PBL (Problem-Based Learning) scenario for Accelerator Physics

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As each working day, since the beginning of the year 2008 (first year of the LHC operation), Mats and Elias are sitting in the CERN Control Centre at the PS[1] control desk. Since several hours the proton beam has been sent to the SPS in preparation for the next LHC fill. Everything is quite, only very small beam losses are observed along the injector chain and the beam has the required quality.

Answering a request from the LHC operators, who want to double the peak luminosity of the previous fill, the intensity is increased in the PSBooster. Mats and Elias are now on the alert, as they immediately note that the intensity transmission in the PS is considerably degraded. By tuning the working point at injection energy they succeed to reduce the beam losses, but not completely, and they still have to check if the transverse emittances remain within the specifications.

Suddenly, a SPS operator comes, complaining about the too large longitudinal beam emittance injected into the SPS. Elias decides to follow-up the previous problem, while Mats is checking both the magnetic field at injection and the Radio-Frequency (RF) peak voltage, whose values indicate 1020 Gauss and 60 kV respectively. Mats realises that one of these two values is definitively not the good one…

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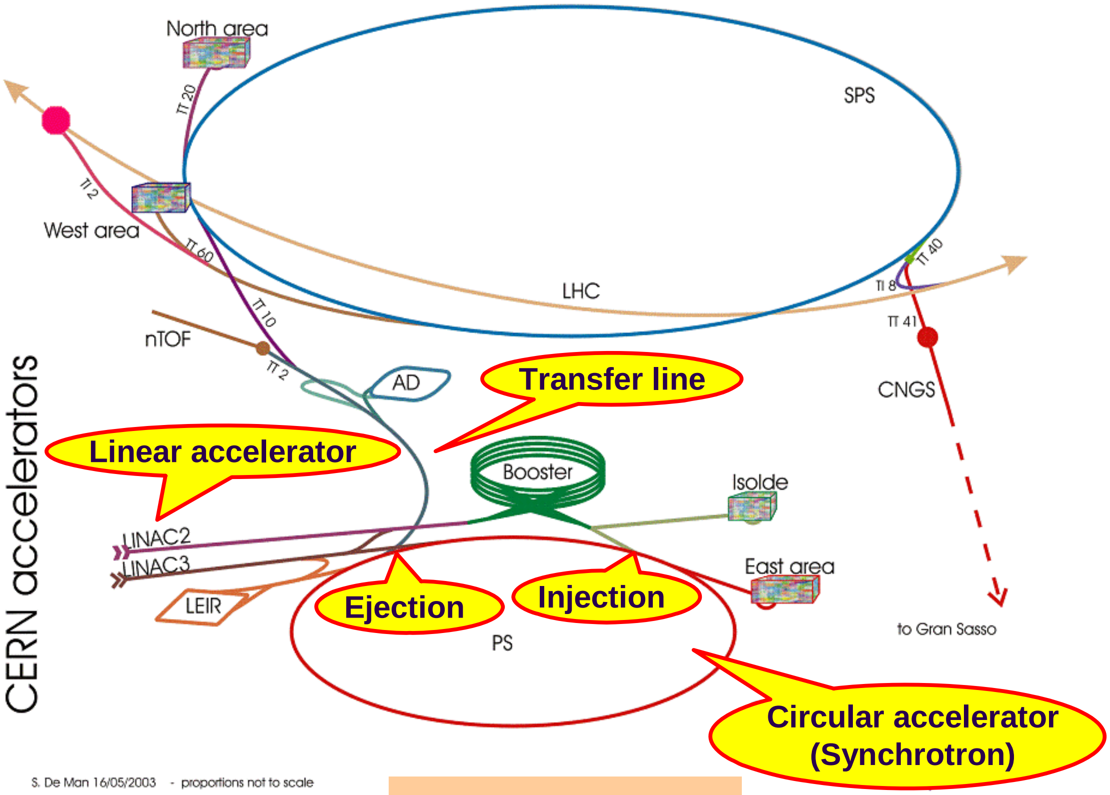
[1] Chain of CERN particle accelerators followed by the proton beam:

LINAC2 = Linear accelerator (from the duoplasmatron source) ⟹ 50 MeV (kinetic energy).
PSBooster = Proton Synchrotron Booster ⟹ 1.4 GeV.
PS = Proton Synchrotron ⟹ 25 GeV.
SPS = Super Proton Synchrotron ⟹ 450 GeV.
LHC = Large Hadron Collider ⟹ 7 TeV.



