Simulations of the longitudinal coupled-bunch instability in the PS

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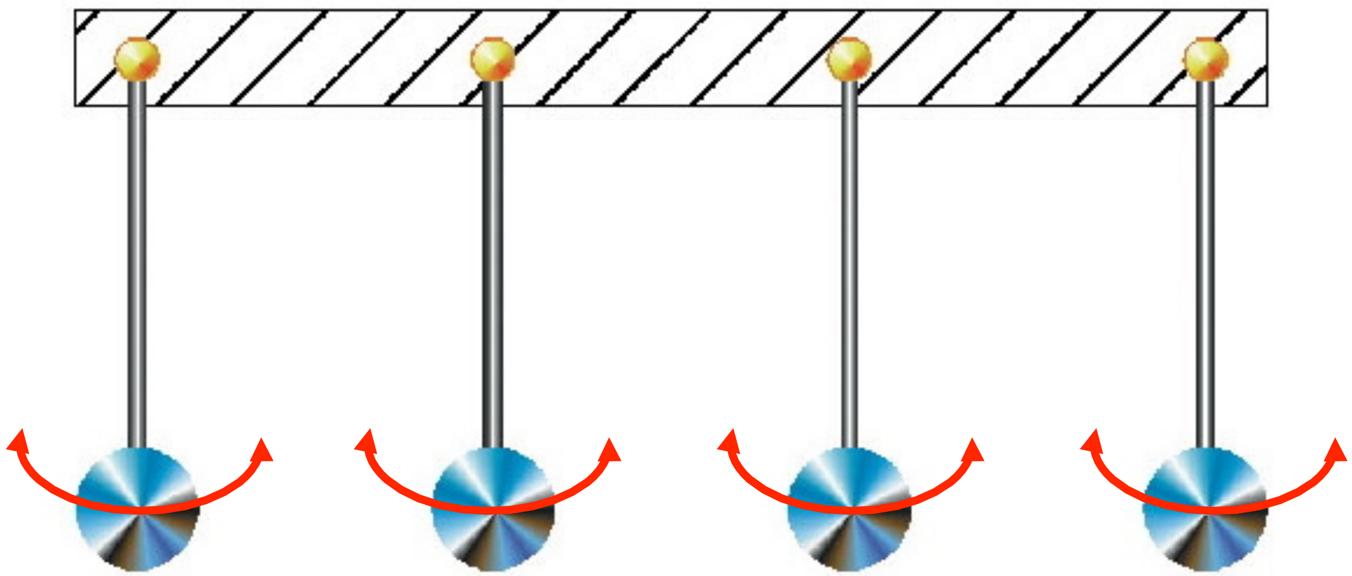
Outline

- Longitudinal coupled bunch instability.
- Description of the Longitudinal Coupled Bunch simulation Code (LCBC).
- Longitudinal bunch-by-bunch feedback system (time domain).
- Applications to PS case: external excitation and frequency domain feedback system.
- Impedance model and comparisons with measurements.
- Conclusions.

Longitudinal coupled bunch instability

- The em fields trapped in machine devices, as HOMs in RF cavities, allow different bunches to influence each other.
- Coherent oscillation modes of a beam can increase with time producing instabilities.
- Simple physical description: coupled harmonic oscillators



