha} \frac{\partial}{\partial q^\alpha}$$

$$\dot{\psi}(t_0) = -X_H \cdot \psi(t_0) = - : H : \psi(t_0) = -[H, \psi(t_0)]$$

- Finite time evolution

$$\psi(t_0 + t) = \exp(- : H : t) \psi(t_0)$$

The Hamiltonian is the generator for translations in time for any function ψ!

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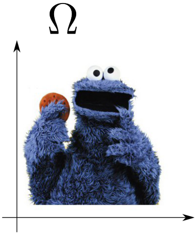

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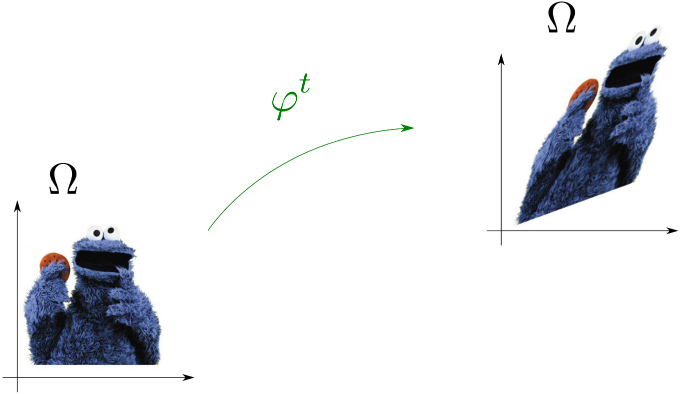

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