Transfer line

Linear accelerator

Ejection

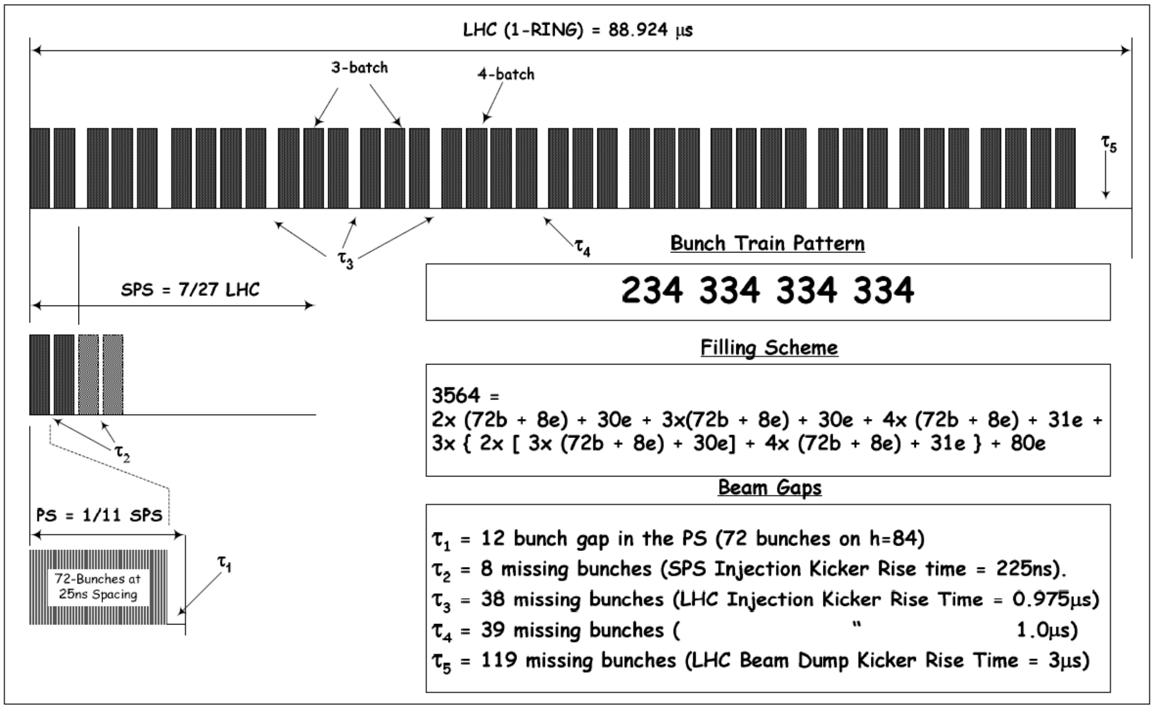
Injection

Circular accelerator (Synchrotron)

