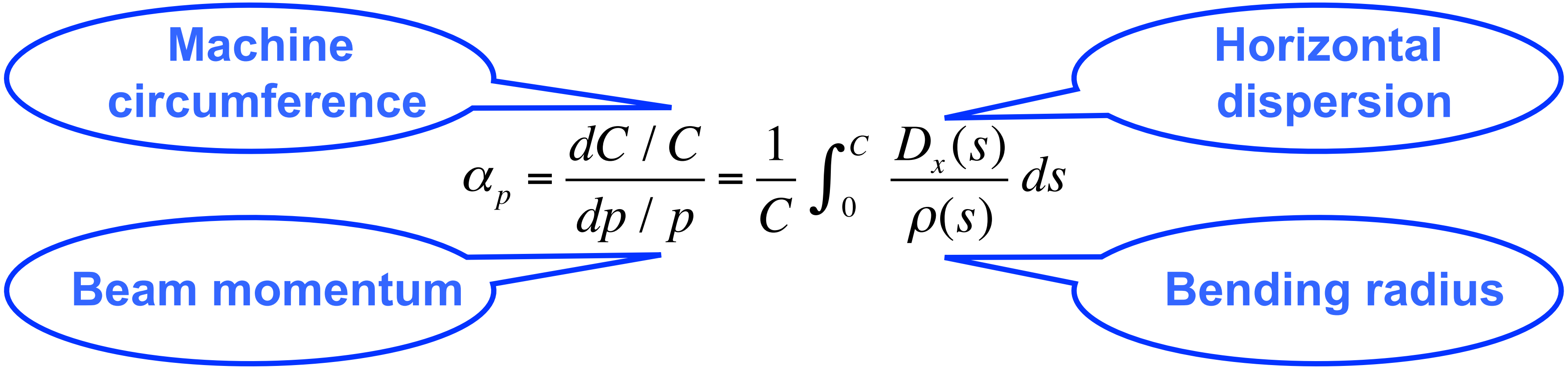
(SOME) EFFECTS NEAR TRANSITION

E. Métral (CERN)

- **Transition energy**
- **Longitudinal beam dynamics "far" below or above transition**
- **Transition crossing (with the example of the CERN PS machine)**
- **Transverse (slow) head-tail instability**
- **Fast (vertical) single-bunch instability**
    - Crossing transition in the CERN PS
    - Injecting just above transition in the CERN SPS
- **Conclusion**

TRANSITION ENERGY (1/3)

◆ Momentum compaction factor



Machine circumference

Horizontal dispersion

$$\alpha_p = \frac{dC/C}{dp/p} = \frac{1}{C}\int_0^C \frac{D_x(s)}{\rho(s)}\,ds$$

Beam momentum

Bending radius

=> Parameter coming from the accelerator lattice

Gamma transition

$$\gamma_t = \frac{1}{\sqrt{\alpha_p}}$$

- In most circular machines, $\alpha_p > 0$
- However, $\alpha_p < 0$ is also possible (e.g. CERN LEAR machine) => Called “Negative Momentum Compaction” (NMC) or “Imaginary γ_t” lattice

TRANSITION ENERGY (2/3)

- Assume $\alpha_p > 0$
  - $dp > 0 \Rightarrow dC > 0$
  - $dp > 0 \Rightarrow dv > 0$

=> What happens to the revolution frequency $f_{rev} = v\,/\,C$?

  - At (very) high energy, $v \approx clight$ and remains constant => f_{rev} ↘
  - At low energy, v increases faster than C => f_{rev} ↗

There is an energy for which the velocity variation is compensated by the trajectory variation (i.e. $df_{rev} = 0$):

=> TRANSITION ENERGY

TRANSITION ENERGY (3/3)

◆ **Slip factor**

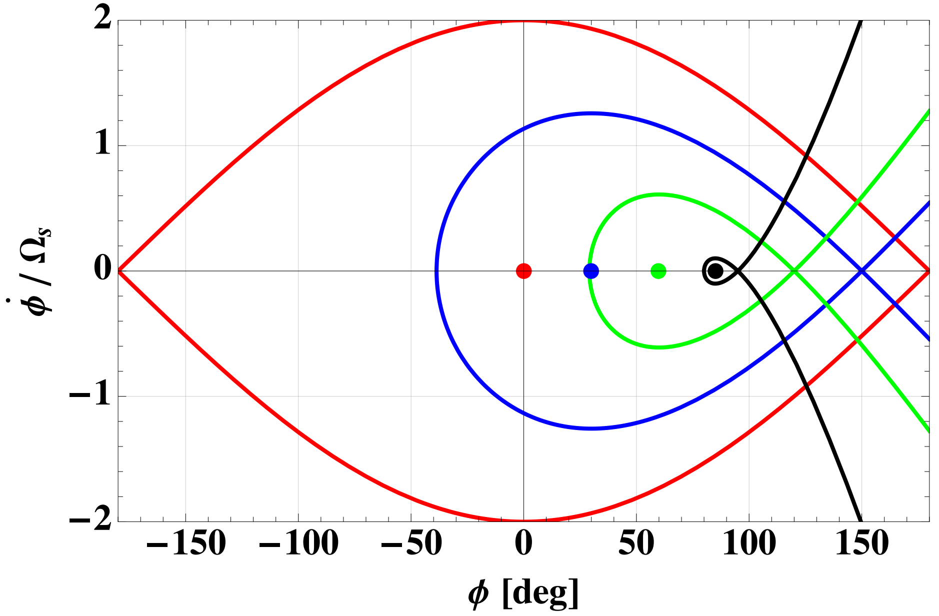
$$\eta = -\frac{df_{rev} / f_{rev}}{dp / p} = \alpha_p - \frac{1}{\gamma^2} = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

Relativistic mass factor of the beam

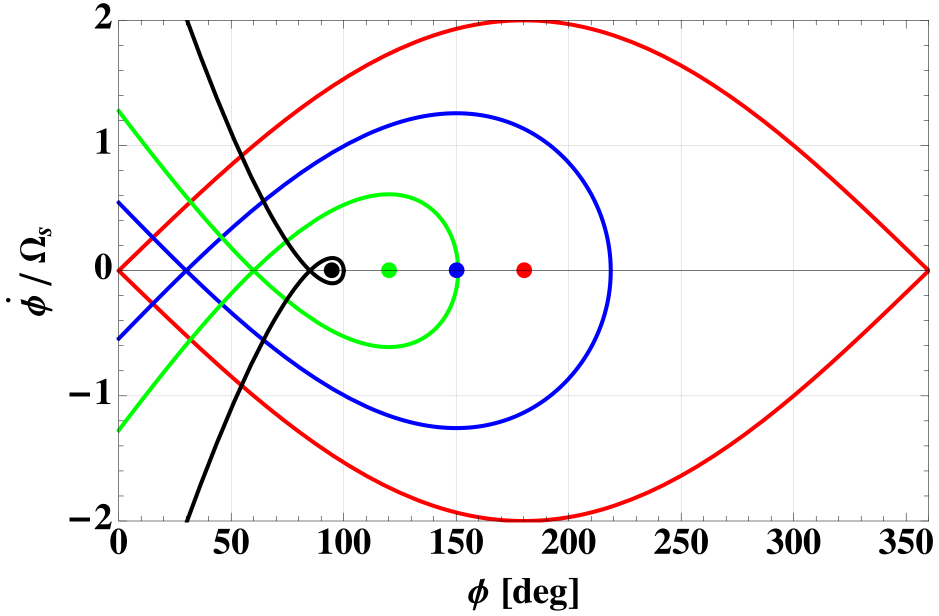
- $\eta < 0$ below transition
- $\eta = 0$ at transition => Isochronous condition
- $\eta > 0$ above transition

LONGITUDINAL BEAM DYNAMICS
"FAR" BELOW OR ABOVE TRANSITION (1/2)

- **(Bucket) separatrices**: Below transition



- Above transition



$\phi_s = 0°$

$\phi_s = 30°$

$\phi_s = 60°$

$\phi_s = 85°$

$$\Omega_s = \Omega_{rev} \left(- \frac{e \hat{V}_{\mathrm{RF}}\, h\, \eta \cos\phi_s}{2\pi \beta^2\, E_{total}} \right)^{1/2}$$

$$\phi_s \Rightarrow \pi - \phi_s$$

$\phi_s = 180°$

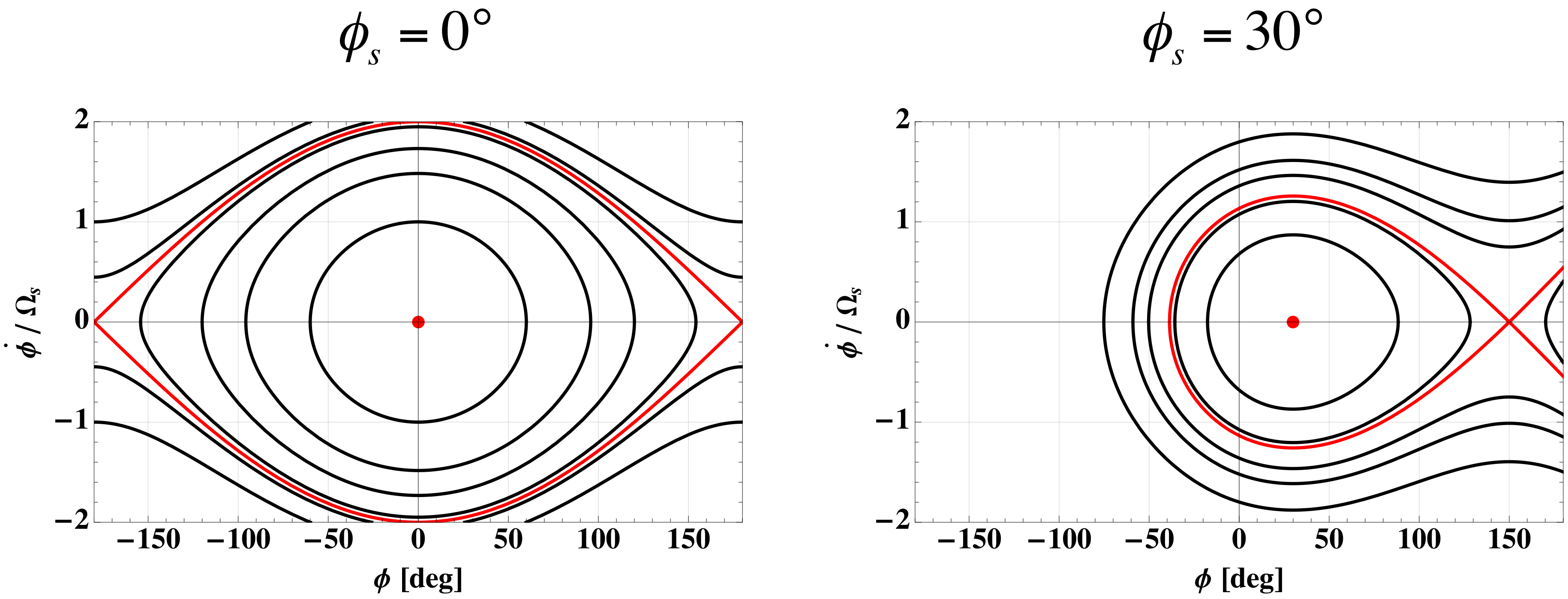
$\phi_s = 150°$

$\phi_s = 120°$

$\phi_s = 95°$

LONGITUDINAL BEAM DYNAMICS
“FAR” BELOW OR ABOVE TRANSITION (2/2)

