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Correction of the Longitudinal Beam Dynamics Examination

1) Fill in the Table using the following formulae

$E_t = E_0 + E_k$

$\gamma = \frac{E_t}{E_0}$ $\beta = \sqrt{\frac{\gamma^2-1}{\gamma^2}}$

$p = \beta\gamma E_0$ $B\rho\,[Tm] = \frac{p\,[GeV/c]}{0.3}$

	E_k	E_t (MeV)	p [MeV/c]	β	γ	$B\rho$ [Tm]
Injection	50 MeV	988,26	310,364	0,31405	1,05329	1,03455
Extraction	1.4 GeV	2'338,26	2141.76	0,91596	2,49212	7,139
used in the following (160 MeV)	160 MeV	1098.26	570,827	0,51976	1,17053	1,90276

2) No! It cannot be considered ultrarelativistic since $\beta = 0{,}3$ at inj. and it's only reaching 0,9 at flat-top

$$f_{rev} = \left(\frac{2\pi R}{\beta c}\right)^{-1} = \begin{cases} 0{,}599'379 \text{ MHz} @ 50 \text{ MeV} \\ 1{,}748'149 \text{ MHz} @ 160 \text{ MeV} \end{cases}$$

3) • $h = 1 \Rightarrow f_{RF} = f_{rev}$

• The RF frequency is increasing by a factor ~2.8!
(as the f_{rev})

• the max bucket length is 2π (stationary) and since there is only $h=1$ bunch in the machine $\Rightarrow L = 2\pi R = 157\,m$

4) $\gamma_{tr} = \frac{1}{\sqrt{\alpha_c}} = 4.156 > \gamma_{ej} > \gamma_{inj}$ $\Rightarrow$ No! The PSB is always below transition

$$\eta \equiv \frac{df/f}{dp/p} = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$$

/ $\eta > 0$ below transition $\Rightarrow$ an increase of energy corresponds to an increase of f_{rev} (β increases faster than path length)

\ $\eta < 0$ above transition $\Rightarrow$ an increase of energy corresponds to a decrease ~~an increase~~ of f_{rev}

5) $\phi_s = 0$ since on the "flat top" $\dot{B} = 0$
If we were above transition $\phi_s = \pi$

6) $\Delta x = D \frac{\Delta p}{p}$ $\Rightarrow$ $\left|\frac{\Delta p}{p}\right|_{max} = 0{,}01 = 1\%$

$|\Delta x| < 3\,cm$, D: $3\,m$

7) $\frac{\Delta f}{f} = \eta \frac{\Delta p}{p}$ since $B = \text{const}$ at extraction

(we cannot say $\Delta R = 0$ and indeed it is not !)

$$\eta = \frac{1}{\gamma_{ej}^2} - \frac{1}{\gamma_{tr}^2} = 0,103113$$

$$\Delta f = \pm \eta \left|\frac{\Delta p}{p}\right|_{max} f_0 = \pm 1,803 \text{ kHz}$$

$$f = f_{rev,ej} \pm \Delta f = \begin{cases} 1,749952 \text{ MHz} \\ 1,746712 \text{ MHz} \end{cases}$$

$f_{rev,ej}$: 1,748149 MHz

(pay attention, if you want to add ± 1.8 kHz you need to compute the f_{rev} with this precision at least !)

8) $N_b^{160\,MeV} = \frac{(\beta\gamma^2)^{160\,MeV}}{(\beta\gamma^2)^{50\,MeV}} N_b^{50} = \frac{0,71213}{0,348} N_b^{50\,MeV} = 2,044\, N_b^{50\,MeV}$

9) $B_{inj} = \frac{B\rho}{\rho}$, $\rho = \frac{L_d}{(2\pi/32)} \simeq 8,24$ m

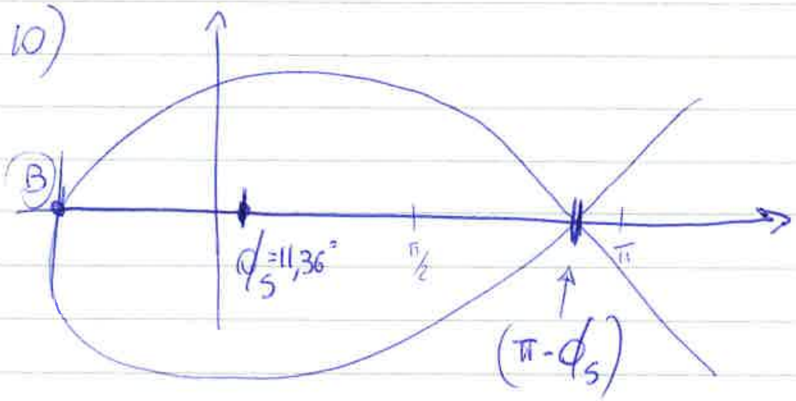
$$B_{inj} = \begin{cases} 0,125 \text{ T @ } 50\,MeV \\ 0,231 \text{ T @ } 160\,MeV \end{cases}$$

10) $\phi_s = \arcsin\left(\frac{2\pi R \dot{(B\rho)}}{V_{rf}}\right) = 0,197 \text{ rad} = 11,36^\circ < 90^\circ$ because we are below tr. (Otherwise we would have chosen the other solution)

$\dot{B\rho} = \frac{d(B\rho)}{dt}$ since $\rho = \text{cst}$

=> continue ...

10)



To compute the max bunch length we need to find the point B.

Let's take the eq of the separatrix and impose $\dot{\phi} = 0$
↑ Slide #80

Need to solve for ϕ :

$$\boxed{\cos\phi + \phi \sin\phi_s = \cos(\pi - \phi_s) + (\pi - \phi_s)\sin\phi_s}$$

11) $T_{rev}^{(160\,MeV)} = \frac{2\pi R}{\beta c} = 1.007\ \mu s$

$\Delta B = \frac{dB}{dt} \times \frac{1}{\rho} \times 100\,T_{rev} = 1.224 \cdot 10^{-4}\ T$

If $R = \text{const} \Rightarrow \frac{\Delta p}{p} = \frac{\Delta B}{B} = \frac{1.224 \cdot 10^{-4}\ T}{0,23094\ T} = 5.3 \cdot 10^{-4}$

$\frac{\Delta f}{f} = \frac{1}{\gamma^2}\frac{\Delta p}{p} = 3.87 \cdot 10^{-4}$ $\quad \Delta f = \frac{\Delta f}{f} \times \frac{1}{T_{rev}^{(160\,MeV)}} = \boxed{380\ Hz}$

12) $\Delta E = \underset{8\,kV}{V_{RF}} \sin\underset{11,36°}{\phi_s} \times 100t = 157.6\ keV \Rightarrow \boxed{E_k = 160.1576\ MeV}$