S. De Man 16/05/2003 - proportions not to scale

LHC beam

Filling scheme for the nominal LHC beam



Bunch Train Pattern

234 334 334 334

Filling Scheme

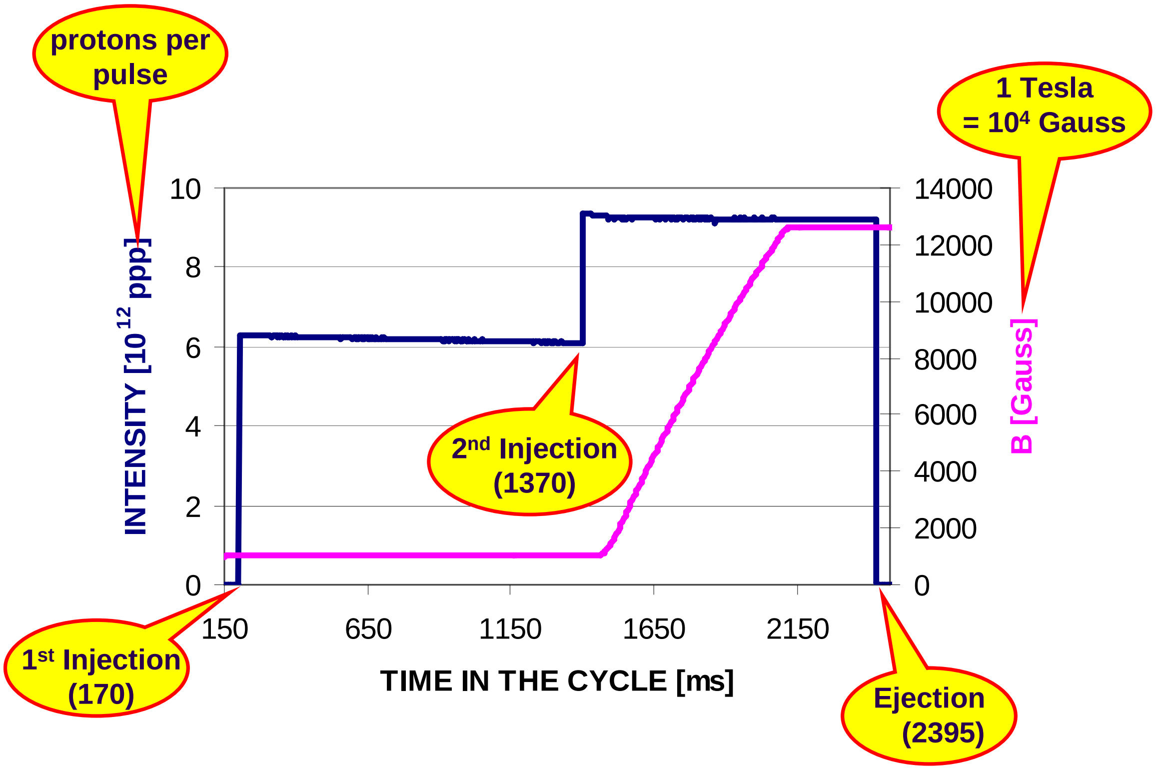
3564 =
2x (72b + 8e) + 30e + 3x(72b + 8e) + 30e + 4x (72b + 8e) + 31e +
3x { 2x [3x (72b + 8e) + 30e] + 4x (72b + 8e) + 31e } + 80e

Beam Gaps

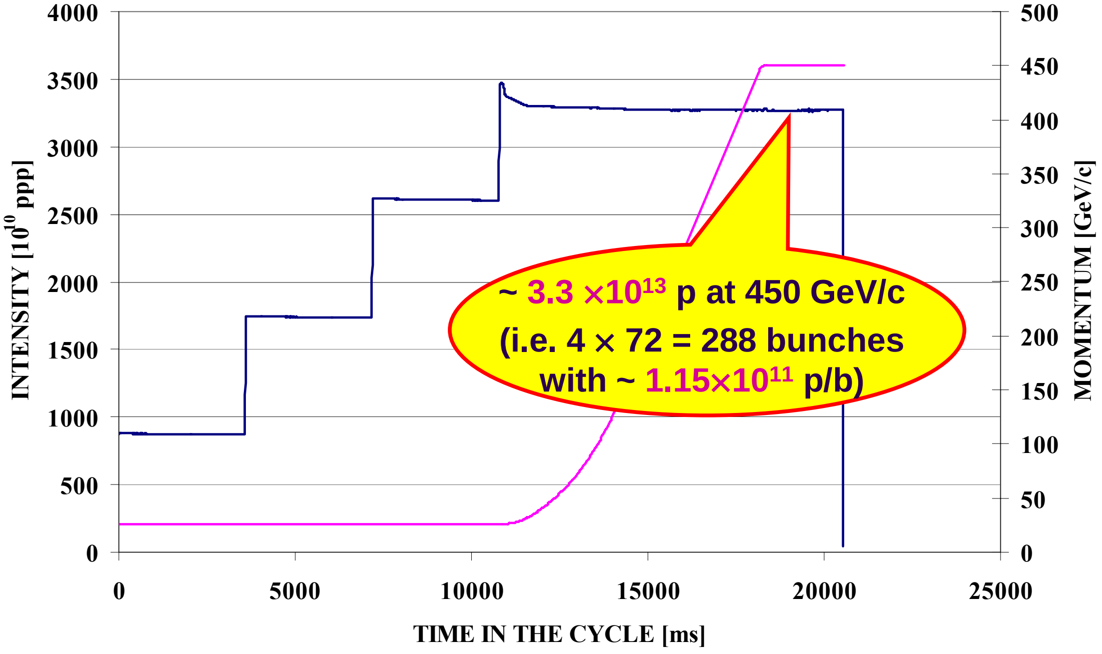
τ_1 = 12 bunch gap in the PS (72 bunches on h=84)
τ_2 = 8 missing bunches (SPS Injection Kicker Rise time = 225ns).
τ_3 = 38 missing bunches (LHC Injection Kicker Rise Time = 0.975μs)
τ_4 = 39 missing bunches (" 1.0μs)
τ_5 = 119 missing bunches (LHC Beam Dump Kicker Rise Time = 3μs)

