(1) \( \gamma_i = \gamma \left( \frac{1}{\gamma_i^2} \right) = \gamma \left( \frac{1}{\frac{4}{5}} \right) = \frac{4}{\gamma} \approx 55,7 \).

- \( \gamma_i = \frac{E_i}{E_0} = \frac{\sqrt{E_0^2 + p_i^2 c^2}}{E_0} = \frac{\sqrt{0,938^2 + 450^2}}{0,938} \approx 479,745 \)

- \( \gamma_i = \frac{4}{\gamma_i^2} - \gamma \approx 3,18 \cdot 10^{-4} \) Already very close to \( \gamma \).

- \( \gamma_c = \frac{E_c}{E_0} = \frac{\sqrt{E_0^2 + p_c^2 c^2}}{E_0} = \frac{\sqrt{0,938^2 + 7000^2}}{0,938} \approx 746,69 \)

- \( \gamma_c = \frac{4}{\gamma_c^2} - \gamma \approx 3,22 \cdot 10^{-4} \)

- \( \gamma_i \leq 0 \) \( \Rightarrow \) \( \gamma \) is always \( \leq 0 \) and therefore LHC does not cross transition.

- As the LHC operates above transition, it means that an accelerated particle will take more time to travel and therefore the revolution period will increase.

(2) \( \gamma_i = \frac{1}{\sqrt{1 - \beta_i^2}} \Rightarrow 1 - \beta_i^2 = \frac{1}{\gamma_i^2} \Rightarrow \beta_i = \sqrt{1 - \frac{1}{\gamma_i^2}} \approx 0,995598 \)

\( \gamma_c = \frac{1}{\sqrt{1 - \beta_c^2}} \Rightarrow \beta_c = \sqrt{1 - \frac{1}{\gamma_c^2}} \approx 1 \)

One can conclude that \( \beta_c = 1 \) from injection all collisions and that one can approximate the velocity of the particles by \( c \) (the speed of light) and that the revolution frequency (and revolution period) is almost constant.
5 = \beta \cdot c = c \cdot \frac{2 \pi \cdot f_{rev}}{v_{circ}} \quad \Rightarrow \quad f_{rev} = \frac{c}{v_{circ}} = \frac{2,997925 \cdot 10^8}{26658,883} \\
= 11,2455 \text{ kHz}

and \quad T_{rev} = \frac{1}{f_{rev}} \approx 88,92 \mu s

\text{1.} \quad f_{ee} = h \cdot f_{rev} = 35640 \times 11245,5 = 400,79 \text{ MHz}

\text{2.} \quad \Delta E_{\text{gain}}^{\text{1 turn}} = \Delta E_{\text{gain}}^{\text{1 turn}} \times T_{rev} \approx 485,38 \text{ keV/turn}

\text{3.} \quad \Delta E_{\text{gain}}^{\text{1 turn}} = e \cdot V_{ke} \cdot \sin \phi_5 \Rightarrow \sin \phi_5 = \frac{\Delta E_{\text{gain}}^{\text{1 turn}}}{e \cdot V_{ke}} \approx 0,03

As we are above transition, beam stability requires \( \gamma \cdot \cos \phi_5 > 0 \)
\( \leq 0 \) \( \Rightarrow \) \( \cos \phi_5 < 0 \)

\Rightarrow \quad \text{If we would have been below transition, one would have had } \phi_5^{\text{st}} = \arcsin (0,03) \approx 0,03 \text{ rad}
\quad = 1,74 \text{ degrees}

But the LHC is operating above transition.

Therefore \( \phi_5 = \pi - \phi_5^{\text{st}} \approx 3,11 \text{ rad} \)
\quad = 178,26 \text{ degrees}

\text{4.} \quad Q_{\phi} = \frac{2 \pi}{N_d} = \frac{2 \pi}{1238} \approx 5,1 \text{ mrad}
\quad \approx 0,93 \text{ degrees}
\[ L_d = \frac{p_d \cdot \alpha_d}{p_d} \Rightarrow \frac{L_d}{\alpha_d} = \frac{14,3}{0,0051} = 2803,93 \]

\[ \Delta E_{\text{gain}} = \frac{e \cdot p_d \cdot B \cdot 2\pi R}{\frac{dB}{dt}} \Rightarrow \frac{B}{dt} = \frac{\Delta E_{\text{gain}}}{e \cdot p_d \cdot \text{Circ}} \]

\[ \approx 6,5 \text{ mT/s} \]

\[ B_i \cdot p_d = 3,3356 \cdot p_i \left[ \text{mV} / \text{cm} \right] = \Rightarrow B_i \approx 0,535 \text{ T} \]

\[ B_c \cdot p_d = 3,3356 \cdot p_c \left[ \text{mV} / \text{cm} \right] = \Rightarrow B_c \approx 8,3 \text{ T} \]

\[ B_c = B_i + \dot{B} \cdot \Delta t = 0,535 + 6,5 \cdot 10^{-3} \times 60 \times 20 = 8,3 \text{ T} \]

\[ \Rightarrow \text{Same result obtained as forecast.} \]

\[ \sin \phi_s = 0 \text{ (as flat top)} \]

\[ \gamma \cos \phi_s > 0 \text{ (for beam stability reasons)} \]

\[ \Rightarrow \cos \phi_s < 0 \]

\[ \phi_s = \frac{\pi}{2} \text{ rad} \]

\[ = 180 \text{ degrees} \]