◆ **Particle trajectories**: Below transition



TRANSITION CROSSING (1/9)

- **“Far”** below or above transition ⇔ **Adiabaticity condition** $\frac{1}{\Omega_s^2}\left|\frac{d\Omega_s}{dt}\right| << 1$

- **“Close”** to transition, the adiabaticity condition is not satisfied => **Non-adiabatic synchrotron motion**
    - When the time is close enough to transition, the particle will not be able to catch up with the rapid modification of the bucket shape
    - **Nonadiabatic time**

$$T_c = \left(\frac{\beta^2 \, E_{rest} \, \gamma_t^4}{4 \pi \, f_{rev}^2 \, \dot{\gamma} \, h \, \hat{V}_{RF} \left| \cos \phi_s \right|} \right)^{1/3}$$

~ 2 ms for the nTOF bunch in the CERN PS

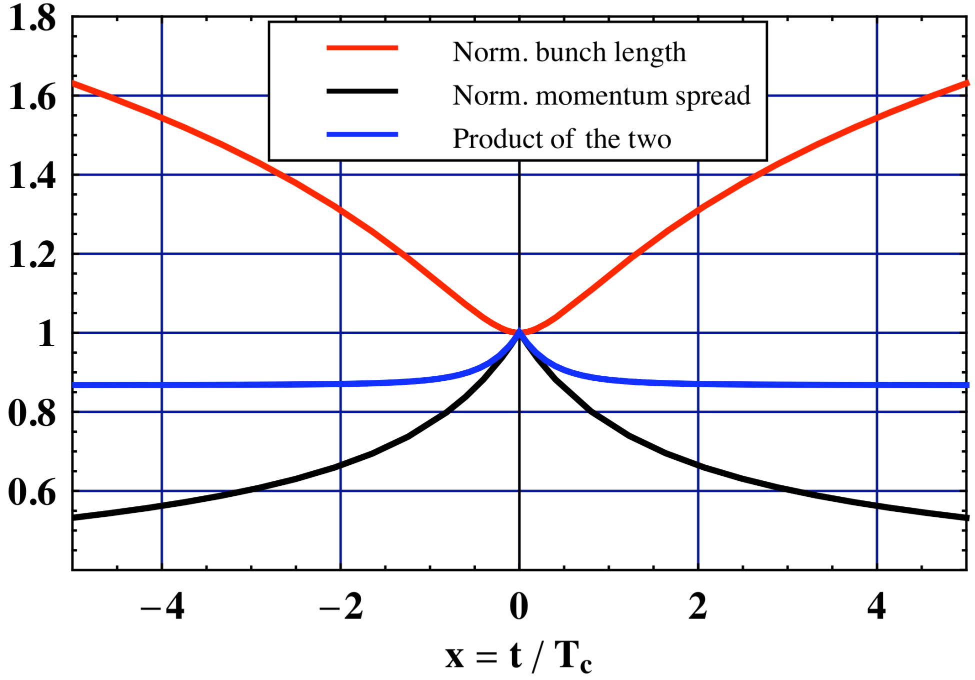
TRANSITION CROSSING (2/9)

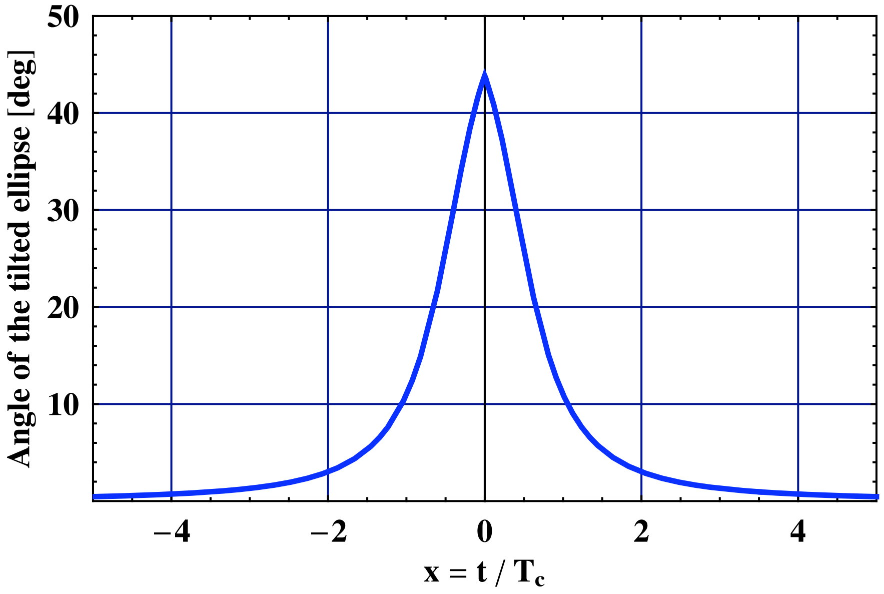
- nTOF bunch in the CERN PS

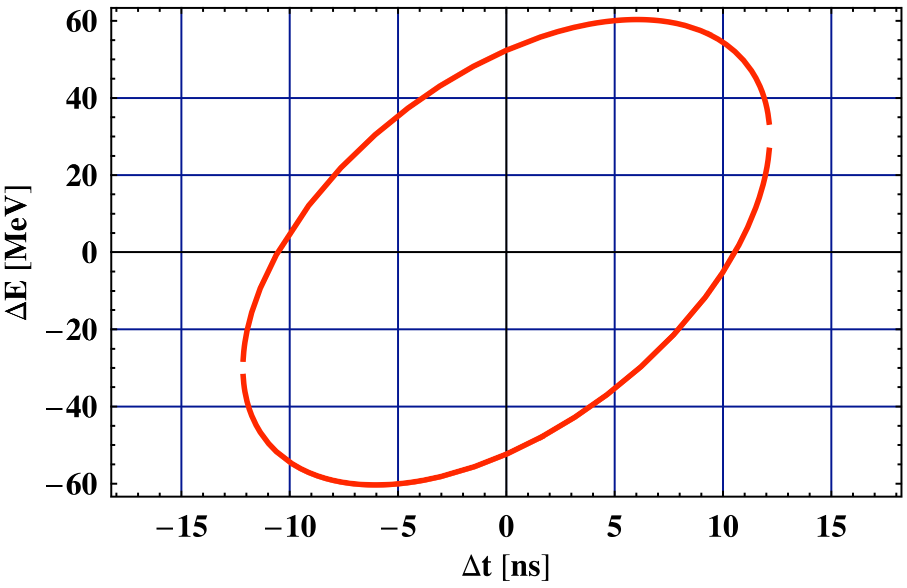
Average machine radius: R [m]	100
Bending dipole radius: ρ [m]	70
$\dot{B}$ [T/s]	2.2
$\hat{V}_{RF}$ [kV]	200
h	8
α_p	0.027
Longitudinal (total) emittance: ε_L [eVs]	2
Number of protons/bunch: N_b [1E10 p/b]	800
Norm. rms. transverse emittance: $\varepsilon^*_{x,y}$ [µm]	5
Trans. average betatron function: $\beta_{x,y}$ [m]	16
Beam pipe [cm × cm]	3.5 × 7
Trans. tunes: $Q_{x,y}$	6.25

=> $\gamma_t \approx 6.1$

TRANSITION CROSSING (3/9)

