TRAINING-WEEK IN ACCELERATOR PHYSICS

Elias Métral

Programme of the week

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<th>Morning (lectures: 2 × 45 min)</th>
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<td>MO 27/05/13</td>
<td>Introduction and luminosity</td>
<td>Exercises on luminosity</td>
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<td>TU 28/05/13</td>
<td>Transverse beam dynamics</td>
<td>Exercises on transverse beam dynamics</td>
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<tr>
<td>WE 29/05/13</td>
<td>Longitudinal beam dynamics</td>
<td>Exercises on longitudinal beam dynamics</td>
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<tr>
<td>TH 30/05/13</td>
<td>Collective effects (space charge, impedances and related instabilities, beam-beam and e-cloud)</td>
<td>Tutorial on MAD-X code (for transverse beam dynamics) + Exercises on collective effects</td>
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<td>FR 31/05/13</td>
<td>Feedback and hand-out of last problem (to be solved after the course)</td>
<td>Reserve time</td>
</tr>
</tbody>
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Introduction

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CONCEPTS AND PREREQUISITES

- **BEAM DYNAMICS** describes the motion of a charged particle beam in an accelerator.

- **LOW-INTENSITY PARTICLE BEAMS** can be modeled by using single-particle dynamics, in which particles are tracked through the external electromagnetic fields (from the guiding and focusing magnets in the transverse planes, RF cavities in the longitudinal plane, etc.) => Classical mechanics (linear and nonlinear), electrodynamics, physical or engineering mathematics and special relativity.

- **HIGH-INTENSITY (and or HIGH-DENSITY) PARTICLE BEAMS** require a more complicated description which involves interactions between the beam particles and between the beam particles and their environment (and/or other particles) => Plasma physics. High-intensity (and or high-density) effects are very important because they usually pose an upper limitation to the number of particles that can be injected into an accelerator.
Example of the LHC p beam in the injector chain
What happens to the particles inside the vacuum chamber?

Reminder: 1 atm = 760 Torr and 1 mbar = 0.75 Torr = 100 Pa

~10^{-10} Torr in the LHC = ~3 million molecules / cm^3

(Φ = 13 cm here)
LAYOUT OF THE LHC

Courtesy W. Herr

IP = Interaction Point

High-luminosity => Higgs boson
⇒ Vertical crossing angle in IP1 (ATLAS) and horizontal one in IP5 (CMS)
(2D) BEAM BRIGHTNESS

$$B = \frac{I}{\pi^2 \varepsilon_x \varepsilon_y}$$

- Beam current
- Transverse emittances

MACHINE LUMINOSITY

$$L = \frac{N_{\text{events/second}}}{\sigma_r} \quad \text{[cm}^{-2} \text{s}^{-1}]$$

- Number of events per second generated in the collisions
- Cross-section of the reaction

- The Luminosity depends only on the beam parameters
  ⇒ It is independent of the physical reaction
- Reliable procedures to compute and measure
⇒ For a Gaussian (round) beam distribution

\[ L = \frac{N_b^2 M f_0 \gamma}{4 \pi \varepsilon_n \beta^*} F_{ca} \]

- Number of particles per bunch
- Number of bunches per beam
- Revolution frequency
- Relativistic mass factor
- Normalized rms transverse beam emittance
- Geometric reduction factor due to the crossing angle at the IP
- \( \beta \)-function at the collision point

◆ PEAK LUMINOSITY for ATLAS&CMS in the LHC = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}
| **Number of particles per bunch** | $N_b$ | $1.15 \times 10^{11}$ |
| **Number of bunches per beam** | $M$ | 2808 |
| **Revolution frequency** | $f_0$ | 11245 Hz |
| **Relativistic velocity factor** | $\gamma$ | 7461 (=> $E = 7$ TeV) |
| **$\beta$-function at the collision point** | $\beta^*$ | 55 cm |
| **Normalised rms transverse beam emittance** | $\varepsilon_n$ | $3.75 \times 10^{-4}$ cm |
| **Geometric reduction factor** | $F_{ca}$ | 0.84 |

$$F_{ca} = 1/\sqrt{1 + \left( \frac{\theta_c \sigma_s}{2 \sigma^*} \right)^2}$$

| **Full crossing angle at the IP** | $\theta_c$ | 285 $\mu$rad |
| **Rms bunch length** | $\sigma_s$ | 7.55 cm |
| **Transverse rms beam size at the IP** | $\sigma^*$ | 16.7 $\mu$m |
INTEGRATED LUMINOSITY

\[ L_{\text{int}} = \int_{0}^{T} L(t) \, dt \]

