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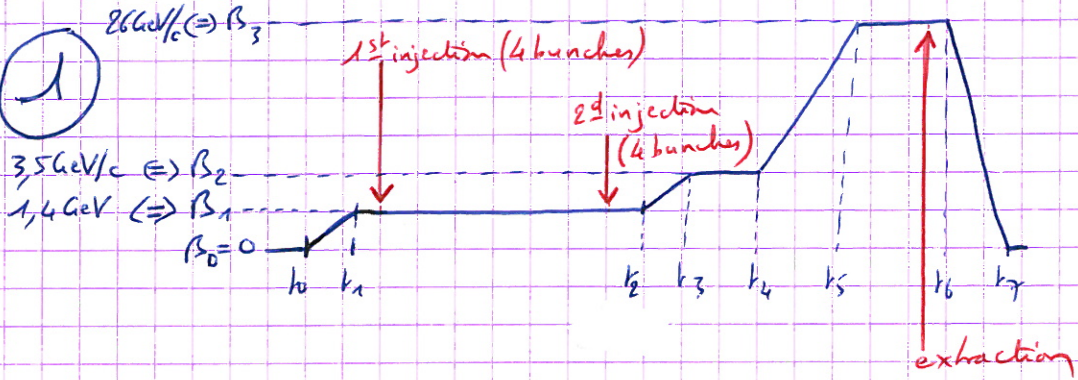
03/02/20[redacted]

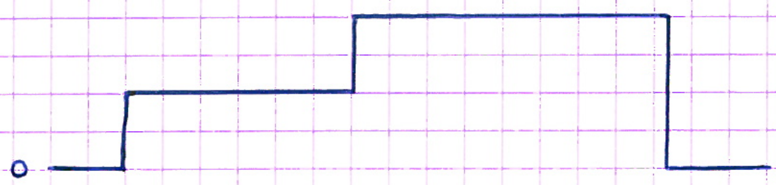
CORRECTIONS of "Longitudinal Beam Dynamics"
The CERN-PS Beam for LHC
JUAS, 03rd February 20[redacted]

(1)



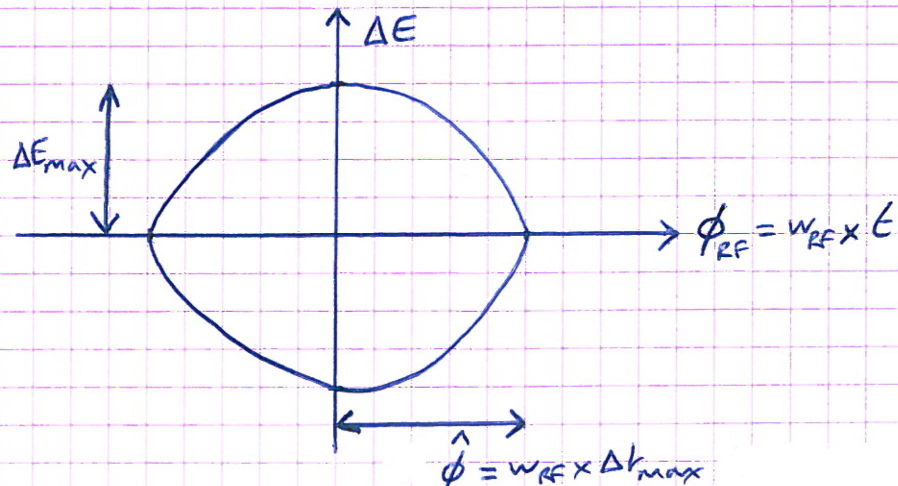
: Variation of the PS magnetic field versus time during the basic cycle.

(2)



: Variation of the PS beam intensity assuming no losses during the basic cycle.

(3) stationary bucket :



$= \pi$ for a stationary bucket

$$\Rightarrow \Delta t_{max} = \frac{\pi}{w_{RF}} = \frac{\pi}{2\pi \times f_{RF}} \quad \text{and} \quad f_{RF} = h \cdot f_{rev}$$

$$\Rightarrow \Delta t_{max} = \frac{1}{2 \cdot h \cdot f_{rev}}$$

Furthermore, for a stationary bucket below transition energy $\phi_s = 0 \Rightarrow F(\phi_s) = 2$