- PS cycle length = 3.6 s.
- SPS cycle length = 21.6 s.
- LHC filling time (for the 2 rings) = 8 min 38 s (= 12 SPS cycles of 21.6 s per beam ⟹ 24 in total, i.e. a filling time of 24 × 21.6 s = 518.4 s).

PS CYCLE



SPS CYCLE



Explanations on the PBL scenario

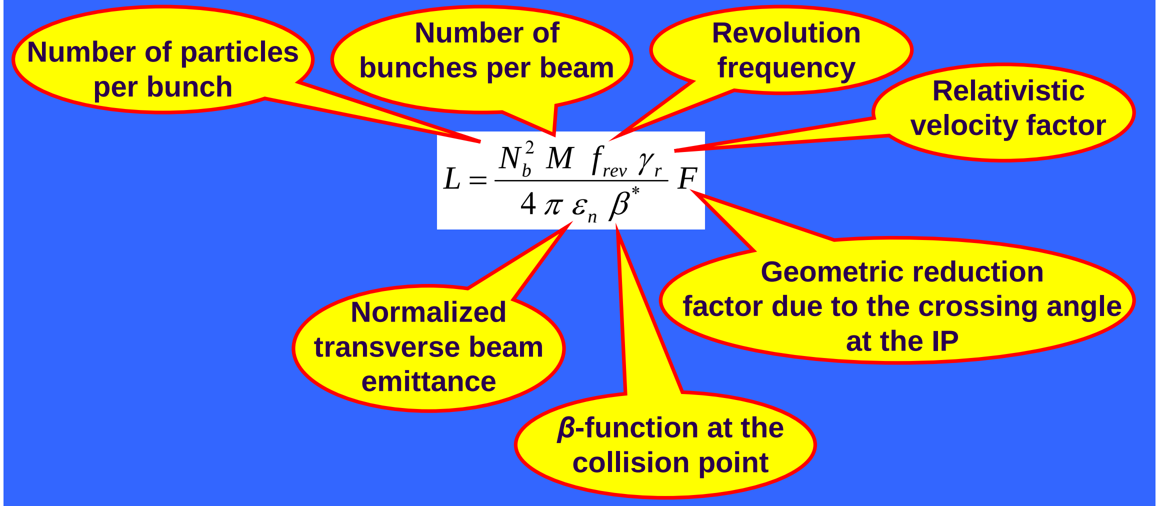
The main aim of this PBL scenario is to introduce the students to the basic concepts of transverse and longitudinal beam dynamics in particle accelerators called synchrotrons. After having understood the fundamental notion of machine (here LHC) luminosity, the students will have to look at the beam parameters required to achieve this luminosity, learning therefore the orders of magnitude. In particular they will have to understand the motion of the particles in both transverse and longitudinal planes, and how these particles can be manipulated.