5. The angular synchronism frequency is given by

\[ \Omega_s = \sqrt{\frac{e \cdot \text{VeC} \cdot \gamma \cdot h \cdot c \cdot \cos \psi_s}{2\pi R_e \cdot E_{sc}}} \]

\[ R = \text{Circ} \frac{2\pi}{2\pi} \]

\[ \approx 4842,89 \text{ m} \]

\[ \approx 144,478 \text{ rad/s} \]

\[ \Rightarrow \text{at top energy} \]

\[ \approx 144,478 \text{ rad/s} \]
\[ p_s = \frac{2s}{2\pi} \approx 23 \text{ Hz} \]

\[ T_s = \frac{1}{p_s} \approx 43.5 \text{ ms} \]

\[ \phi_s = \frac{p_s}{f_{\text{rev}}} \approx 2 \times 10^{-3} \]

A synchrotron oscillation is performed in \( \frac{1}{\phi_s} \approx 689 \) turns of the LHC.

7.1) Clockwise, as beam 2 is moving anti-clockwise and it will go slower than beam 1 to perform a LHC turn as it is accelerated and we are above transition.

7.2) \[ \gamma_c = \frac{\Delta p_{\text{rev}}}{\Delta p} = \frac{\Delta p_{\text{rev}}}{\frac{\Delta p}{p}} = \gamma_{\text{rev}} \frac{\Delta p}{p} \]

\[ = -3.28 \times 10^{-4} \cdot 11245.5 \cdot 10^{-4} \]

\[ \approx -0.36 \text{ mHz} \]

\[ \Delta f_{\text{rev}} = \Delta f_{\text{rev}} \cdot h = -12.9 \text{ Hz} \]

\[ \Delta p = \frac{\Delta \text{Circ}}{\text{Circ}} \]

\[ = \Delta \text{Circ} = \Delta p \cdot \text{Circ} \frac{\Delta p}{p} \]

\[ = 3.225 \times 10^{-4} \times 26658.388 \times 10^{-5} \]

\[ \approx 859.7 \text{ mm} \]

\[ \Delta R = \frac{\Delta \text{Circ}}{2\pi} \approx 136.8 \text{ mm} \]
As bunch 1 is moving clockwise and bunch 2 is moving anti-clockwise with the same speed, when bunch 1 is at IPs, bunch 2 has to be at IP3 if they want to collide at IP2 ⇒ it means that bunch 2 has to be shifted by a quarter of the LHC circumference compared to bunch 1 (and the initial situation where the 2 bunches collided in IP1).

7.4) Let's call n the number of turns needed for bunch 2 to be shifted by a quarter of the LHC circumference and \( \Delta \text{Trv} \) the shift in revolution period for bunch 2 with the higher momentum \( \left( \frac{\Delta p}{p} = 10^{-4} \right) \)

\[ n \cdot |\Delta \text{Trv}| = \frac{\text{Trv}}{4} \]

if one wants bunch 2 to be shifted by a quarter of the LHC

\[ n = \frac{1}{4 \cdot |\Delta \text{Trv}|} \text{ and } |\Delta \text{Trv}| = |\Delta \text{Trv}| = |\gamma \cdot \frac{\Delta p}{p}| \]

\[ n = \frac{1}{4 |\gamma \cdot \frac{\Delta p}{p}|} \text{ and } n = \frac{\Delta k \cdot p \text{eV}}{\text{Trv}} = \Delta k \text{eV} \cdot \text{Trv} \]

\[ \Delta k \text{eV} = \frac{1}{4 \text{eV} / |\gamma \Delta p / p|} \]

\[ \frac{\Delta p}{p} = 10^{-4} \]

\[ \gamma = \gamma_c \]

\[ \Delta k \text{eV} = \frac{1}{4.11265.5 \cdot 3.22 \cdot 10^{-4} \cdot 10^{-4}} \approx 1 \text{ min} 29.5 \]

7.5) Bunch 2 would have moved anticlockwise and it would have taken
3 times more time to have bunches 1 and 2 colliding in IP2 (6 min 29.5) only. The other (faster) solution would have been to decelerate bunch 2 by using a momentum offset of $10^{-4}$ and in this case the time would have been the same (11 min 29s).

7.6) To collide in IP8 it will take 3 times more time, i.e. 34 min 28.5 (starting from the situation of collision in IP1 and IP5).
- The other method is to decelerate bunch 2 (momentum offset of $10^{-4}$) and in this case it takes the same time as for bunch 2 and 1 to collide in IP2 = 11 min 29s.
- If we start from the situation where bunches 2 and 1 collide in IP2, it takes 2 x 11 min 29.5 = 22 min 58.5 in both cases (+ and $-10^{-4}$).

7.7) To go faster we need to use a higher momentum offset. The problem is that it leads to a larger DR and therefore possible particle losses (due to interaction with the vacuum chamber or other equipments close to the beam).