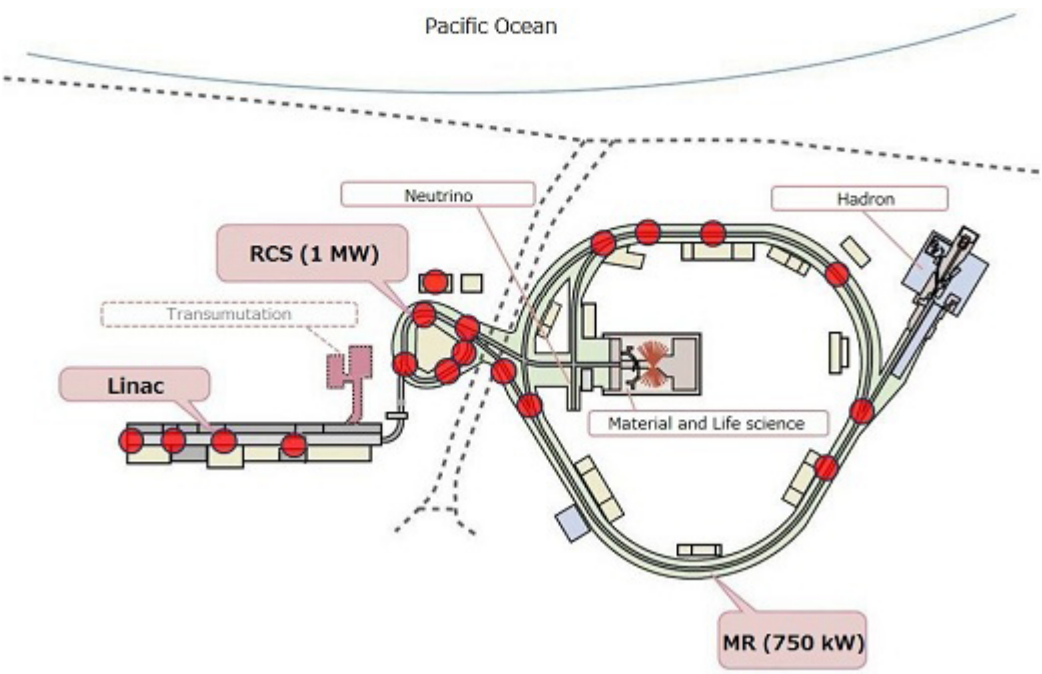
JUAS session 2017 February 8th 2017

Longitudinal beam dynamics examination

(1h30 – Free access to lecture notes and paper documents)

Context (not needed to answer the questions):

We consider a ring with similar parameters to the Main Ring (MR) of the Japan Proton Accelerator Research Complex (J-PARC). Its proton beam is used in 3 experimental facilities to study materials science, life science, hadron physics, neutrinos and transmutation. It was damaged by the Great East Japan earthquake on March 11th 2011 but the complex could restart after several months of repair. The Nobel Prize in physics was attributed in 2015 to Takaaki Kajita, a member of the T2K collaboration, which uses the neutrino beam generated in the J-PARC facility.



Questions :

With a circumference of 1567.5 m, the Main Ring accelerates protons from 3 GeV (kinetic injection energy) to 50 GeV (kinetic top energy) in 1.9 s.

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1) Assuming that the ring contains 96 dipole magnet and the field at injection energy is 0.143 T:
   a. Compute the bending radius and the magnetic length of a dipole. What proportion of the ring is made of dipoles?
   b. Assuming the magnetic length of a dipole does not change with energy, show that the magnetic field needed at top energy is 1.9 T.
   c. How does the revolution frequency change between injection and top energy?
   d. The harmonic number is h=9. How does the RF frequency change between injection and top energy?

2) The horizontal and vertical betatron tunes are respectively Qx=22.4 and Qy=20.2. We assume (until question 4 included) that the gamma transition is close to the horizontal tune, as for usual FODO cells.

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a. Explain physically what the transition energy is.
b. Compute the momentum compaction factor and the slippage factor at injection and top energy. Is the beam energy crossing transition energy during the cycle?
c. What needs to be done when crossing transition from the point of view of longitudinal dynamics?

3) An effective RF voltage of $\hat{V}_{RF} = 80$ kV is used at injection energy (no acceleration).

a. Compute the synchrotron tune Q_s (for small amplitude particles). How many machine turns does it take for small amplitude particles to perform a turn in the longitudinal phase space? How does the synchrotron tune qualitatively vary with amplitude in this phase space?
b. Do a schematic drawing of the bucket in the longitudinal phase space at injection: for instance in energy ΔE as a function of phase as below and in the course (ΔE = E-E_s with E the energy and E_s the energy of the synchronous particle). Indicate the synchronous phase and the maximum acceptance in terms of energy and phase.

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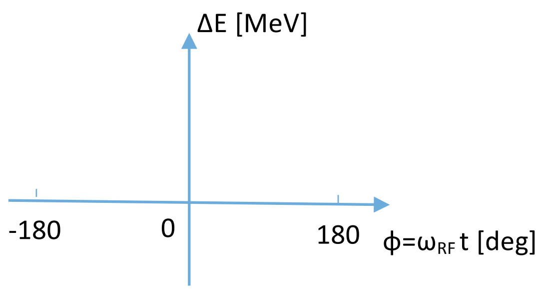

c. Do a similar schematic drawing of the bucket at top energy with that same voltage, and compute the acceptance. Is there more acceptance – in relative and absolute terms – at injection or top energy?
d. A bunch of maximum energy spread +/-5 MeV is injected into the MR. In a similar bucket as in question 3)b):
   - draw qualitatively what happens if the bunch is well matched.
   - draw qualitatively what happens if that same bunch is now injected with a global energy error of +5 MeV.
   - What order of magnitude of energy error is allowed before particles from this bunch are lost from the bucket?

4) We assume that the accelerating gap is 35 mm long, and that the magnetic field increase is linear with time along the ramp (1.9 s).

   a. What is the minimum effective RF voltage $\hat{V}_{RF} = V_{RF}\, T_a$ (with V_{RF} the voltage applied along the RF cavity gap and T_a the transit time factor) that allows for a non-zero longitudinal acceptance during the ramp?

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   b. The maximum effective RF voltage $\hat{V}_{RF}$ specified in the design report is 280 kV. How much RF voltage V_{RF} should be applied across the gap to reach that specification (assuming the voltage is constant along the gap)? Does the transit time factor change along the acceleration from injection to top energy?

   c. We assume that the maximum voltage (280 kV) is used along the ramp. Do a schematic drawing of the accelerating bucket very close to injection and top energy in the longitudinal phase space (as in question 3)b)) and compute (or estimate from the slides) the energy error and phase acceptances for both cases.

5) In reality, contrary to the assumption in question 2), the MR of JPARC was designed to be operated with a negative momentum compaction factor: α_p = - 0.001.

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   a. How much is the relativistic gamma transition in this case? What is the slippage factor at injection and top energy? Discuss the result.

   b. Why would you design a machine with a negative momentum compaction factor?

   c. With $\hat{V}_{RF}$ =80 kV, compute the synchrotron tunes and acceptance at injection and top energy.

Physical constants:

- Elementary charge: e = $1.60\ 10^{-19}$ C
- Proton mass: $m_p = 1.67\ 10^{-27}$ kg
- Speed of light: c = $3.00\ 10^{8}$ m/s