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Corrections of the Longitudinal Beam Dynamics Examination

(1) In the six cases we see the longitudinal phase space $(\phi, \Delta E)$, with the separatrix (line separating the stable region and the unstable region) in red and the particles (each dot represents one particle) in blue.
The separatrix (defining the RF bucket) is symmetric ~~and~~ extends from -180 deg ($-\pi$) to $+180$ deg ($+\pi$), which means that it is the case of a stationary bucket (i.e. no acceleration/deceleration). This can also be seen by the fact that the synchronous phase $\phi_s = 0$. Furthermore, $\phi_s = 0$ means that we are below transition as beam stability requires $\eta \cos\phi_s > 0$: as $\phi_s = 0$, $\cos\phi_s = 1$ and $\eta \cos\phi_s = \eta$. Therefore $\eta > 0$ and as $\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$, it means that $\gamma < \gamma_{tr}$.
Below transition, a higher energy means a higher revolution frequency (or shorter revolution period) and therefore a particle gaining energy will arrive earlier $\Rightarrow$ the particles are oscillating anti-clockwise.

a) The bunch is too long in the RF bucket $\Rightarrow$ It oscillates and after a $\frac{1}{4}$ of a synchrotron period it leads to the right picture. We start to see a "S" shape due to the fact that the particles at the head and tail are oscillating with a smaller synchrotron frequency.

b) Same case as a) but after many turns. And if we wait more, the beam will occupy a much larger area (emittance) than the initial one.

c) The bunch is injected with a phase error and it oscillates. Here again all the particles are not oscillating with the same frequency. If we wait more, the beam will occupy also a much larger area than the initial one (and will become a "ring" and not a "disk" as in b)).

d) Same as c) but starting from an energy error.

e) The energy spread of the bunch is too big as it is larger than the energy acceptance of the bucket $\Rightarrow$ The particles outside of the bucket are being lost.

f) The distribution of particles remains constant with time $\Rightarrow$ The beam is "matched" longitudinally.

↳ Conclusion: The case f) is what we are aiming for when a bunch is injected into a synchrotron, which means that:

i) There should be no phase error → Not as c)

ii) There should be no energy error → Not as d)

iii) The bunch should be injected inside the RF bucket, i.e. there should be a sufficiently large energy acceptance and length acceptance → Not as e)

iv) The bunch should be "matched" (as in f) and not as in a) and b)), i.e. the ratio between the height and width of the bunch should correspond to the RF bucket.

2) a) $p_i = 26\ \mathrm{GeV}/c$

i) $f_{rev} = \dfrac{v_i}{C}$ with $v_i = \beta_i \cdot c$, $\beta_i = \sqrt{1 - \dfrac{1}{\gamma_i^2}}$, $\gamma_i = \dfrac{E_{t_i}}{E_0}$

and $E_{t_i} = \sqrt{E_0^2 + p_i^2 c^2}$

$\Rightarrow E_{t_i} = 26\ \mathrm{GeV}$

$\gamma_i = 27{,}74$

$\beta_i = 0{,}99935 \simeq 1$

$v_i \simeq c$

and $f_{rev} \simeq 43{,}351\ \mathrm{kHz}$

ii) $\left.\begin{array}{l} f_{RF} = h \cdot f_{rev} \\ h = 4620 \end{array}\right\} \Rightarrow f_{RF} \simeq 200\ \mathrm{MHz}$

$\gamma_t = 22{,}8$

iii) $\eta_i = \dfrac{1}{\gamma_i^2} - \dfrac{1}{\gamma_{tr}^2} = \dfrac{1}{\gamma_i^2} - \alpha_p = -6{,}2 \cdot 10^{-4}$

→ Above transition

iv) $Q_{s_i} = \sqrt{\dfrac{e V_{RF} h}{2\pi \beta_i^2 E_{t_i}} \eta_i \cos\phi_{s_i}}$

$\eta_i < 0 \Rightarrow \cos\phi_{s_i} < 0 \Rightarrow \phi_{s_i} = \pi$ as there is no acceleration/deceleration (injection plateau at 26 GeV/c).

$\Rightarrow Q_{s_i} = \sqrt{\dfrac{3.10^6 \times 4620 \times 6{,}2.10^{-4}}{2\pi \times 1^2 \times 26.10^9}} \simeq 7{,}3.10^{-3}$

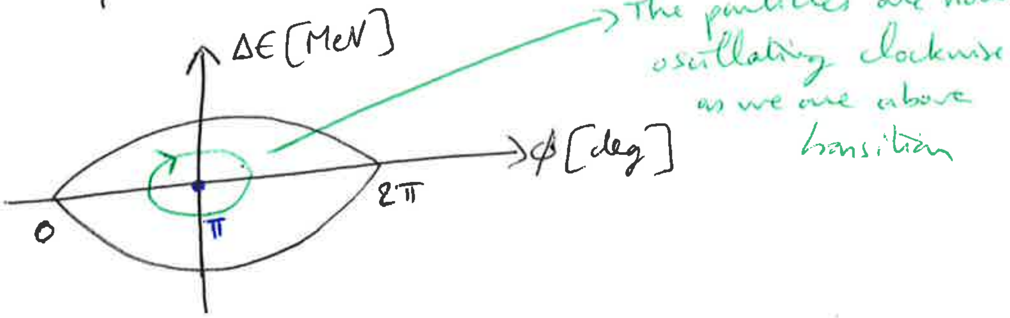
v) $n = \dfrac{1}{Q_{s_i}} \simeq 138$ turns

vi) $\tau_b^{max} = \dfrac{1}{f_{RF}} \simeq 5\,ns$

$\Delta E^{max} = \sqrt{\dfrac{2\beta_i^2 E_{t_i} e V_{RF}}{\pi h |\eta|}} = \sqrt{\dfrac{2 \times 1^2 \times 26.10^9 \times 3.10^6}{\pi . 4620 . 6{,}2.10^{-4}}}$

