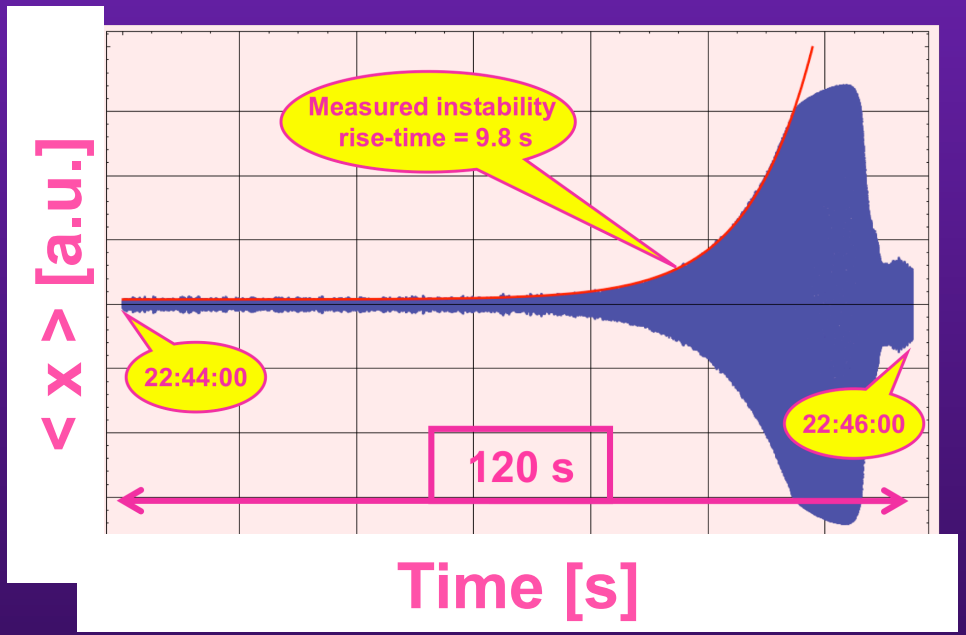
HEAD-TAIL INSTABILITY RISE TIME IN TIME AND FREQUENCY DOMAIN

Elias Métral

- **Procedures in time and frequency domain**
- **Application to the LHC HT SBI observed in 2010**

TIME DOMAIN

- Plot the transverse beam position vs. time
- Look at the very beginning of the instability, where one should see an exponential growth (perturbative approach)
- Do the fit: exponential or linear fit in log plot
- The instability rise time is defined by the time needed for the amplitude (of the envelope) to be multiplied by Exp[1] ≈ 2.7
- Example of the LHC HT SBI observed on 17/05/2010:

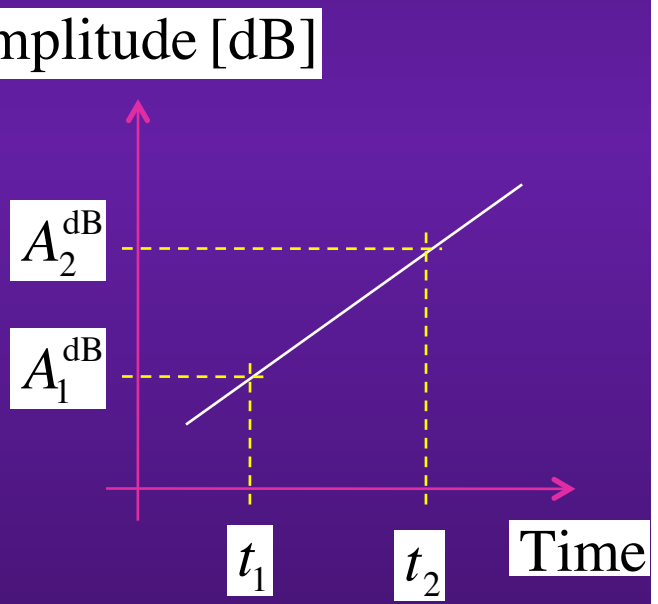


FREQUENCY DOMAIN (1/2)

- Make the FFT of the transverse beam position vs. time and look at it in log scale (dB)
- Look at the unstable line (unstable mode) which starts to grow
- The instability rise time is given by the time needed for the amplitude (of the unstable line) to be increased by ~ 9 dB
- DEM:

$$A^{\mathrm{dB}} = 20 \,\mathrm{Log}\, A \qquad \mathrm{Log}\, A = \frac{\ln A}{\ln 10}$$

$$A_2^{\mathrm{dB}} - A_1^{\mathrm{dB}} = \Delta A^{\mathrm{dB}} = 20 \,\mathrm{Log}\, A_2 - 20 \,\mathrm{Log}\, A_1$$
$$= \frac{20}{\ln 10} \ln \frac{A_2}{A_1} = \frac{20}{\ln 10} \frac{t_2 - t_1}{\tau}$$



$$A_1 = A_0 \, e^{\frac{t_1}{\tau}} \qquad A_2 = A_0 \, e^{\frac{t_2}{\tau}}$$

Instability rise-time

FREQUENCY DOMAIN (2/2)

- The instability rise time is equal to $\tau = t_2 - t_1$, when

$$A_2^{\mathrm{dB}} - A_1^{\mathrm{dB}} = \Delta A^{\mathrm{dB}} = \frac{20}{\ln 10} \approx 8.7\ \mathrm{dB} \approx 9\ \mathrm{dB}$$

- Example of the LHC HT SBI observed on 17/05/2010:

  ~ 24 dB in 24 s => ~ 9 dB in ~ 9 s

  => Instability rise time ≈ 9 s (similar to time domain!)