

# TUTORIALS for the course of “Collective Effects in Beam Dynamics” (EM) – 1<sup>st</sup> day

## SPACE CHARGE

1) How would the transverse force of page 11 be modified in the case of a particle moving with the same velocity but in opposite direction as the bunch? What would be its value when  $\gamma \rightarrow \infty$ , compared to the case of the space charge interaction (i.e. for a particle moving in the same direction as the bunch)?

2) Compute numerically the value of the maximum incoherent space charge tune shift using the formula of page 15 for the case of a LHC-type bunch at the entrance of the SPS:

- Radius of the SPS machine:  $R = 1100 \text{ m}$ .
- Number of protons in the bunch:  $N_b = 1.3 \times 10^{11}$ .
- Beam momentum:  $p = 26 \text{ GeV}/c$ .
- Normalized rms beam emittance:  $\epsilon_{x,rms}^{norm} = \epsilon_{y,rms}^{norm} = 3 \mu\text{m}$ .
- Full ( $4\sigma$ ) bunch length:  $\tau_b = 4 \text{ ns}$ .

Knowing that in synchrotrons, one usually try to run with a maximum incoherent space charge tune shift of  $\sim -0.3$  to avoid overlapping with too many resonances (inducing emittance blow-ups and/or beam losses), do you think that space charge is a limitation in the SPS?

3) Derive the equation of the transverse space charge tune shift of page 17 and check that it is indeed the same result as in page 15.

4) Derive the nonlinear space charge tune shift of page 27 starting from the general equations of page 18 for the electric fields. Check that the maximum transverse space charge tune shift is 80% of the one for a bi-Gaussian bunch.

## ENVELOPE EQUATIONS

1) Make the full derivation of the 1-dimensional transverse beam envelope equation of page 7.

2) Check that the transverse tune shift deduced from the 1-dimensional envelope equation of page 9 gives the same result (with a factor 1/2) as the one obtained in the “space charge” course with  $a = \sqrt{2} \sigma_x$ .

3) Derive the expression of the nonadiabatic time  $T_c$  of page 25, starting from the equations of page 22.

## WAKE FIELDS AND IMPEDANCES

- 1) Show that the 3 relations of page 18 are true.
- 2) Show that the 2 definitions of the transverse impedance given on page 21 give the same result as the one of page 20 for the case of the transverse space charge impedance.
- 3) Derive the last equation of page 23, giving the transverse impedance of a ring-shaped beam expressed with the longitudinal component of the electric field.
- 4) If a first resonance is observed near  $\sim 200$  MHz, what is the approximate radius of the object which could generate it?
- 5) Derive the general relations between the transverse components of the electromagnetic fields and the longitudinal ones in a source-free region (page 54).
- 6) Derive the longitudinal source terms of page 56, remembering that the longitudinal part of the electric field must be continuous at  $r = a$ .

Hints:

a) Integrate from  $r = a - \delta$  to  $r = a + \delta$ , where  $\delta$  is vanishingly small, to determine the constant value.

b) Use the relation  $I_1'(x) K_1(x) - I_1(x) K_1'(x) = \frac{1}{x}$ .