This scenario is a vivid description of the daily stint of the people working in the CERN Control Centre. The first paragraph sets the scene. The second paragraph deals with the motion of the particles in the transverse planes. No numerical result is required to solve the problem. Instead, explanations of the different mechanisms which can lead to these observations should be given. Finally, the longitudinal beam dynamics is discussed in the third paragraph. Here, to solve completely the problem, the students will have to make some computations to see which of the two figures given is wrong and what the correct value should be.

I summarized below the basics concepts for the machine luminosity, the transverse beam dynamics, the longitudinal beam dynamics and the collective effects.

1) LUMINOSITY

⇒ For a Gaussian (round) beam distribution



$$L = \frac{N_b^2 \, M \, f_{rev} \, \gamma_r}{4 \, \pi \, \varepsilon_n \, \beta^*} F$$

◆ PEAK LUMINOSITY for ATLAS&CMS in the LHC = $L_{peak} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

◆ INTEGRATED LUMINOSITY $L_{\text{int}} = \int_0^T L(t)\, dt$

⇒ The real figure of merit = $L_{\text{int}} \, \sigma_{event} = \text{number of events}$

◆ LHC integrated Luminosity expected per year: [80-120] fb^{-1}

Reminder: 1 barn = 10^{-24} cm^2
and femto = 10^{-15}

- Understand the fundamental notion of **machine luminosity**, which is the number of events per second generated in the collisions divided by the cross-section for the event under study:
    - The Luminosity depends only on the beam parameters ⟹ It is independent of the physical reaction.
    - Reliable procedures to compute and measure.
- Understand that the physicists want to maximize the number of events per year, i.e. the challenge for the accelerator physicists is to maximize the integrated machine luminosity.
- Understand how the machine luminosity depends on the beam parameters, in particular it goes with
    - The number of particles per bunch square (⟹ What is a bunch? ⟹ Longitudinal beam dynamics),
    - The number of bunches per beam (There are 2 colliding beams in the LHC ⟹ Where? How?),
    - The beam energy,
    - The revolution frequency of the particles,
    - The inverse of the normalized transverse beam emittance (⟹ What is the normalized transverse emittance of a bunch? ⟹ Transverse beam dynamics),
    - The inverse of the β-function at the collision point (⟹ What is the β-function? ⟹ Transverse beam dynamics).
- This will then bring the students to the basic concepts of longitudinal (see Section 2) and transverse (see Section 3) beam dynamics, which should be understood. The collective effects could be briefly mentioned (see Section 4).

2) INTRODUCTION TO TRANSVERSE BEAM DYNAMICS

- **Design orbit** (in the centre of the vacuum chamber).
- **Lorentz force** (from magnetic fields) ⟹ The motion of particle beams under the influence of the Lorentz force is called **Beam Optics**.
- **Lattice** = Arrangement of magnets along the design orbit.
- **Dipoles** (constant force) ⟹ **Guide** the particles along the design orbit.
- **Quadrupoles** (linear force) ⟹ **Confine** the particles in the vicinity of the design orbit.
- **Beam rigidity** (relation between magnetic field, curvature radius of the dipoles and the beam momentum).
- **Betatron oscillation** in x (and in y) ⟹ **Betatron tune** $\boldsymbol{Q_x}$ (and $\boldsymbol{Q_y}$) = Number of betatron oscillations per machine revolution (>> 1).
- **Twiss parameters** (α, β, γ) define the **ellipse in phase space** (x, $x' = \mathrm{d}\,x / \mathrm{d}\,s$) at azimuthal position s.
- **β-function** reflects the size of the beam and depends only on the lattice.
- Matrix Formalism ⟹ Transfer and Twiss matrices.
- **Physical transverse beam emittance** = Measure of the spread in phase space of the points representing beam particles ⟹ Must be smaller than the mechanical acceptance.
- **Normalized transverse beam emittance** ⟹ The normalized emittance is conserved during acceleration (in the absence of collective effects…), and the physical emittance decreases inversely proportionally to the beam momentum.

- **Higher order multipoles** from imperfections (nonlinear force) $\Longrightarrow$ **Resonances** excited in the tune diagram and the working point (Q_x, Q_y) should not be close to most of the resonances to avoid emittance blow-up or particle loss.
- Nonlinearities reduce the acceptance $\Longrightarrow$ **Dynamic aperture** (Largest oscillation amplitude which is stable in the presence of nonlinearities).