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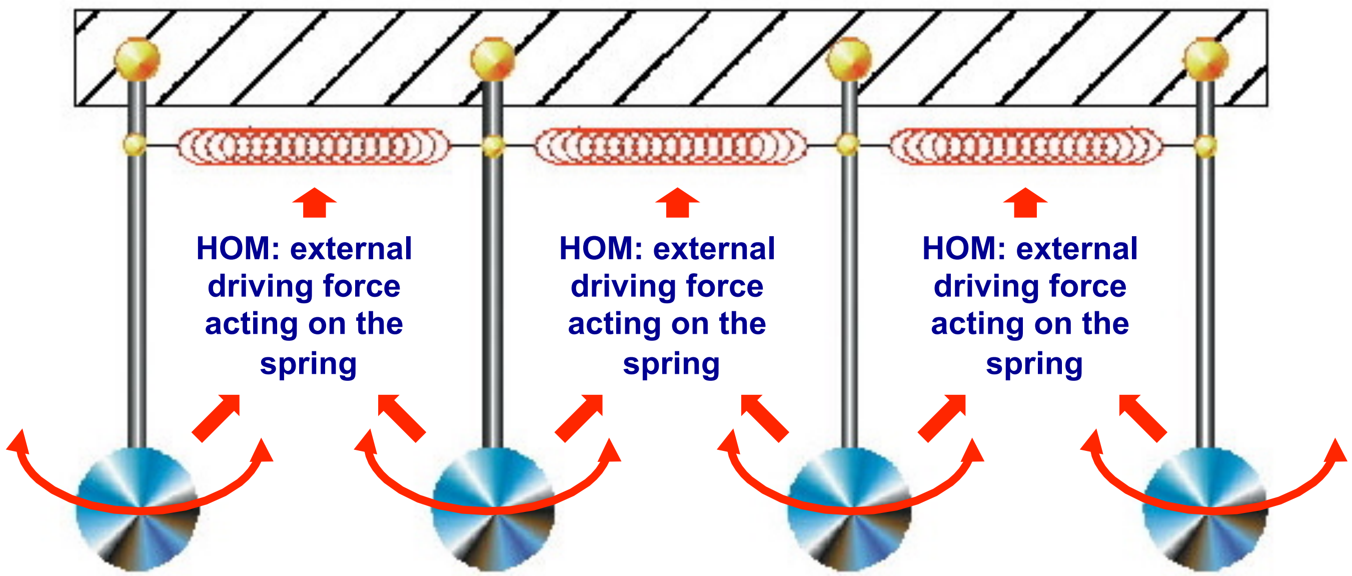
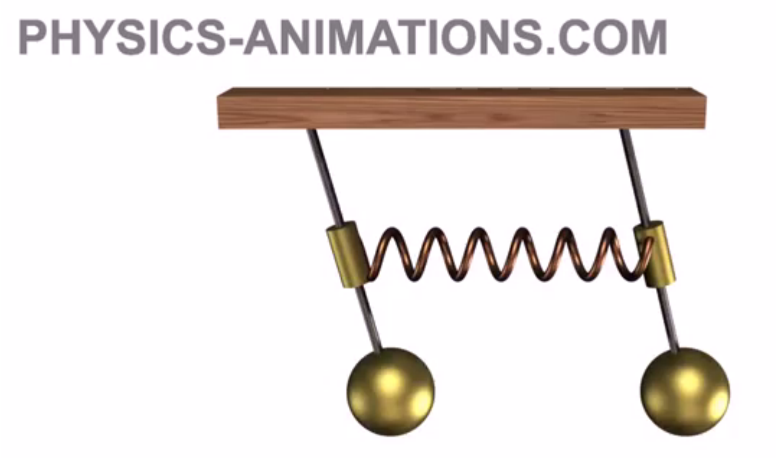
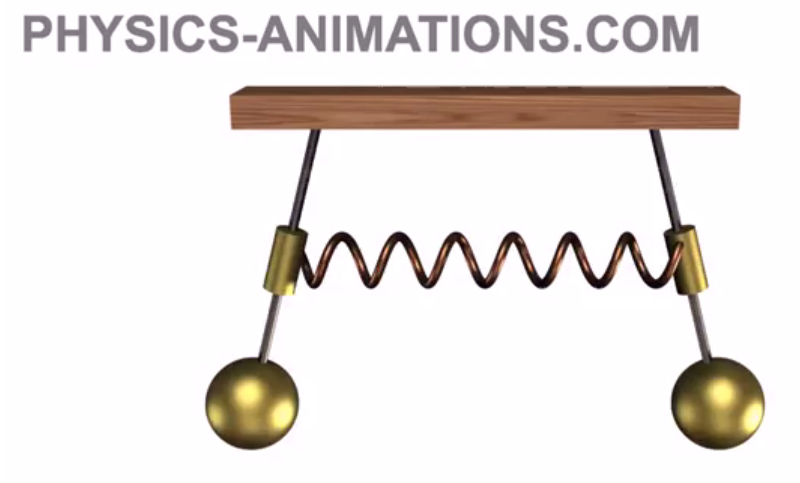
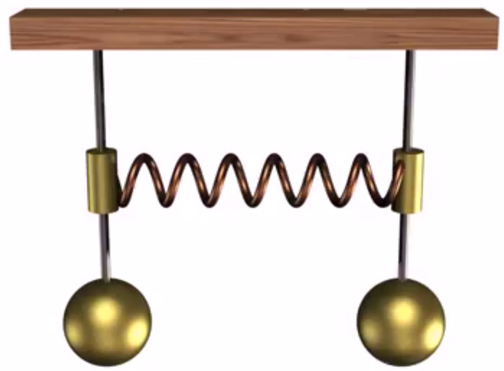


Longitudinal coupled bunch instability

- Coherent oscillation modes: example with two oscillators, modes ‘0’ and ‘π’







Longitudinal coupled bunch instability

- Theoretical description: see e.g.: A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, John Wiley & Sons, (1993).
- Starting from the linearized Vlasov equation and working in the frequency domain, we end up with the following eigenvalue system

$$(\Omega - m\omega_s)\sigma_m(q\omega_o - \Omega) = -i\frac{2\pi m c e^2 N_p N_b}{T_o^2}\sum_{l=-\infty}^{\infty} i^{(m-l)} \sum_{p=-\infty}^{\infty} \frac{Z_{//}\left[(N_b p - \mu)\omega_o - \Omega\right]}{(N_b p - \mu)\omega_o - \Omega}$$

$$\sigma_l\left[(N_b p - \mu)\omega_o - \Omega\right] F_m\left[(N_b p - \mu)\omega_o - \Omega, q\omega_o - \Omega\right]$$

with m=azimuthal mode of the perturbation ψ_1 of the stationary distribution function, μ is the oscillation mode

$$\psi_1(\hat{z}, \phi; t) = e^{i\Omega t}\sum_{m=-\infty}^{\infty} R_m(\hat{z}) e^{-im\phi}$$

$$\sigma_m(\omega) = \int_o^{\infty} R_m(\hat{z}) J_m\left(\frac{\omega}{c}\hat{z}\right)\hat{z}\, d\hat{z} \qquad F_m(\omega, \omega') = \int_o^{\infty} J_m\left(\frac{\omega}{c}\hat{z}\right) J_m\left(\frac{\omega'}{c}\hat{z}\right)\frac{\partial \psi_o(\hat{z})}{\partial \hat{z}} d\hat{z}$$

Longitudinal coupled bunch instability

- The eigenvalue system is valid for equally spaced bunches (but theory can be developed also for uneven fills: see e.g.: S. Prabhakar, et al. *Curing coupled-bunch instabilities with uneven fills*, Phys. Rev. Lett. 2001 Mar 5;86(10):2022-5).
- To solve the problem we suppose there is no coupling between different azimuthal modes => only l=m remains on the RHS and each azimuthal mode can be studies independently from the others.
- Of particular interest is the case in which there is a single high quality resonator as source of impedance close to the frequency $(N_b p_1 - \mu_1)\omega_0$

$$\Omega^{(\mu_1)} = m\omega_s - i\frac{2\pi mce^2 N_p N_b}{T_o^2}\frac{Z_{//}\left[\left(N_b p_1 - \mu_1\right)\omega_o - m\omega_s\right]}{\left(N_b p_1 - \mu_1\right)\omega_o - m\omega_s}$$

$$F_m\left[\left(N_b p_1 - \mu_1\right)\omega_o - m\omega_s, \left(N_b p_1 - \mu_1\right)\omega_o - m\omega_s\right]$$