⇒ The real figure of merit = \[ L_{\text{int}} \, \sigma_r = \text{number of events} \]

LHC integrated Luminosity expected per year (~10^7 s): [80-120] fb\(^{-1}\)

Reminder: 1 barn = 10^{-24} cm\(^2\) and femto = 10^{-15}
The total proton-proton cross section at 7 TeV is ~ 110 mbarns:

- Inelastic \( \sigma_{\text{in}} = 60 \text{ mbarns} \)
- Single diffractive \( \sigma_{\text{sd}} = 12 \text{ mbarns} \)
- Elastic \( \sigma_{\text{el}} = 40 \text{ mbarns} \)

The cross section from elastic scattering of the protons and diffractive events will not be seen by the detectors as it is only the inelastic scatterings that give rise to particles at sufficient high angles with respect to the beam axis.

Inelastic event rate at nominal luminosity = \( 10^{34} \times 60 \times 10^{-3} \times 10^{-24} \) = 600 millions / second per high-luminosity experiment.
- The bunch spacing in the LHC is 25 ns ➔ Crossing rate of 40 MHz

- However, there are bigger gaps (for the kickers) ➔ Average crossing rate = number of bunches × revolution frequency = 2808 × 11245 = 31.6 MHz

- (600 millions inelastic events / second) / (31.6 × 10^6) = 19 inelastic events per crossing

- Total inelastic events per year (~10^7 s) = 600 millions × 10^7 = 6 × 10^{15} ~ 10^{16}

- The LHC experimental challenge is to find rare events at levels of 1 in 10^{13} or more ➔ ~ 1000 Higgs events in each of the ATLAS and CMS experiments expected per year
Single-particle trajectory

In the middle of the vacuum chamber

One particle

Circular design orbit

ACCELERATOR MODEL
WALL CURRENT MONITOR = Device used to measure the instantaneous value of the beam current

For the vacuum + EM shielding

Load

Ferrite rings

Induced or wall current

⇒ High-frequency signals do not see the short circuit

Longitudinal bunch profiles for a LHC-type beam in the PS

A Wall Current Monitor

 Courtesy J. Belleman

WALL CURRENT MONITOR IN SS3 OF THE PS
(Transverse) beam POSITION PICK-UP MONITOR

⇒ Horizontal beam orbit measurement in the PS

6 spikes observed as $Q_x \approx 6.25$
POSITION PICK-UP IN SS5 OF THE PS

Correction QUADRUPOLE

Correction DIPOLE

BEAM LOSS MONITOR
FAST WIRE SCANNER

⇒ Measures the transverse beam profiles by detecting the particles scattered from a thin wire swept rapidly through the beam.

HORIZONTAL PROFILE

Gaussian fit

\[
e_x(\sigma_x) \equiv \frac{\sigma_x^2}{\beta_x}
\]

Courtesy S. Gilardoni
FAST WIRE SCANNER IN SS54 OF THE PS

- SCINTILLATORS ⇒ Producing light
- Light conductors
- PHOTOMULTIPLIER ⇒ Converting light into and electrical signal

Motor of the wire scanner
BEAM LOSS MONITOR

3.42 \times 10^{13} \text{ ppp @ 14 GeV/c}
(Monday 27/09/04 12:47)

PS record intensity (CNGS beam)

Saturation at 255
=> Could be much higher

Detector = ACEM (Aluminum Cathod Electron Multiplier), placed at the beginning of the SS
⇒ Similar to a Photomultiplier and it is cheap, robust and radiation resistant
SUMMARY OF LECTURE ON TRANSVERSE BEAM DYNAMICS

- Design orbit **in the centre of the** vacuum chamber
- Lorentz force: $\vec{F} = e\left( \vec{E} + \vec{v} \times \vec{B} \right)$
- Dipoles (constant force) ⇒ Guide the particles along the design orbit
- Quadrupoles (linear force) ⇒ Confine the particles in the vicinity of the design orbit
- Betatron oscillation in $x$ (and in $y$) ⇒ Tune $Q_x$ (and $Q_y$) $>> 1$
- Twiss parameters define the ellipse in phase space ($x, x' = d x / d s$)
- $\beta$-function reflects the size of the beam and depends only on the lattice
- Beam emittance must be smaller than the mechanical acceptance
- Higher order multipoles from imperfections (nonlinear force) ⇒ Resonances excited in the tune diagram and the working point ($Q_x, Q_y$) should not be close to most of the resonances
- Nonlinearities reduce the acceptance ⇒ Dynamic aperture
- Injection and extraction (septum and kicker)
- Betatron and dispersion matching (between a circular accelerator and a transfer line)
SUMMARY OF LECTURE ON LONGITUDINAL BEAM DYNAMICS