3) INTRODUCTION TO LONGITUDINAL BEAM DYNAMICS

- **RF cavities** are used to accelerate (or decelerate) the particles.
- **Transition** energy and sinusoidal voltage $\Longrightarrow$ Use of the **Lorentz force** from electric fields.
- **Harmonic number** = Number of RF **buckets** (stationary or accelerating).
- **Bunched beam** (instead of an unbunched or continuous beam).
- **Synchrotron oscillation** around the synchronous particle in the longitudinal direction $\Longrightarrow$ **Synchrotron tune Q_z** = Number of synchrotron oscillations per machine revolution (<< 1).
- **Stable phase** ϕ_s below transition and $\pi - \phi_s$ above transition.
- **Ellipse in phase space** (Δt, ΔE).
- **Longitudinal beam emittance** must be smaller than the **bucket acceptance**.

4) COLLECTIVE EFFECTS

- As the beam intensity increases, the beam cannot be considered as a collection of noninteracting single particles anymore. In addition to the single-particle phenomena, collective effects become important:
    - (Direct) **space charge** = Interaction between the particles (without the vacuum chamber) $\Longrightarrow$ Coulomb repulsion + magnetic attraction.
    - **Wake field** = Electromagnetic field generated by the beam interacting with its surroundings (vacuum pipe, etc.) $\Longrightarrow$ **Impedance** = Fourier transform of the wake field.
    - **Beam-Beam** = Interaction between the 2 counter-rotating beams $\Longrightarrow$ Coulomb repulsion + magnetic repulsion.
    - **Electron cloud**.

References

Many papers and books have been published on accelerator physics during the last forty years. Instead of giving you an impossible exhaustive list, I would suggest the following references. The first and main reference for this course is the book from E. Wilson (former Head of the CERN Accelerator School). In Ref. [2], you can find the proceedings of all the CERN accelerator schools, which are given each year in a different European town. As it might be difficult to choose which papers to start with, I then propose, for instance, Refs. [3] and [4] to approach (with more mathematics than in Ref. [1]) the basics concepts of transverse and longitudinal beam dynamics. Finally, Refs. [5] and [6] are the 2006 CERN Summer Student Lectures I gave last year with a colleague of mine and the LHC design report respectively.

[1] E. Wilson, An Introduction to Particle Accelerators (252 pages), Oxford University Press, 2001 (Reprinted in 2006).

[2] CERN Accelerator Schools (**http://cas.web.cern.ch/cas/**).

[3] M. Martini, An Introduction to Transverse Beam Dynamics in Accelerators, (**http://doc.cern.ch/archive/electronic/cern/preprints/ps/ps-96-011.pdf**).

[4] L. Rinolfi, Longitudinal Beam Dynamics (Application to synchrotron), (**http://doc.cern.ch/archive/electronic/cern/preprints/ps/ps-2000-008.pdf**).

[5] S. Gilardoni and E. Métral, 2006 CERN Summer Student Lectures (**http://agenda.cern.ch/askArchive.php?base=agenda&categ=a062758&id=a062758/transparencies**).

[6] LHC Design Report (**http://ab-div.web.cern.ch/ab-div/Publications/LHC-DesignReport.html**).

What could happen during the week…?

- ***Monday***: Identification of the problem. Understand the notion of **machine luminosity**, and how it depends on the beam parameters. Understand the **beam rigidity** and have an idea of the basic parameters (dimension, energy, number of bunches, repetition rate…) of the different accelerators in the injectors chain needed to reach the required luminosity in the LHC.
- ***Tuesday and Wednesday***: Introduction to **transverse beam dynamics** to understand what is the **working point**, the **transverse beam emittance** and the **resonances** (which can lead to **emittance blow-up** and/or **beam loss**). The effect of space charge could also be mentioned, as it is an important phenomenon in low-energy machines such as the PS.
- ***Thursday and Friday***: Introduction to **longitudinal beam dynamics** to understand what is the **longitudinal beam emittance** and **matching** (here the longitudinal beam emittance is blown-up due to a longitudinal mismatch between the PSB and the PS).