Longitudinal coupled bunch instability

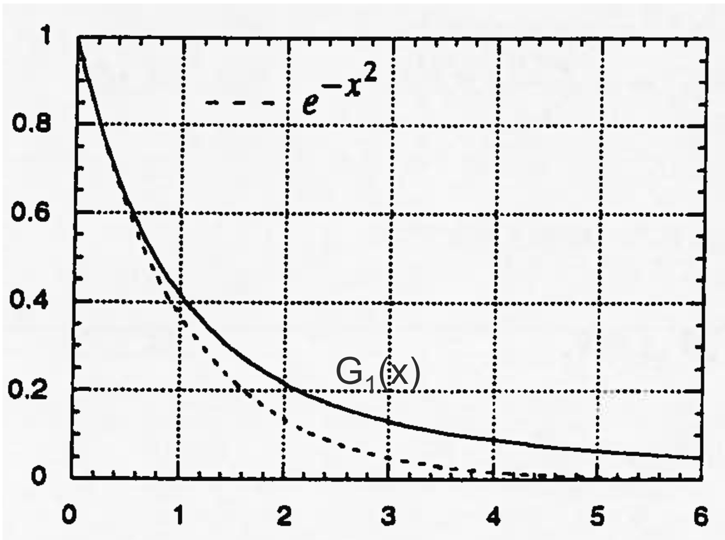
- The imaginary part of Ω gives the growth or damping rate depending on the sign

$$-\mathrm{Im}[\Omega] = \alpha = \frac{1}{\tau} = -\frac{mc\eta e^2 N_p N_b}{2E_o T_o L_o \omega_s}\left[(N_b p_1 - \mu_1)\omega_o - m\omega_s\right]$$
$$\mathrm{Re}\left[Z_{//}\left(N_b p_1 \omega_o - \mu_1 \omega_o - m\omega_s\right)\right] G_m(x)$$

$p_1<0$ ➔instability (above transition)

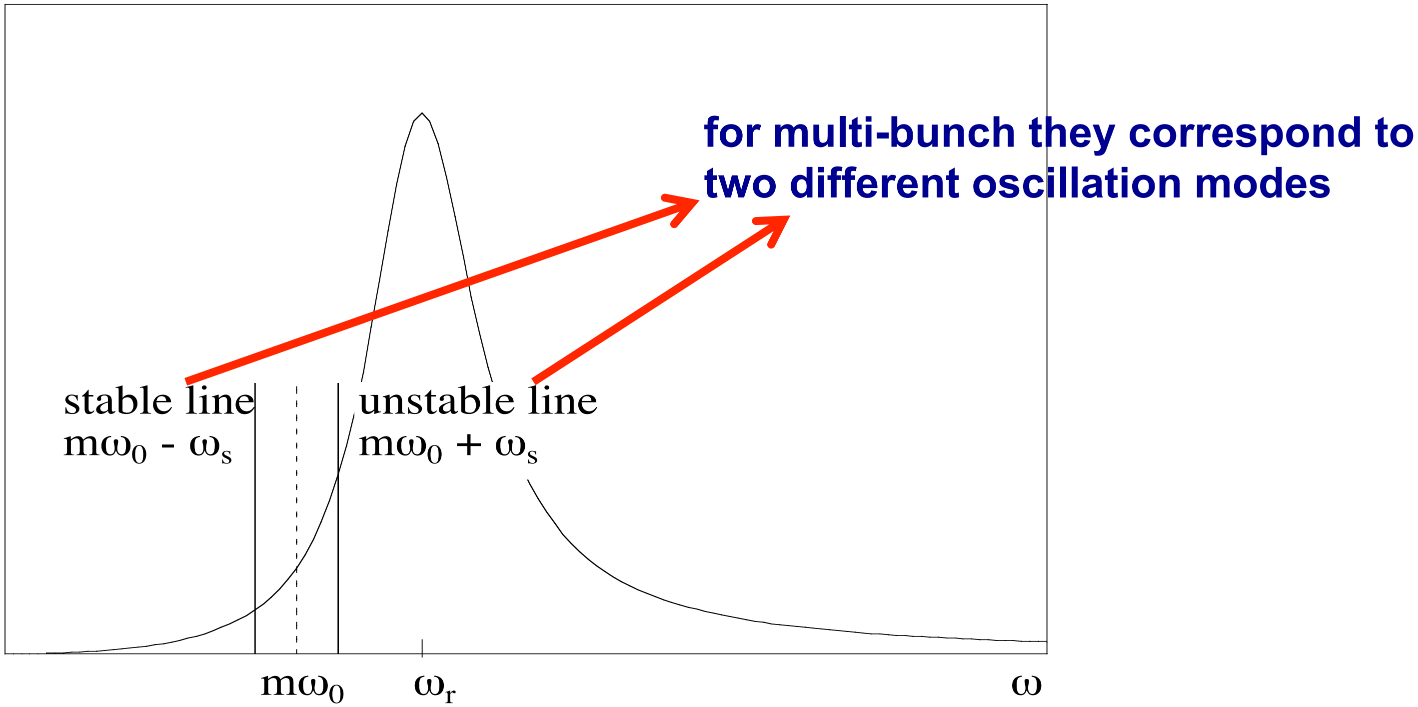
$$G_m(x) = \frac{2}{x^2} e^{-x^2} I_m\left(x^2\right)$$

$$x = \frac{\left[(N_b p_1 - \mu_1)\omega_o - m\omega_s\right]\sigma_z}{c}$$



Longitudinal coupled bunch instability

- Above transition energy positive sidebands of the beam spectrum, evaluated at multiples of ω_0 plus ω_s are unstable, while the negative sidebands, evaluated at multiple of ω_0 minus ω_s are stable.



