Tevatron Luminosity Evolution Model and its Application to the LHC

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Content

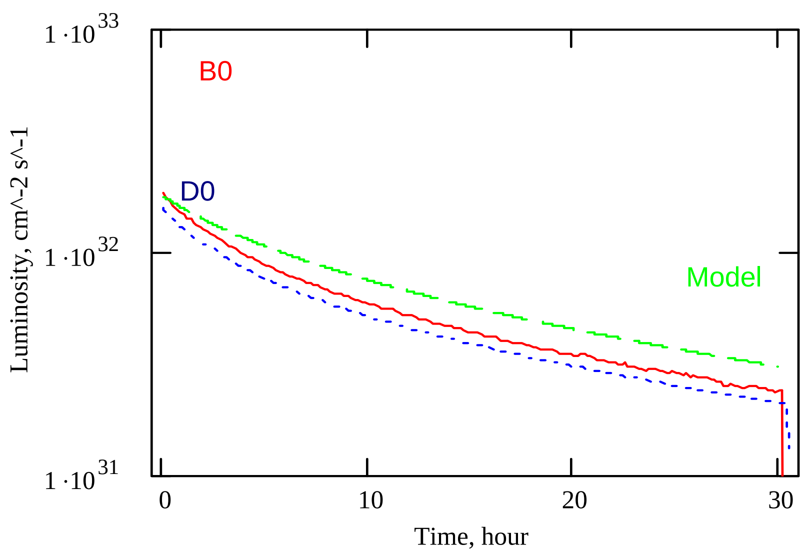
- Model prediction for Tevatron
- Model prediction for LHC
- Conclusions
- Bolts and nuts
  - Emittance growth due to transverse noise (hump, transverse damper, etc.)
  - IBS
  - IBS for longitudinal degree of freedom
  - RF noise

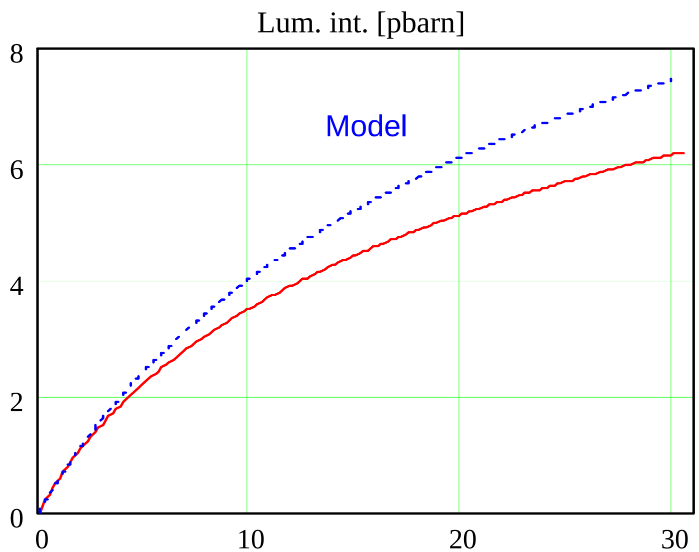
Luminosity Evolution Model

- Luminosity evolution in Tevatron is driven by
  - Single and multiple intrabeam scattering (IBS)
  - Elastic and non-elastic scattering on the residual gas
  - Elastic and non-elastic scattering on counter-rotating beam
  - RF noise
  - Transverse noise (E or B field noise, quad motion, etc.)
  - Beam-beam effects
- The model is based on ODEs ($N_{p,a}$, $\varepsilon_{x,y(p,a)}$, $\sigma_{s(p,a)}$)
  - Details of evolution for longitudinal distribution came from parameterization of solution of integro-differential equation describing single and multiple IBS
- The model
  - was the base for the luminosity evolution scenario (2003)
  - helps in understanding of the beam-beam effects

Influence of Beam-beam effects on beam parameters evolution

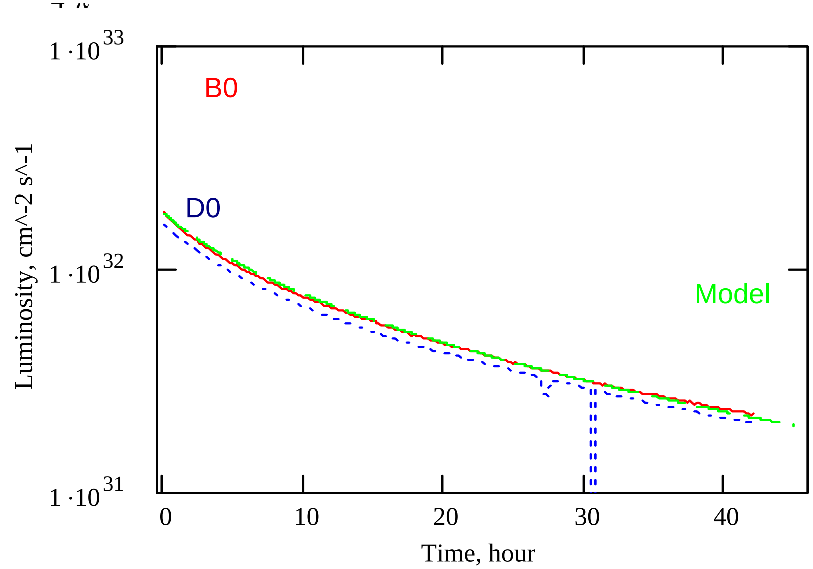
Store 4581

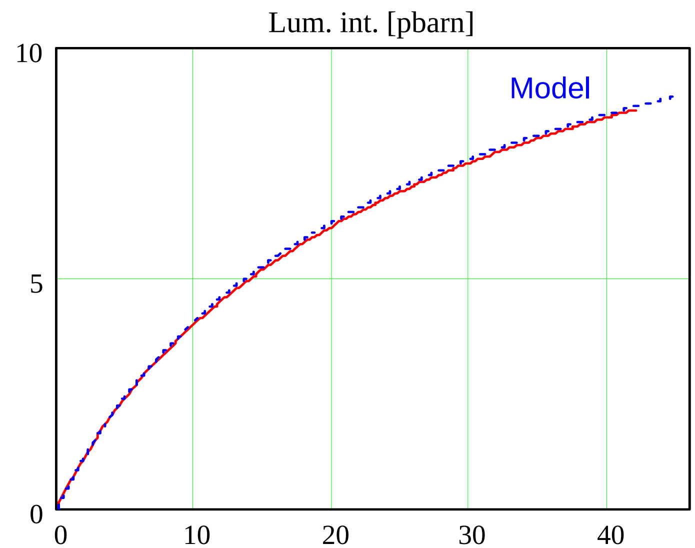




Before shutdown (Jan. 6, 06)

Store 4859





After shutdown (July. 27, 06)

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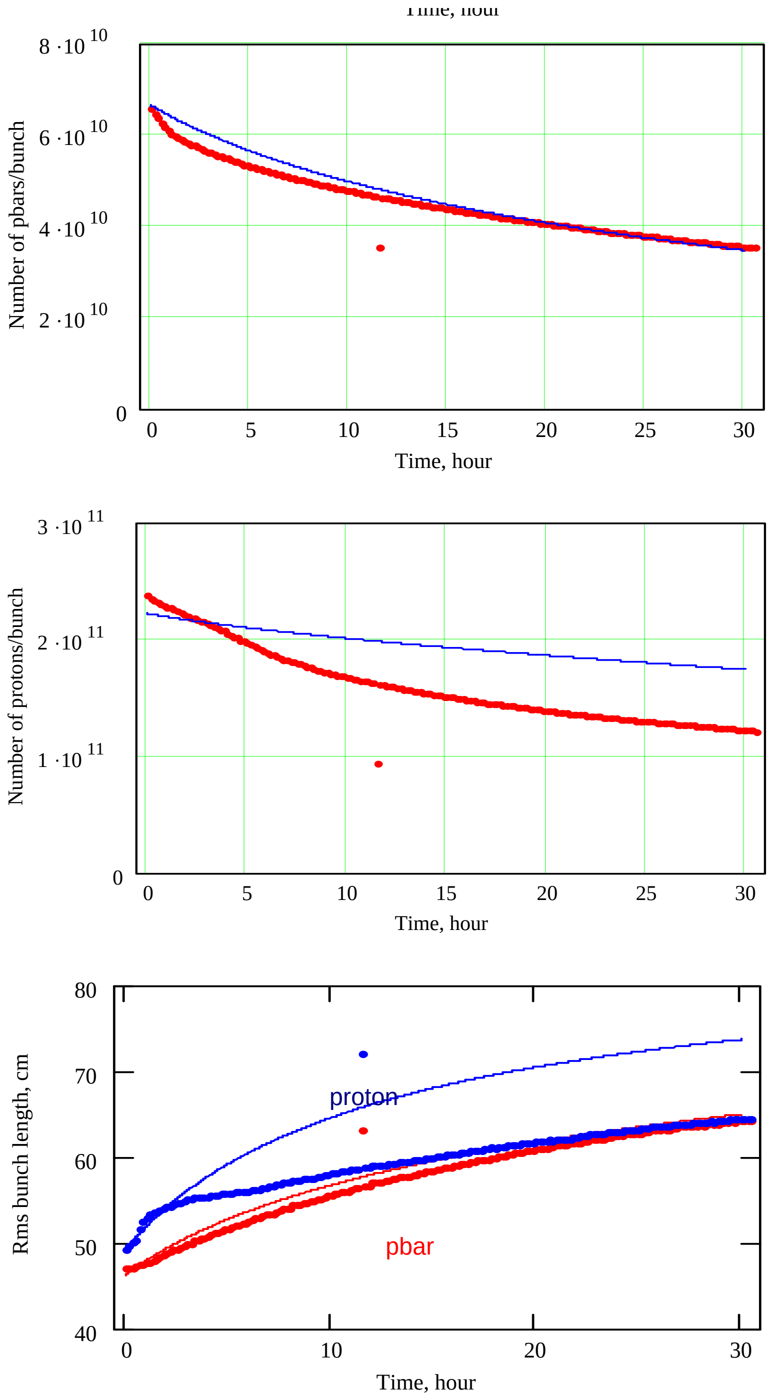
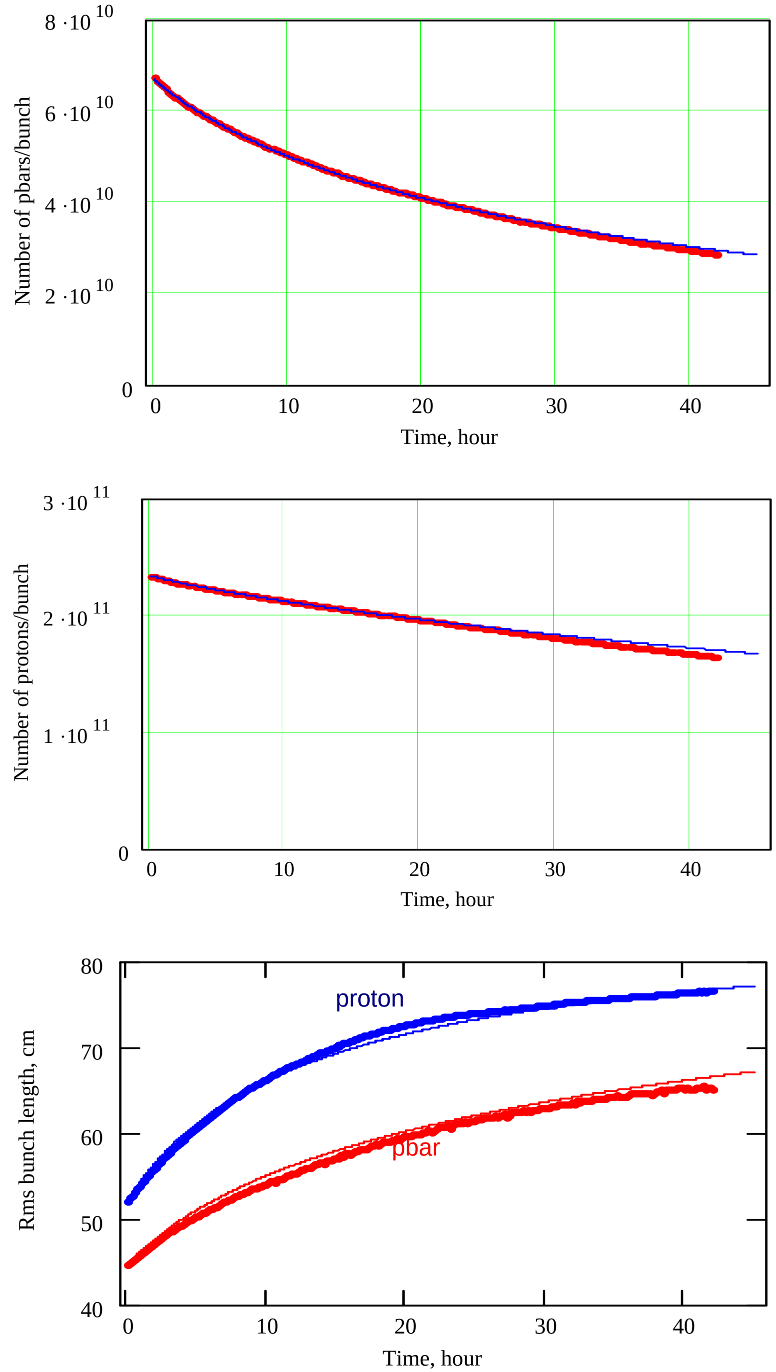


