LONGITUDINAL BEAM DYNAMICS EXAMINATION

Consider a proton synchrotron with the following characteristics.

Injection kinetic energy	$E_{k,inj} = 30$ GeV
Average machine radius	$R = 5000$ m
Number of dipoles	$N_d = 800$
Effective length of a dipole	$L_d = 4$ m
Energy gain per turn	$\Delta E_{turn} = 600$ keV
RF voltage	$V_{rf} = 1$ MV
Ejection magnetic field	$B_{ej} = 1.2$ Tesla
RF frequency	$f_{rf} = 9.5493$ MHz
Total bunch length at ejection	$t_b = 10$ ns

1) What are the bending angle and radius of curvature of a dipole?
2) Compute the magnetic field at injection?
3) Compute the rate of increase of the magnetic field dB/dt.
4) What is the minimum value of the RF voltage? Is the RF voltage given in the table sufficient?
5) Compute the synchronous phase?
6) The transition energy is reached after 10 s of acceleration. What is the value of γ_t?
7) If the RF voltage is switched off, what is the variation of the revolution frequency df_{rev} for a variation of the magnetic field $dB/B = 10^{-3}$?
8) What is the value of the synchronous phase above transition? Why?
9) How long does it take to accelerate from injection to ejection?
10) What is the ejection energy?
11) Assuming that the beam is on the injection magnetic plateau, in which direction does the revolution frequency change if the beam momentum is decreased? What about the ejection magnetic plateau?
12) What are the values of the slip factor η at injection, transition and ejection?
13) Compute the harmonic number h. What are the values of the synchrotron period at injection, transition and ejection?
14) At ejection, what is the azimuthal phase extension $\Delta\theta$ (in radians) and the RF phase extension $\Delta\phi$ (in radians) of the bunch? Compute the bunch momentum spread Dp/p at ejection (the bunch is matched in the RF bucket). Deduce the longitudinal bunch emittance in eV.s (assuming an elliptic area).
15) Comment about the beam behavior when the RF voltage is switched off, both far from and at transition.

N.B.: The proton rest energy is $E_0 = 0.938$ GeV and the velocity of light is $c \approx 3\ 10^8$ m/s.