Longitudinal Coupled Bunch simulation Code (LCBC)

- The simulation code considers bunches as point charges (no internal structure)

DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, June 23, 1993

Note: **G-19**

A TIME DOMAIN SIMULATION CODE OF THE LONGITUDINAL MULTIBUNCH INSTABILITIES

M. Bassetti, A. Ghigo, M. Migliorati, L. Palumbo, M. Serio

Longitudinal Coupled Bunch simulation Code (LCBC)

- Its a time domain simulation code that has been developed to investigate the effect of a bunch-by-bunch feedback system on the longitudinal coupled bunch instability in DAΦNE.
- The code tracks the longitudinal dipole motion of all the bunches and it includes the bunch-by-bunch feedback, the effects of the HOMs, the synchrotron radiation, the fast RF feedback …
- The core of the algorithm can be divided into three main parts:
- 1) propagation of all the bunches in the ring
- 2) interaction with longitudinal FB
- 3) interaction with RF cavity: fundamental mode (fast RF feedback) and HOMs.

Longitudinal Coupled Bunch simulation Code (LCBC)

- propagation in the ring

$$\left(\Delta\varphi_n\right)_o = \left(\Delta\varphi_n\right)_i + \frac{2\pi h\eta}{E_o}\left(\varepsilon_n\right)_i$$

$$\left(\varepsilon_n\right)_o = \left(1 - D\right)\left(\varepsilon_n\right)_i - U_o - U_{bb}$$

- interaction with longitudinal FB

$$\left(\Delta\varphi_n\right)_o = \left(\Delta\varphi_n\right)_i$$

$$\left(\varepsilon_n\right)_o = \left(\varepsilon_n\right)_i + \Delta E_{fb}$$

- interaction with RF cavity

$$\left(\Delta\varphi_n\right)_o = \left(\Delta\varphi_n\right)_i$$

takes into account the fundamental theorem of beam loading

$$\left(\varepsilon_n\right)_o = \left(\varepsilon_n\right)_i + e\hat{V}\cos\left(\Delta\varphi_n\right) + \sum_m v_m\left(t_n\right) + \frac{1}{2}\sum_m \Delta V_m$$

Longitudinal Coupled Bunch simulation Code (LCBC)

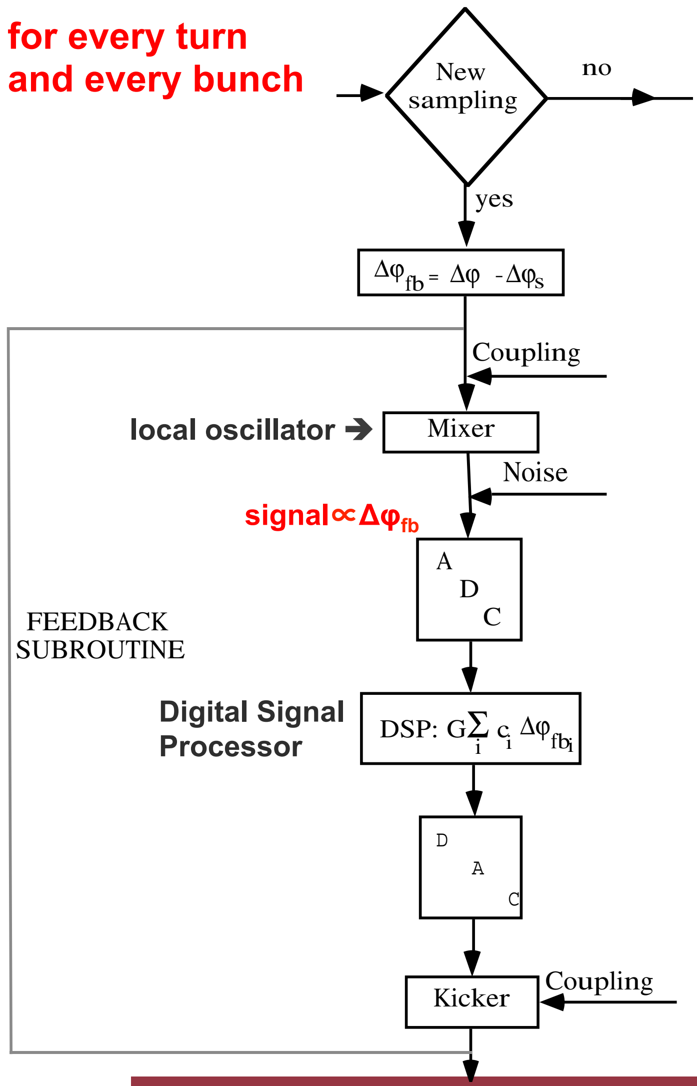
- The wake voltage for each HOM between the passage of two bunches executes free oscillations

$$\begin{pmatrix} v_m(t) \\ i_m(t) \end{pmatrix} = \exp(-\alpha_m t) \begin{pmatrix} \cos(\beta_m t) - \frac{\alpha_m}{\beta_m}\sin(\beta_m t) & -\frac{\omega_m R_{sm}}{\beta_m Q_m}\sin(\beta_m t) \\ \frac{\omega_m Q_m}{\beta_m R_{sm}}\sin(\beta_m t) & \cos(\beta_m t) + \frac{\alpha_m}{\beta_m}\sin(\beta_m t) \end{pmatrix} \begin{pmatrix} v_m(t_o) \\ i_m(t_o) \end{pmatrix}$$

- When a bunch ‘n’ crosses the cavity, it induces a kick ΔV_m to the m-th HOM voltage $v_m(t)$

$$v_m(t) = v_m(t) + \Delta V_m \qquad \Delta V_m = -\frac{\omega_{rm} R_{sm}}{Q_m} Q_{bn}$$

Longitudinal bunch-by-bunch feedback system



- The bunch-by-bunch feedback system allows to damp the individual motion of each bunch independently of the cause, thus uncoupling its motion from that of the other bunches.