Store 4859



Model parameters

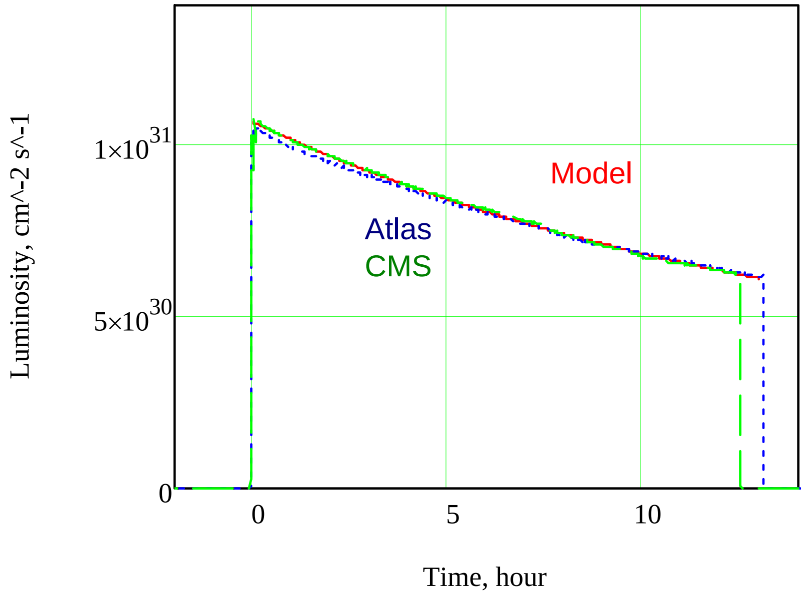
- Cross-section of nuclear interaction in IPs - 69 mbarn
  - Inelastic - 60 mbarn
  - Elastic - 15 mbarn;
- Beam life-time due to interaction with res. gas - 480 hour
- Spectral density of RF phase noise - $4.2\cdot10^{-11}$ rad^2/Hz
- Amplification factor of IBS - 1.3 (?)

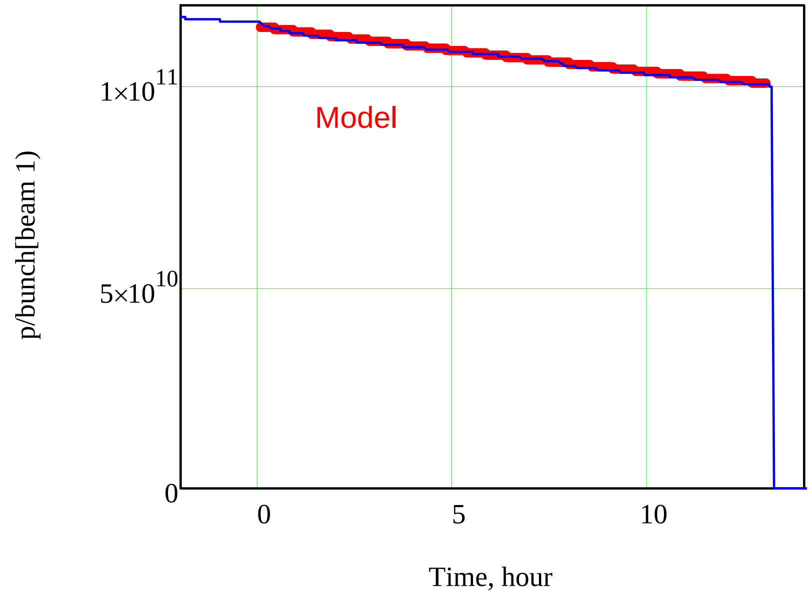
- Since end of August of 2006 the model is applied for analysis of each store and results are available on the web

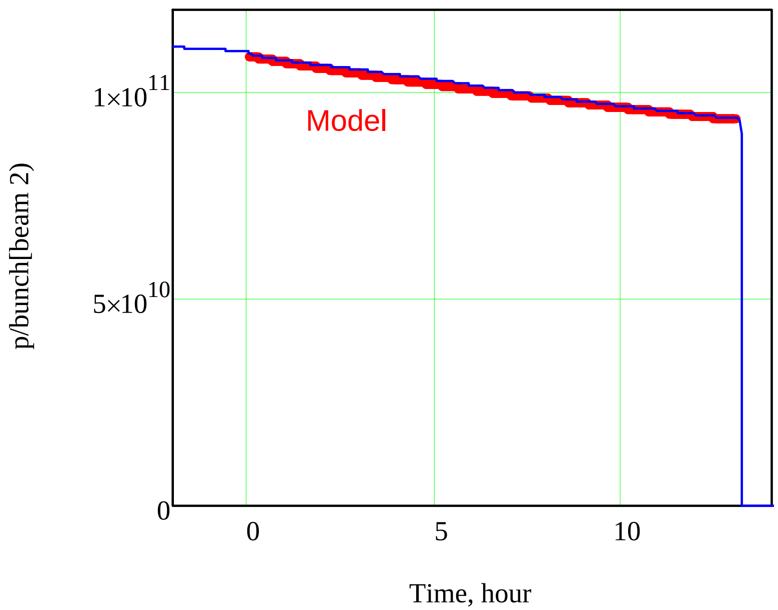
Present status

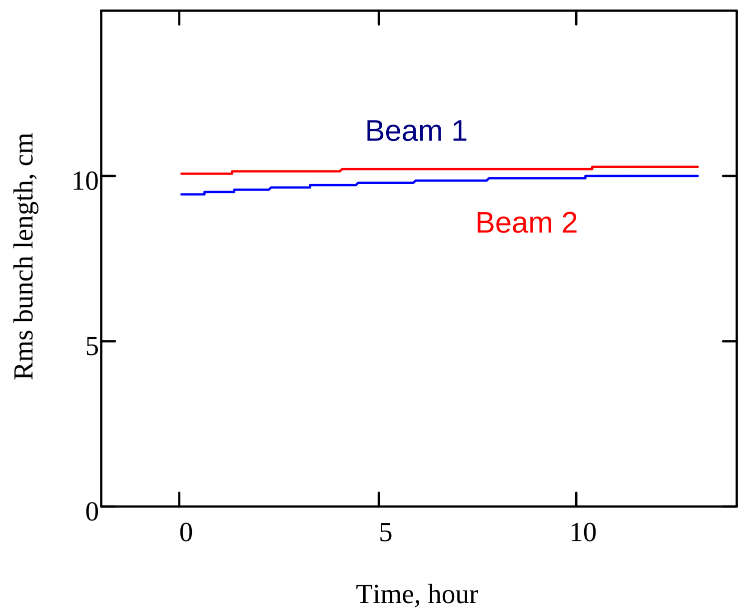
- About 10% of luminosity integral is lost due to beam-beam
- IBS is the main mechanism causing fast luminosity decrease
  - Presently, there are no means to reduce IBS in Tevatron
- About 40% of pbars are burned in luminosity
  - It is the second leading reason of luminosity decrease

