Longitudinal beam dynamics examination

(1h30 – Free access to lecture notes and paper documents)

This exam is composed of two independent exercises totalling 82 points. The marks will be normalized to 20

Exercise 1: The Large Electron Positron Collider (LEP) → 42 pts

The Large Electron Positron collider (LEP) at CERN reached a beam energy of 104.5 GeV in 2001, before being shut down. The main parameters of LEP are described below:

Circumference	26658.9 m
Bending radius	3.026 km
Momentum compaction factor	1.4e-4
RF frequency	352 209 188 Hz

1. Compute the relativistic factor gamma and the magnetic field in the LEP dipoles at the beam energy of 104.5 GeV (4 pts).

2. What is the maximum number of bunches that could be stored in LEP? How is this number referred to in the course? (2 pts)

3. In the absence of synchrotron radiation and acceleration:
   - What is the phase of the synchronous particle? (4 pts)
   - Consider a particle with a higher momentum than the synchronous particle, is its revolution frequency larger or lower than the revolution frequency of the synchronous particle? (2 pts)
   - Compute the height of the RF bucket assuming a peak effective RF voltage of 100 MV (2 pts).
   - Draw qualitatively the bucket in that case, with a matched bunch that would have a bunch length of 12 mm (6 pts).
   - Is it reasonable to assume that all particles oscillate with the same synchrotron tune in these conditions? (2 pts)

4. We now account for synchrotron radiation. From the JUAS synchrotron radiation course (Rasmus Ischebeck, Machine Physics 1, slide 11), the energy loss per turn $(\Delta E)_{turn}$ due to synchrotron radiation for relativistic electrons can be expressed as $(\Delta E)_{turn} = \frac{e^2\gamma^4}{3\varepsilon_0\rho}$, where ε_0 is vacuum permittivity, γ is the relativistic gamma factor and ρ is the bending radius.

- How much energy does a LEP electron lose per turn? How much does it represent in percentage of its total energy? (2 pts)
- Assuming that the energy loss per turn from synchrotron radiation needs to be compensated by the RF cavities, express the RF voltage as a function of gamma and the phase of the synchronous particle (for the case of no acceleration). (4 pts)
- What is the consequence on the synchronous phase φ_s? (2 pts)
- The peak voltage of a LEP copper cavity is 3 MV, and its accelerating gap is 277 mm. Compute the transit time factor of the LEP cavity. How many such cavities do you need to produce an effective voltage of 3.6 GV? What is the phase of the synchronous particle in that case (in degrees)? (6 pts)
- Assuming that the power dissipated in a cavity is $P_{diss}=V^2/R_s$ with V the peak RF voltage in the cavity, and R_s the shunt impedance of the cavity (which is assumed independent of the RF voltage). How does this dissipated power vary with the beam energy? Is this dependence favourable for increasing the energy in a lepton machine? What machine parameter would you change to reach higher energy? (6 pts)

<u>Exercise 2: answer the questions below</u> (a: 14 pts, b: 12 pts, c: 14 pts) → 40 pts

a. Draw a stationary bucket below transition and:
   - Give the minimum and maximum phase for particles inside the bucket(2 pts)
   - Locate the synchronous particle (2 pts)
   - Locate the separatrix (2 pts)
   - Draw the trajectory of a particle inside this bucket in phase space and its direction.
     1. In which parts of its trajectory does the particle arrive later than the synchronous particle at the RF cavity? (1 pt)
     2. In which parts of its trajectory does the particle arrive earlier than the synchronous particle at the RF cavity? (1 pt)
     3. In which parts of its trajectory does the particle need to travel a larger circumference than the synchronous particle? (1 pt)
     4. In which parts of its trajectory does the particle need to travel a smaller circumference? (1 pt)
   - Draw the trajectory of a particle outside of the bucket and its direction. (2pts)
   - Is the synchrotron tune of a particle near the separatrix smaller or larger than the synchrotron tune of the synchronous particle? (2pts)
   - What happens to the beam if the RF voltage is switched off? (2 pts)

b. A bunch of particles inside the bucket defined in (a) is accelerated up to transition
   - What happens to the particles outside of the bucket when acceleration starts? (2 pts)

- Describe qualitatively what happens to the bunch and to the bucket during acceleration and close to transition (4 pts)
- What needs to be done from RF point of view when crossing transition? What happens if nothing is done? Which parameter changes sign at transition energy? (4 pts)
- What happens to the beam if the RF voltage is switched off when the beam is at transition energy? (2 pts)

c. The beam energy is now above transition. Draw a stationary bucket and:
   - Give the minimum and maximum phase for particles inside the bucket; (2 pts)
   - Locate the synchronous particle; (2 pts)
   - Locate the separatrix; (2 pts)
   - Draw the trajectory of a particle inside this bucket in phase space and its direction. (2 pts)
     1. In which parts of its trajectory does the particle arrive later than the synchronous particle at the RF cavity? (1 pt)
     2. In which parts of its trajectory does the particle arrive earlier than the synchronous particle at the RF cavity? (1 pt)
     3. In which parts of its trajectory does the particle need to travel a larger circumference than the synchronous particle? (1 pt)
     4. In which parts of its trajectory does the particle need to travel a smaller circumference? (1 pt)
   - Draw the trajectory of a particle outside of the bucket and its direction. (2 pts)
   - Is the synchrotron tune of a particle near the separatrix smaller or larger than the synchrotron tune of the synchronous particle? (2 pts)
   - What happens to the beam if the RF voltage is switched off? (2 pts)

Physical constants:

- Elementary charge: $e = 1.60\ 10^{-19}$ C
- Electron mass: $m_e = 9.11\ 10^{-31}$ kg
- Proton mass: $m_p = 1.67\ 10^{-27}$ kg
- Speed of light: $c = 3.00\ 10^{8}$ m/s
- Vacuum permittivity: $\varepsilon_0 = 8.85\ 10^{-12}$ F/m