- The coefficients c_i of the DSP perform the filtering algorithm, after which the feedback correction information converted by a fast digital to analog converter and then amplified with a power amplifier and fed to a longitudinal kicker.

Longitudinal bunch-by-bunch feedback system

The simulation code allowed to:

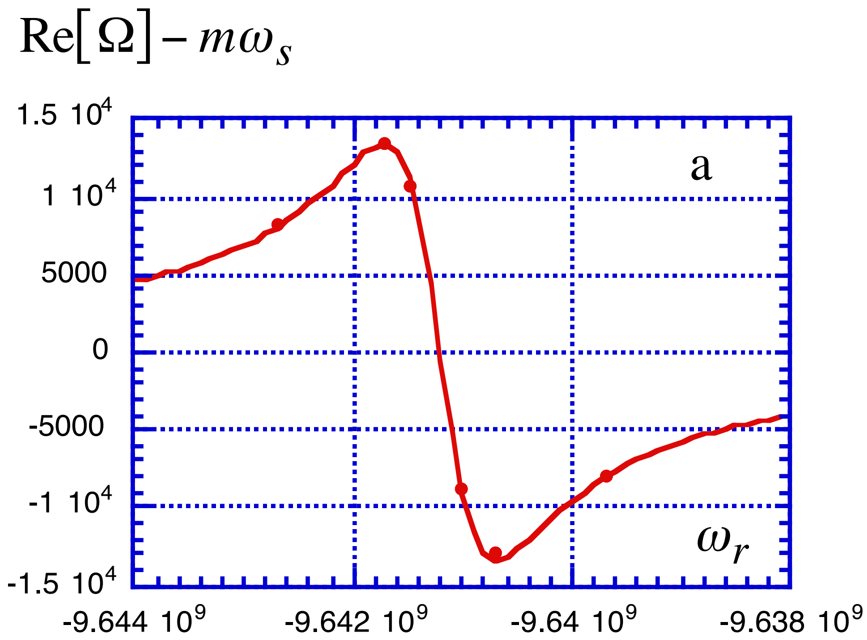
- estimate the number of samplings (down-sampling) to properly reconstruct the synchrotron oscillation;

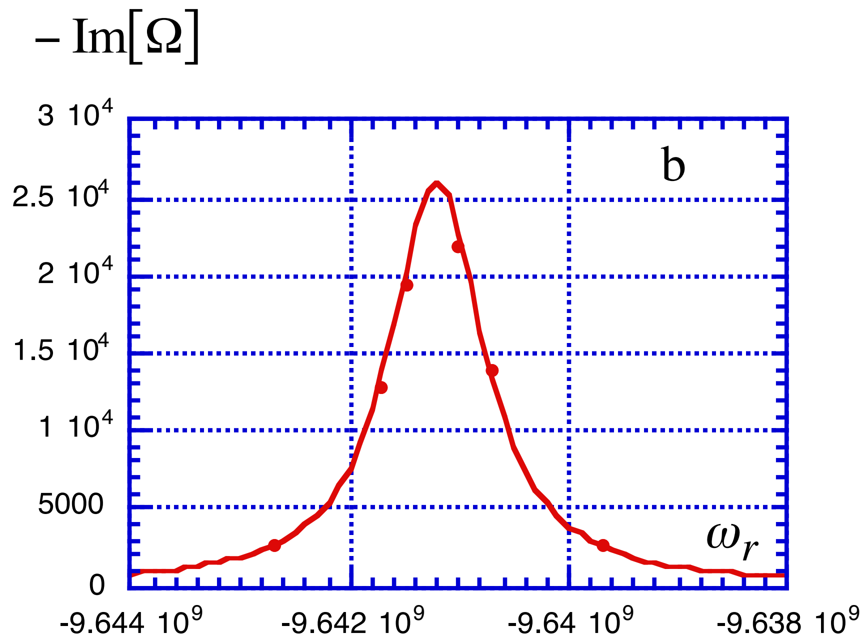
- compare different filters to be used in the DSP (i.e. the values of the coefficients c_i): delay line, high and low pass, resonant filters. The best performance of the feedback was obtained with the derivative filter (coefficients allowing to reconstruct the bunch phase synchrotron oscillations shifted by $\pi/2$);

- evaluate the total power needed by the longitudinal kicker to damp the coupled bunch instabilities in DAΦNE.