- RF cavities are used to accelerate (or decelerate) the particles
- Transition energy and sinusoidal voltage ⇒ $\vec{F} = e \left( \vec{E} + \vec{v} \times \vec{B} \right)$
- Harmonic number = Number of RF buckets (stationary or accelerating)
- Bunched beam (instead of an unbunched or continuous beam)
- Synchrotron oscillation around the synchronous particle in $z$ ⇒ Tune $Q_z << 1$
- Stable phase $\Phi_s$ below transition and $\pi - \Phi_s$ above transition
- Ellipse in phase space $(\Delta t, \Delta E)$
- Beam emittance must be smaller than the bucket acceptance
- Bunch splittings and rotation very often used
SUMMARY OF LECTURE ON COLLECTIVE EFFECTS (1/2)

- (Direct) space charge = Interaction between the particles (without the vacuum chamber) ⇒ Coulomb repulsion + magnetic attraction
  - Tune footprint in the tune diagram ⇒ Interaction with resonances
  - Disappears at high energy
  - Reduces the RF bucket below transition and increases it above

- Wake fields = Electromagnetic fields generated by the beam interacting with its surroundings (vacuum pipe, etc.) ⇒ Impedance = Fourier transform of the wake field
  - Bunched-beam coherent instabilities
    - Coupled-bunch modes
    - Single-bunch or Head-Tail modes (low and high intensity)
  - Beam stabilization
    - Landau damping
    - Feedbacks
    - Linear coupling between the transverse planes
SUMMARY OF LECTURE ON COLLECTIVE EFFECTS (2/2)

- **Beam-Beam** = Interaction between the 2 counter-rotating beams
  - Coulomb repulsion + magnetic repulsion
  - Crossing angle, head-on and long-range interactions
  - Tune footprint in the tune diagram ⇒ Interaction with resonances
  - Does not disappear at high energy
  - PACMAN effects ⇒ Alternate crossing scheme
  - Coherent modes ⇒ Possible loss of Landau damping

- **Electron cloud**
  - Electron cloud build-up ⇒ Multi-bunch single-pass effect
  - Coherent instabilities induced by the electron cloud
    - Coupled-bunch
    - Single-bunch
  - Tune footprint in the tune diagram ⇒ Interaction with resonances
  - Does not disappear at high energy
**REMINDERS: (1) RELATIVISTIC EQUATIONS**

\[
\gamma = \frac{E_{\text{total}}}{E_{\text{rest}}} = \frac{m}{m_0} = \frac{1}{\sqrt{1 - \beta^2}}
\]

\[
\beta = \frac{v}{c}
\]

\[
\vec{p} = m \vec{v}
\]

For a particle of charge \(e\)

\[
E_{\text{total}}^2 = E_{\text{rest}}^2 + p^2 c^2
\]

\[
\frac{d \vec{p}}{dt} = \vec{F} = e \left( \vec{E} + \vec{v} \times \vec{B} \right)
\]
(2) MOST IMPORTANT 4-VECTORS & INVARIANTS
AND LORENTZ SCALAR PRODUCT

- **4-dimensional radius vector**
  \[ \vec{X} = (c \ t, \vec{x}) \]

- **4-velocity**
  \[ \vec{V} = c \frac{d \vec{X}}{ds} = \gamma \left( c, \frac{d \vec{X}}{d\tau} \right) \]

- **4-momentum (energy-momentum vector)**
  \[ \vec{P} = \left( \frac{E}{c}, \vec{p} \right) = \gamma m_0 \left( c, \vec{v} \right) = m_0 \vec{V} \]

- **Current vector**
  \[ \vec{J} = \rho \left( c, \vec{v} \right) = \frac{\rho}{\gamma} \vec{V} \]

with \( \rho = \rho_0 \) the density in the rest system of the volume element considered.