Application of Luminosity Evolution Model to LHC

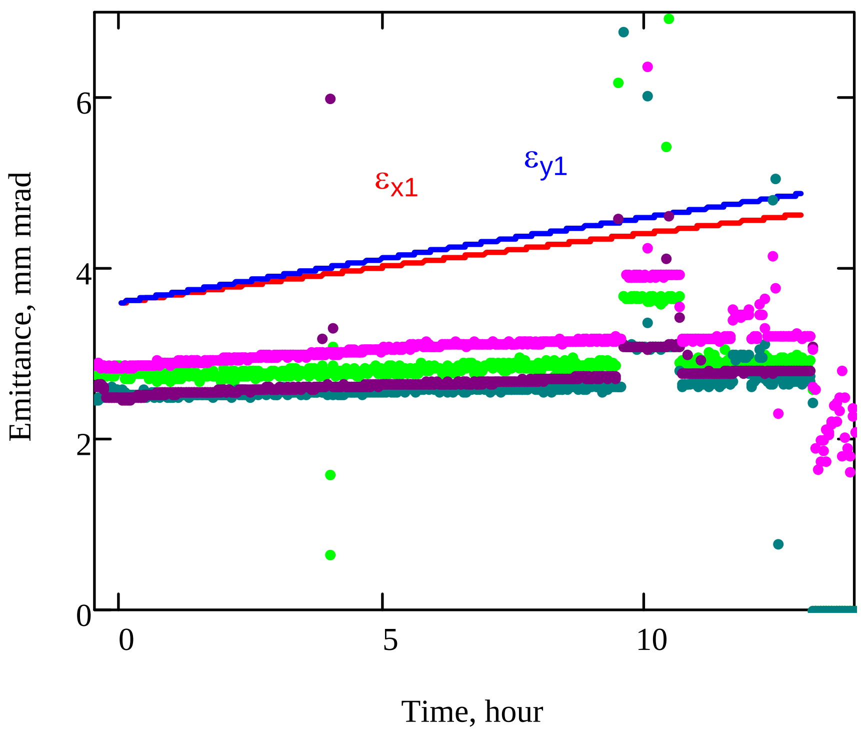


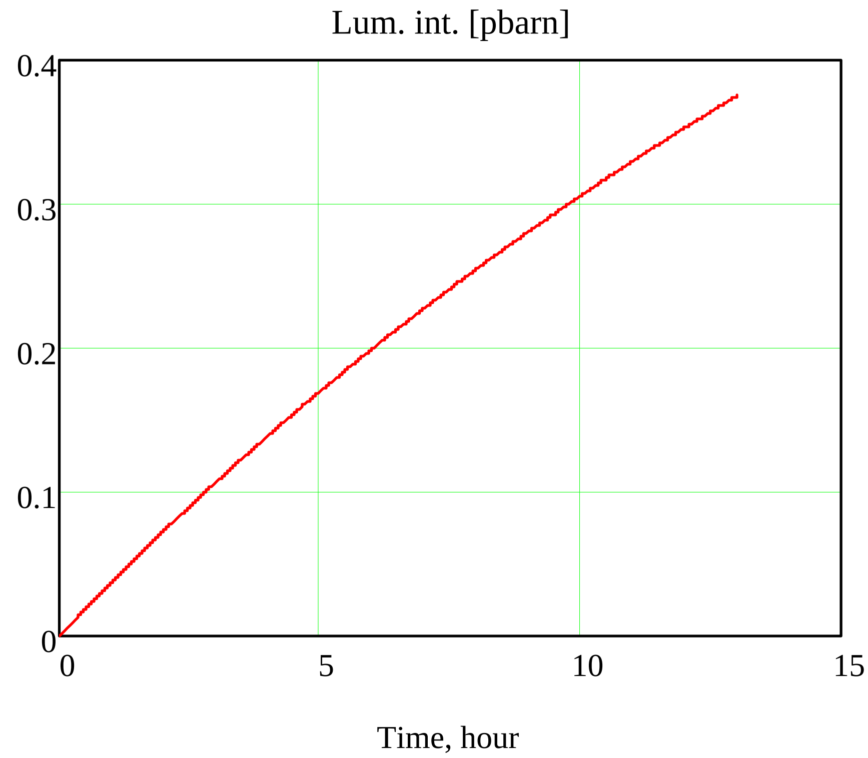


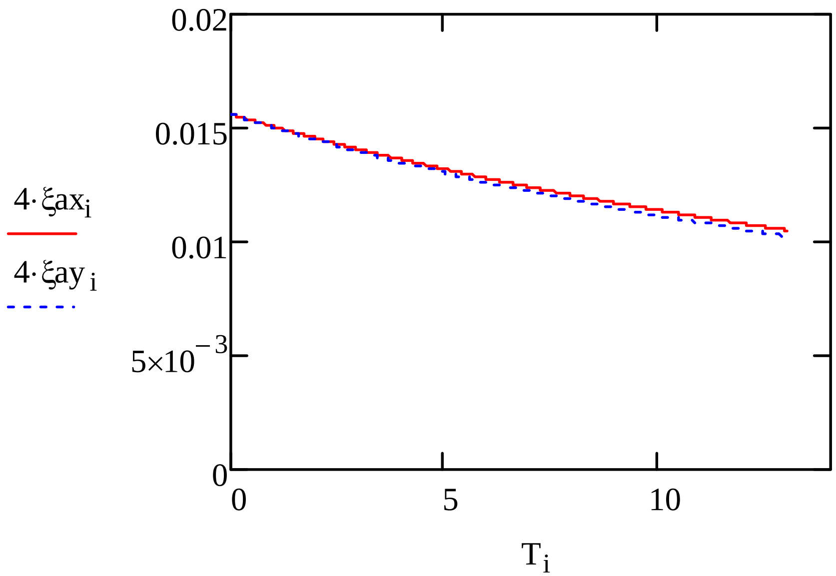


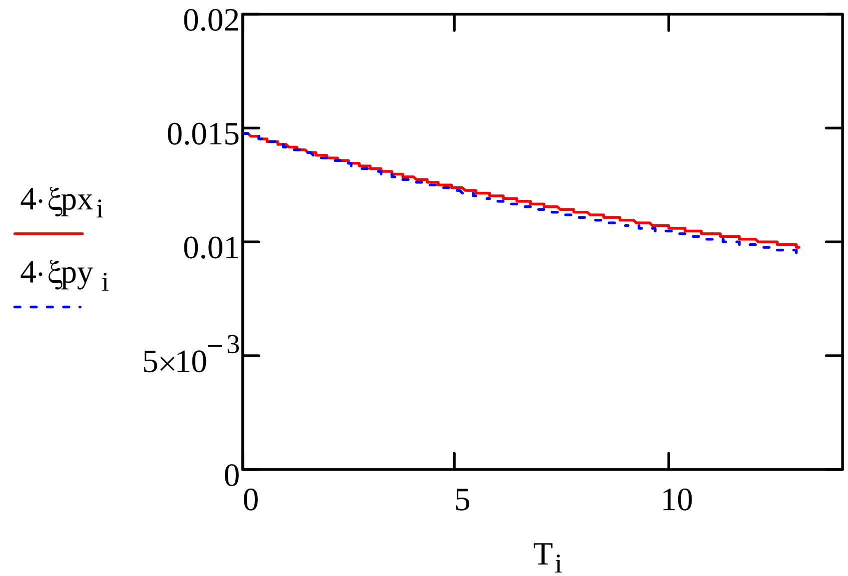


Fill 1303


Emittance, mm mrad
ε_{x1}
ε_{y1}
0
2
4
6
0
5
10
Time, hour


Lum. int. [pbarn]
0
0.1
0.2
0.3
0.4
0
5
10
15
Time, hour


$4\cdot\xi ax_i$
$4\cdot\xi ay_i$
0
5×10^{-3}
0.01
0.015
0.02
0
5
10
T_i


$4\cdot\xi px_i$
$4\cdot\xi py_i$
0
5×10^{-3}
0.01
0.015
0.02
0
5
10
T_i

- Bunch length does not grow because it is already too long
  - Looks like the beam-beam effects additionally limit longitudinal acceptance (not included in the model)
- Transverse size measurements are not consistent with the luminosity change
  - Looks like more than 50% of the size comes from background (diffraction, ?)
  - In the model the total growth rate was redistributed between horizontal and vertical degrees of freedom in almost equal parts
    - It has a little effect the luminosity evolution

The base of calculations

- IBS is very well understood and predictable
  - Growth rates at the store beginning
    - $T_x = \varepsilon_x/(d\varepsilon_x/dt)$ = 74 & 84 hour (Beam 1 & Beam 2)
      - It is about 25% of the observable $\perp$ growth rate
      - The rest is coming from other sources
        - Gas scattering can make only minor contribution
        - Noise on the betatron sidebands is the only plausible explanation
    - $T_s = \varepsilon_s/(d\varepsilon_x/dt)$ = 17.5 & 22 hour (Beam 1 & Beam 2)
      - There is almost no growth observed, i.e. bunch is clipped by longitudinal losses
      - It is build in into the model which predicts a decrease of growth rates to: 46 & 95 hour basing on the intensity loss

The base of calculations (continue)

- There are no quite clear picture of the RF phase noise
  - If one believes to Ref 1[†] the rms bunch growth rate is ~0.043 rad^2/hour ($d\sigma/dt$=15 ps/hour for σ=0.45 ns); than it is too large and need to be improved by about an order of magnitude
    - Note that the voltage increase amplifies the growth rate ($d\sigma_s^2/dt \propto V$) to even larger value
    - Note also that this growth rate can be IBS driven but details required for an estimate are not present in the publication
    - The RF phase noise in Tevatron drives the growth rate of ~0.002 rad^2/hour and is not negligible for the Tevatron luminosity evolution
  - To make it simple the Tevatron spectral density of the RF noise of $4.2\cdot10^{-11}$ rad^2/Hz was used in simulations;
    - it drives the growth rate $5.4\cdot10^{-4}$ rad^2/hour @ 5.5 MeV

[†] "LHC beam diffusion dependence on RF noise: models and measurements," T. Mastorides, et.al. IPAC-10

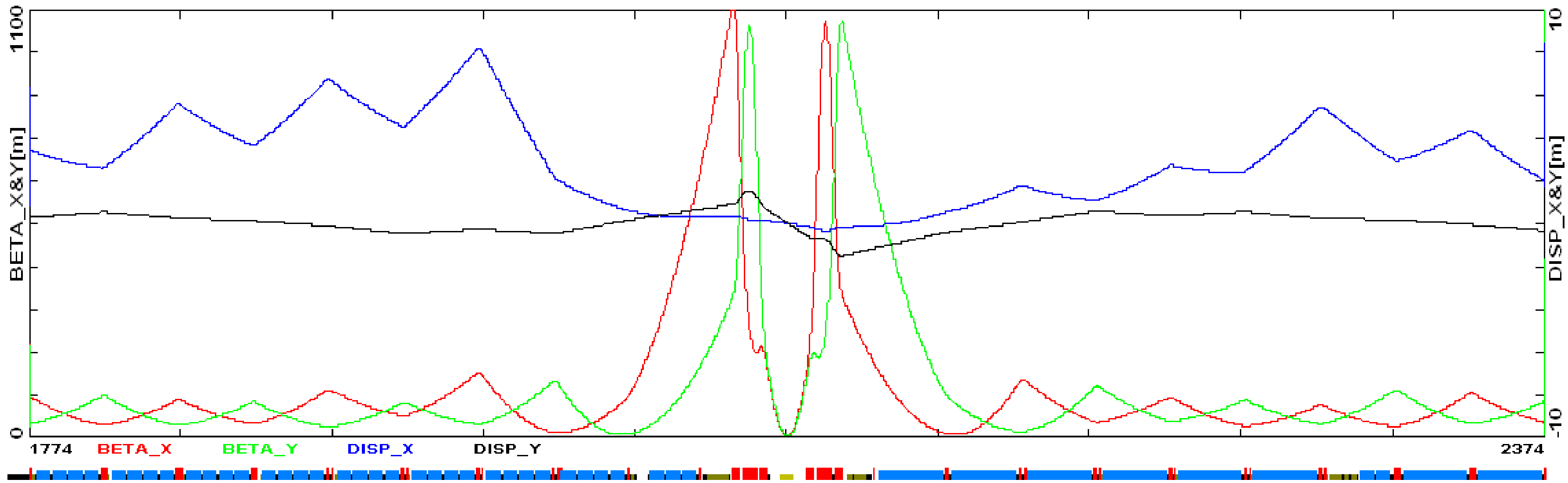
The major conclusions for the LHC fill 1303

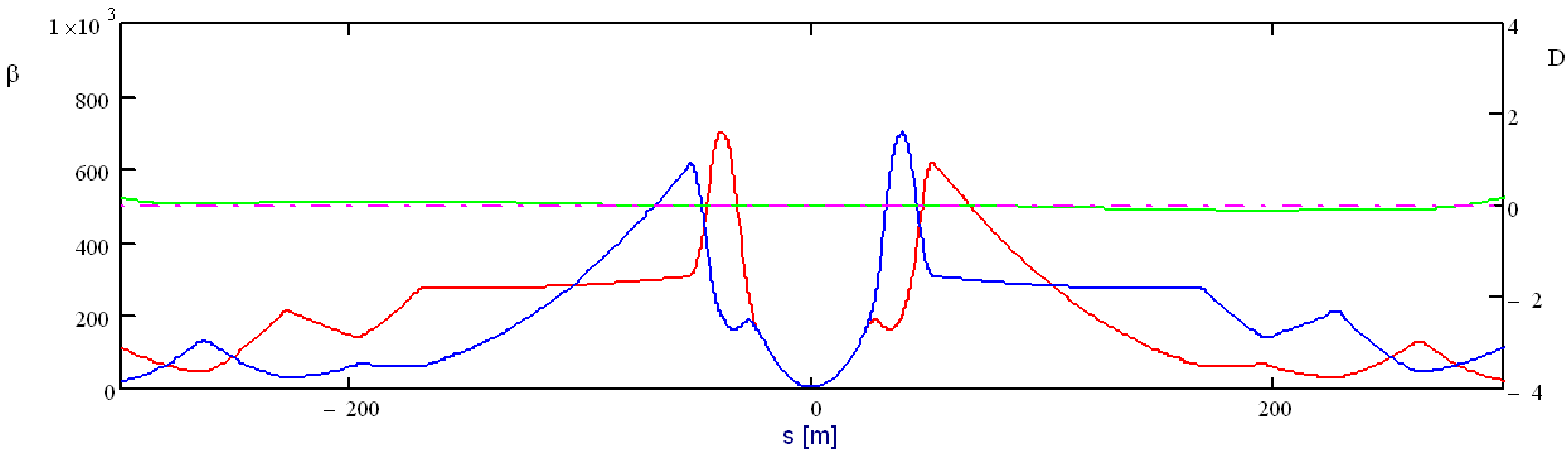
- Luminosity lifetime – 20 hour
- Intensity loss times
  - Total 94 & 72 hour (Beam 1 & Beam 2)
    - 267 & 253 hour (Beam 1 & Beam 2) due to luminous loss

      For $\sigma_{interation}$=90 mbarn (need a more accurate number)
    - 170 & 112 hour (Beam 1 & Beam 2) due to longitudinal heating and clipping
- Beam loss is dominated by the longitudinal loss
  - Beam-beam loss is important for some bunches but does not dominate the average
- The transverse emittance growth is dominated by transverse noise at betatron sidebands: feedback and hump

