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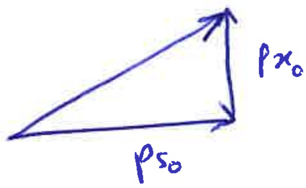
Physical explanation of the "Adiabatic damping"

↳ See for instance the nice explanation from Dugan (USPAS2002 course)

* Question: The Courant–Snyder "invariant" emittance ε decreases if we accelerate the particles. This is called "adiabatic damping".

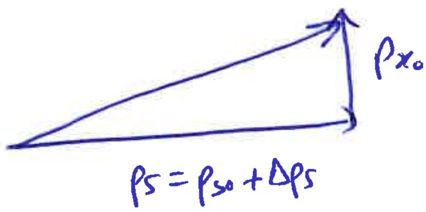
⇒ Physical explanation?

Let's assume a particle with momentum $p_0 \longrightarrow p_0^2 = p_{x0}^2 + p_{y0}^2 + p_{s0}^2$ and look at the motion for instance in the x-plane:



$$x_0' = \frac{p_{x0}}{p_{s0}}$$

If the particle is accelerated (longitudinally), $p_{s0} \longrightarrow p_{s0} + \Delta p_s$, but p_{x0} remains the same ⇒ the slope changes



$$x' = \frac{p_{x0}}{p_s}$$

$$x' = x_0' + \Delta x' = \frac{p_{x0}}{p_s} = \frac{p_{x0}}{p_{s0} + \Delta p_s} = \frac{p_{x0}}{p_{s0}\left(1 + \frac{\Delta p_s}{p_{s0}}\right)} = \frac{x_0'}{1 + \frac{\Delta p_s}{p_{s0}}}$$

$$\simeq x_0' \left(1 - \frac{\Delta p_s}{p_{s0}}\right)$$

$$\Longrightarrow \Delta x' = - x_0' \cdot \frac{\Delta p_s}{p_{s0}}$$

If now one considers a beam of particles, all with the same emittance ε_x but randomly distributed in phases, at an azimuthal point s_0 around the accelerator where the ellipse is not tilted, i.e. where $\alpha_x(s_0) = -\frac{\beta_x'(s_0)}{2} = 0 \Longrightarrow$ one has:

$$x(s_0) = \sqrt{\varepsilon_x \beta_x(s_0)} \cos\left(\mu_x(s_0) - \varphi_x\right)$$

$$\text{and} \quad x'(s_0) = -\sqrt{\frac{\varepsilon_x}{\beta_x(s_0)}} \sin\left(\mu_x(s_0) - \varphi_x\right) \quad \text{as } \alpha_x(s_0) = 0$$

The Courant-Snyder invariant emittance ε_x is given by (for a given particle)

$$\varepsilon_x = \beta_x(s_0)\, x'(s_0)^2 + \gamma_x(s_0)\, x^2(s_0) \quad \text{with } \gamma_x(s_0) = \frac{1 + \alpha_x(s_0)^2}{\beta_x(s_0)}$$

$$= \frac{1}{\beta_x(s_0)} \quad \text{here}$$

In the presence of acceleration, the change of the slope $x'_0 \longrightarrow x' = x'_0 + \Delta x'$ leads to a change of the emittance ε_x :

$$\Delta \varepsilon_x = 2\,\beta_x(s_0)\, x'(s_0)\, \Delta x'(s_0) \quad \text{(from the previous equation)}$$

$$= -2\,\underbrace{\beta_x(s_0)\, x'^2(s_0)}_{\varepsilon_x \sin^2(\mu_x(s_0) - \varphi_x)} \frac{\Delta p_s}{p_{s0}} \quad \text{(from last equation of page 1)}$$

$$\Rightarrow \Delta \varepsilon_x = -2\,\varepsilon_x \sin^2\left(\mu_x(s_0) - \varphi_x\right) \frac{\Delta p_s}{p_{s0}}$$

Averaging over all the particles to get the emittance of the beam, one gets:

$$\langle \Delta \varepsilon_x \rangle = -\varepsilon_x \frac{\Delta p_s}{p_{s0}} \quad \text{as } \langle \sin^2 \rangle = \frac{1}{2}$$

$$\Rightarrow \frac{\langle \Delta \varepsilon_x \rangle}{\varepsilon_x} = -\frac{\Delta p_s}{p_{s0}} \Rightarrow \boxed{\frac{\varepsilon_x(p_s)}{\varepsilon_{x0}} = \frac{p_{s0}}{p_s}}$$

$\Rightarrow$ The Courant-Snyder invariant emittance ε_x is then a decreasing function of the momentum $p_s = mv$
$= \gamma m_0 \beta c$

$\Rightarrow$ ε_x decreases with $\beta\gamma$ (similar result obtained for ε_y)

Defining the normalized emittance $\varepsilon_{n_{x,y}}$ by $\varepsilon_{n_{x,y}} = \beta\gamma\, \varepsilon_{x,y}$,

the normalized emittances do not change during acceleration.

transverse