Proper time
Lorentz scalar product

\[( u_1 u_2 ) = u_{1\mu} u_{2}^{\mu} \]

\[= u_{1}^{0} u_{2}^{0} - u_{1}^{1} u_{2}^{1} - u_{1}^{2} u_{2}^{2} - u_{1}^{3} u_{2}^{3} \]

with

\[ u^{\mu} = ( u^{0}, u^{1}, u^{2}, u^{3} ) \]

the contravariant 4-vector

and

\[ u_{\mu} = ( u^{0}, - u^{1}, - u^{2}, - u^{3} ) \]

the covariant 4-vector

Invariants

\[ X^2 = X_{\mu} X^{\mu} = ( c t )^2 - \vec{x}^2 \]

\[ V^2 = c^2 \]

\[ P^2 = m_0^2 c^2 \]

\[ J^2 = \left( \frac{\rho}{\gamma} \right)^2 c^2 = \rho_0^2 c^2 \]
(3) ENERGY, MOMENTUM AND VELOCITY OF ONE PARTICLE SEEN FROM THE REST SYSTEM OF ANOTHER ONE

- Consider 2 particles: 1 and 2, with rest mass $m_{01}$ and $m_{02}$

- The 3 invariants are $P_1^2 = m_{01}^2 c^2$, $P_2^2 = m_{02}^2 c^2$ and $P_1 P_2$ (or $(P_1 + P_2)^2$ or $(P_1 - P_2)^2$)

- Total Centre of Mass (CM) energy squared

\[ s = c^2 (P_1 + P_2)^2 = E_{CM}^2 \]

\[ \Rightarrow \sqrt{s} = E_{CM} \]
Making the computation in the rest system of particle 1, one can show the 3 following invariant expressions

- The energy of particle 2 seen from particle 1 is

\[ E_{21} = \frac{P_1 P_2}{m_{01}} \]

- The momentum of particle 2 seen from particle 1 is

\[ \vec{p}_{21}^2 = \frac{E_{21}^2}{c^2} - m_{02}^2 c^2 = \frac{(P_1 P_2)^2 - m_{01}^2 m_{02}^2 c^4}{m_{01}^2 c^2} \]

- The relative velocity (symmetric in 1 and 2) is

\[ v_{21}^2 = \frac{\vec{p}_{21}^2 c^4}{E_{21}^2} = c^2 \left( \frac{(P_1 P_2)^2 - m_{01}^2 m_{02}^2 c^4}{(P_1 P_2)^2} \right) \]
It can also be shown (using the relation given in the Useful relations)

\[
(\vec{v}_1 \times \vec{v}_2)^2 = \vec{v}_1^2 \vec{v}_2^2 - (\vec{v}_1 \cdot \vec{v}_2)^2
\]

that

\[
v_{21} = \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - \left(\frac{\vec{v}_1 \times \vec{v}_2}{c^2}\right)^2}
\]

\[
= \frac{\sqrt{1 - \frac{\vec{v}_1 \cdot \vec{v}_2}{c^2}}}{1 - \frac{\vec{v}_1 \cdot \vec{v}_2}{c^2}}
\]
(4) LORENTZ FORCE

\[ \vec{F} = e \left( \vec{E} + \vec{v} \times \vec{B} \right) \]

- **Cartesian (x,y,s)**
  \[ F_x = e \left( E_x - v B_y \right) \]
  \[ F_y = e \left( E_y + v B_x \right) \]
  \[ F_s = e E_s \]

- **Cylindrical (r,θ,s)**
  \[ F_r = e \left( E_r - v B_\theta \right) \]
  \[ F_\theta = e \left( E_\theta + v B_r \right) \]
  \[ F_s = e E_s \]
(5) LORENTZ TRANSFORM

\[ \begin{align*}
x &= x' \\
y &= y' \\
s &= \gamma (s' + vt') \\
t &= \gamma \left( \frac{v}{c^2} s' + t' \right)
\end{align*} \]

- **Velocity of** \( R' \) **with respect to** \( R \)

\[ \vec{v} \]

\[ R = (0, x, y, s) \quad R' = (0, x', y', s') \]

- **Length contraction (in** \( R \))

\[ ds = \frac{ds'}{\gamma} \quad \text{for} \quad dt = 0 \]