Longitudinal Coupled Bunch simulation Code (LCBC)

- Some benchmarks: synchrotron frequency shift and growth rate due to a single HOM versus ω_r: theory vs simulations





Longitudinal bunch-by-bunch feedback system

Benchmark: if the longitudinal feedback works in the linear regime, given g the feedback gain in V/rad, its characteristic damping rate is

$$\frac{1}{\tau_{fb}} = \frac{\eta h \omega_o^2 e}{4\pi E_o \omega_s} g$$

$\eta = 2.56 \times 10^{-2}$;

$h = 84$;

$\omega_0 = \frac{2\pi}{2.09584994 \times 10^{-6}}$;

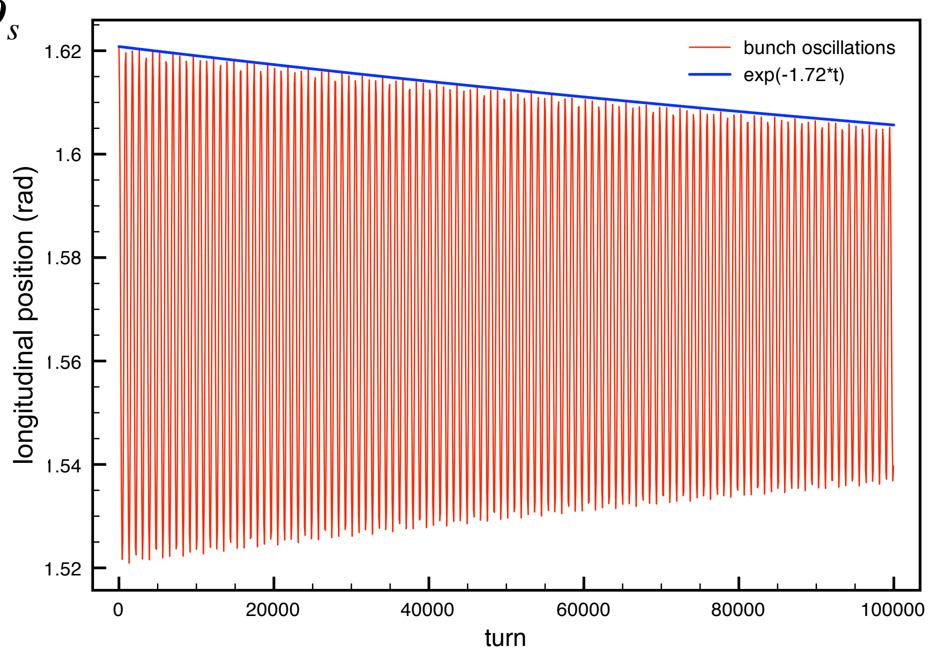
$E_0 = 2.60169 \times 10^{10}$;

$\omega_s = 3.43668741 \times 10^{3}$;

$g = 100$;

$\alpha = \frac{\eta\, h\, \omega_0^2}{4\pi E_0 \omega_s} g$

1.7201 s^{-1}



Application to PS case

- Example of benchmark with PS parameters: a single HOM in resonance with an unstable oscillation mode

$c = 3 \times 10^8;$

$e = 1.6 \times 10^{-19};$

$\eta = 2.1665 \times 10^{-2};$

$N_p = 7;$

$N_b = 4 \times 10^{11};$

$L_0 = 628;$

$\omega_s = 1660;$

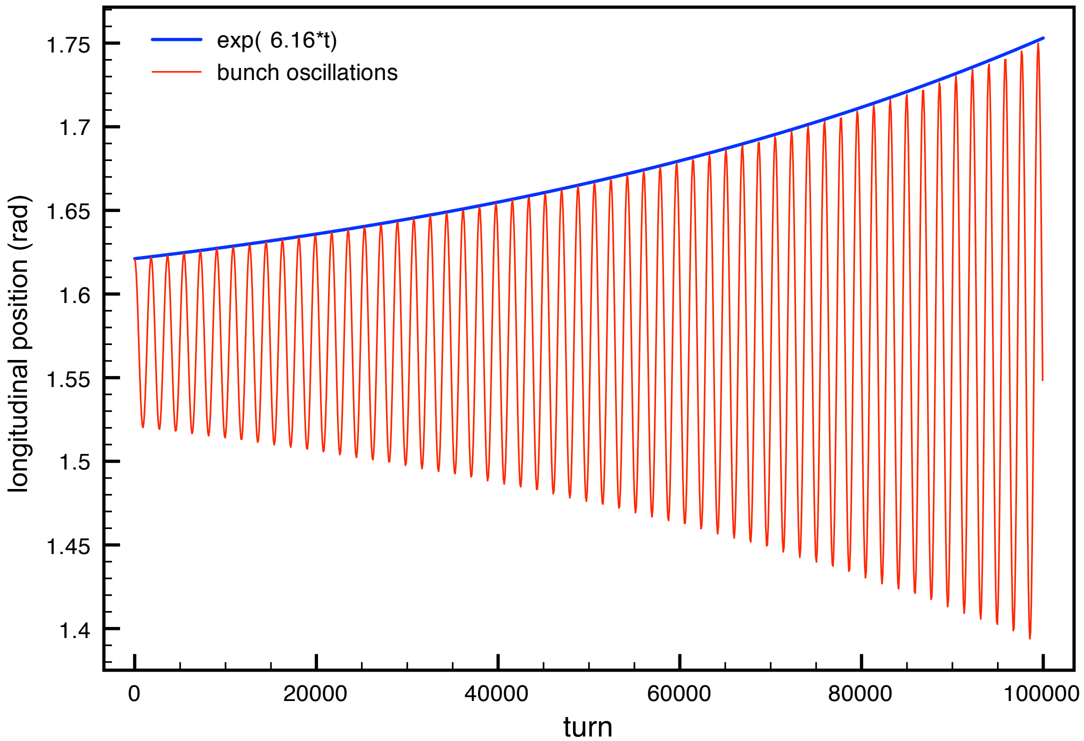
$\omega_r = 2.4 \times 10^7;$

$R_s = 5000;$

$E_0 = 1.3 \times 10^{10};$

$$\alpha = \frac{c^2 \eta e N_p N_b}{2 L_0^2 E_0 \omega_s} \omega_r R_s$$

6.15828 s^{-1}



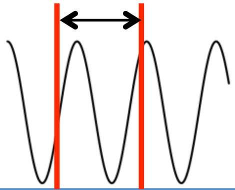
Application to PS case: frequency domain FB

