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Correction of the final exercise for the one-week training course in Accelerator physics (Lund 2013)

1) $N_d = \underbrace{8}_{\text{8 arcs}} \times \underbrace{23}_{\text{23 FODO cells}} \times \underbrace{6}_{\text{3 dipoles / half-cell}} + \underbrace{8}_{\text{8 arcs}} \times \underbrace{2}_{\text{2 DS at both ends of arc}} \times \underbrace{8}_{\text{4×2 dipoles each}}$

$$\Rightarrow \boxed{N_d = 1232}$$

2) $2\pi = N_d . \theta_d \Rightarrow \theta_d = \frac{2\pi}{N_d} = \frac{2\pi}{1232} \Rightarrow \boxed{\theta_d = 5{,}1 \text{ mrad}}$

$$l_d = \rho . \theta_d \Rightarrow \rho = \frac{l_d}{\theta_d} = \frac{14{,}3}{5{,}1.10^{-3}} \Rightarrow \boxed{\rho = 2803{,}93 \text{ m}}$$

3) $\boxed{f = f_F} = \frac{1}{K_q . l_q}$ $\quad \boxed{f_D = -f_F}$

$\boxed{K_q = \frac{e}{p_0} G_q} = \frac{1}{B\rho} G_d$ with G_d = gradient of the arc quadrupoles

$B\rho = \frac{p_0}{e}$ = beam rigidity

$p_0 = 7$ TeV/c

$$\begin{cases} G_d = 205 \text{ T/m} \\ B\rho = 3{,}3356 \times p_0 (\text{GeV/c}) = 3{,}3356 \times 7000 \\ [\text{T.m}] \end{cases}$$

$$\Rightarrow \boxed{B\rho = 23\,349{,}2 \text{ T.m}}$$

$$\Rightarrow K_q = \frac{205}{23\,349{,}2} \text{ m}^{-2} \Rightarrow \boxed{K_q = 8{,}78.10^{-3} \text{ m}^{-2}}$$

and $f = \frac{1}{8{,}78.10^{-3} \times 3{,}1} \Rightarrow \boxed{f = 36{,}74 \text{ m}}$

4) 1) $L = 3 \cdot l_d' \Rightarrow l_d' = \frac{L}{3} = \frac{53{,}45}{3} \Rightarrow \boxed{l_d' = 17{,}82\ m}$

The bending angle of a dipole remains the same as the number of dipoles remains the same BUT

$\rho' = \frac{l_d'}{\theta_d} = \frac{17{,}82}{5{,}1 \cdot 10^{-3}} \Rightarrow \boxed{\rho' = 3493{,}5\ m}$

2) $\frac{L}{2f} = \sin\left(\frac{\mu}{2}\right) \Rightarrow \boxed{\mu = 2\ \mathrm{ArcSin}\left[\frac{L}{2f}\right]}$

$\Rightarrow \mu = 2\ \mathrm{ArcSin}\ \frac{53{,}45}{2 \times 36{,}74} \Rightarrow \boxed{\begin{aligned}\mu &= 1{,}63\ \mathrm{rad} \\ &= 93{,}3\ \mathrm{deg}\end{aligned}}$

3) $\beta_{QF/QD} = 2L \cdot \frac{1 \pm \sin\left(\frac{\mu}{2}\right)}{\sin\mu} \Rightarrow \boxed{\begin{aligned}\beta_{QF} &= 184{,}97\ m \\ \beta_{QD} &= 29{,}19\ m\end{aligned}}$

4) $\left.\begin{aligned} D_{QF/QD} &= \frac{4f^2}{\rho'}\left(1 \pm \frac{L}{4f}\right) \\ \frac{L}{2f} &= \sin\left(\frac{\mu}{2}\right)\end{aligned}\right\} \Rightarrow D_{QF/QD} = \frac{4}{\rho'} \times \left(\frac{L}{2\sin(\mu/2)}\right)^2 \left(1 \pm \frac{\sin(\mu/2)}{2}\right)$