- **Time dilatation (in** \( R \))

\[ dt = \gamma dt' \quad \text{for} \quad ds' = 0 \]
(6) MAXWELL EQUATIONS

- **Differential forms**
  
  \[ \text{div} \ \vec{E} = \frac{\rho}{\varepsilon} \]  
  \[ \text{div} \ \vec{H} = 0 \]  
  \[ \text{rot} \ \vec{E} = - \mu \frac{\partial \vec{H}}{\partial t} \]  
  \[ \text{rot} \ \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \]  

  with
  
  \[ \vec{B} = \mu \ \vec{H} \]  
  \[ \vec{D} = \varepsilon \ \vec{E} \]  
  \[ \vec{J} = \rho \vec{v} + \sigma \vec{E} \]

- **Integral forms**
  
  Gauss’s law for electric charge
  
  \[ \iiint \text{div} \ \vec{E} \ dV = \iint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon} \iiint \rho \ dV \]

  Gauss’s law for magnetic charge
  
  \[ \iiint \text{div} \ \vec{H} \ dV = \iint \vec{H} \cdot d\vec{S} = 0 \]

  Faraday’s and Lenz law
  
  \[ \iiint \text{rot} \ \vec{E} \cdot d\vec{S} = \oint \vec{E} \cdot d\vec{s} = - \mu \iiint \frac{\partial \vec{H}}{\partial t} \cdot d\vec{S} \]

  Ampere’s law
  
  \[ \iiint \text{rot} \ \vec{H} \cdot d\vec{S} = \oint \vec{H} \cdot d\vec{s} = \iint \vec{J} \cdot d\vec{S} + \varepsilon \iiint \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S} \]

Maxwell equations valid in homogeneous, isotropic, continuous media
(7) NABLA, GRAD, ROT, DIV and LAPLACIAN OPERATORS

- **Cartesian \((x,y,s)\)**
  \[
  \vec{\nabla} = \begin{bmatrix}
  \frac{\partial}{\partial x} \\
  \frac{\partial}{\partial y} \\
  \frac{\partial}{\partial s}
  \end{bmatrix}
  \]

- **Cylindrical \((r,\theta,s)\)**
  \[
  \vec{\nabla} = \frac{1}{r} \begin{bmatrix}
  \frac{\partial}{\partial r} \\
  \frac{\partial}{\partial \theta} \\
  \frac{\partial}{\partial s}
  \end{bmatrix}
  \]
  Also noted \(\text{curl} \, \vec{E} \) or \(\vec{\nabla} \times \vec{E}\)

\[
\text{grad} \, \rho \equiv \vec{\nabla} \rho = \begin{bmatrix}
  \frac{\partial \rho}{\partial x} \\
  \frac{\partial \rho}{\partial y} \\
  \frac{\partial \rho}{\partial s}
  \end{bmatrix}
\]

\[
\text{rot} \, \vec{E} \equiv \vec{\nabla} \times \vec{E} = \begin{bmatrix}
  \frac{\partial E_y}{\partial s} - \frac{\partial E_z}{\partial y} \\
  \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \\
  \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}
  \end{bmatrix}
\]

\[
\text{div} \, \vec{E} \equiv \vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial s}
\]

\[
\Delta \rho \equiv \nabla^2 \rho = \text{Laplacian operator}
\]

\[
\Delta \rho = \frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial s^2}
\]

\[
\text{div} \, \vec{E} = \frac{1}{r} \frac{\partial}{\partial r} \left( r E_r \right) + \frac{1}{r} \frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_z}{\partial s}
\]

\[
\Delta \rho = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \rho}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \rho}{\partial \theta^2} + \frac{\partial^2 \rho}{\partial s^2}
\]
Consider a surface separating two media “1” and “2”. The following boundary conditions can be derived from Maxwell equations for the normal ($\perp$) and parallel ($\parallel$) components of the fields at the surface.

1. $\vec{E}_\parallel^1 = \vec{E}_\parallel^2$
2. $\vec{H}_\parallel^1 - \vec{H}_\parallel^2 = \vec{K}$
3. $D_\perp^1 - D_\perp^2 = \Sigma$
4. $B_\perp^1 = B_\perp^2$

where $\Sigma$ is the surface charge density and $\vec{K}$ is the surface current density.
(9) USEFUL RELATIONS / NOTIONS

- Gaussian distribution
  \[ \lambda(s) = \frac{q}{\sqrt{2\pi} \sigma_s} e^{-\frac{s^2}{2\sigma_s^2}} \]

- Equation of motion (and solutions) of an harmonic oscillator (which will be very often used) => The best way to keep something under control (i.e. stable) is to make it oscillate!