- **A frequency domain longitudinal feedback, instead of the bunch-by-bunch one has been implemented in LCBC.**

- **While tracking the bunches, the code carries out a mode analysis to obtain amplitude, frequency and phase of the modes into which the phase oscillation of all the bunches can be decomposed.**

- **For each selected mode to be damped, the code applies a kick proportional to the FB gain (V/rad) and mode amplitude (rad) with a proper phase.**

Application to PS case: frequency domain FB



$$\phi_b = A_b \sin\left(\omega_s t + \theta_b\right) = \sum_{\mu=0}^{N_b-1} a_\mu \sin\left(\omega_s t + \frac{2\pi}{N_b}\mu b + \theta_\mu\right)$$

RF phase of each bunch in a given time window → Reconstruction of amplitude and phase oscillation for each bunch → Mode analysis → Select the mode to damp: a_μ, θ_μ → Amplitude (proportional to a_μ) and phase of the energy kick → Same kick amplitude for all the bunches (but different phase)

$$\Delta V_{fb} = -\frac{g}{\omega_s}\frac{d\phi_b}{dt} = -g a_\mu \cos\left(\omega_s t + \frac{2\pi}{N_b}\mu b + \theta_\mu\right)$$

equation of motion of a bunch in presence of the feedback

$$\ddot{\phi}_b + \frac{\omega_s}{V_{RF}} g\dot{\phi}_b + \omega_s^2 \phi_b = 0$$

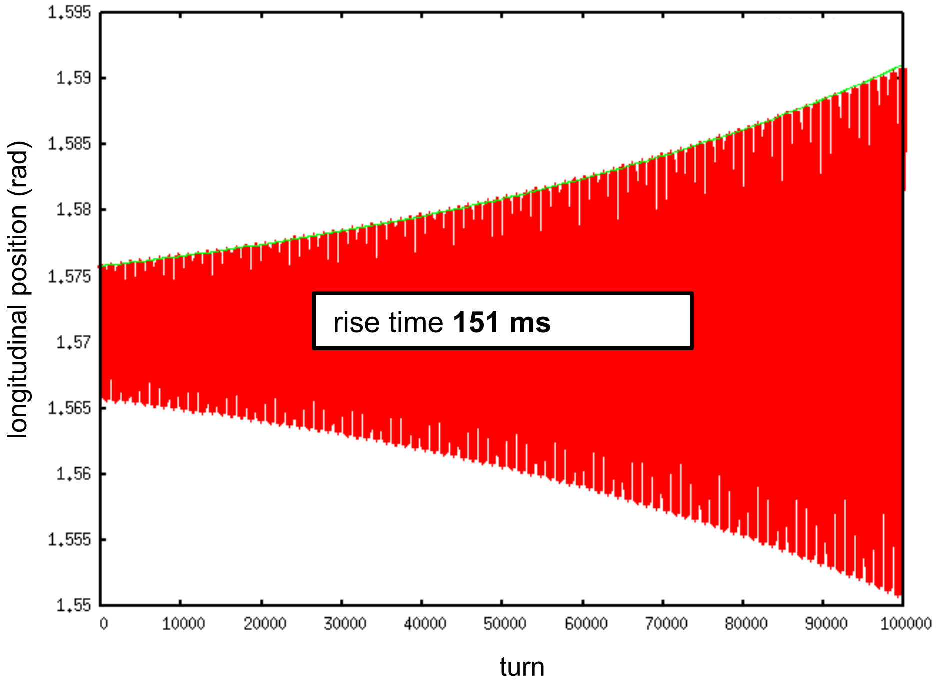
FB damping rate →

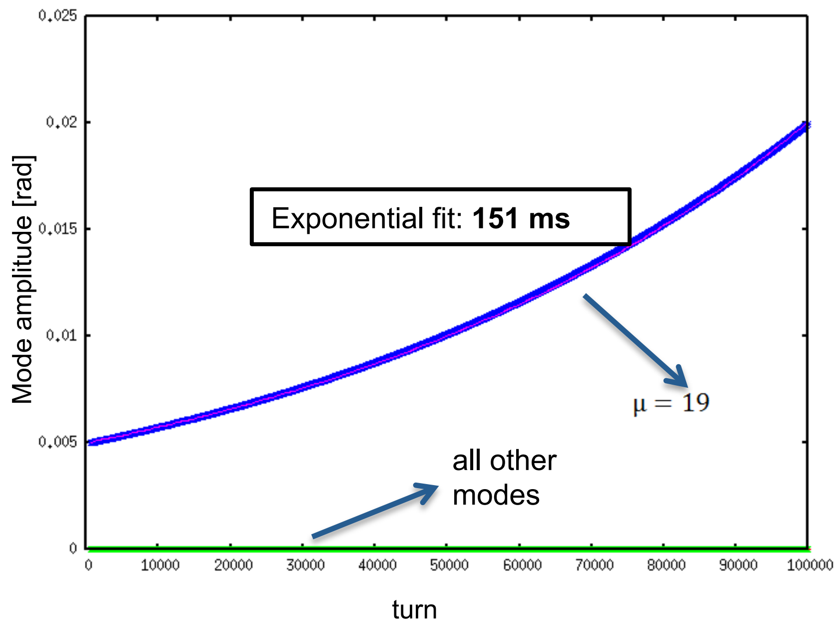
$$\alpha_{fb} = -\frac{1}{2}\frac{\omega_s}{V_{RF}} g = -\frac{1}{2}\frac{\omega_{RF}\eta}{\omega_s T_0 \left(E_0 / e\right)} g$$