$= \frac{L^2}{\rho'} \cdot \frac{\left(1 \pm \frac{\sin(\mu/2)}{2}\right)}{\sin^2\left(\frac{\mu}{2}\right)}$

and $L = \rho' \cdot \theta \rightarrow 3\theta_d$

$\Rightarrow \frac{L}{\rho'} = \theta$

$\Rightarrow \boxed{D_{QF/QD} = L \cdot \theta \cdot \frac{\left(1 \pm \frac{\sin(\mu/2)}{2}\right)}{\sin^2(\mu/2)}}$

with θ the angle of all the dipoles of a half-cell ($\theta = 3\theta_d$).

$\Rightarrow D_{QF/QD} = 53{,}45 \times (3\theta_d) \times \frac{\left(1 \pm \frac{\sin\left(\frac{1{,}63}{2}\right)}{2}\right)}{\sin^2\left(\frac{1{,}63}{2}\right)} \Rightarrow \boxed{\begin{aligned}D_{QF} &= 2{,}11\ m \\ D_{QD} &= 0{,}98\ m\end{aligned}}$

$\boxed{D_y = 0}$ as there is no vertical bending

(5) $\Delta\mu_{ArcCells} = \mu \times 23 \times 8 \Rightarrow \boxed{\Delta\mu_{ArcCells} = 299{,}735 \text{ rad}}$

and $\boxed{\Delta Q_{ArcCells} = \frac{\Delta\mu_{ArcCells}}{2\pi} = 47{,}7042}$

(6) $Q_x = 64{,}28$
$Q_y = 59{,}31$

$$\Rightarrow \begin{cases} \Delta Q_x^{\text{insertions}} = 64{,}28 - 47{,}7042 \\ \Delta Q_y^{\text{insertions}} = 59{,}31 - 47{,}7042 \end{cases}$$

$$\Rightarrow \boxed{\begin{array}{l} \Delta Q_x^{\text{insertions}} = 16{,}5758 \\ \hline \Delta Q_y^{\text{insertions}} = 11{,}6058 \end{array}}$$

On average, $\boxed{\begin{array}{l} \Delta Q_x^{1\,\text{insertion}} = \dfrac{\Delta Q_x^{\text{insertions}}}{8} = 2{,}072 \\ \hline \Delta Q_y^{1\,\text{insertion}} = \dfrac{\Delta Q_y^{\text{insertions}}}{8} = 1{,}451 \end{array}}$

(7) $\varepsilon_{n_x} = \beta\gamma\,\varepsilon_x = 3{,}75\,\mu m$
and same thing in y

$\varepsilon_x = \frac{\sigma_x^2}{\beta_x}$

$$\Rightarrow \sigma_x = \sqrt{\beta_x \cdot \varepsilon_x} = \sqrt{\beta_x \cdot \frac{3{,}75 \cdot 10^{-6}}{\beta\gamma}}$$

$$\left.\begin{array}{l} \gamma = \dfrac{E_t}{E_0} = \dfrac{\sqrt{E_0^2 + p^2}}{E_0} = 7462{,}69 \\ \\ \beta \simeq 1 \end{array}\right\} \beta\gamma \simeq 7462{,}69$$

and $\beta_x = \beta_{QF}$ at the F quad and β_{QD} at the D quad
$\beta_y = \beta_{QD}$ at the F quad and β_{QF} at the D quad

$$\Rightarrow \boxed{\begin{array}{l} \sigma_x^{QF} = \sigma_y^{QD} \simeq 0{,}3 \text{ mm} \\ \hline \sigma_x^{QD} = \sigma_y^{QF} \simeq 0{,}12 \text{ mm} \end{array}}$$

(8) See the 2 pictures attached: thin and thick cases

Reminder: 1) For the thin lens case, the length of the quads are reduced to say $\boxed{1\,cm}$ → therefore the strength has to be increased by $\boxed{\frac{3{,}1}{0{,}01}}$ to keep the integrated strength constant. One can also replace the 3 dipoles by 1.