- MKSA units are used here, whereas CGS units can be found in several books and publications => Conversion from CGS to MKSA
  \[ \frac{4\pi}{c} = Z_0 = 120\pi \Omega \]
  \[ \frac{e^2}{m_0 c^2} = r_0 = \text{Classical radius of the particle} \]

- The engineer convention is also adopted (\[ e^{j\omega t} \]) instead of the physicist’s one (\[ e^{-i\omega t} \])
- **Transposition of the product of 2 matrices**
  \[(A B)^t = B^t A^t\]

- **Inversion of a 2 × 2 matrix**
  
  \[M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\]

  \[M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\]

- \[\int_{-\infty}^{+\infty} e^{-a t^2} \, dt = \sqrt{\frac{\pi}{a}}\]

- \[\int_{-\infty}^{+\infty} e^{-(a t^2 + b t + c)} \, dt = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a} - c}\]
\[ \cos(a + b) = \cos a \cos b - \sin a \sin b \]

\[ \cos(a - b) = \cos a \cos b + \sin a \sin b \]

\[ \sin(a + b) = \sin a \cos b + \sin b \cos a \]

\[ \sin(a - b) = \sin a \cos b - \sin b \cos a \]

- **Rotation (by an angle + \( \Phi / 2 \)) matrix**

\[
R = \begin{bmatrix}
\cos \frac{\Phi}{2} & -\sin \frac{\Phi}{2} \\
\sin \frac{\Phi}{2} & \cos \frac{\Phi}{2}
\end{bmatrix}
\]

\[ (\vec{v}_1 \times \vec{v}_2)^2 = \vec{v}_1^2 \vec{v}_2^2 - (\vec{v}_1 \cdot \vec{v}_2)^2 \]

\[
\int_0^s \frac{dt}{1 + t^2} = \arctan s
\]
(10) Units of physical quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>unit</th>
<th>SI unit</th>
<th>SI derived unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance</td>
<td>F (farad)</td>
<td>m$^{-2}$ kg$^{-1}$s$^{4}$A$^{2}$</td>
<td>C/V</td>
</tr>
<tr>
<td>Electric charge</td>
<td>C (coulomb)</td>
<td>As</td>
<td></td>
</tr>
<tr>
<td>Electric potential</td>
<td>V (volt)</td>
<td>m$^{2}$ kg s$^{-3}$A$^{-1}$</td>
<td>W/A</td>
</tr>
<tr>
<td>Energy</td>
<td>J (joule)</td>
<td>m$^{2}$ kg s$^{-2}$</td>
<td>Nm</td>
</tr>
<tr>
<td>Force</td>
<td>N (newton)</td>
<td>m kg s$^{-2}$</td>
<td>N</td>
</tr>
<tr>
<td>Frequency</td>
<td>Hz (hertz)</td>
<td>s$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Inductance</td>
<td>H (henry)</td>
<td>m$^{2}$ kg s$^{-2}$A$^{-2}$</td>
<td>Wb/A</td>
</tr>
<tr>
<td>Magnetic flux</td>
<td>Wb (weber)</td>
<td>m$^{2}$ kg s$^{-2}$A$^{-1}$</td>
<td>Vs</td>
</tr>
<tr>
<td>Magnetic flux density</td>
<td>T (tesla)</td>
<td>kg s$^{-2}$A$^{-1}$</td>
<td>Wb/m$^{2}$</td>
</tr>
<tr>
<td>Power</td>
<td>W (watt)</td>
<td>m$^{2}$ kg s$^{-3}$</td>
<td>J/s</td>
</tr>
<tr>
<td>Pressure</td>
<td>Pa (pascal)</td>
<td>m$^{-1}$ kg s$^{-2}$</td>
<td>N/m$^{2}$</td>
</tr>
<tr>
<td>Resistance</td>
<td>Ω (ohm)</td>
<td>m$^{2}$ kg s$^{-3}$A$^{-2}$</td>
<td>V/A</td>
</tr>
</tbody>
</table>
### (11) Fundamental physical constants