Hint to solve the longitudinal problem

The longitudinal emittance is the same between 2 consecutive synchrotrons if it is expressed in eV.s. Therefore the conservation of the ratio $\Delta E_{\max} / \Delta t_{\max}$, between the PSB and PS buckets, provides a perfect longitudinal matching for a stationary bucket. We remind one of the expressions of the bucket (half) height:

$$\Delta E_{\max} = \beta_s \sqrt{\frac{e\hat{V}_{RF}\ E_s}{\pi|\eta|h} F(\Phi_s)}\,,$$

where $F(\Phi_s) = 2\cos(\Phi_s) - (\pi - 2\Phi_s)\sin(\Phi_s)$. Here, s stands for the synchronous particle, Φ is the RF phase, β is the relativistic velocity factor, e is the proton charge, $\hat{V}_{RF}$ is the peak RF voltage, E is the total proton energy, η is the slip (or off-momentum) factor and h is the harmonic number. The expression of the RF phase extension is $\hat{\phi} = \omega_{RF}\ \Delta t_{\max}$, where ω_{RF} is the angular RF frequency and $\Delta t_{\max}$ is the bucket (half) length in second (for a stationary bucket).

Numerical values to solve the longitudinal problem

$R_{PS} = 100\,\mathrm{m}$	PS radius
$R_{PSB} = R_{PS} / 4$	PSB radius
$h_{PS} = 7$	PS harmonic number
$h_{PSB} = 1$	PSB harmonic number
$\hat{V}_{RF}^{PSB} = 8\ \mathrm{kV}$	Peak RF voltage at the ejection of the PSB
$\alpha_p^{PS} = \gamma_{tr,PS}^{-2} = 0.027$	PS momentum compaction factor
$\alpha_p^{PSB} = \gamma_{tr,PSB}^{-2} = 0.0617$	PSB momentum compaction factor
$\rho_{PS} = 70\,\mathrm{m}$	PS curvature radius
$c = 2.997925 \times 10^8\ \mathrm{m/s}$	Speed of light
$E_0 = 0.938\,\mathrm{GeV}$	Proton rest energy

Solution of the longitudinal problem

1) Are the 1020 Gauss OK?

- This is the very important formula of the beam rigidity which has to be used here. It is given by

$$B[\mathrm{T}]\,\rho[\mathrm{m}] = 3.3356\ p[\mathrm{GeV/c}] .$$

The numerical application yields B = 1020 Gauss. The value given is good and therefore this is the RF voltage which is not the good one. Why? If the longitudinal emittance of the beam sent to the SPS is too large and the RF voltage at PS injection is not the good one, it means that the blow-up of the longitudinal emittance is due to a longitudinal mismatch between the PSB and the PS…

2) We know that the 60 kV is not the good value. What is the good one?

- In a stationary bucket (as it is the case in the PS at injection), the synchronous phase below transition energy is $\Phi_s = 0$. Therefore, the bucket (half) height $\Delta E_{\max}$ is given by the formula with F = 2. Concerning the bucket (half) length $\Delta t_{\max}$, it is also given by the formula with $\hat{\phi} = \pi$ in a stationary bucket.
- The longitudinal matching condition between the PSB and PS is given by

$$\left(\frac{\Delta E_{\max}}{\Delta t_{\max}}\right)_{PS} = \left(\frac{\Delta E_{\max}}{\Delta t_{\max}}\right)_{PSB} .$$

As the beam energy is the same at extraction of the PSB and injection in the PS, this leads to

$$\hat{V}_{RF}^{PS} = \hat{V}_{RF}^{PSB} \times \left|\frac{\eta_{PS}}{\eta_{PSB}}\right| \times \frac{h_{PSB}}{h_{PS}} \times \left(\frac{R_{PS}}{R_{PSB}}\right)^2 .$$

The numerical application gives $\hat{V}_{RF}^{PS} = 25\,\mathrm{kV}$. The RF voltage should therefore be 25 kV and not 60 kV. With 60 kV the beam coming from the PSB is not matched longitudinally. It will start to oscillate in the RF bucket finding after some time a new matching condition but with a larger longitudinal emittance, as observed by the SPS operator…