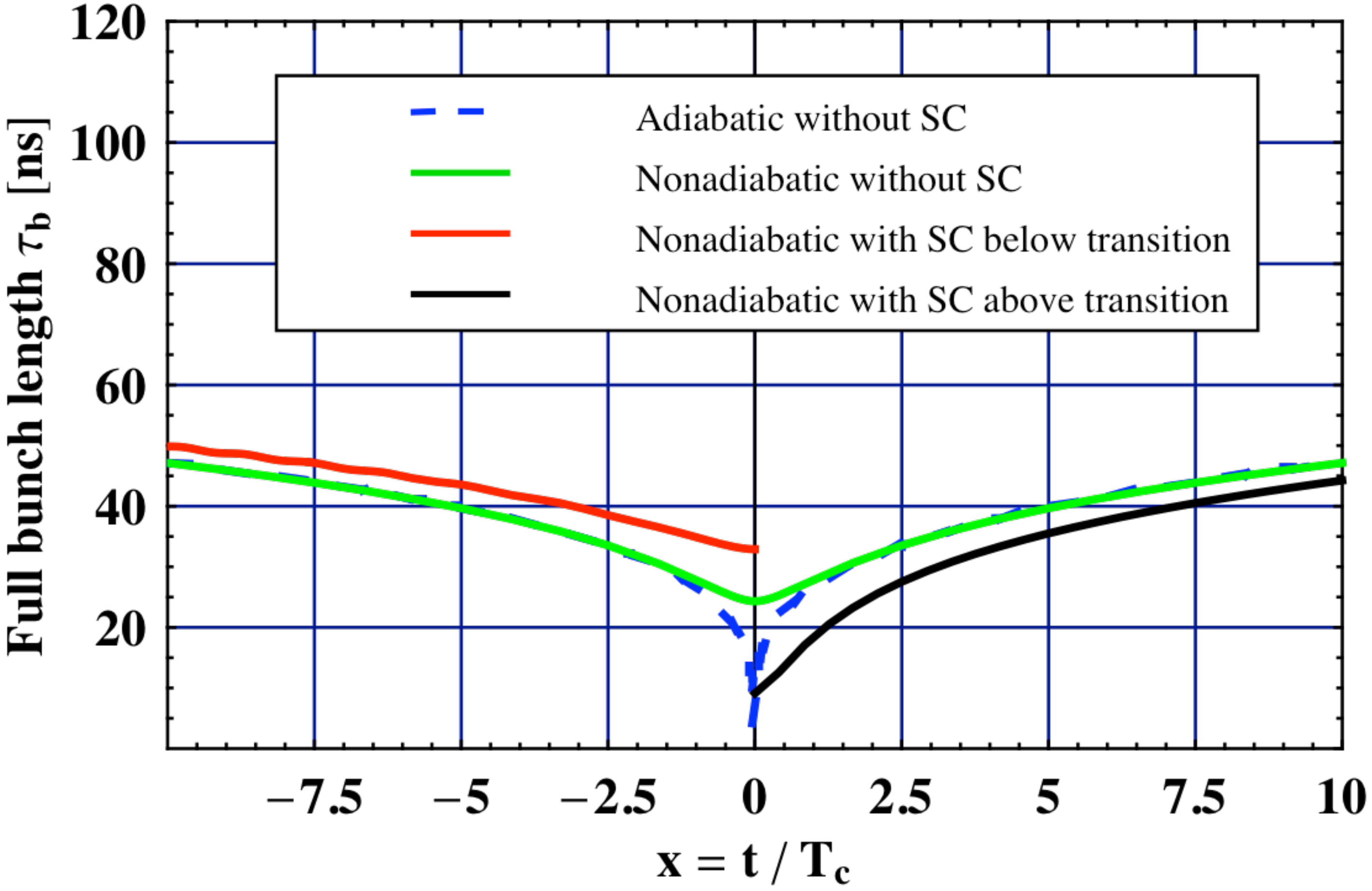






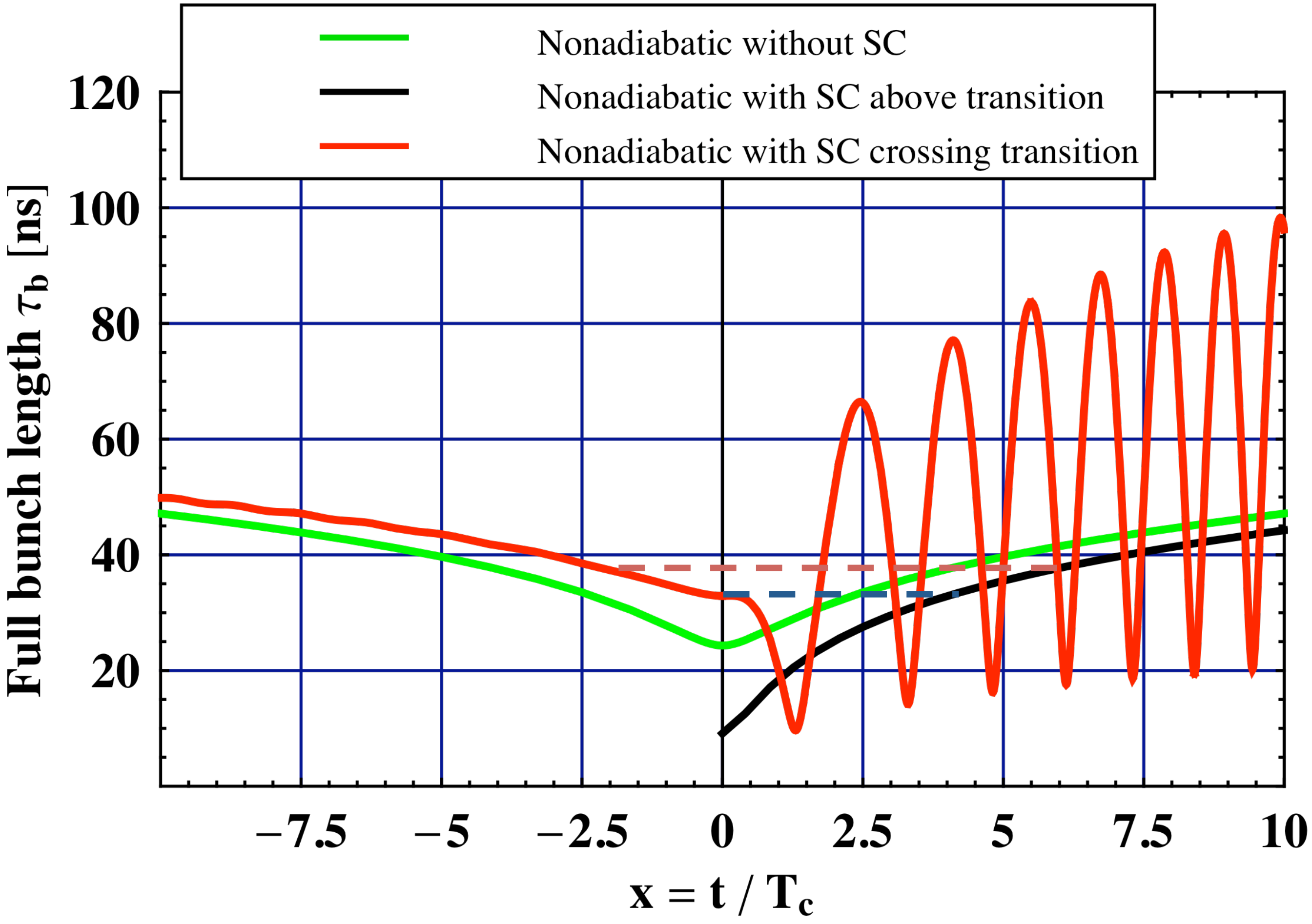
TRANSITION CROSSING (4/9)

- **Longitudinal mismatch** (due to the longitudinal Space Charge): **IN STATIC**



TRANSITION CROSSING (5/9)

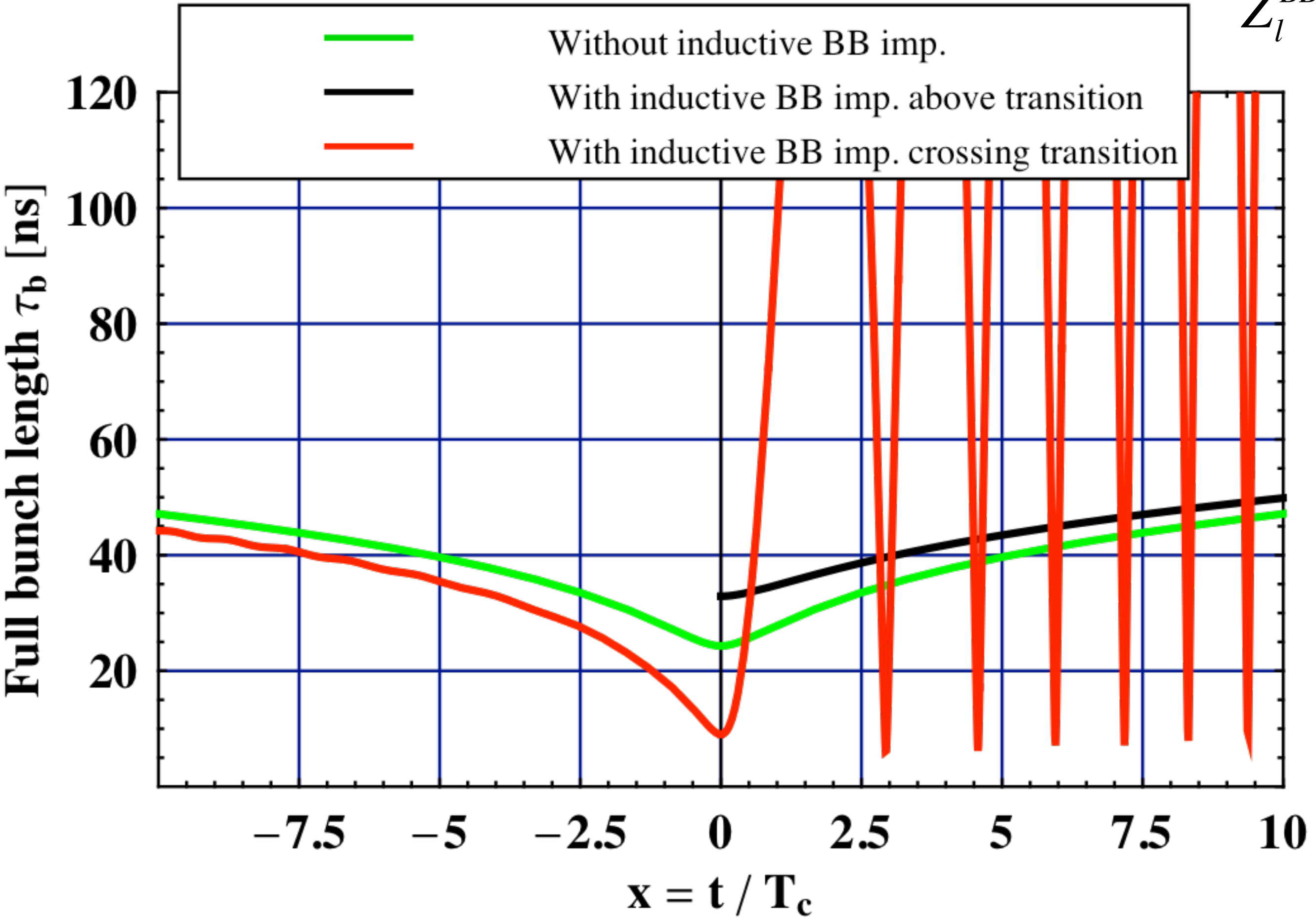
- **Longitudinal mismatch** (due to the longitudinal Space Charge): **IN DYNAMIC (i.e. crossing transition)**



TRANSITION CROSSING (6/9)

- Longitudinal mismatch (**due to the inductive part of the longitudinal Broad-Band impedance**): IN DYNAMIC (i.e. crossing transition)

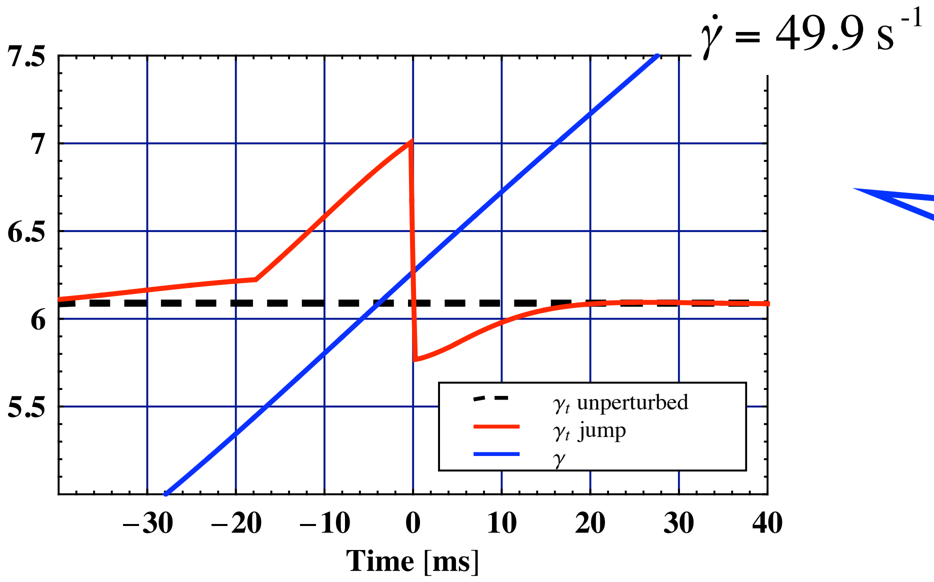
$Z_l^{BB} \, / \, n = j \, 20 \, \Omega$



TRANSITION CROSSING (7/9)