<table>
<thead>
<tr>
<th>Physical constant</th>
<th>symbol</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avogadro’s number</td>
<td>$N_A$</td>
<td>$6.0221367 \times 10^{23}$</td>
<td>/mol</td>
</tr>
<tr>
<td>atomic mass unit ($\frac{1}{12} m(C^{12})$)</td>
<td>$m_u$ or $u$</td>
<td>$1.6605402 \times 10^{-27}$</td>
<td>kg</td>
</tr>
<tr>
<td>Boltzmann’s constant</td>
<td>$k$</td>
<td>$1.380658 \times 10^{-23}$</td>
<td>J/K</td>
</tr>
<tr>
<td>Bohr magneton</td>
<td>$\mu_n = e\hbar/2m_e$</td>
<td>$9.2740154 \times 10^{-24}$</td>
<td>J/T</td>
</tr>
<tr>
<td>Bohr radius</td>
<td>$a_0 = 4\pi\varepsilon_0\hbar^2/m_e c^2$</td>
<td>$0.529177249 \times 10^{-10}$</td>
<td>m</td>
</tr>
<tr>
<td>classical radius of electron</td>
<td>$r_e = e^2/4\pi\varepsilon_0 m_e c^2$</td>
<td>$2.81794092 \times 10^{-15}$</td>
<td>m</td>
</tr>
<tr>
<td>classical radius of proton</td>
<td>$r_p = e^2/4\pi\varepsilon_0 m_p c^2$</td>
<td>$1.5346986 \times 10^{-18}$</td>
<td>m</td>
</tr>
<tr>
<td>elementary charge</td>
<td>$e$</td>
<td>$1.60217733 \times 10^{-19}$</td>
<td>C</td>
</tr>
<tr>
<td>fine structure constant</td>
<td>$\alpha = e^2/2\varepsilon_0 hc$</td>
<td>$1/137.0359895$</td>
<td></td>
</tr>
<tr>
<td>$m_u c^2$</td>
<td></td>
<td>$931.49432$</td>
<td>MeV</td>
</tr>
<tr>
<td>mass of electron</td>
<td>$m_e$</td>
<td>$9.1093897 \times 10^{-31}$</td>
<td>kg</td>
</tr>
<tr>
<td>$m_u c^2$</td>
<td></td>
<td>$0.51099906$</td>
<td>MeV</td>
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<tr>
<td>mass of proton</td>
<td>$m_p$</td>
<td>$1.6726231 \times 10^{-27}$</td>
<td>kg</td>
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<tr>
<td>$m_p c^2$</td>
<td></td>
<td>$938.27231$</td>
<td>MeV</td>
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<tr>
<td>mass of neutron</td>
<td>$m_n$</td>
<td>$1.6749286 \times 10^{-27}$</td>
<td>kg</td>
</tr>
<tr>
<td>$m_p c^2$</td>
<td></td>
<td>$939.56563$</td>
<td>MeV</td>
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<td>molar gas constant</td>
<td>$R = N_A k$</td>
<td>$8.314510$</td>
<td>J/mol K</td>
</tr>
<tr>
<td>neutron magnetic moment</td>
<td>$\mu_n$</td>
<td>$-0.96523707 \times 10^{-26}$</td>
<td>J/T</td>
</tr>
<tr>
<td>nuclear magneton</td>
<td>$\mu_p = e\hbar/2m_u$</td>
<td>$5.0507866 \times 10^{-27}$</td>
<td>J/T</td>
</tr>
<tr>
<td>Planck’s constant</td>
<td>$\hbar$</td>
<td>$6.626075 \times 10^{-34}$</td>
<td>J s</td>
</tr>
<tr>
<td>permeability of vacuum</td>
<td>$\mu_0$</td>
<td>$4\pi \times 10^{-7}$</td>
<td>N/A$^2$</td>
</tr>
<tr>
<td>permittivity of vacuum</td>
<td>$\varepsilon_0$</td>
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<td>F/m</td>
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<tr>
<td>proton magnetic moment</td>
<td>$\mu_p$</td>
<td>$1.41060761 \times 10^{-26}$</td>
<td>J/T</td>
</tr>
<tr>
<td>proton $g$ factor</td>
<td>$g_p = \mu_p/\mu_N$</td>
<td>$2.792847386$</td>
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<tr>
<td>speed of light (exact)</td>
<td>$c$</td>
<td>$299792458$</td>
<td>m/s</td>
</tr>
<tr>
<td>vacuum impedance</td>
<td>$Z_0 = 1/\varepsilon_0 c = \mu_0 c$</td>
<td>$376.7303$</td>
<td>$\Omega$</td>
</tr>
</tbody>
</table>
REFERENCES (1/2)


REFERENCES (2/2)


[16] E. Métral’s web page [http://emetral.web.cern.ch/emetral/] where several courses (with some exercises, exams and corrections) can be found