The matching condition between PSB and PS

$$\Rightarrow \left(\frac{\Delta E_{max}}{\Delta t_{max}}\right)_{PS} = \left(\frac{\Delta E_{max}}{\Delta t_{max}}\right)_{PSB}$$

$$\Rightarrow \frac{\Delta E^{PS}_{max}}{\Delta E^{PSB}_{max}} = \frac{\Delta t^{PS}_{max}}{\Delta t^{PSB}_{max}}$$

$$\Rightarrow \beta^{PS} \times \sqrt{\frac{e.\hat{V}_{RF}^{PS}.E_s^{PS}.2}{\pi.|\eta^{PS}|.h_{PS}}} \times \frac{1}{\beta^{PSB} \times \sqrt{\frac{e.\hat{V}_{RF}^{PSB}.E_s^{PSB}.2}{\pi.|\eta^{PSB}|.h_{PSB}}}}$$

$$= \frac{1}{2.h_{PS}.f_{rev}^{PS}} \times \frac{1}{\frac{1}{2.h_{PSB}.f_{rev}^{PSB}}}$$

The energy is the same at extraction of the PSB and injection in the PS $\Rightarrow$ $\beta^{PS} = \beta^{PSB}$

$E_s^{PS} = E_s^{PSB}$

$$\Rightarrow \sqrt{\frac{\hat{V}_{RF}^{PS}}{\hat{V}_{RF}^{PSB}} \times \left|\frac{\eta^{PSB}}{\eta^{PS}}\right| \times \left|\frac{h_{PSB}}{h_{PS}}\right|} = \frac{h_{PSB}}{h_{PS}} \times \frac{f_{rev}^{PSB}}{f_{rev}^{PS}}$$

$$\Rightarrow \boxed{\hat{V}_{RF}^{PS} = \hat{V}_{RF}^{PSB} \times \left|\frac{\eta^{PS}}{\eta^{PSB}}\right| \times \frac{h_{PSB}}{h_{PS}} \times \left(\frac{f_{rev}^{PSB}}{f_{rev}^{PS}}\right)^2}$$

A.N.: • $\hat{V}_{RF}^{PSB} = 8\,kV$

• $\eta^{PS} = \alpha_p^{PS} - \gamma^{-2}$ and $\alpha_p^{PS} = 0{,}027$

$\gamma = \frac{0{,}938 + 1{,}4}{0{,}938} = 2{,}493$

$\Rightarrow \eta^{PS} = -0{,}134$

- $\eta^{PSB} = \alpha_1^{PSB} - \gamma^{-2}$

$$= 0{,}0617 - \frac{1}{2{,}493^2}$$

$$= -0{,}09926$$

$$\Rightarrow \left|\frac{\eta^{PS}}{\eta^{PSB}}\right| = 1{,}35$$

- $h_{PSB} = 1$
- $h_{PS} = 8$

$$\Big\} \Rightarrow \frac{h_{PSB}}{h_{PS}} = \frac{1}{8}$$

- $v = \beta . c = R . w_{rev}$ and $w_{rev} = 2\pi f_{rev}$

$$\Rightarrow f_{rev} = \frac{\beta . c}{2\pi R}$$

$$\Rightarrow f_{rev}^{PS} = \frac{\beta . c}{2\pi R^{PS}} \text{ and } f_{rev}^{PSB} = \frac{\beta . c}{2\pi R^{PSB}}$$

$$\Rightarrow \frac{f_{rev}^{PSB}}{f_{rev}^{PS}} = \frac{R^{PS}}{R^{PSB}} = 4 \Rightarrow \left(\frac{f_{rev}^{PSB}}{f_{rev}^{PS}}\right)^2 = 16$$

Therefore: $\hat{V}_{RF}^{PS} = 8\,kV \times 1{,}35 \times \frac{1}{8} \times 16 = 21{,}6\,kV$

$$\Rightarrow \boxed{\hat{V}_{RF}^{PS} = 21{,}6\,kV}$$

4 • By definition, $f_s = \frac{1}{2\pi}\sqrt{\frac{e . \hat{V}_{RF} . \eta . h . c^2}{2\pi E_s R_s^2} \cos\phi_s}$

$e = 1{,}6 . 10^{-19}$

$\hat{V}_{RF} = 21{,}6\,kV$

$\eta = -0{,}134$

$h = 8$

$c = 3 . 10^8$

$\phi_s = 0$

$E_s = 1{,}4 + 0{,}938 = 2{,}338$ GeV

$R_s = 100$ m

$$\Rightarrow \boxed{f_s = 599{,}45 \text{ Hz} \simeq 600 \text{ Hz}}$$

- $Q_s = \dfrac{f_s}{f_{rev}} = \dfrac{599{,}45}{\left(\beta c / 2\pi R\right)}$ and $\beta = \sqrt{1 - \dfrac{1}{\gamma^2}} = 0{,}916$