$\simeq 132$ MeV

- From page 62, when $\begin{cases} \eta < 0 \\ \text{stationary bucket} \end{cases} \Rightarrow \phi_s = \pi$



b) Acceleration up to $p_e = 450$ GeV/c

- $E_{te} = \sqrt{E_0^2 + p_e^2 c^2} \simeq 450$ GeV

- $\Delta t = \dfrac{E_{te} - E_{ti}}{dE/dt} = \dfrac{450 - 26}{78} \simeq 5{,}4\,s$

- $\left.\begin{aligned} \Delta E_{turn} &= \frac{dE}{dt} \times T_{rev} \\ T_{rev} &= \frac{1}{f_{rev}} \simeq 23\,\mu s \end{aligned}\right\} \Rightarrow \Delta E_{turn} \simeq 1{,}8$ MeV

- $\phi_{s,acc} = \pi - \phi_{s,acc}^{BT}$

$$= \pi - \mathrm{Arcsin}\left(\frac{\Delta E_{turn}}{e V_{RF}}\right)$$

$\simeq 2{,}5$ rad

$\simeq 143{,}1$ deg $\longrightarrow \cos(\phi_{s,acc}) = -0{,}8$

- $B_e \rho\,[\mathrm{Tm}] = 3{,}3356\, p_e\,[\mathrm{GeV/c}]$

$$\Rightarrow \rho = \frac{3{,}3356 \times 450}{2{,}03} \simeq 739{,}4 \text{ m}$$

$$\theta_d = \frac{2\pi}{N_d} \simeq 8{,}4 \cdot 10^{-3} \text{ rad}$$

$$L_d = \rho \cdot \theta_d \simeq 6{,}2 \text{ m}$$

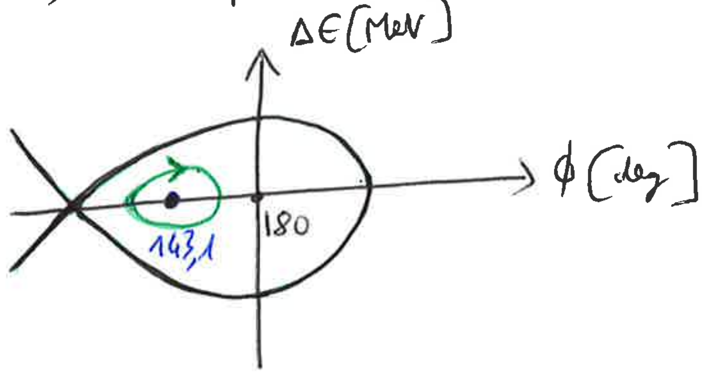
- $$\frac{dB}{dt} = \frac{V_{RF} \cdot \sin\phi_{s,acc}}{C\rho} \simeq 0{,}35 \text{ T/s}.$$

- $B_i \rho\,[\mathrm{Tm}] = 3{,}3356\, p_i\,[\mathrm{GeV/c}]$

$\Rightarrow B_i \simeq 0{,}12$ T

- From page 62, when $\left|\begin{array}{l} - \eta < 0 \\ - \text{acceleration} \end{array}\right.$ $\Rightarrow \phi_s \in [90°, 180°]$

↳ we computed above $143{,}1$ deg.



c) Deceleration down to $p_{low} = 20$ GeV/c

- $\eta_{low} = \dfrac{1}{\gamma_{low}^2} - \alpha_p$ with $\gamma_{low} = \dfrac{E_{low}}{E_0} = \dfrac{\sqrt{E_0^2 + p_{low}^2 c^2}}{E_0}$

$\simeq 21{,}3$

$\Rightarrow \eta_{low} \simeq 2{,}7 \cdot 10^{-4} \longrightarrow$ Below transition

- Yes, the particles will cross transition energy as at 26 GeV/c, $\eta < 0$ and at 20 GeV/c, $\eta > 0$.

↳ The transition energy is the energy that corresponds to $\eta = 0$, i.e. this is the energy for which the velocity variation is compensated by the trajectory variation and therefore the revolution frequency remains constant (despite a change of beam momentum).

- From page 62, when $\eta < 0$ (i.e. still above transition), ~~deceleration~~ deceleration

$$\Rightarrow \phi_{s_1} \in [180°, 270°]$$

and when $\eta > 0$ (i.e. below transition), deceleration

$$\Rightarrow \phi_{s_2} \in [270°, 360°]$$

↳ If we want to use the same RF voltage, deceleration rate = acceleration rate, then one has:

$$\phi_{s_1} = 360 - \phi_{s,acc} \simeq 217 \text{ deg} \rightarrow \cos(\phi_{s_1}) = -0{,}8$$

and $\phi_{s_2} = 360 + (180 - \phi_{s_1}) \simeq 323 \text{ deg} \simeq -37 \text{ deg} \rightarrow \cos(\phi_{s_2}) = +0{,}8$