◆ **Remedies**

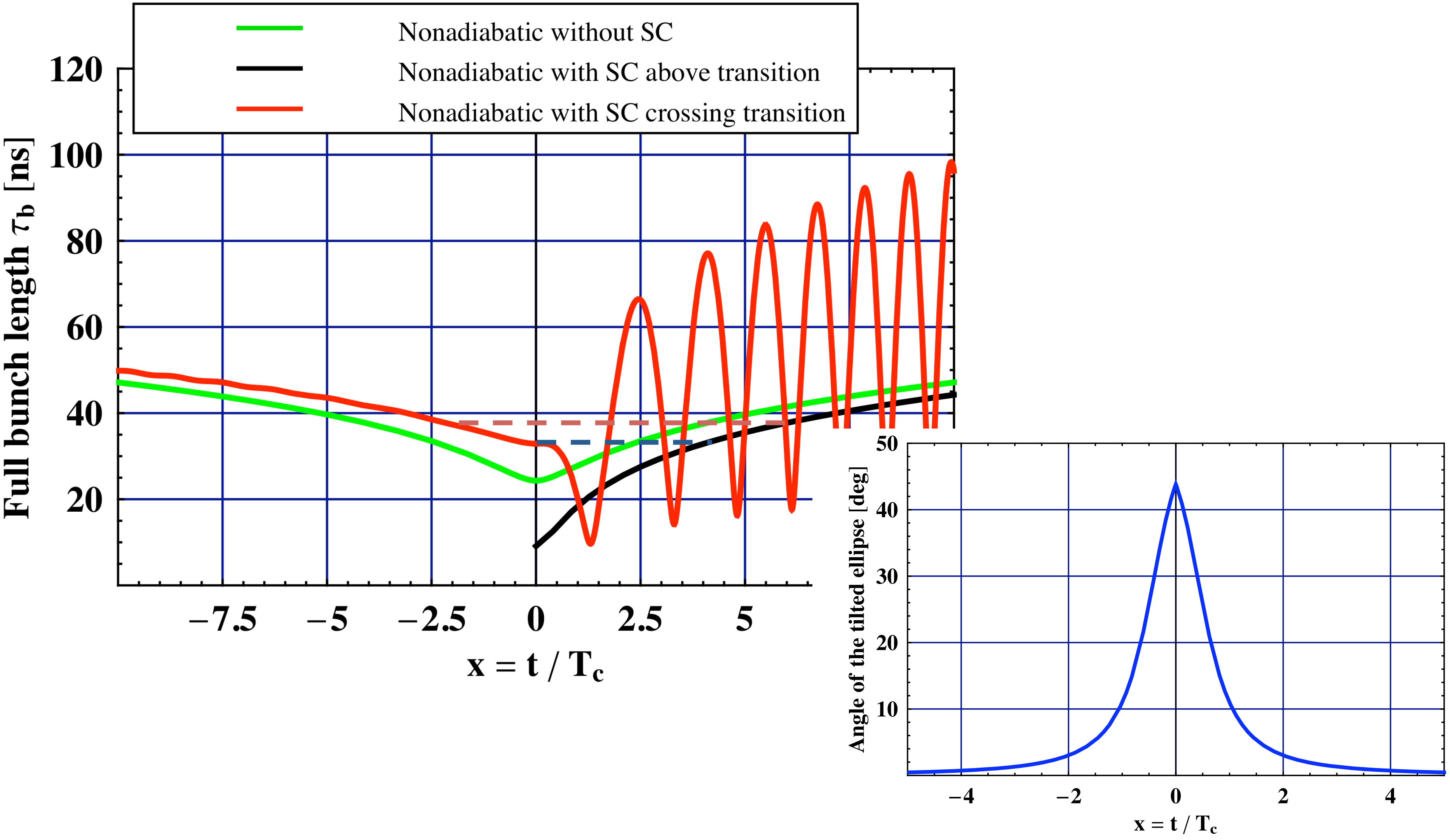
- Avoid crossing transition in the design phase => $\alpha_p < 0$
- If transition crossing cannot be avoided, the **"γ_t jump"** is the only (known) method to overcome all the intensity limitations => Artificial increase of the transition crossing speed by means of fast pulsed quadrupoles (at non zero dispersion locations)



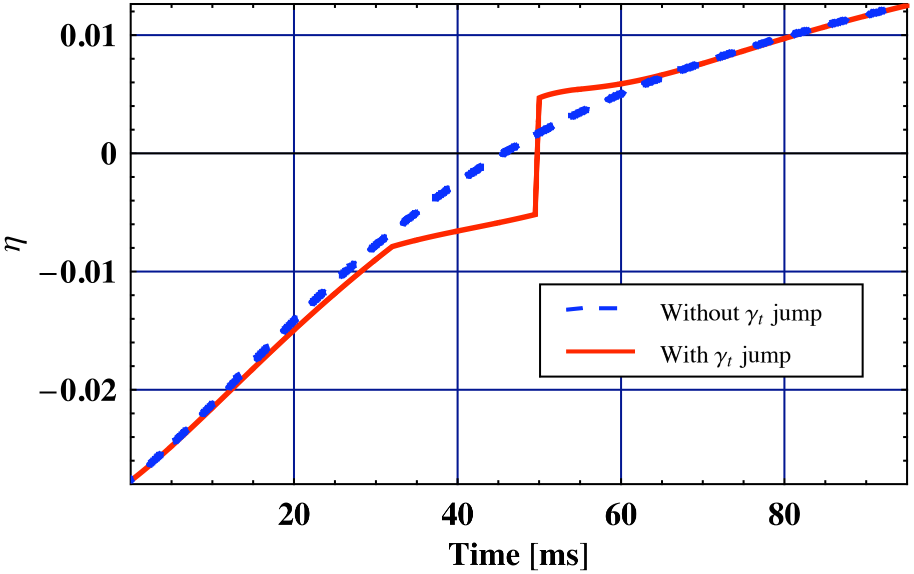
Effective crossing speed ~ 50 times faster with the γ_t jump

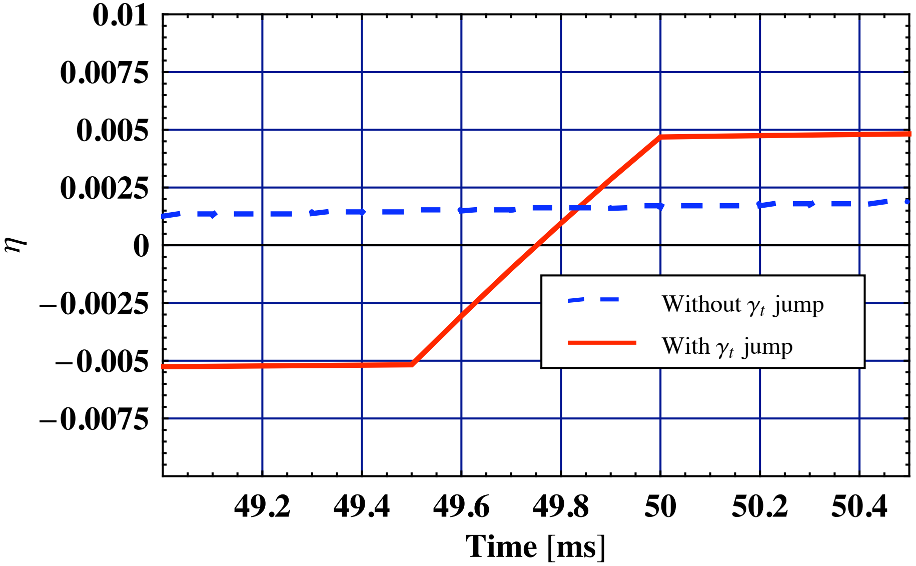
TRANSITION CROSSING (8/9)

- Asymmetric or symmetric γ_t jump?



TRANSITION CROSSING (9/9)





TRANSVERSE (SLOW) HEAD-TAIL INSTABILITY (1/2)

- If the **sign of the chromaticity** (which is equal to ~ -1 for an uncorrected machine like the PS) is not changed (in both transverse planes) above transition, a (single-bunch) head-tail instability may develop
  - This instability can be damped through Landau damping using octupoles, which introduce some amplitude detunings. This method was first used in the past to stabilize the PS beams
  - However, the better method of changing the sign of the chromaticities (and keeping them to small positive values) by acting on the optics with sextupoles was then adopted, and it has become a routine operation at the CERN PS for many years

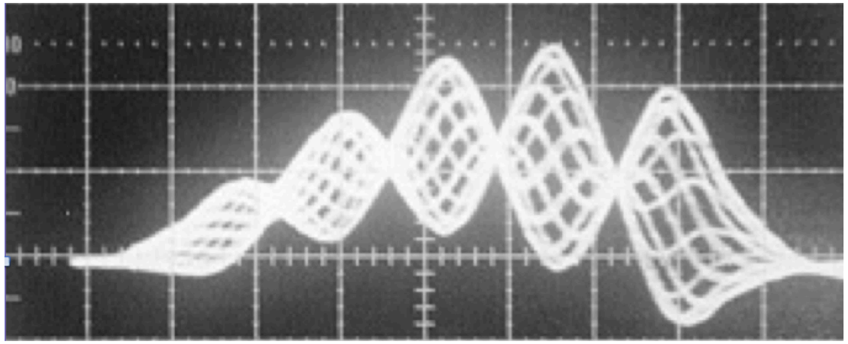
=> **The chromatic frequency should be (slightly) positive to avoid the head-tail mode 0 (most critical) from developing**

$$f_{\xi_y} = Q_y \, f_{rev} \, \frac{\boxed{\xi_y}}{\eta}$$

TRANSVERSE (SLOW) HEAD-TAIL INSTABILITY (2/2)

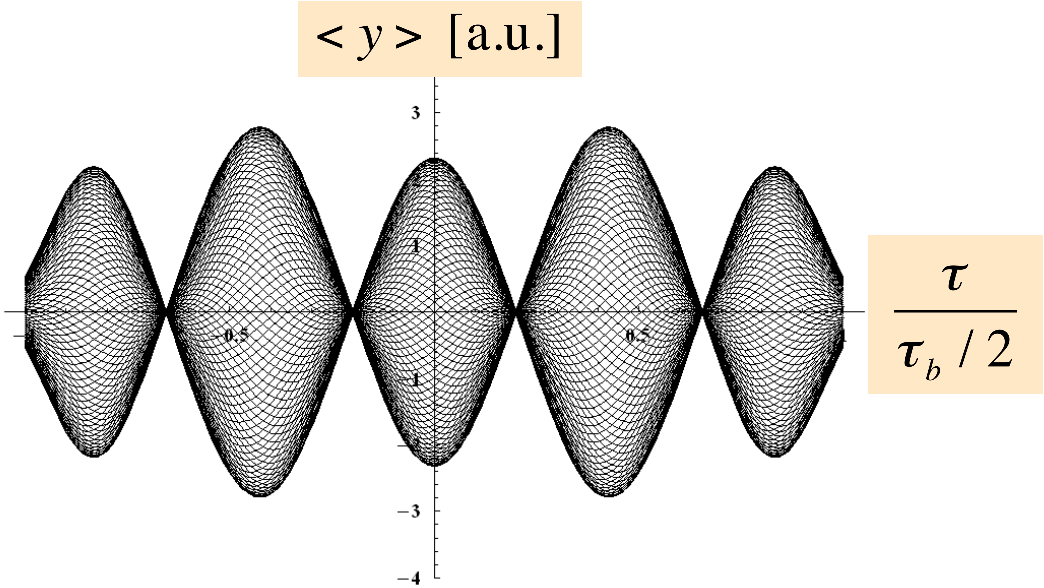
- Example of transverse (slow) head-tail instability observed in the CERN PS at injection (below transition)
  - Consecutive traces at a pick-up superimposed
  - **Standing-wave patterns with** 4 **nodes** (called “mode 4”)

Measurements



Analytical prediction

(for a bunch going through the centre of the pick-up)



FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (1/14)

- A **simple formula** is obtained from 5 seemingly diverse formalisms (in the absence of space charge and transverse feedback), **assuming i) a Broad-Band impedance and ii) the long-bunch regime**:
  - Coasting-Beam approach with peak values
  - Fast Blow-Up
  - Beam Break-Up
  - Post Head-Tail
  - Transverse Mode-Coupling Instability (TMCI)

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (2/14)