*LHS scenario with small β**

Optics comparison near IPs





Tevatron (β^*=35 cm) and LHC (β^*=350 cm) β-functions

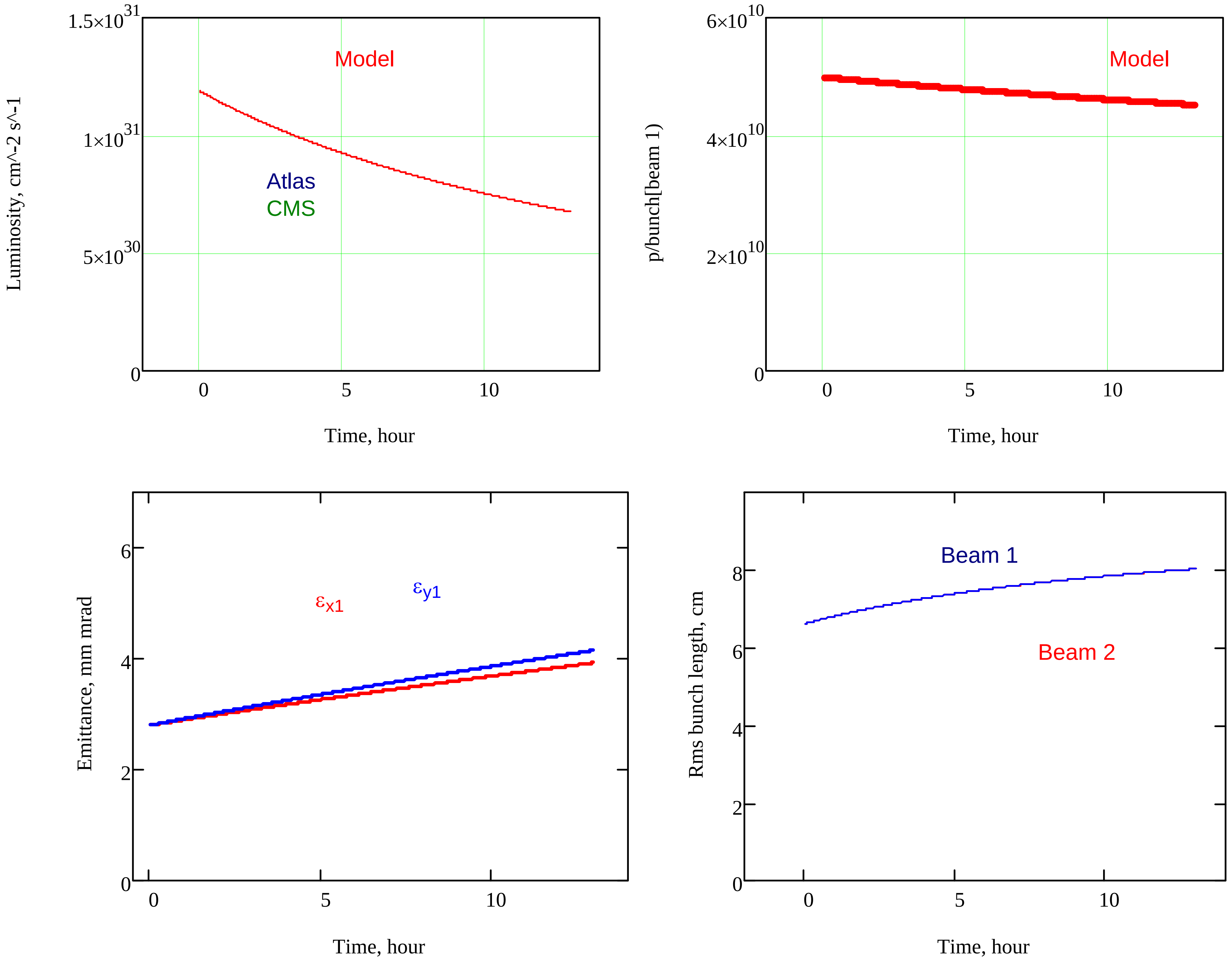
IP optics comparison

	Tevatron (Typical)	LHC (Fill 1303)
β^* [cm]	28	350
β_{max} [m]	1250	700
Rms emittance [mm mrad]	3-6	3.6-5
Momentum spread	$(1.2\text{-}1.5)10^{-4}$	$(1.\text{-}1.06)10^{-4}$
Maximum rms size in IP [mm]	2.65	0.93
Distance to full beam separation [m]	60	~80
Chromaticity per IP	22	~5

- If the machine can accept the same maximum rms beam size as the Tevatron the LHC β^* can be reduced from 3.5 m to 0.43 m
  - This will increase the maximum β-function to 5.6 km and the chromaticity contribution per IP to ~40
- If the we want to match the IP chromaticity to Tevatron (22)
  - Then, β^*=87 cm, the maximum β-function is 2.8 km and the maximum rms size is 1.86 mm
- Taking into account that the optimal operation requires a smaller emittance and the LHC momentum spread is lower than for the Tevatron

  β^*=0.8 m looks as a good goal

Luminosity and the beam parameters evolution for low β^* scenario



Comparison of fill 1303 to the low β* operation

	Fill 1303	Low β^* scenario
Peak luminosity, 10^{30} $cm^{-2}s^{-1}$	10.6	11.9
Collisions per crossing	2.7	3
Number of colliding bunches	32	32
Luminosity lifetime, hour	19.4	17.6
Beam intensity lifetime, hour	78	110
Beam intensity lifetime due to luminosity loss, hour	340	138
Number of particles, 10^{11}	1.15	0.5
Norm. rms emittance [mm mrad]	3.6	2.8
Initial bunch length	9.8	6.6
Initial rms momentum spread, 10^{-4}	1.05	0.88
RF voltage, MeV	5.5	7.5
Beam-beam tune shift, ξ	0.015	0.009
β^*[cm]	350	80

Conclusions

- Operation at low β^* allows one to have the same luminosity with less than half intensity per bunch and, consequently, lower total intensity
- Improvements of the damper noise by about factor of 3 (in amplitude) should leave the IBS as the major source of emittance growth
  - It should be the high priority item for next few months
- Luminosity lifetime will be slightly lower for small β^* scenario but still quite acceptable for 15 hour stores
  - However
    - Beam-beam tune shift is ~1.5 smaller
    - Smaller intensity results in a larger margin for all kinds of instabilities
    - Smaller intensity reduces the load on collimators if something will go wrong
    - Smaller initial and final transverse emittances partially compensate an increase of maximum β-function in IP quads

The model constituents

Emittance growth due to transverse noise

$$\frac{d\varepsilon}{dt} = \frac{16\pi^2\overline{\Delta\nu^2}}{g^2}\left(\left(\frac{d\varepsilon}{dt}\right)_0 + \frac{f_0 g^2}{2\beta_{BPM}}\overline{x_{BPM}{}^2}\right)$$

$$\left(\frac{d\varepsilon_{x,y}}{dt}\right)_0 = \beta_{x,y}\left(\frac{el}{Pc}\right)^2 \frac{\omega_0{}^2}{4\pi}\sum_{n=-\infty}^{\infty} S_{\delta B}\left(\left(\nu - n\right)\omega_0\right)$$

- The growth of the feedback system gain, g, does not affect the emittance growth
- For a collider the tune spread is dominated by the beam-beam tune shift

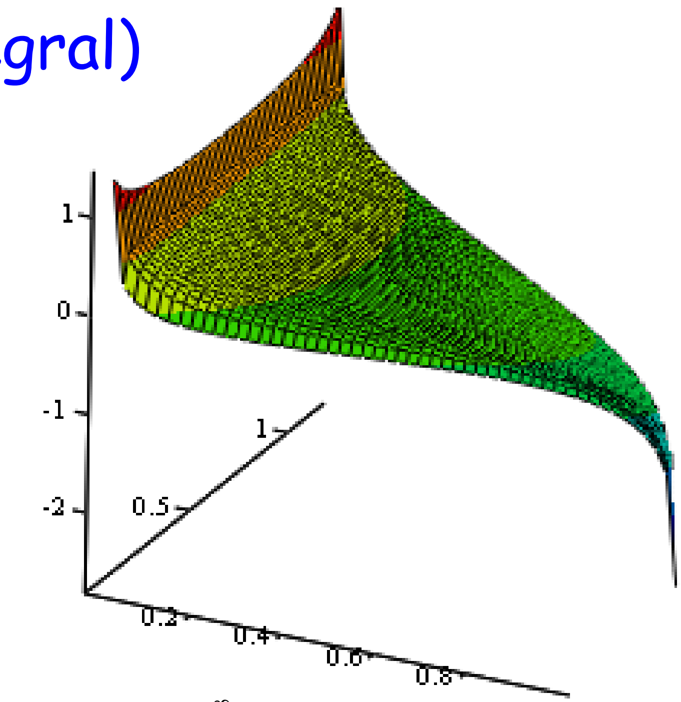
$$\sqrt{\overline{\Delta\nu^2}} \simeq 0.2\xi_{tot}$$

- Observed emittance growth corresponds to the effective noise of ~0.2 μm for 2 systems (H&V)

IBS for Gaussian Bunch

- IBS for Gaussian bunch is well understood (Bjorken, Mtingwa)
  - Quite complicated in the general case
  - Much simpler for ultra relativistic case
  - Smooth lattice approximation yields the result which is ~10% different
- The general case theory is based on the theory of temperature relaxation in plasma (Landau collision integral)

$$\boxed{\frac{d}{dt}\begin{pmatrix}\sigma_{vx}{}^2\\ \sigma_{vy}{}^2\\ \sigma_{vz}{}^2\end{pmatrix}=\frac{(2\pi)^{3/2}\,n r_0{}^2 c^4 L_c}{\sqrt{\sigma_{vx}^2+\sigma_{vy}^2+\sigma_{vz}^2}}\begin{pmatrix}\Psi\left(\sigma_{vx},\sigma_{vy},\sigma_{vz}\right)\\ \Psi\left(\sigma_{vy},\sigma_{vz},\sigma_{vx}\right)\\ \Psi\left(\sigma_{vz},\sigma_{vx},\sigma_{vy}\right)\end{pmatrix}}$$



$$L_c=\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right),\qquad \rho_{\min}=r_0c^2/\overline{\mathrm{v}^2}\,,$$

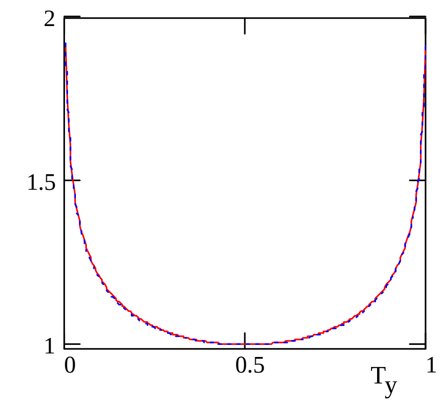
$$\rho_{\max}=\sqrt{\overline{\mathrm{v}^2}/4\pi n r_0 c^2}\,,$$

$$\overline{\mathrm{v}^2}=\sigma_{vx}^2+\sigma_{vy}^2+\sigma_{vz}^2$$

$$\Psi(x,y,z)=\frac{\sqrt{2}r}{3\pi}\left(y^2\,\mathrm{R_D}\left(z^2,x^2,y^2\right)+z^2\,\mathrm{R_D}\left(x^2,y^2,z^2\right)-2x^2\,\mathrm{R_D}\left(y^2,z^2,x^2\right)\right),\qquad \mathrm{R_D}(x,y,z)=\frac{3}{2}\int_0^\infty\frac{dt}{\sqrt{(t+x)(t+y)(t+z)^3}}$$

Dalitz plot for temperatures

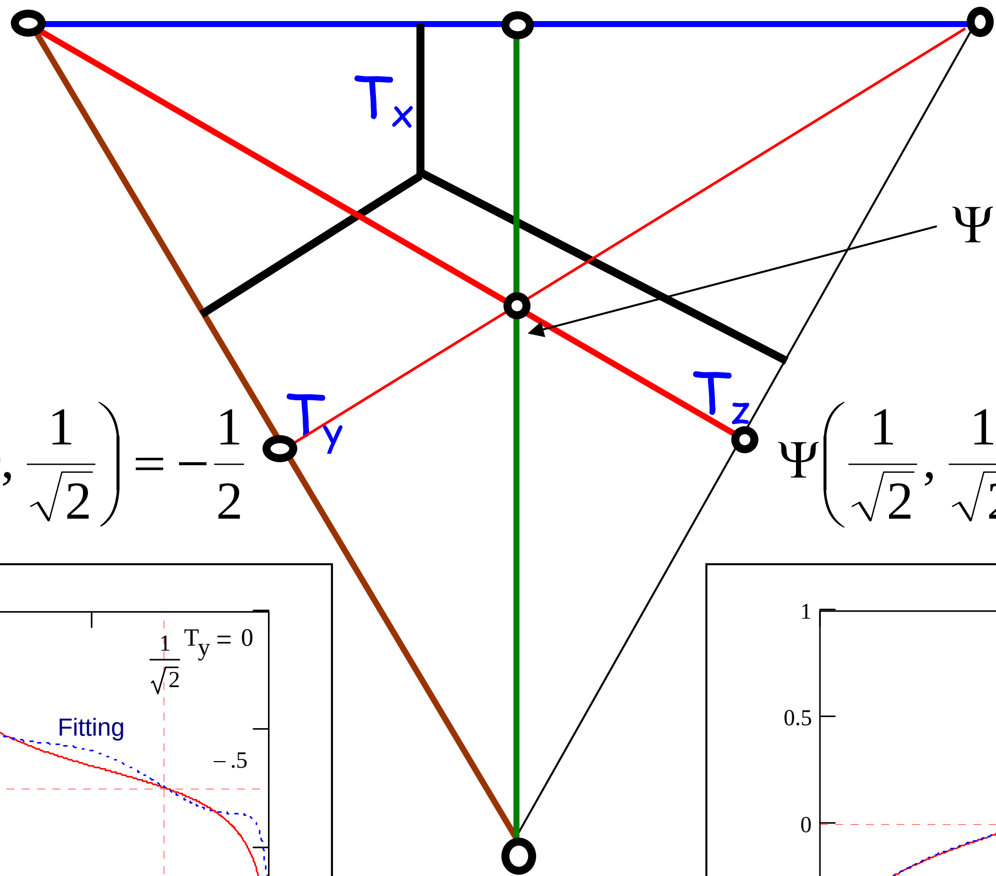
$T_x = x^2, T_y = y^2, T_z = z^2,$



$T_x = 0$ (Along blue line)