2) The angle of the bending magnet has to be put to 3 times the one of 1 magnet, as there are 3 dipoles in fact

$\Rightarrow L_d = 53{,}45\,m$ and $\boxed{\theta = 15{,}3\ mrad}$

(9) 1) 2) See pictures attached.

3) $\mu \ll 1$:

$$\beta_{QF/QD} = 2L \cdot \frac{1 \pm \sin\frac{\mu}{2}}{\sin\mu} \simeq \frac{2L}{\mu} \Rightarrow \boxed{\frac{\beta_{QF/QD}}{2L} \simeq \frac{1}{\mu}}$$

$$D_{QF/QD} = L\cdot\theta \cdot \frac{1 \pm \frac{\sin\mu/2}{2}}{\sin^2\mu/2} \simeq L\cdot\theta \frac{1}{\left(\frac{\mu}{2}\right)^2} \simeq \frac{4L\theta}{\mu^2}$$

$$\Rightarrow \boxed{\frac{D_{QF/QD}}{2L\theta} \simeq \frac{2}{\mu^2}}$$

or $D_{QF/QD} \simeq \frac{4L\theta}{\mu^2} = \left(\frac{2L}{\mu}\right)^2 \times \frac{\theta}{L}$ and $L = \rho \cdot \theta$

↓ Removing the ' for simplicity.

$$\Rightarrow \boxed{D_{QF/QD} \simeq \frac{\beta^2_{QF/QD}}{\rho}}$$

4) $Q_x = \frac{1}{2\pi}\int_0^C \frac{ds}{\beta_x(s)} = \frac{2\pi R}{2\pi\beta_x}$ if $\beta_x = \text{constant}$

$$\Rightarrow \boxed{Q_x = \frac{R}{\beta_x}}$$

5) $\rho = R$ $\left(\frac{5}{7}\right.$

$$\left.\begin{array}{l} D_x = \dfrac{\beta_x^2}{\rho} \\ Q_x = \dfrac{R}{\beta_x} \end{array}\right\} \Rightarrow \left.\begin{array}{l} D_x = \dfrac{1}{\rho} \times \dfrac{R^2}{Q_x^2} \\ R = \rho \end{array}\right\} \Rightarrow \boxed{D_x = \frac{\rho}{Q_x^2}}$$

6) $$\alpha_p = \frac{1}{C}\int_0^C \frac{D_x(s)}{\rho(s)}\,ds = \frac{1}{C}\int_0^C \frac{1}{Q_x^2}\,ds = \frac{1}{Q_x^2}$$

$$\Rightarrow \boxed{\alpha_p = \frac{1}{Q_x^2}}$$

$$\alpha_p = \frac{1}{\gamma_{tr}^2} \Rightarrow \gamma_{tr} = \frac{1}{\sqrt{\alpha_p}} = Q_x$$

$$\Rightarrow \boxed{\gamma_{tr} \simeq Q_x}$$

7) Therefore, if one wants to modify γ_{tr} (increase or decrease its value), one should play on the horizontal tune (i.e. on the phase advance μ, i.e. on $\frac{L}{2f}$ for a FODO cell)

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1) $$\xi = \xi_x = \xi_y = -\frac{2}{\mu}\tan\left(\frac{\mu}{2}\right)$$

$$\simeq -1{,}3 \Rightarrow \boxed{\xi = \xi_x = \xi_y \simeq -1{,}3}$$

2) $$S_{01}^{total} = -\frac{4\pi}{D_{x1}\,l_s}\left(\frac{\beta_{y2} Q_x^{Arc}\Delta\xi_x + \beta_{x2} Q_y^{Arc}\Delta\xi_y}{\beta_{x1}\beta_{y2} - \beta_{x2}\beta_{y1}}\right)$$