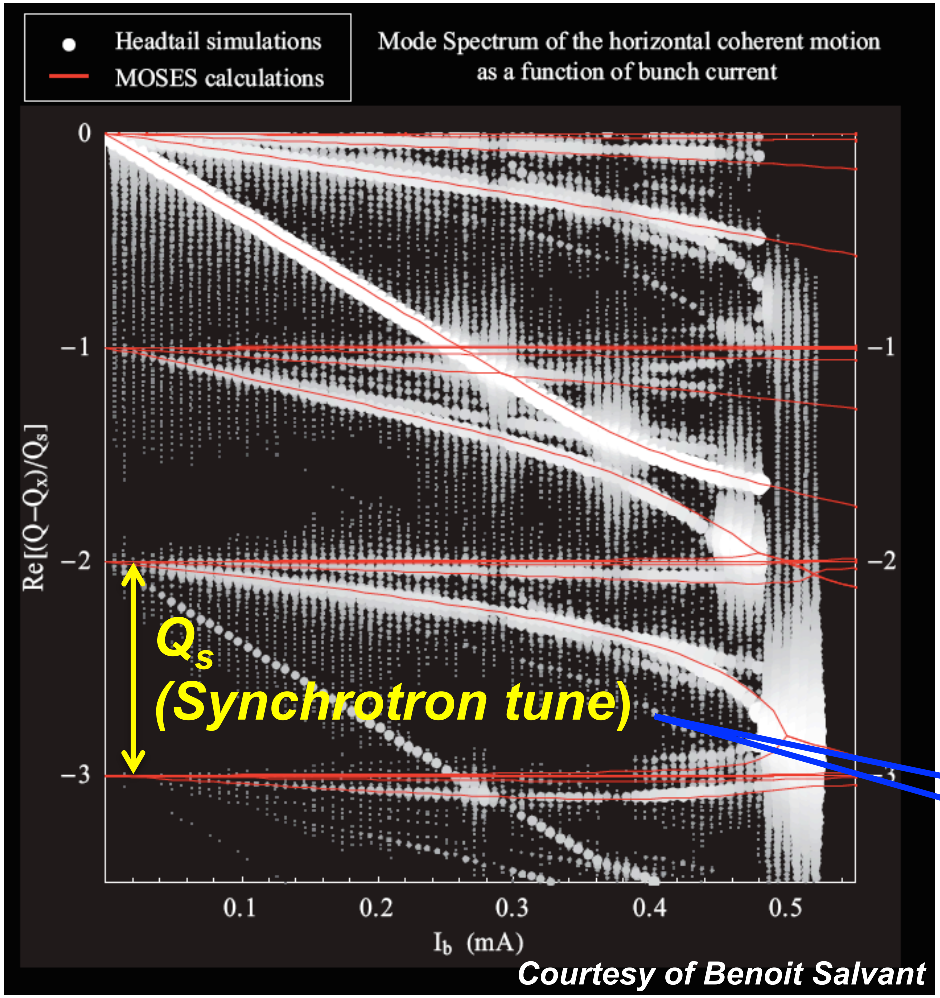
- Example of TMCI

Bunch coherent motion $\propto e^{\frac{t}{\tau_{instab}}}$

Synchrotron period

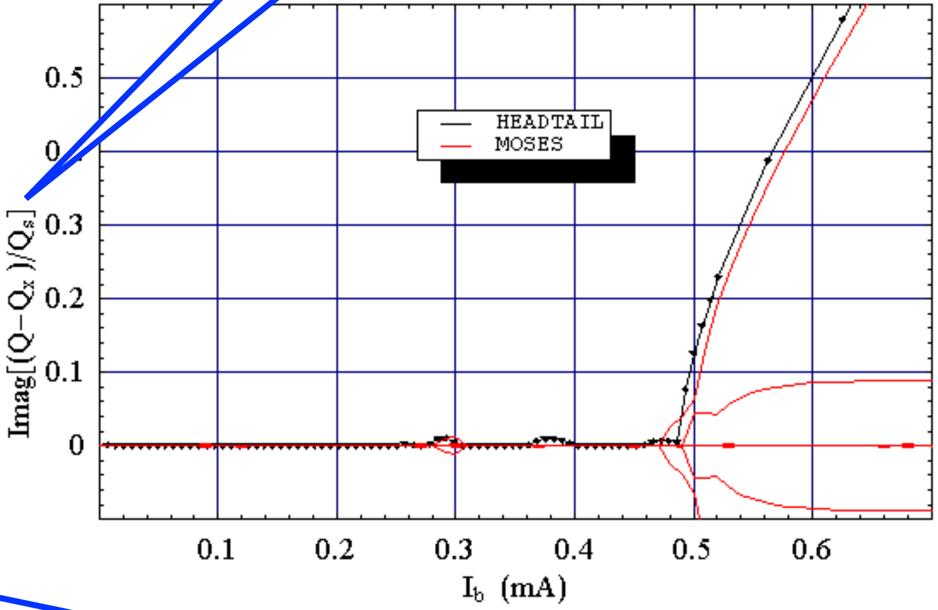
$$= \frac{T_s}{2\,\pi\,\tau_{instab}}$$

Instability rise-time



Q_s (Synchrotron tune)

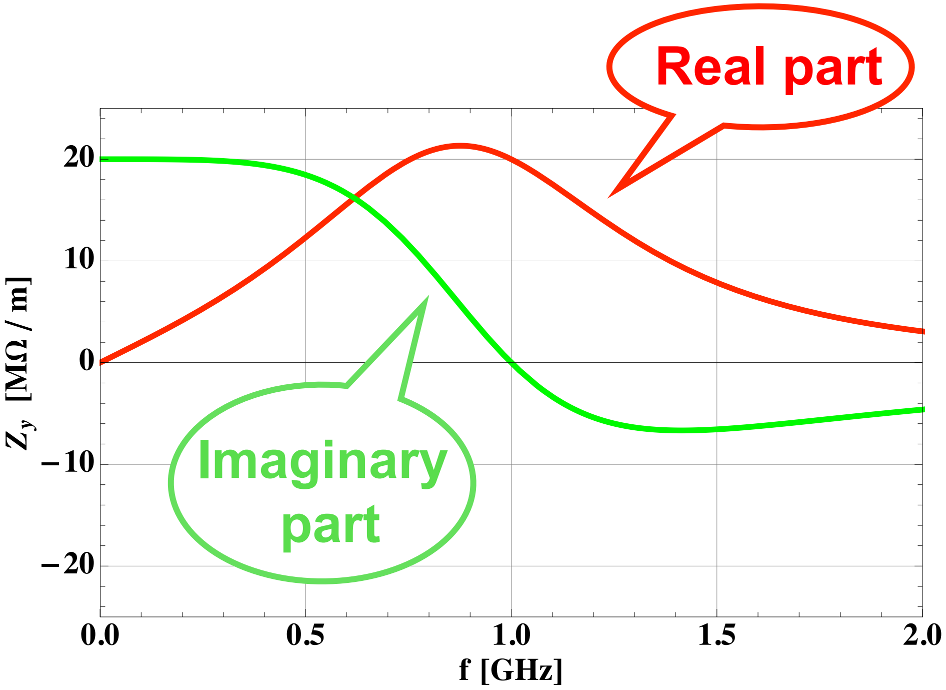
Courtesy of Benoit Salvant



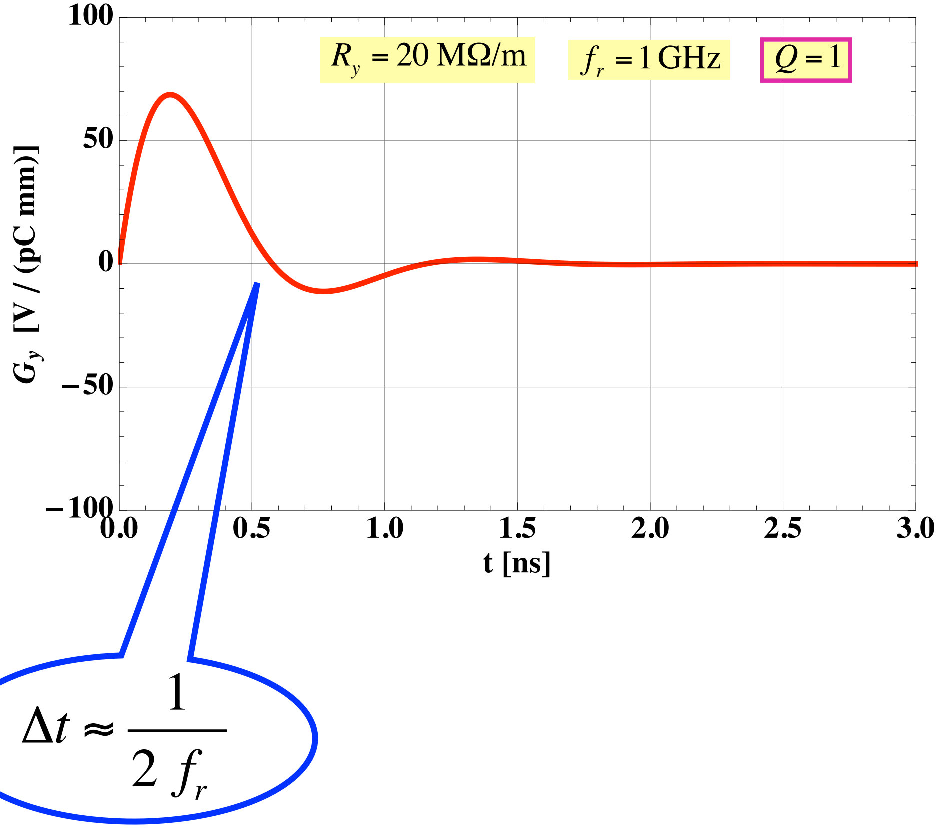
Q_s→0 approaching transition

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (3/14)

- i) 1st assumption: **Broad-Band impedance**


Real part

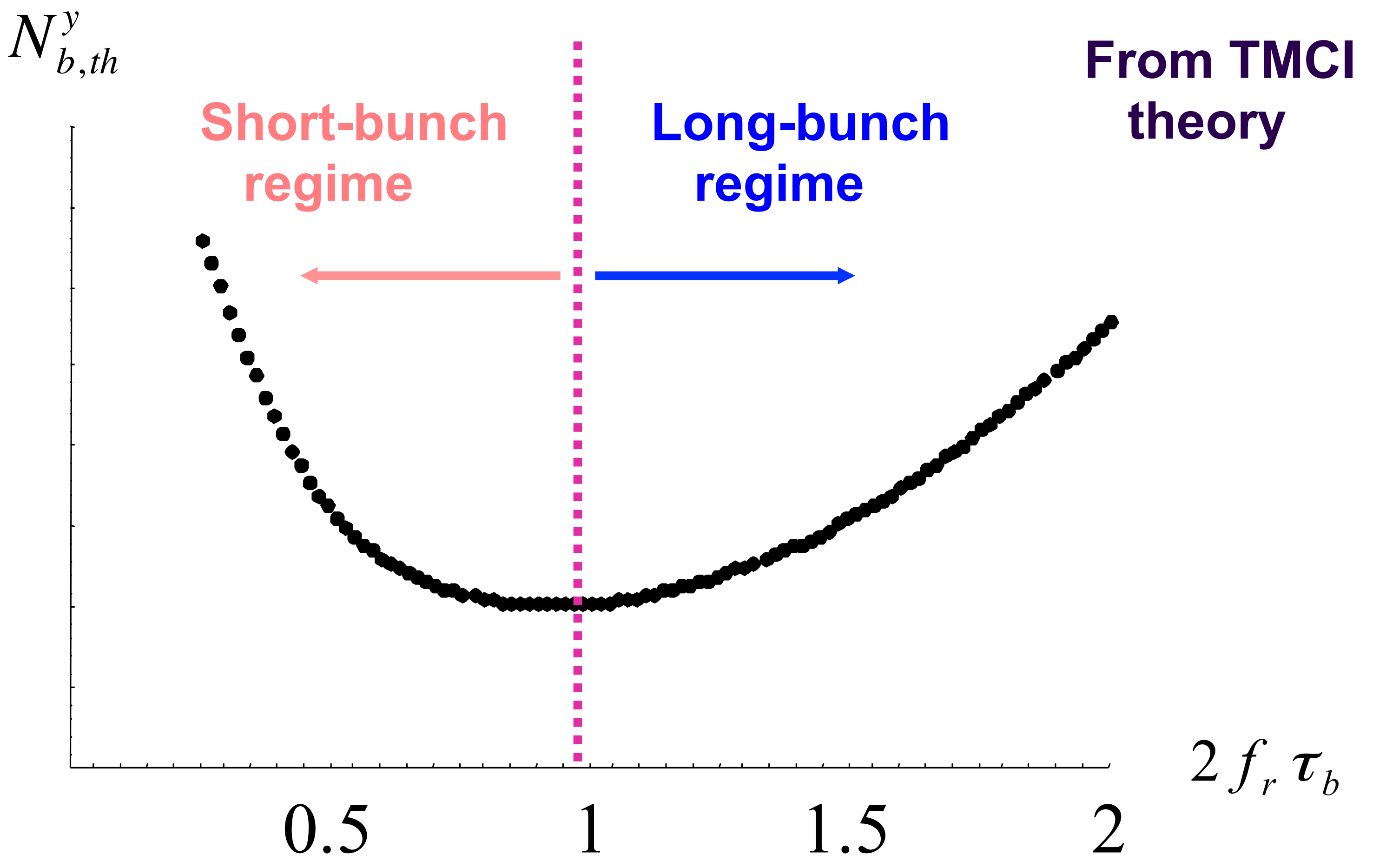
Imaginary part



$$\Delta t \approx \frac{1}{2\, f_r}$$

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (4/14)