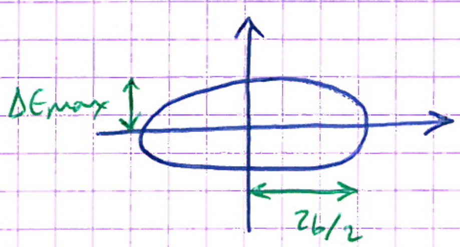
$$= \frac{599{,}45}{\left(0{,}916 \times 3.10^8 / 2\pi \times 100\right)}$$

$$\Rightarrow \boxed{Q_s = 1{,}37 . 10^{-3} \ll 1}$$

5 $\left(\dfrac{\Delta p}{p_0}\right)_{max} = 2.10^{-3} \Rightarrow \Delta E_{max} = \beta . c . p_0 \times \dfrac{\Delta p}{p_0}$ [eV]

$$= 0{,}916 \times 3.10^8 \times \sqrt{\left[(1{,}4+0{,}938)^2 - 0{,}938^2\right]} \times \frac{10^9}{3.10^8} \times 2.10^{-3}$$

$$\Rightarrow \Delta E_{max} = 3923389{,}575 \text{ eV}$$



$$\text{Emittance} = \pi \times \Delta E_{max} \times \frac{\tau_b}{2} = \varepsilon_\ell$$

$$\Rightarrow \tau_b = \frac{2 . \varepsilon_\ell}{\pi . \Delta E_{max}} = \frac{2 \times 1}{\pi \times 3923389{,}575}$$

$$= 162{,}3 \text{ ns}$$

$$\boxed{\tau_b = 162{,}3 \text{ ns} \simeq 160 \text{ ns}}$$

→ This is the same order of magnitude as what was given in the Introduction

(6) $B.\rho\,[T.m] = 3{,}3356 \times p\,[GeV/c]$ and $\rho = 70\,m$

- At injection: $E_c = 1{,}4\ GeV \Rightarrow p = 2{,}14\ GeV/c$

$$\Rightarrow \boxed{B_{inj} = 0{,}1\ T}$$

- At ejection: $p = 26\ GeV/c \Rightarrow \boxed{B_{ej} = 1{,}24\ T}$

(7) Assuming a constant orbit during the acceleration, one has

$$\frac{dp}{p} = \gamma^2 \frac{df}{f} + \gamma^2 \frac{dR}{R} \quad \text{and } dR = 0$$

$$\Rightarrow \boxed{\frac{dp}{p} = \gamma^2 \frac{df}{f}}$$

Therefore: <u>when the momentum p increases between the injection and the ejection energies, the frequency f increases also</u>

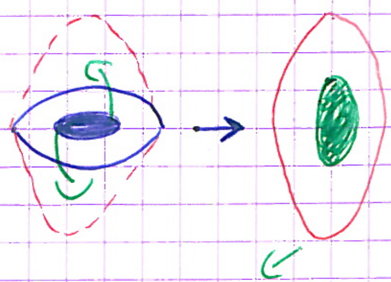
(8) <u>The minimum value for the SPS acceptance is the emittance of the bunch at 26 GeV/c, i.e. 0,3 eV.s.</u>

(9) • $f_{RF} = h\,f_{rev}$ with $h = 84 \Rightarrow f_{RF} = 84\,f_{rev} = 84 . \frac{\beta c}{2\pi R \beta s} \simeq 477{,}5\ kHz \times 84$

$\Rightarrow f_{RF} \simeq 40\ MHz$ and $T_{RF} = \frac{1}{f_{RF}} \simeq 25\ ns$

$\Rightarrow$ Bucket length $\simeq 25\ ns$.

- Bunch rotation, i.e. non adiabatic $\Leftarrow$ voltage increase



To be extracted like this, after $\frac{1}{4}$ of synchrotron period.

(10) • $\dot{B} = 1\ T/s \Rightarrow (\Delta E)_{turn} = 2\pi R \rho e \dot{B} \simeq 44\ keV/turn$

$p_1 = 3{,}5\ GeV/c \to p_2 = 26\ GeV/c \Rightarrow E_1 = \sqrt{p_1^2c^2 + E_0^2} = 3{,}62\ GeV$

$E_2 = \sqrt{p_2^2c^2 + E_0^2} = 26\ GeV$

$\Rightarrow \Delta E = E_2 - E_1 = 22{,}4\ GeV$

$\Rightarrow \Delta t = \frac{\Delta E}{(\Delta E)_{turn}} \simeq 509146\ turns \simeq 509146 \times T_{rev}^{\,2{,}1\,\mu s} \simeq 1\ s$

- $\left.\begin{array}{l}(\Delta E)_{turn} = e\hat{V}_{RF} \sin\phi_s \\ \hat{V}_{RF} = 100\ kV\end{array}\right\} \Rightarrow \sin\phi_s \simeq \left|\begin{array}{l}26\ deg \text{ at } 5\ GeV/c, \text{ i.e. before transition} \\ 154\ deg \text{ at } 10\ GeV/c, \text{ i.e. after transition.}\end{array}\right.$