1) Fill in the Table using the following formulae

\[ E_t = E_e + E_r \]
\[ x = \frac{E_t}{E_e} \]
\[ \beta = \sqrt{\frac{E_t - E_e}{E_e}} \]
\[ p = \beta \sqrt{E_e} \]
\[ B_g[T_m] = \frac{p}{0.3} \]

<table>
<thead>
<tr>
<th></th>
<th>( E_r )</th>
<th>( E_t ) (GeV)</th>
<th>( p ) [MeV/c]</th>
<th>( \beta )</th>
<th>( x )</th>
<th>( B_g[T_m] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection</td>
<td>50 MeV</td>
<td>388.26</td>
<td>310.366</td>
<td>0.31605</td>
<td>1.05323</td>
<td>1.03455</td>
</tr>
<tr>
<td>Extraction</td>
<td>1.6 GeV</td>
<td>2333.26</td>
<td>2141.76</td>
<td>0.91596</td>
<td>2.49212</td>
<td>7.138</td>
</tr>
<tr>
<td>(160 MeV)</td>
<td>160 MeV</td>
<td>1038.26</td>
<td>570.827</td>
<td>0.51376</td>
<td>1.17053</td>
<td>1.90276</td>
</tr>
</tbody>
</table>

2) No! It cannot be considered ultrarelativistic since \( \beta = 0.3 \) at inj. and it's only reaching 0.9 at flat-top

\[ f_{ew} = \left( \frac{2\pi R}{\beta c} \right)^{-1} = \begin{cases} 0.593 & 378 \text{ MHz at 50 GeV} \\ 1.748145 & 160 \text{ MeV} \end{cases} \]
3) \( b = 1 \) \( \Rightarrow \) \( f_{FE} = 4 f_{HR} \)

- The RF frequency is increasing by a factor \( \sim 2.8 \)
- The max bucket length is \( 2 \tau \) (stationary) and since there is only \( b = 1 \) bunch in the machine \( \Rightarrow \) \( L = 2 \tau R = 157 \text{ m} \)

4) \( \chi = \frac{1}{\sqrt{2} c} = 4.156 > \chi_{ij} > \chi_{ij} \) \( \Rightarrow \) No! The PSB is always

Below Transition:

\[
\eta = \frac{d f / f}{d p / p} = \frac{4}{x^2} - \frac{4}{x_{inj}^2}
\]

\[\eta > 0 \quad \text{below transition} \Rightarrow \text{an increase of energy corresponds}
\]
\[\text{to an increase of flux (Bernstein)}
\]
\[\eta < 0 \quad \text{above transition} \Rightarrow \text{an increase of energy corresponds}
\]
\[\text{to a decrease of flux}
\]

5) \( \phi = 0 \) since on the "flat top" \( \vec{B} = 0 \)

\[\phi \text{ we were above transition } \phi = \pi \]

6) \( \Delta x = \frac{D \Delta \phi}{P} \)

\[\Delta x \leq 3 \text{ cm} \]

\[\Rightarrow \left| \frac{\Delta \phi}{P} \right| = 0.01 = 1 \%
\]
7) \( \Delta f = n \frac{\Delta B}{B} \) since \( B = \text{const.} \) at extraction

\( q^i = \frac{v_{e}^i}{v_{n}^i} - \frac{v_{e}^j}{v_{n}^j} = 0.103113 \)

\( \Delta f = \pm \frac{\Delta B}{B} \frac{f_0}{f_{\max}} = \pm 1.803 \text{ kHz} \)

\( f = f_{\text{max}} \pm \Delta f = \left\{ \begin{array}{l} 1.769352 \text{ MHz} \\ 1.768712 \text{ MHz} \end{array} \right. \)

1.768712 MHz (pay attention, if you want to add \( \pm 1.8 \text{ kHz} \), you need to compute the few with this precision at least \( B \)).

8) \( N_b = \frac{\left( B^2 \right)^{10/7}}{20/7} \left( \frac{0.71213}{0.348} \right) N_{50 \text{ MVN}} = 2.044 N_{50 \text{ MVN}} \)

9) \( B_{\text{inj}} = \frac{B_0}{P} \), \( P = \frac{L_i}{(2\pi/3)} \approx 8.24 \text{ m} \)

\( B_{\text{inj}} = \left\{ \begin{array}{l} 0.125 \text{ T @ 50 MVN} \\ 0.231 \text{ T @ 160 MVN} \end{array} \right. \)

10) \( \phi_s = \arcsin \left( \frac{2\pi R B_s}{V_{\text{ref}}} \right) = 0.197 \text{ rad} = 11.36^\circ < 90^\circ \) because \( B_s \) below \( \Theta \).

\( B_s = \frac{dB_s}{dt} \) since \( g \approx 0 \).

Alternatively we could have chosen the other solution.

Continue...
To compute the max bunch length we need to find the point $B$.

Let's take the eq of the separatrix and impose $\phi = 0$

Need to solve for $\phi$:

$$\cos \phi + \phi \sin \phi = \cos (\pi - \phi_5) + (\pi - \phi_5) \sin \phi_5$$

$$(10) \quad T_{new}^{(10MeV)} = \frac{2\pi R}{\gamma c} = 1.007 \mu s$$

$$\Delta B = \frac{dB}{dt} \times \frac{1}{\gamma} \times 100 T_{new} = 1.824 \cdot 10^{-4} \text{T}$$

If $R = \text{const}$ then $\frac{\Delta f}{f} = \frac{\Delta B}{B} = \frac{1.824 \cdot 10^{-4} \text{T}}{0.23084 \text{T}} = 5.3 \cdot 10^{-4}$

$$\Delta f = 3.87 \cdot 10^{-4} \quad \Delta f = \Delta f \cdot \frac{1}{T_{new}^{(10MeV)}} = 380 \text{ Hz}$$

$$(12) \quad \Delta E = V_{RF} \Delta m/\gamma = 157.6 \text{ GeV} \Rightarrow \sqrt{E_E} = 160.1576 \text{ GeV}$$