- ii) 2nd assumption: **long-bunch regime**



FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (5/14)

- Simple formula (with the 2 assumptions):

$$N_{b,th}^{y} \propto \boxed{\frac{f_r}{\left|Z_y\right|}}\,\boxed{\left|\eta\right| Q_y}\,\boxed{\varepsilon_L}\,\boxed{\left(1+\frac{f_{\xi_y}}{f_r}\right)}$$

- Increase the chromatic frequency
- Chromaticity jump in case transition has to be crossed

Increase the beam longitudinal emittance (when possible)

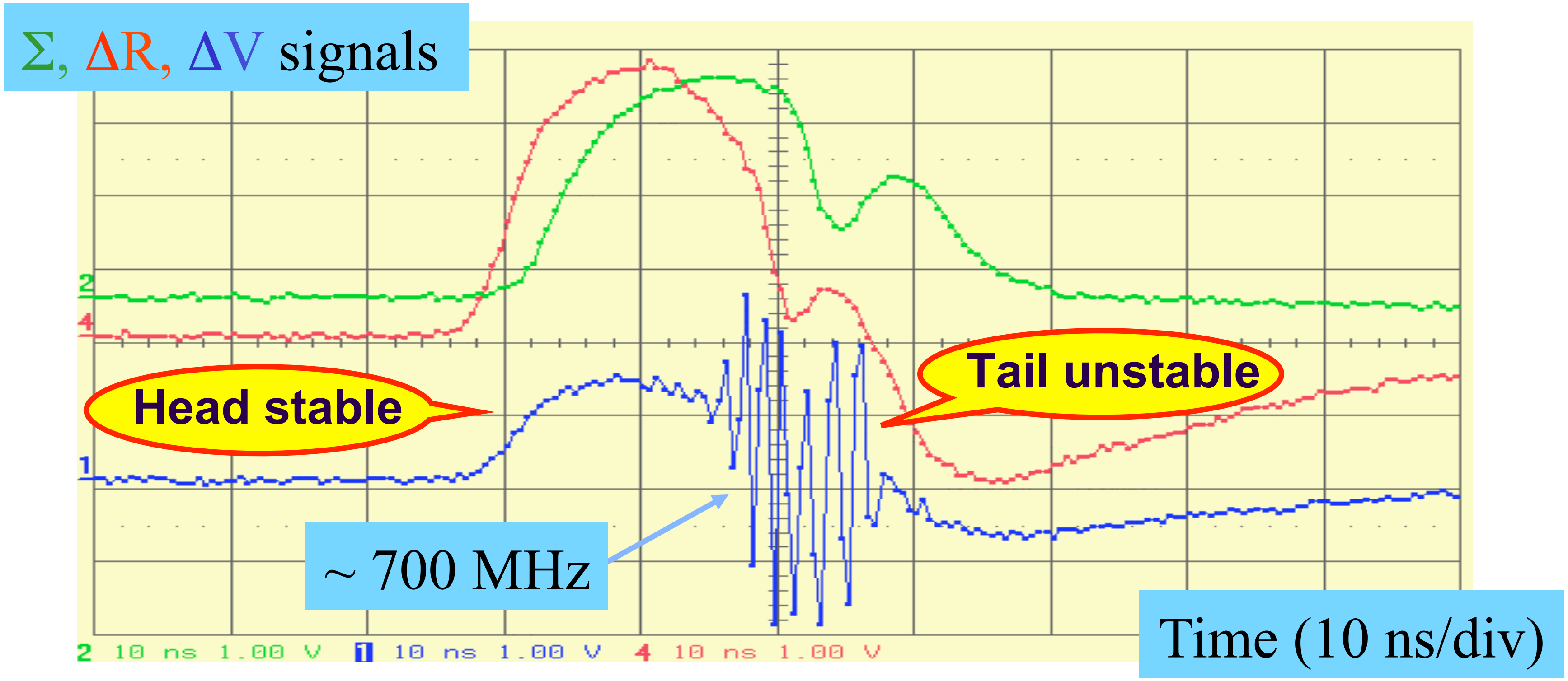
Try to decrease the impedance and/or increase the resonance frequency => Impedance reduction campaign

Change the optics to increase the betatron tune (decrease the beta function at critical impedances) and/or go further away from transition => New optics needed

* *No dependence on Q_s!*

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (6/14)

- **In the PS**: even in the presence of the γ_t jump, together with the change of the sign of both chromaticities when transition is crossed, a fast vertical single-bunch instability is observed (with the nTOF bunch) when no longitudinal emittance blow-up is applied before transition


Σ, ΔR, ΔV signals

Head stable

Tail unstable

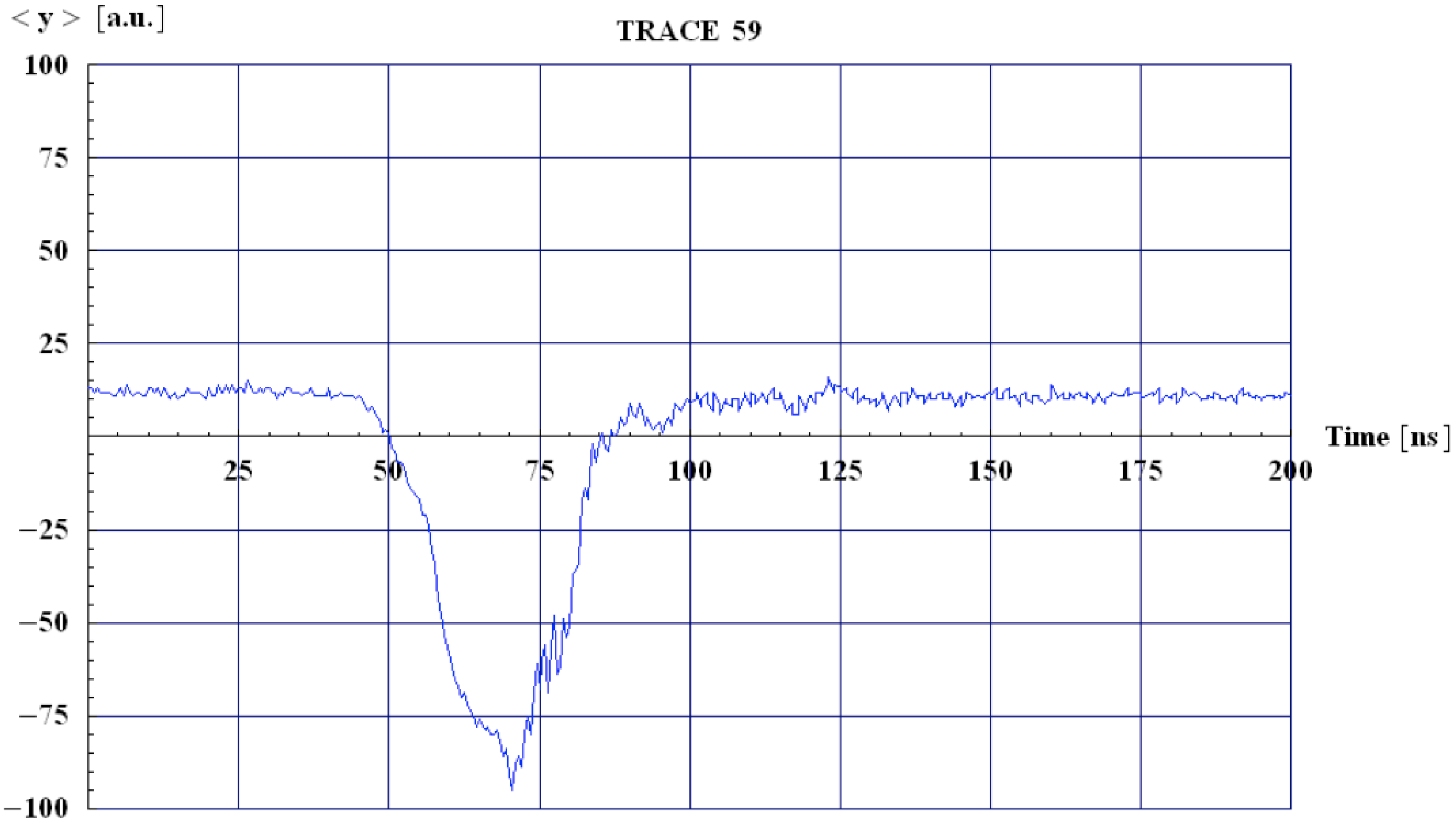
~ 700 MHz

Time (10 ns/div)

=> Instability suppressed by increasing the longitudinal emittance

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (7/14)