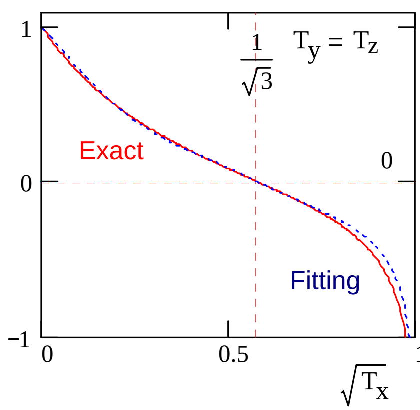
$\Psi \propto -\ln(x^2+z^2)$ $\qquad$ $\Psi\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 1$ $\qquad$ $\Psi \propto -\ln(x^2+z^2)$

$$\Psi\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = 0$$

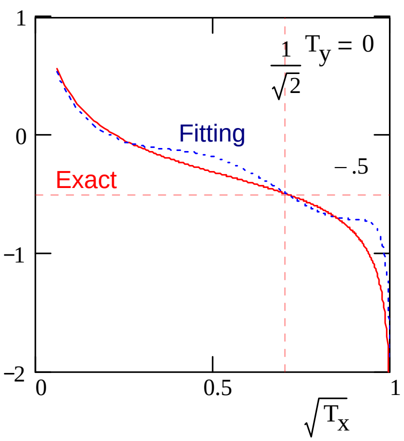


$\Psi\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$ $\qquad$ $\Psi\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) = -\frac{1}{2}$

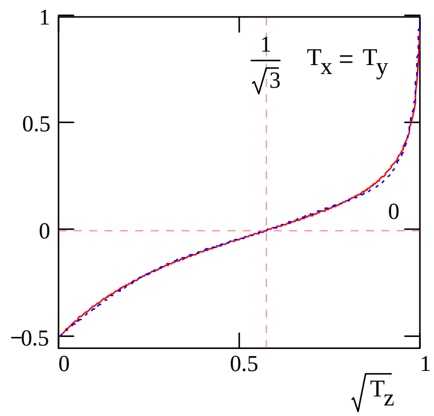
$\Psi \propto \ln(y^2+z^2)$



$T_y = T_z$ (Along green line)



$T_y = 0$ (Along brown line)



(Along red line)

IBS for Gaussian Bunch

- The general case theory (uncoupled but both dispersions are present) does not look compact even in the matrix definition

$$\boldsymbol{\Sigma} = (\gamma\beta c)^2 \mathbf{G}^T \boldsymbol{\Xi}^{-1} \mathbf{G}$$

$$\frac{d\varepsilon_k}{dt} = \frac{N r_0^{\,2} c^2}{4\sqrt{2}\sigma_z \beta^2 \gamma^4 \sqrt{\varepsilon_x \varepsilon_y}} \left\langle \frac{L_c \sum_{i,j=1}^{3} \mathbf{B}_{ij}^k \mathbf{R}_{ij}}{\sqrt{\beta_x \beta_y F_D \,\mathrm{tr}(\boldsymbol{\Sigma})}} \right\rangle_s , \quad \boldsymbol{\Xi} = \begin{pmatrix} \beta_x/\varepsilon_x & 0 & -\beta_x \Phi_x/\varepsilon_x \\ 0 & \beta_y/\varepsilon_y & -\beta_y \Phi_y/\varepsilon_y \\ -\beta_x \Phi_x/\varepsilon_x & -\beta_y \Phi_y/\varepsilon_y & \boldsymbol{\Xi}_{33} \end{pmatrix}$$

$$\boldsymbol{\Xi}_{33} = 1/\sigma_p^{\,2} + A_x/\varepsilon_x + A_y/\varepsilon_y$$

$\mathbf{R} = \left(\mathbf{G}^{-1}\right)^T \mathbf{T} \boldsymbol{\Psi}_{IBS}\left(\mathbf{T}^T \boldsymbol{\Sigma} \mathbf{T}\right) \mathbf{T}^T \mathbf{G}^{-1}$, $\mathbf{T}$ reduces Σ to its diagonal form $\boldsymbol{\sigma}$

$$\boldsymbol{\Psi}_{IBS}(\boldsymbol{\sigma}) = \mathrm{diag}\left(\Psi(\sigma_{11},\sigma_{22},\sigma_{33}), \Psi(\sigma_{22},\sigma_{33},\sigma_{11}), \Psi(\sigma_{33},\sigma_{11},\sigma_{22})\right)$$

$$\mathbf{B}^x = \begin{pmatrix} \beta_x & 0 & \Phi_x \beta_x \\ 0 & 0 & 0 \\ \Phi_x \beta_x & 0 & A_x \end{pmatrix}, \mathbf{B}^y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \beta_y & \Phi_y \beta_y \\ 0 & \Phi_y \beta_y & A_y \end{pmatrix}, \mathbf{B}^z = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \beta_y & \Phi_y \beta_y \\ 0 & \Phi_y \beta_y & \beta_z \end{pmatrix}, \mathbf{G} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}$$

$$\Phi_x = D_x' + \alpha_x D_x/\beta_x , \qquad \Phi_y = D_y' + \alpha_y D_y/\beta_y ;$$

$$A_x = \left(D_x^{\,2} + (\beta_x \Phi_x)^2\right)/\beta_x , \quad A_y = \left(D_y^{\,2} + (\beta_y \Phi_y)^2\right)/\beta_y ;$$

$$F_D = 1 + D_x^{\,2} \sigma_p^{\,2}/(\varepsilon_x \beta_x) + D_y^{\,2} \sigma_p^{\,2}/(\varepsilon_y \beta_y) \quad ;$$

IBS for Gaussian Bunch with pan-cake distribution

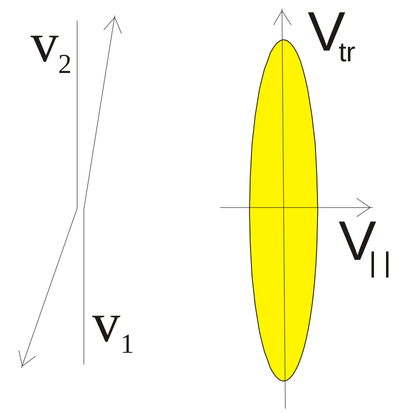
- No coupling, zero vertical dispersion

$$\frac{d}{dt}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \sigma_p{}^2\end{pmatrix}=\frac{Nr_0{}^2c}{4\sqrt{2}\beta^3\gamma^3\sigma_z}\left\langle\frac{L_c}{\sigma_x\sigma_y\theta_\perp}\begin{pmatrix}\Psi(0,\theta_x,\theta_y)A_x+\Psi(\theta_x,\theta_y,0)\beta_x/\gamma^2\\ \Psi(\theta_y,\theta_x,0)\beta_y/\gamma^2\\ \Psi(0,\theta_x,\theta_y)\end{pmatrix}\right\rangle_s$$

where $\theta_\perp=\sqrt{\theta_x{}^2+\theta_y{}^2}$, $\theta_x{}^2=\varepsilon_x/\beta_x\left(1+\sigma_p{}^2\left(\Phi_x\beta_x\right)^2/\sigma_x{}^2\right)$, $\theta_y{}^2=\varepsilon_y/\beta_y$

- For ultra-relativistic case

$$\boxed{\frac{d}{dt}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \sigma_p{}^2\end{pmatrix}=\frac{Nr_0{}^2c}{4\sqrt{2}\beta^3\gamma^3\sigma_z}\left\langle\frac{L_c}{\sigma_x\sigma_y\theta_\perp}\begin{pmatrix}\Psi(0,\theta_x,\theta_y)A_x\\ 0\\ \Psi(0,\theta_x,\theta_y)\end{pmatrix}\right\rangle_s}$$



$$\Psi(0,x,y)\simeq1+\frac{\sqrt{2}}{\pi}\ln\left(\frac{x^2+y^2}{2xy}\right)-0.055\left(\frac{x^2-y^2}{x^2+y^2}\right)^2,$$

$$A_x=\frac{D_x{}^2+\left(\beta_xD'_x+\alpha_xD_x\right)^2}{\beta_x},\quad L_c=\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right),\qquad \rho_{\min}=r_0/\left(\theta_\perp\beta\gamma\right)^2,$$

$$\rho_{\max}=\min\left(\sigma_x,\sigma_y,\gamma\sigma_z,\theta_\perp\beta\gamma/\sqrt{4\pi nr_0}\right)$$

Growth rates of H and L emittances are directly related

IBS in the LHC

- The ultra-relativistic case approximation overestimates the growth rates: 3% at the 3.5 GeV & 23% at the 3.5 GeV
- Coulomb logarithm, L_c, is ~ 25 => accuracy of logarithmic approximation ~5%

Emittance growth time due to IBS ($N=1.1\cdot10^{11}$, $\sigma_s=10$ cm)

	350 GeV, $\sigma_{\Delta p/p}=2.95\cdot10^{-4}$			3.5 TeV, $\sigma_{\Delta p/p}=1.16\cdot10^{-4}$		
$\varepsilon_x=\varepsilon_y$ [mm mrad]	2.5	3	3.5	2.5	3	3.5
τ_x [hour]	14.4	21.7	30.8	38.6	58.5	83.2
τ_y [hour]	$-1\cdot10^3$	$-2\cdot10^3$	$-3\cdot10^3$	$-3\cdot10^5$	$-5\cdot10^5$	$-7\cdot10^5$
τ_s [hour]	4.34	5.45	6.64	17.7	22.5	27.5

where: $$\tau_{x,y,s}=\frac{\varepsilon_{x,y,s}}{d\varepsilon_{x,y,s}/dt}$$

the general case theory is used and averaging is performed for the present best optics file for Beam 1

- Note that X-Y coupling redistributes the horizontal emittance growth between two transverse degrees of freedom

IBS in the Smooth Optics Approximation

- LHC has comparatively smooth optics and the smooth optics approximation yields remarkably close results

$$\boxed{\frac{d}{dt}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \sigma_p{}^2\end{pmatrix}=\frac{Nr_0{}^2cL_c}{4\sqrt{2}\beta^3\gamma^3\sigma_x\sigma_y\sigma_z\theta_\perp}\begin{pmatrix}\langle A_x\rangle_s\\ 0\\ 1\end{pmatrix}}$$

$$\sigma_x=\sqrt{\frac{\varepsilon_x R_0}{\nu_x}+\left(\frac{\sigma_p R_0}{\nu_x{}^2}\right)^2}\ ,\quad \sigma_y=\sqrt{\frac{\varepsilon_y R_0}{\nu_y}}\ ,$$

$$\langle A_x\rangle_s=\left\langle\frac{D_x{}^2+\left(\beta_x D'_x+\alpha_x D_x\right)^2}{\beta_x}\right\rangle_s\approx\frac{R_0}{\nu_x{}^3}$$

- For the LHC $\langle A_x\rangle_s=2.29\ \mathrm{cm}$ and at 3.5 TeV the smooth optics approximation yields 18% larger growth rates
- Corrected smooth lattice approximation is used in the luminosity evolution modes

4. IBS in Non-linear Longitudinal Well

Diffusion equation

- In the case $v_{\|} << v_x, v_y$ the friction in the Landau collision integral can be neglected
- Diffusion equation

$$D(I) = \oint D(p)\,p\,dq \Big/ \oint p\,dq$$

1D: $\dfrac{\partial f}{\partial t} = \dfrac{1}{2}\dfrac{\partial}{\partial p}\left(D(p)\dfrac{\partial f}{\partial p}\right)$ $\Rightarrow$ 2D: $\boxed{\dfrac{\partial f}{\partial t} = \dfrac{1}{2}\dfrac{\partial}{\partial I}\left(I\dfrac{D(I)}{\omega(I)}\dfrac{\partial f}{\partial I}\right)}$

  - I is the action and ω is the frequency for dimensionless Hamiltonian of synchrotron motion: $H = \dfrac{p^2}{2} + 2\left(\sin\dfrac{\varphi}{2}\right)^2$
  - Diffusion coefficient depends on distribution, (I)

$$\boxed{D(I) = 4L_c\tilde{A}\left(\oint n(\varphi)\,p\,d\varphi \Big/ \oint p\,d\varphi\right)} \qquad \tilde{A} = \pi^2\sqrt{\frac{\pi}{2}}\frac{(\alpha - 1/\gamma^2)e^4 q^2}{eV_0 m_p c\beta\gamma^2 C}\left\langle \frac{N}{\sigma_1\sigma_2}\frac{\Psi(\sigma_p/\gamma,\theta_1,\theta_2)}{\sqrt{\theta_1^2+\theta_2^2+(\sigma_p/\gamma)^2}}\right\rangle_s$$

Here: $n(\varphi) = \int f\left(I(p,\varphi)\right)dp$, $\displaystyle\int_{-\pi}^{\pi} n(\varphi)\,d\varphi = 1$