$S_{01}^{total} \rightarrow$ For all sextupoles of 1 family

$l_s = 0{,}1\,m$

$$S_{02}^{total} = \frac{4\pi}{D_{x2}\,l_s}\left(\frac{\beta_{y1} Q_x^{Arc}\Delta\xi_x + \beta_{x1} Q_y^{Arc}\Delta\xi_y}{\beta_{x1}\beta_{y2} - \beta_{x2}\beta_{y1}}\right)$$

with
$$\begin{cases} D_{x1} = D_x^{QF} = 2{,}11\,m \\ \beta_{y2} = \beta_y^{QD} = \beta_x^{QF} = 184{,}97\,m \\ \mu_x^{Arc} = \mu_y^{Arc} = 47{,}7042 \\ \Delta\xi_x = \Delta\xi_y = -1{,}3 \\ \beta_{x2} = \beta_x^{QD} = \beta_y^{QF} = 29{,}19\,m \\ \beta_{x1} = \beta_x^{QF} = 184{,}97\,m \\ \beta_{y1} = \beta_y^{QF} = 29{,}19\,m \\ D_{x2} = D_x^{QD} = 0{,}98\,m \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} S_{01} = \dfrac{S_{01}^{total}}{8\times 23} \simeq 0{,}13\,m^{-3} \\ S_{02} = \dfrac{S_{02}^{total}}{8\times 23} \simeq -0{,}28\,m^{-3} \end{cases}}$$

11 $\gamma_{tr} = 55{,}68 \Rightarrow \gamma > \gamma_{tr}$ in the LHC (from injection till collision)

$\Rightarrow$ The LHC operates always above transition at injection and collision.

$\Rightarrow \boxed{\phi_s = 180\ deg}$

12 $\left.\begin{array}{l} f_{rev} = 11{,}245\ kHz \\ h = 35640 \end{array}\right\} \Rightarrow f_{RF} = h.f_{rev} \Rightarrow \boxed{f_{RF} \simeq 400{,}8\ MHz}$

$$\text{Bucket length} = \frac{C}{h} = \frac{2\pi R}{h}$$ and $C = 2\pi R = 26\,658{,}883\ m$

$$\Rightarrow \boxed{\begin{array}{l} \text{Bucket length} \simeq 74{,}8\ cm \\ \simeq 2{,}5\ ns \end{array}}$$

(13) $\Delta t = 20\,\text{min} = 20 \times 60 = 1200\,s$

$p_i = 450\ GeV/c \Rightarrow B_i = \frac{3{,}3356 \cdot p_i}{\rho} \simeq 0{,}535\ T$

$p_c = 7000\ GeV/c \Rightarrow B_c = \frac{3{,}3356 \cdot p_c}{\rho} \simeq 8{,}327\ T$

$$\Rightarrow \dot{B} = \frac{dB}{dt} = \frac{B_c - B_i}{\Delta t} \simeq 6{,}5\,mT/s$$

$$\Rightarrow \boxed{\dot{B} \simeq 6{,}5\,mT/s}$$

Below Transition

$$\phi_s^{BT} = \arcsin\left(2\pi \rho R \frac{\dot{B}}{\hat{V}_{RF}}\right) \simeq 0{,}03$$

Above

$$\Rightarrow \phi_s = \phi_s^{AT} = \pi - \phi_s^{BT} = \pi - 0{,}03 \simeq 3{,}11\ \text{rad}$$

$$\Rightarrow \boxed{\phi_s \simeq 178{,}26\ \text{deg}}$$

(14) The luminosity would be reduced by ~ 20% (see picture of page 22 of the course on Lumi) ⟶ Assuming here no crossing angle.

(15) We lost ~ 10% in luminosity (see page 18 of the course on Lumi).

(16) One should decrease the preparation time t_p to 4,6 h and my for ~ 10-11 h (plot of page 28 of the course on Lumi to be redone to find t_p such that $\frac{\langle L \rangle}{L_{peak}} = \frac{1}{2}$. The t_r can then be deduced).