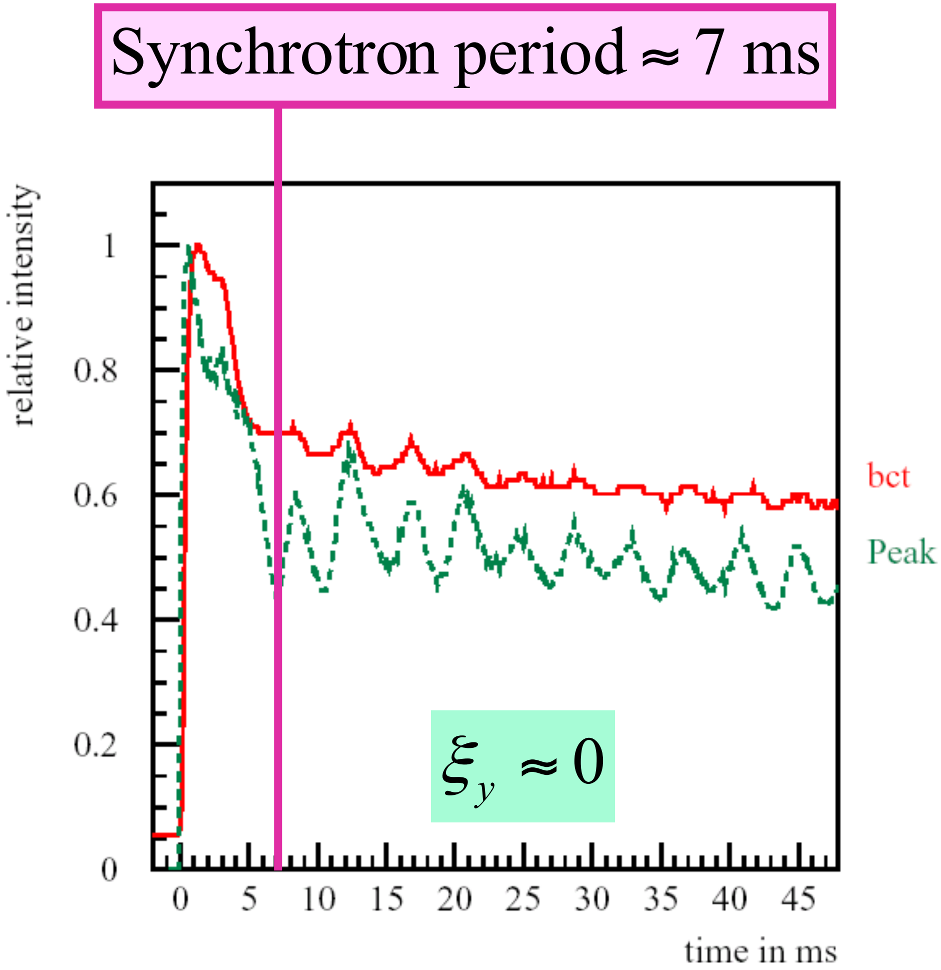
=> Similar observation on other beams (e.g. below with the beam for the Antiproton Decelerator) when the longitudinal emittance is too small



Courtesy of Rende Steerenberg

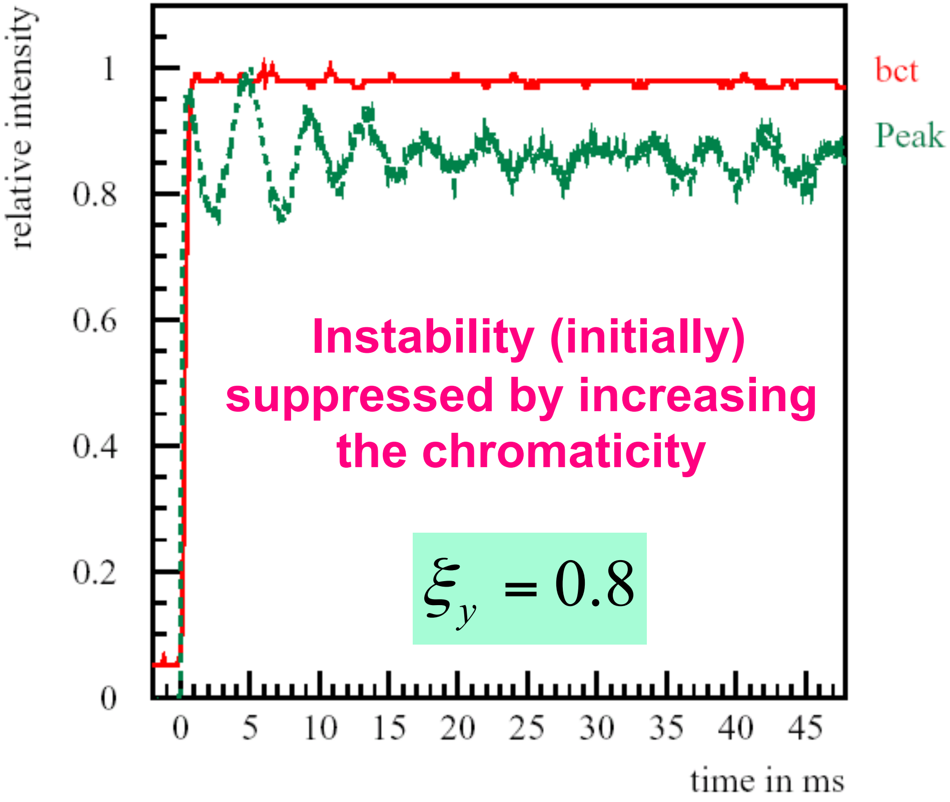
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (8/14)

- **In the SPS**



Synchrotron period ≈ 7 ms

$\xi_y \approx 0$



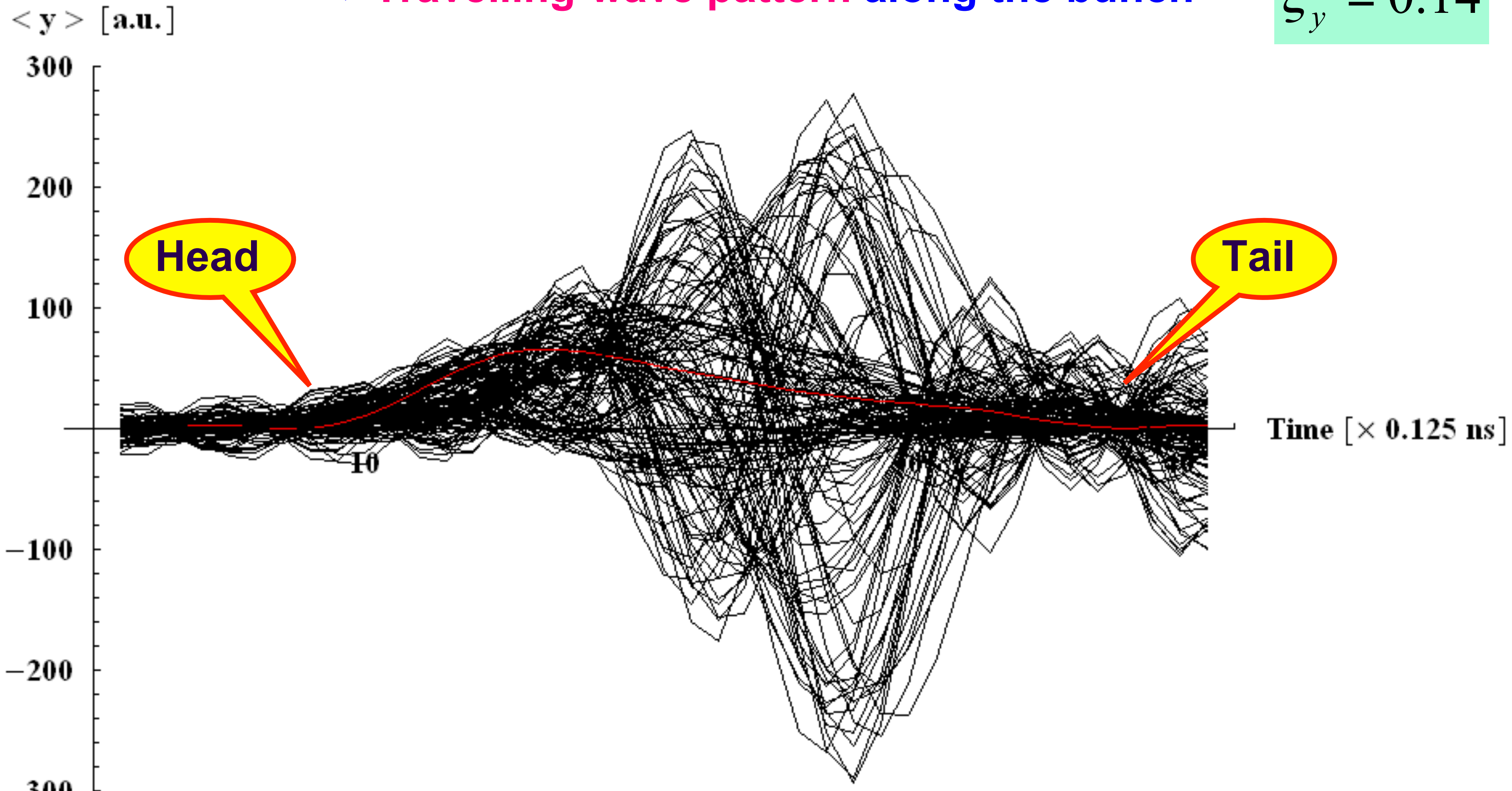
Instability (initially) suppressed by increasing the chromaticity

$\xi_y = 0.8$

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (9/14)

⇒ Travelling-wave pattern along the bunch

$\xi_y = 0.14$



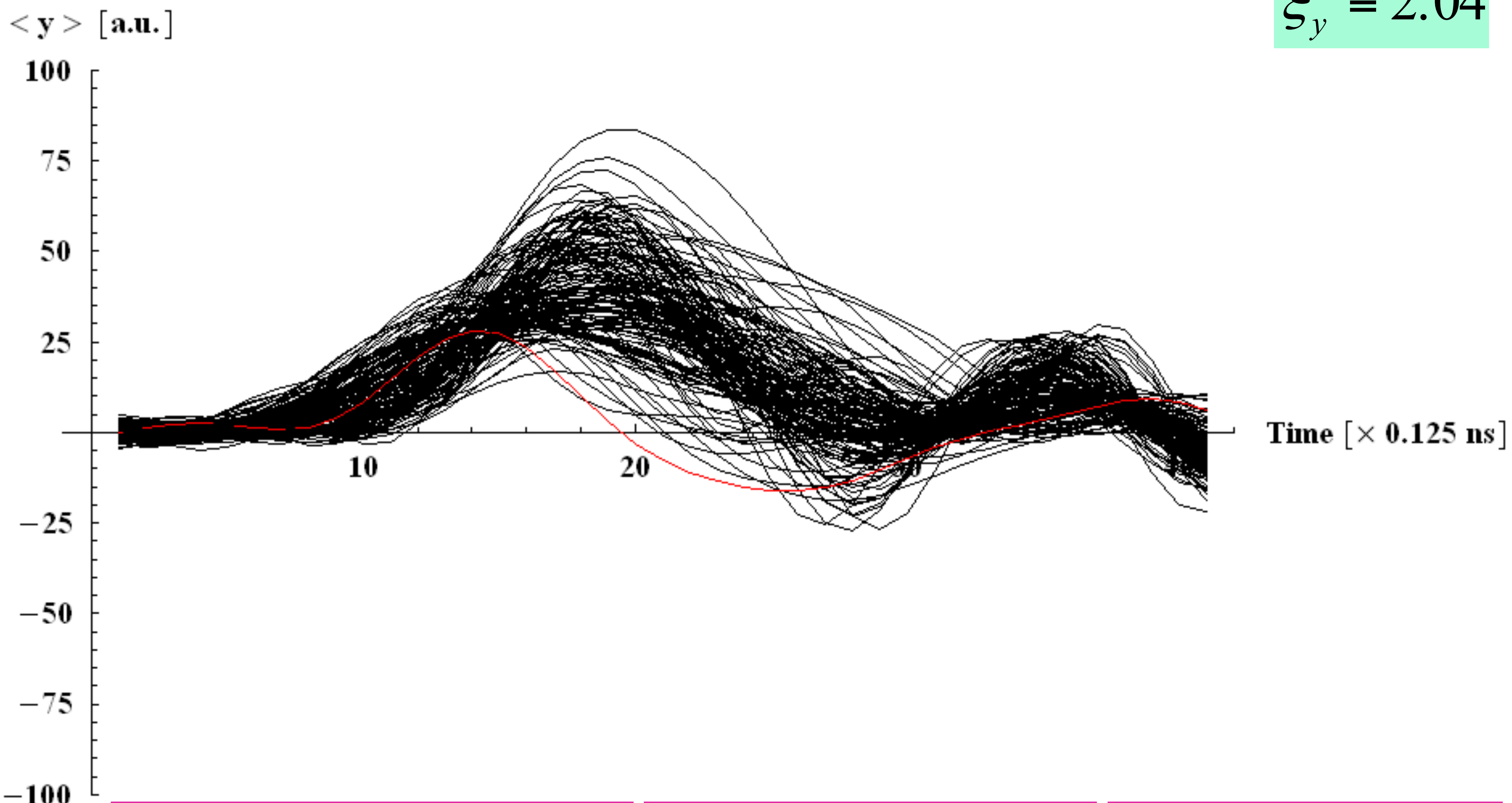
1st trace (in red) = turn 2

Last trace = turn 150

Every turn shown

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (10/14)