α - momentum compaction, q - harmonic number

V_0 - RF voltage, C - ring circumference

Simultaneous treatment of single and multiple scattering

- Boltzmann type equation
  - For $\mathrm{v}_{\parallel} << \mathrm{v}_{\perp}$ one can write for Coulomb scattering in long. direction

$$\frac{\partial f}{\partial t} = \left\langle \tilde{A} \int n(\varphi) \frac{f(p+q) - f(p)}{|q|^3} \mathrm{d}q \right\rangle_{period} = \left\langle \tilde{A} \int n(\varphi) \frac{f(I') - f(I)}{|p - p'|^3} \delta(\varphi - \varphi') \mathrm{d}I' d\psi d\psi' \right\rangle_{period}$$

- After simplification we obtain

$$\boxed{\frac{\partial f(I,t)}{\partial t} = \tilde{A} \int_0^\infty W(I,I') \big(f(I',t) - f(I,t) \big) \mathrm{d}I'}$$

$$W(I,I') = \frac{2\omega\omega'}{\pi} \int_0^{\min(a,a')} \frac{d\varphi}{pp'} n(\varphi) \left[\frac{1}{|p-p'|^3} + \frac{1}{|p+p'|^3} \right] \xrightarrow{E' \ge E}$$

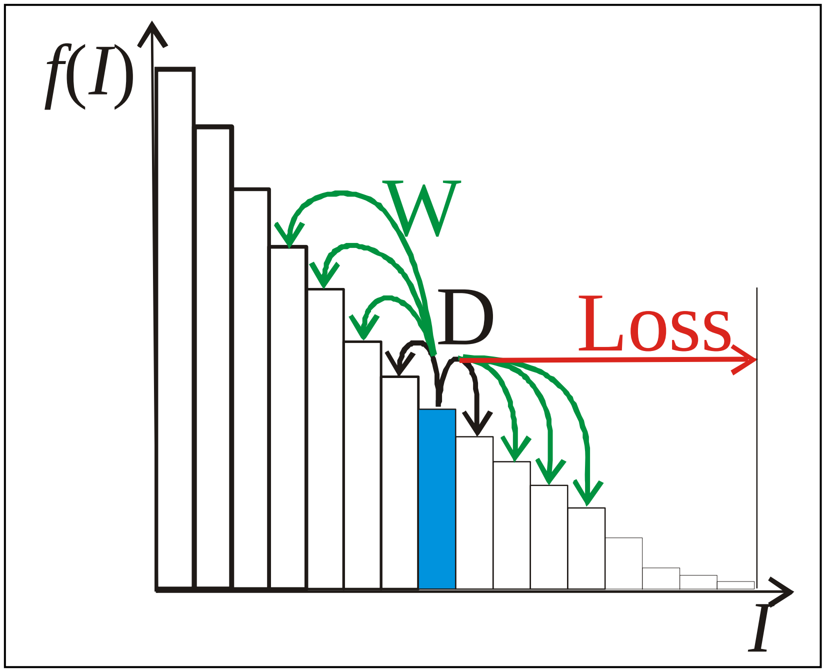
$$\frac{\omega\omega'}{\pi (E - E')^3} \left[(E - E') \int_0^a n(\varphi) \frac{dx}{p} + 2 \int_0^a n(x) p \, dx \right] \quad .$$

$a \equiv a(I)$ is the motion amplitude

- The kernel is symmetric: $W(I,I') = W(I',I)$,
- The kernel divergence needs to be limited at the minimum action change corresponding to the maximum impact parameter

Numerical model

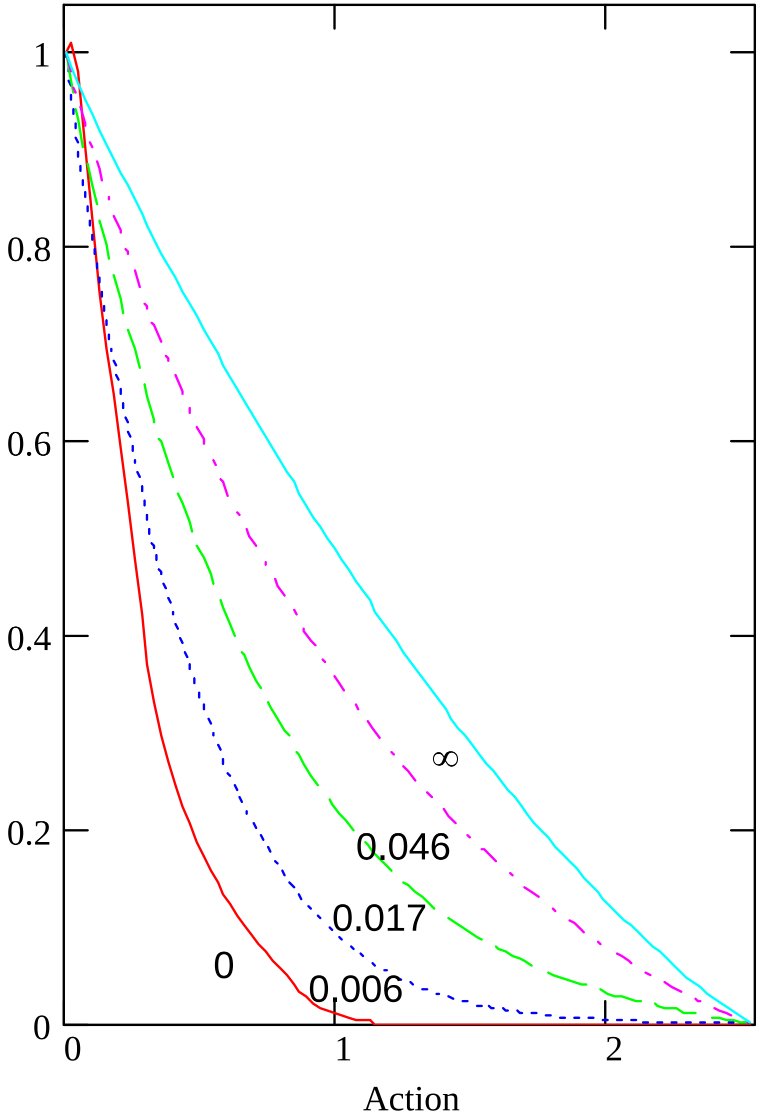
- Set of bins
  - Transition probabilities
    - Nearby bins – diffusion equation to resolve divergence of $W(I,I')$
    - Far away bins – transition probabilities are described by $W(I,I')$
    - Particle loss outside bucket need to be added



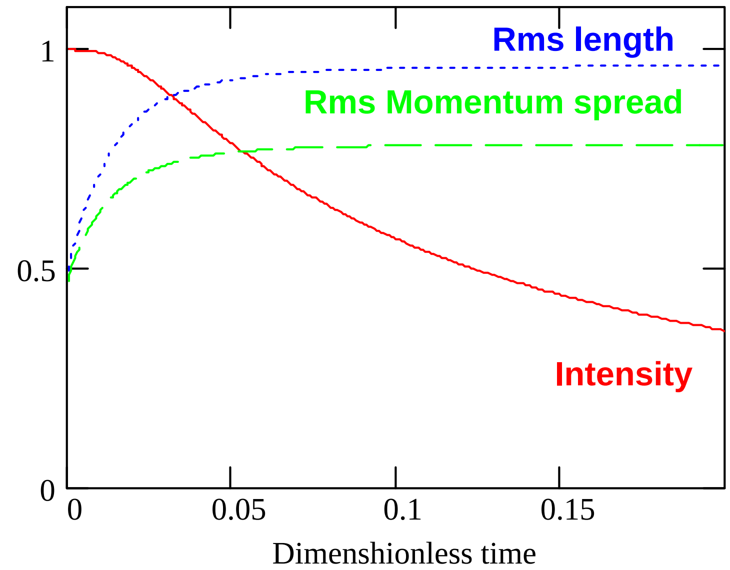
- In matrix form

$$\mathbf{f}_{n+1} = \mathbf{f}_n + \mathbf{W}\mathbf{f}_n \Delta t$$

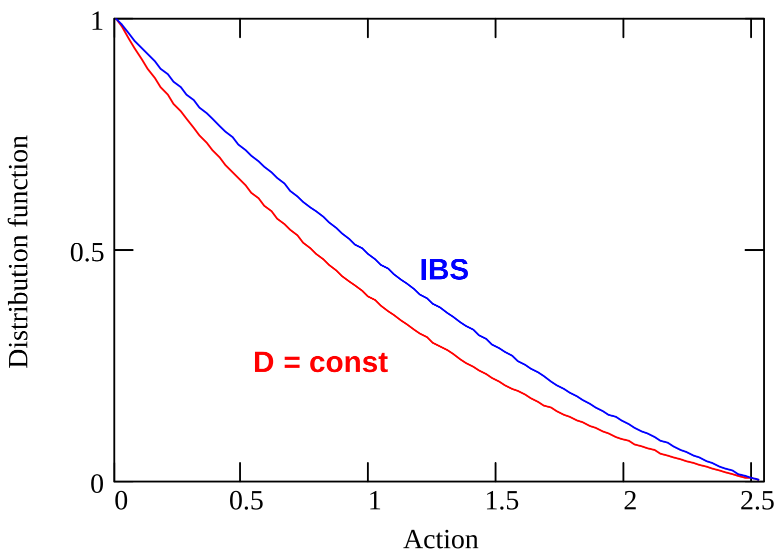
W – is matrix of transition probabilities. It is a symmetric matrix



Dependence of longitudinal distribution on time for IBS. Measured initial distribution is used



Dependencies: $N(t)/N_0$, $\sqrt{\overline{\overline{\varphi^2(t)}}}$, $\sqrt{\overline{\overline{p^2(t)}}}$



2. Bunch lengthening due to RF phase noise

Theoretical description

$$\ddot{x} + \Omega_s^{\,2}\sin(x - \psi(t)) = 0 \quad \Rightarrow \quad \ddot{x} + \Omega_s^{\,2}\sin(x) = \Omega_s^{\,2}\cos(x)\psi(t)$$

Action - $I = \frac{1}{2\pi}\oint p\,dx$ Frequency - $\omega \equiv \omega(I) = 2\pi\left(\oint \frac{dx}{p}\right)^{-1}$

Introduce the diffusion coefficient using the following form of diff. eq.

$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial}{\partial I}\left(I\frac{D(I)}{\omega(I)}\frac{\partial f}{\partial I}\right)$$

where diffusion coefficient is

$$D(I) = \frac{\omega}{I}\frac{d}{dt}\overline{\delta I^2} = 2\pi\Omega_s^{\,2}\sum_{n=0}^{\infty} C_n(I)P(n\omega(I)) \quad ,$$

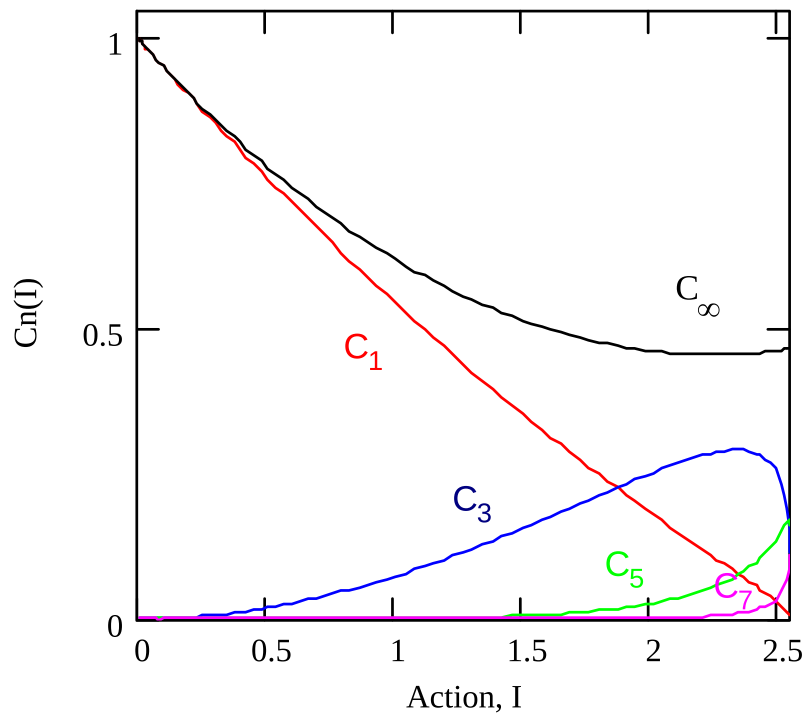
and the spectral density is normalize

$$\overline{\psi(t)^2} = \int_{-\infty}^{\infty} P(\omega)d\omega \quad .$$

For the white noise, $P(\omega) = P_0$, it yield:

$$D(I) = 2\pi\Omega_s^{\,2}P_0 C_\infty(I)$$

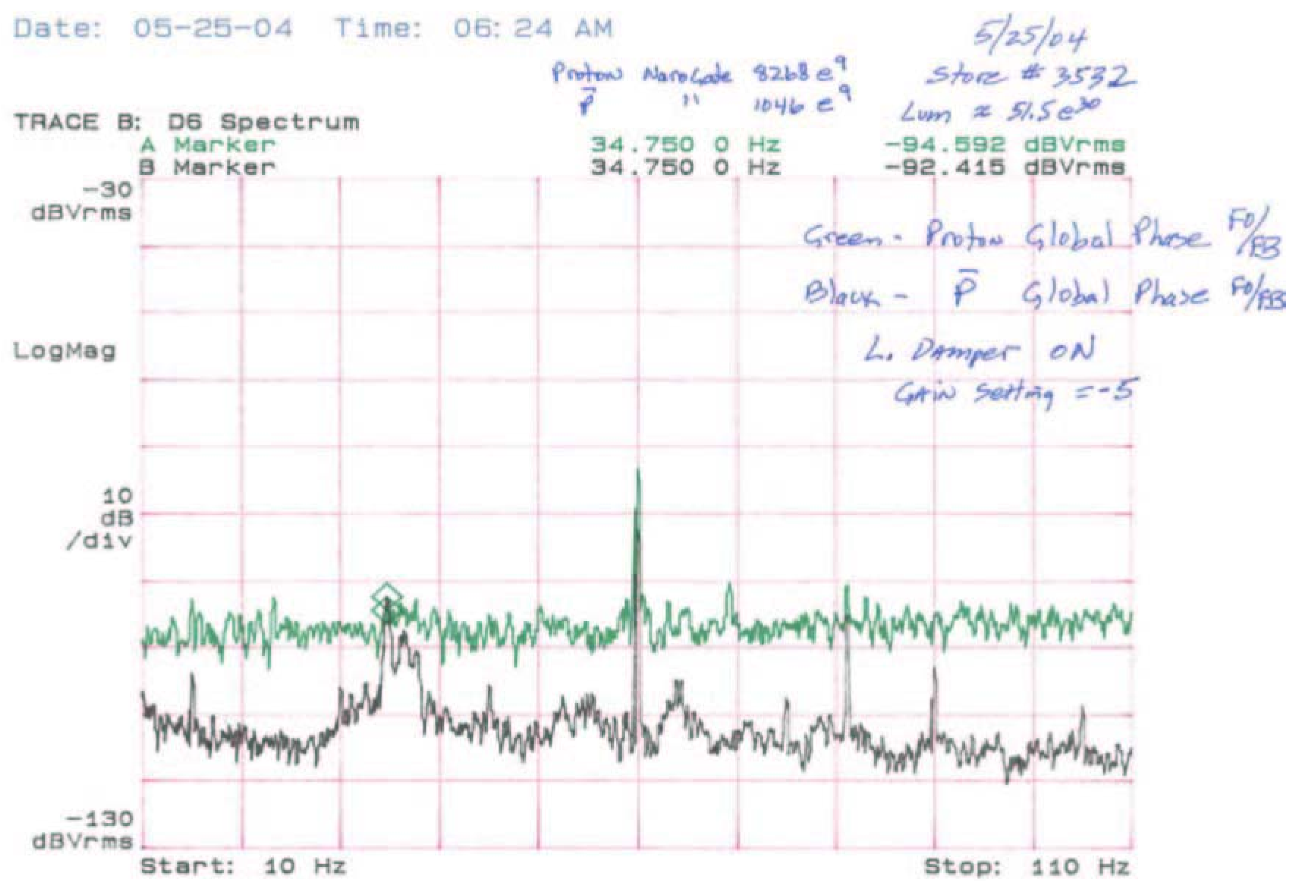
where $C_\infty(I) = \sum_{n=0}^{\infty} C_n(I)$



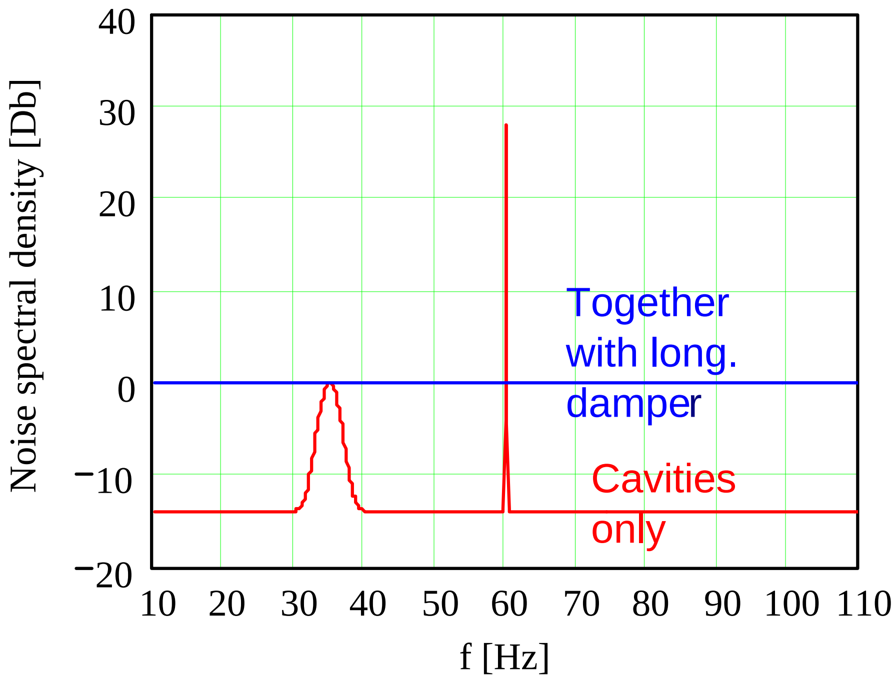
For all even n, $C_n(I) = 0$

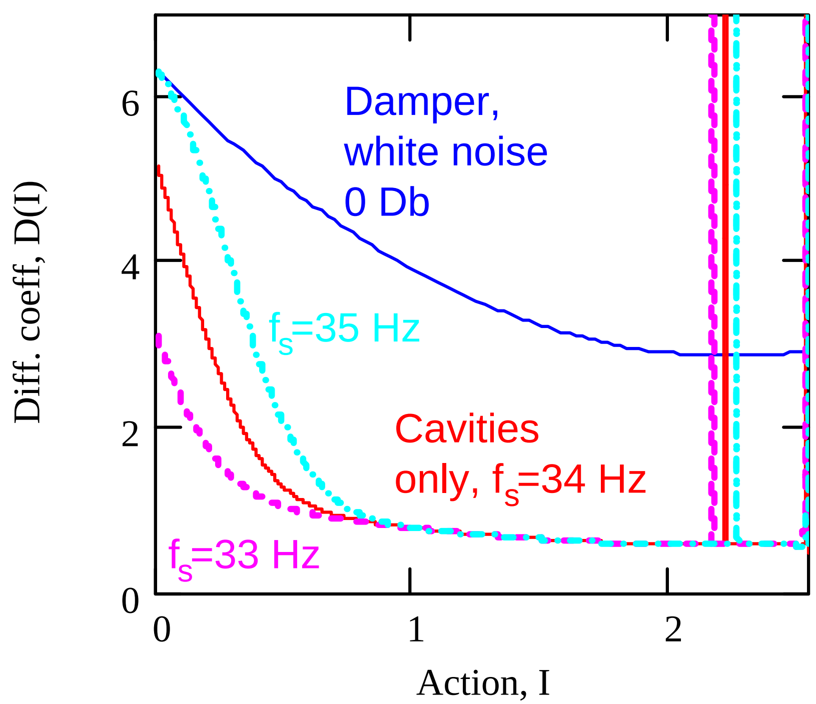
Direct measurement of RF noise performed by John Reid

- Microphonics - cavity mechanical resonances are at synchrotron frequency
  - Phase feedback suppresses microphonics by more than 20 Db
- Longitudinal damper is too noisy
  - Damper "white" noise hides mechanical resonances



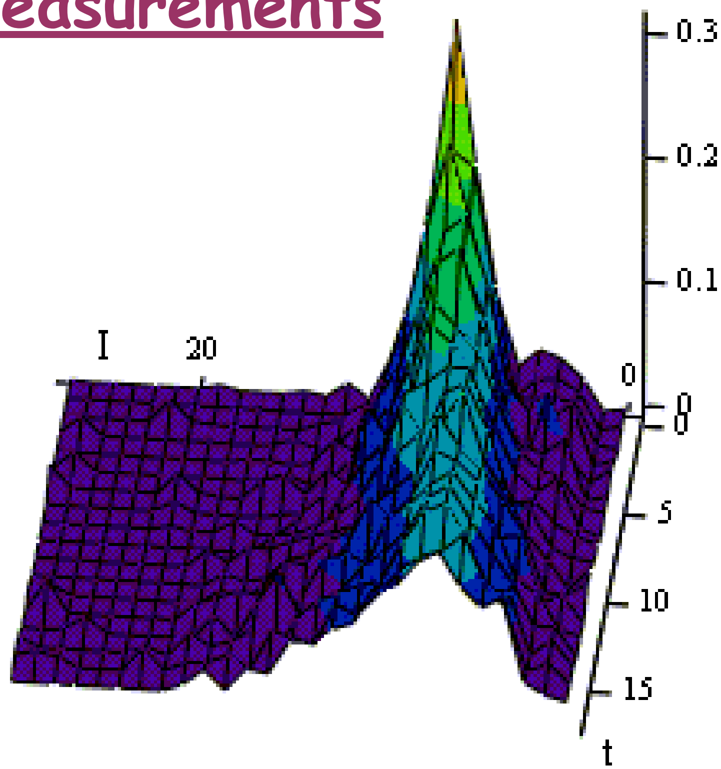
Dependence of Diffusion on the Action

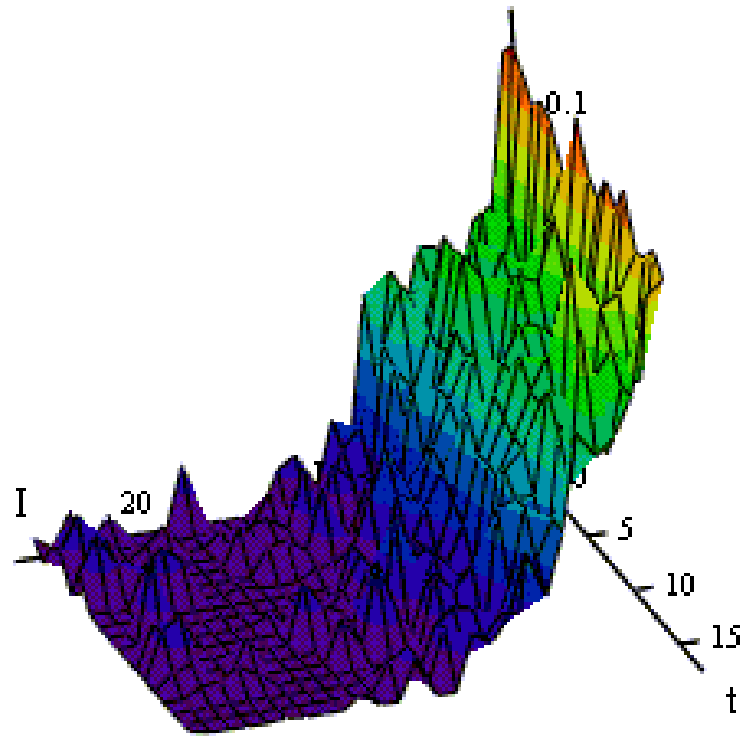




- Small changes of Synchrotron frequency (RF voltage) can significantly change diffusion coefficient if longitudinal damper is off
- The only detailed experimental data we have are for the case when the damper is on