$\xi_y = 2.04$



1st trace (in red) = turn 2

Last trace = turn 150

Every turn shown

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (11/14)

- **γ_t was recently modified in the SPS to increase the TMCI intensity threshold** above the foreseen intensities for the future upgrade

- Simple rough estimate of γ_t for machines made of simple FODO cells:
  - Approximating the machine radius by the bending radius, yields

$$D_x \approx \frac{\rho}{Q_x^2}$$

  - Inserting this in the definition of α_p (and then expressing γ_t) yields

$$\gamma_t \approx Q_x$$

=> If one wants to modify γ_t, (increase or decrease its value) one should modify the horizontal tune

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (12/14)

- TMCI intensity threshold with the old (Q26) optics at injection: **~ 1.7 10^{11} p/b**

- Predictions going from Q26 to the new (Q20) optics:

  - **Q26:** $|\eta| Q_y = 0.62\,10^{-3} \times 26.13 \approx 0.0162$ $\gamma_t = 22.8$
  - **Q20:** $|\eta| Q_y = 1.80\,10^{-3} \times 20.13 \approx 0.0362$ $\gamma_t = 18$

=> A **gain of a factor** 0.0362 / 0.0162 **≈ 2.2** in the intensity threshold was expected

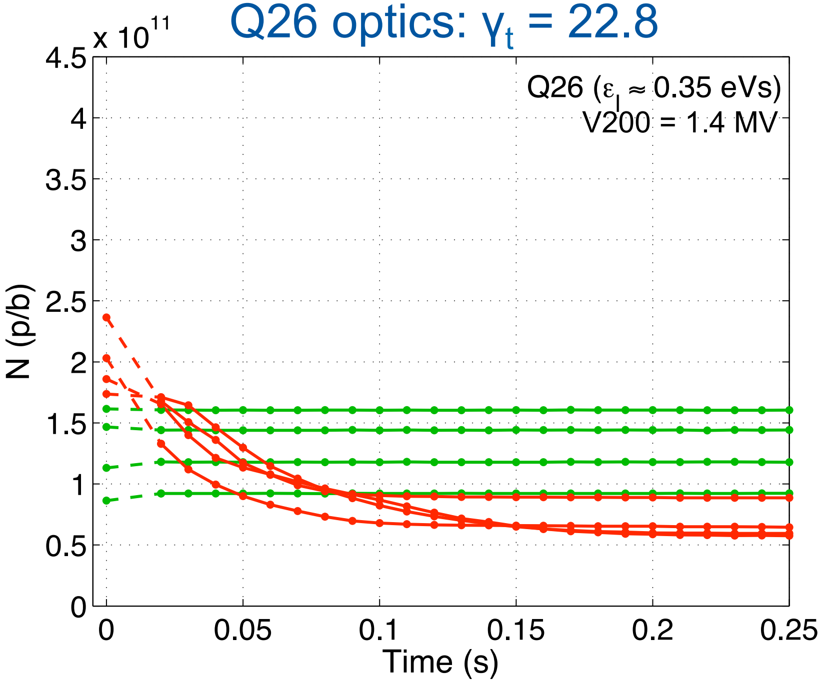
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (13/14)

- Measurements

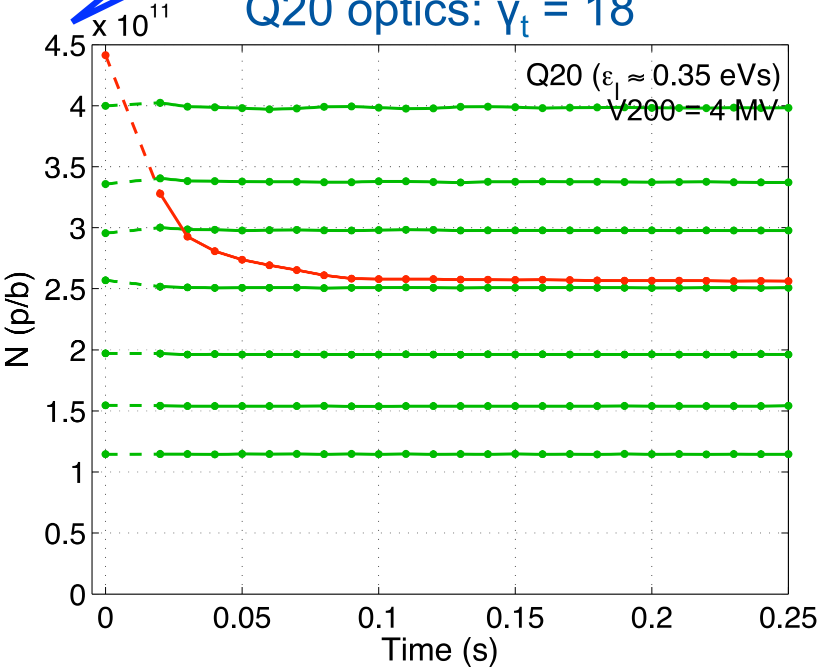
=> Good agreement with simple formula

Gain of a factor
4.5 / 1.7 ≈ **2.6**

Q26 optics: $\gamma_t = 22.8$



Q20 optics: $\gamma_t = 18$

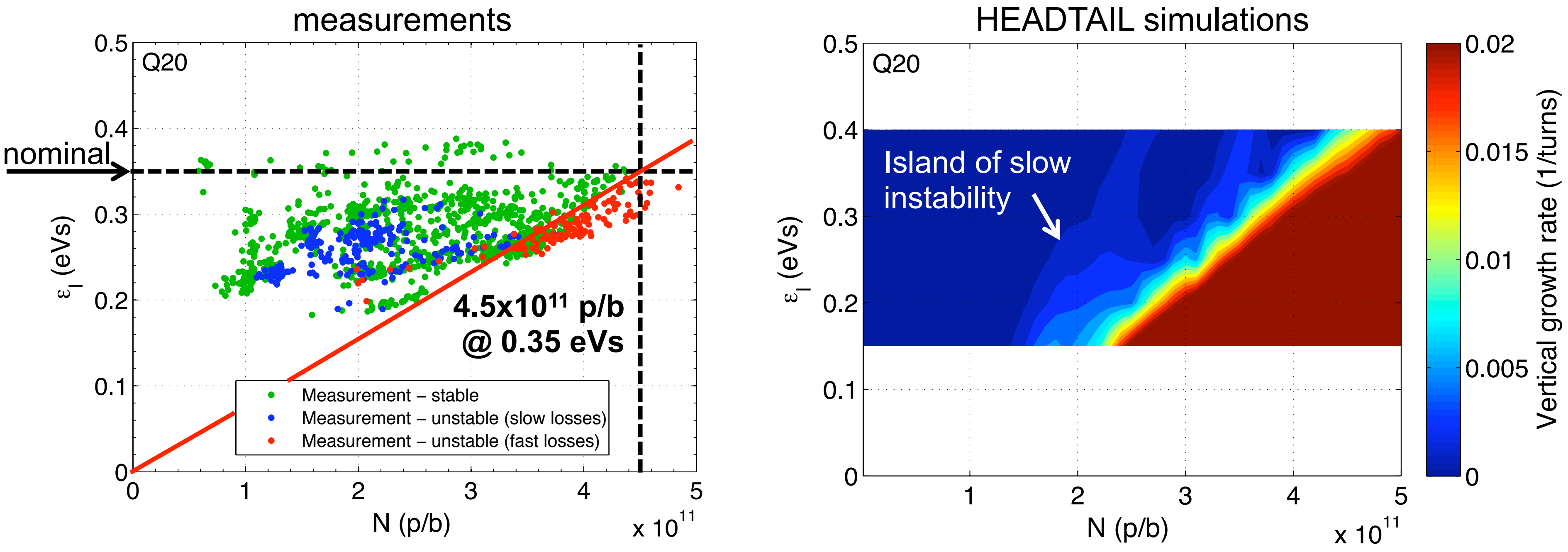


Courtesy of Benoit Salvant et al.

Courtesy of Hannes Bartosik et al.

FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (14/14)

- Very good agreement between measurements and simulations



Courtesy of Hannes Bartosik et al.

=> Intensity threshold with the new (Q20) optics: **~ 4.5 10^{11} p/b**

CONCLUSION (1/2)

- Stability of synchrotron motion (when crossing transition): $\boxed{\phi_s \Rightarrow \pi - \phi_s}$

- Head-Tail instability (when crossing transition): change the sign of the chromaticity (both planes) => Positive chromatic frequency

$$\boxed{f_{\xi_y} = Q_y \, f_{rev} \, \frac{\xi_y}{\eta} > 0}$$

- Transverse Mode-Coupling Instability => With the 2 assumptions: i) Broad-Band impedance and ii) long-bunch regime *(effects of space charge and transverse feedback still under discussion…)*

  - **Impedance reduction**
  - **Increase longitudinal emittance** (as in the PS)
  - **Increase the |slip factor|** (as in the SPS) **and/or the tune**
  - **Increase the chromatic frequency** (below or above transition)
  - **"Chromaticity jump"** when transition needs to be crossed => Not only the sign needs to be changed (for Head-Tail reason) but the shape could be optimised (for TMCI reason)

CONCLUSION (2/2)

- Increasing the |slip factor| also helps for i) the **Longitudinal Mode-Coupling Instability** and ii) the **fast single-bunch electron cloud instability**

- Attractive operation of synchrotrons under an isochronous or quasi-isochronous condition to **(naturally) achieve very short bunches**

  => Requires

  - An accurate control of the first high-order component of the momentum compaction factor to provide the necessary momentum acceptance

$$C(\delta) = C_0 \left[1 + \alpha_0 \, \delta \left(1 + \alpha_1 \, \delta + \alpha_2 \, \delta^2 + \ldots \right) \right] \qquad \delta = \Delta p / p$$

  - Effective ways to damp all the collective instabilities

REFERENCES

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- Detailed studies for the CERN PS => Sandra Aumon, High Intensity Beam Issues in the CERN PS, CERN-THESIS-2012-261 (http://cds.cern.ch/record/1517412/files/CERN-THESIS-2012-261_2.pdf).
  Supervisor: Simone Gilardoni

- Detailed studies for the CERN SPS => Hannes Bartosik, Beam Dynamics and Optics Studies for the LHC Injectors Upgrade, CERN-THESIS-2013-25 (http://cds.cern.ch/record/1644761/files/CERN-THESIS-2013-257.pdf).
  Supervisor: Yannis Papaphilippou