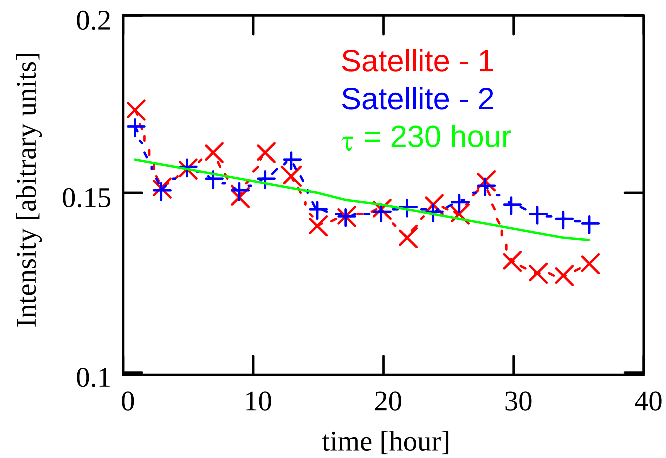
Measurements

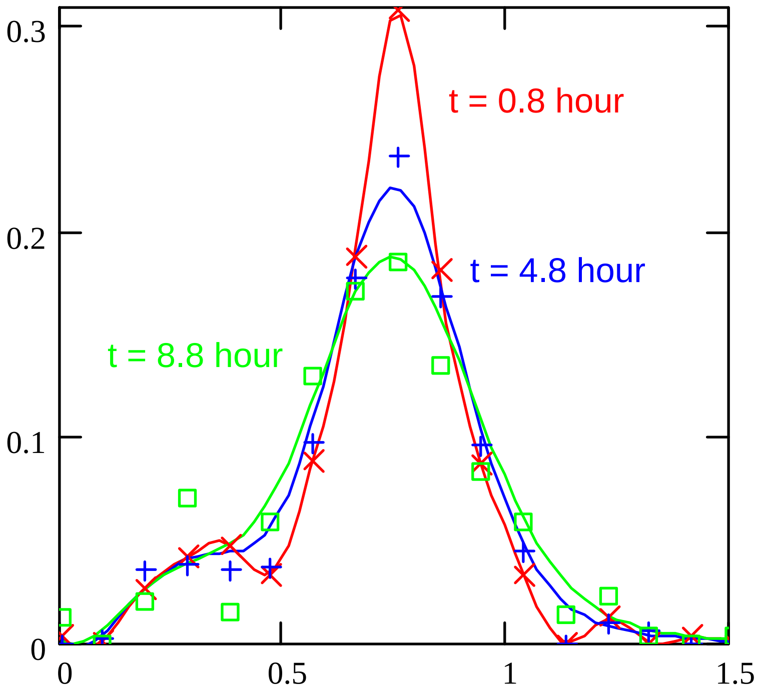


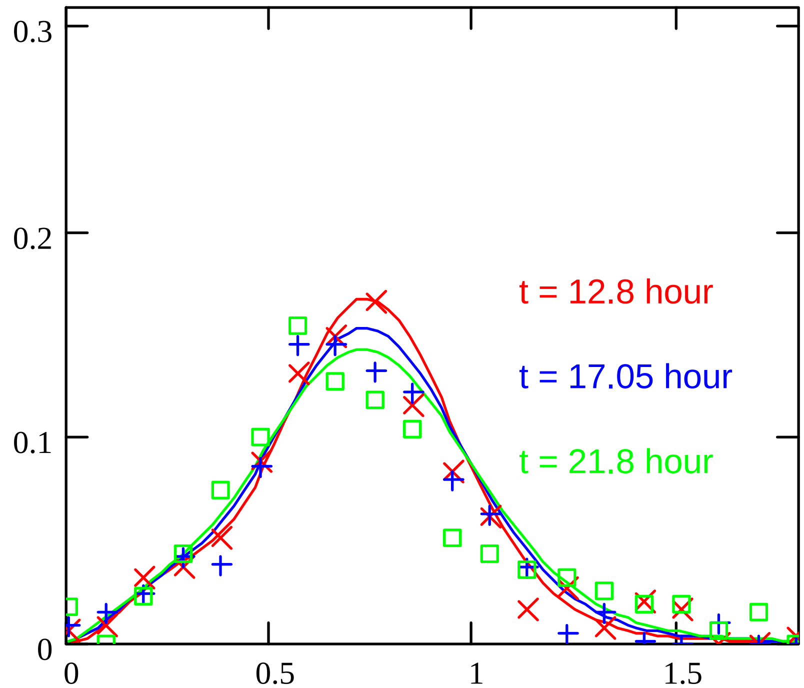


Development of distribution function on time for leading satellites: (-2) – left, and (-1) – right. Time scale [0, 15] corresponds to 37 hours of store time

- Both distributions are corrected for the satellite lifetime of 230 hours
- Longitudinal damper is on –
  Spectral density of RF phase noise is close to the white noise.







Measured and computed distribution functions for satellite (-2) of Store 3678;

RF noise spectral density - $42 \cdot 10^{-12}$ rad^2/Hz, growth rate $d\overline{\phi^2}/dt = 1.87 \cdot 10^{-3}$ rad^2/hour

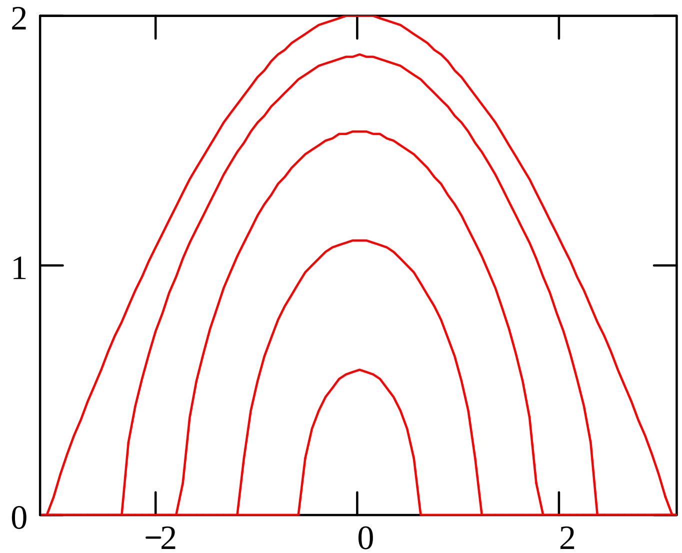
Previous estimate - *$50 \cdot 10^{-12}$* rad^2/Hz *(DoE June 2003 Review)*

Beam lifetime is 230 hour versus >360 of vacuum lifetime

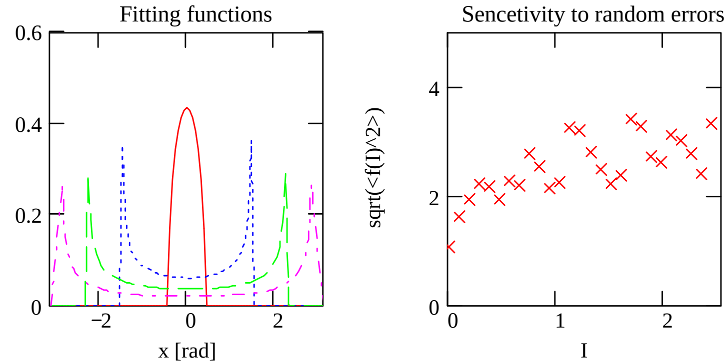
- Diffusion at small amplitudes is described well
  - That allows estimate the noise spectral density with better than 20% accuracy
- Diffusion at large amplitudes is not described well: the peak of distribution function is moved in for the measured distribution but not for computed one
- Beam-beam effects kill particles with large synchrotron amplitudes which, consequently, limits RF bucket size

Restoration of longitudinal distribution from signal of resistive wall monitor

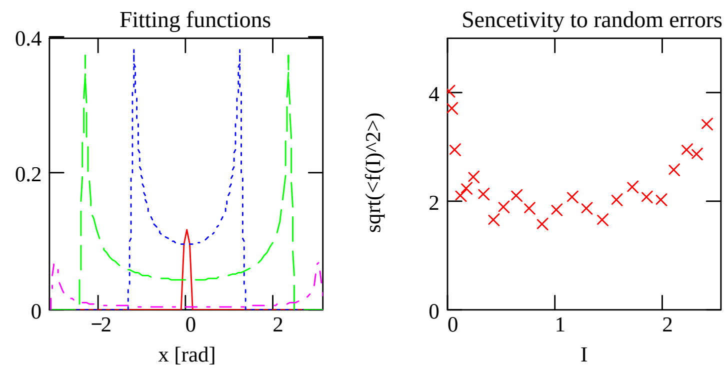
- The method is suggested by Alvin Tollestrup



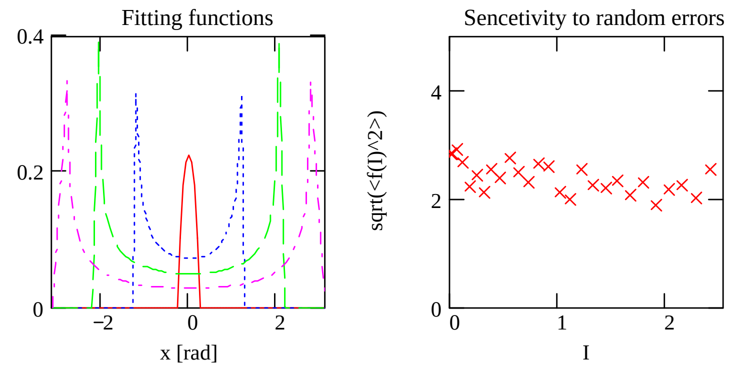
- Further improvements
  - Optimized binning
  - Constrained fit ($f(I)>0$)
  - Fitting for the baseline



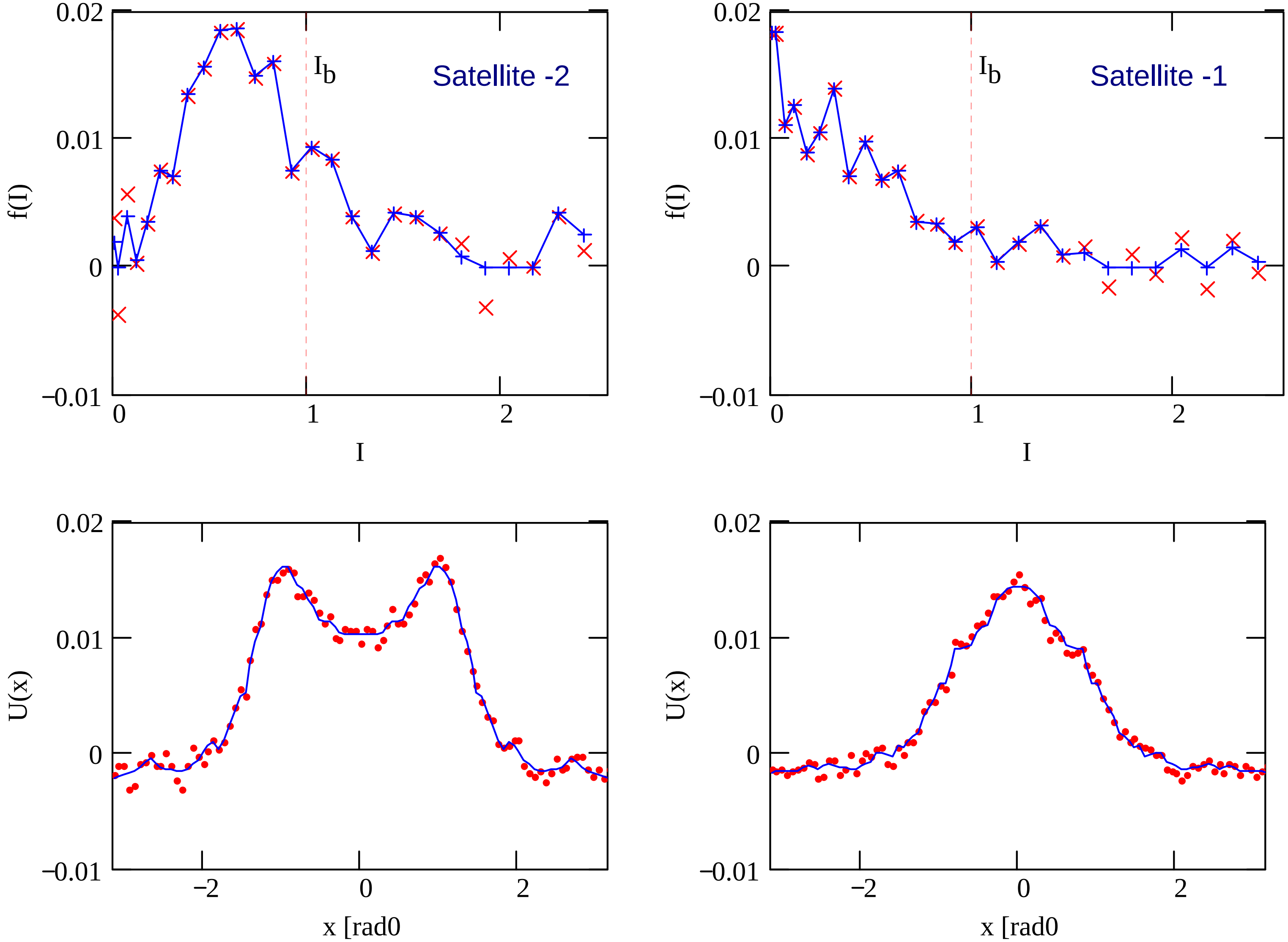
Equal ΔI binning



Equal Δt binning



Optimal binning $I_j = I_{\max}(j/N)^{1.4}$



Top – Distr. functions for satellites (-1) and (-2) with constraint and linear fits

Bottom – results of constraint fit for satellites (-1) and (-2)