Application to PS case: frequency domain FB

Benchmark of the frequency domain FB:

- **We have first excited with a HOM a coherent oscillation mode (e.g. μ=19 in h=21 with 21 bunches) and evaluated the rise time.**



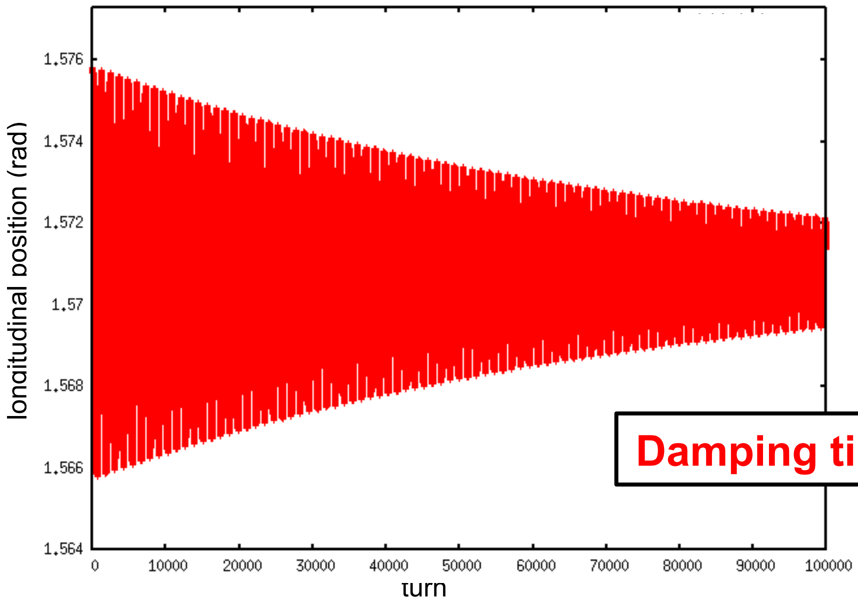


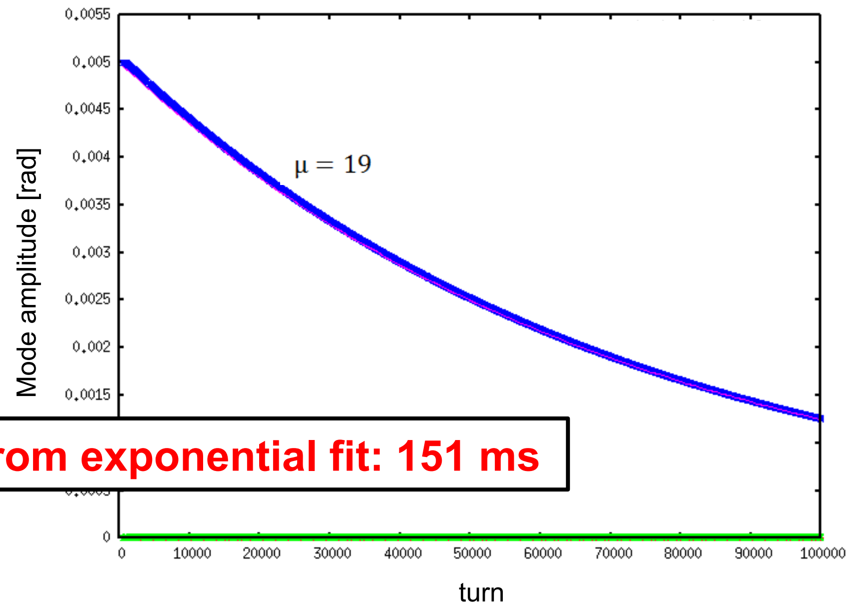
- **To damp this mode we need a gain of g= 877 V/rad**

Application to PS case: frequency domain FB

Benchmark:

- **We then used the FB (without the HOM) with the previous obtained gain to check if it correctly damped the right coherent oscillation mode with the right damping time.**



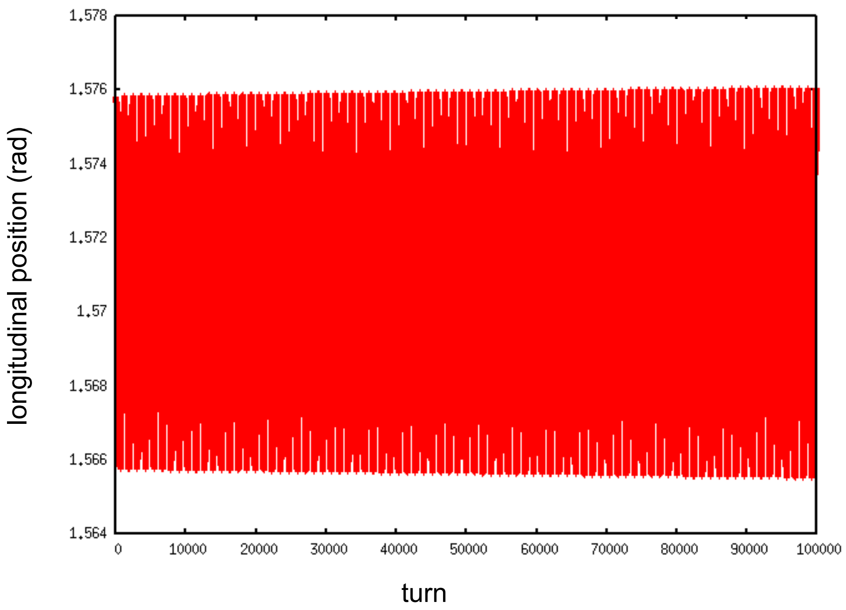


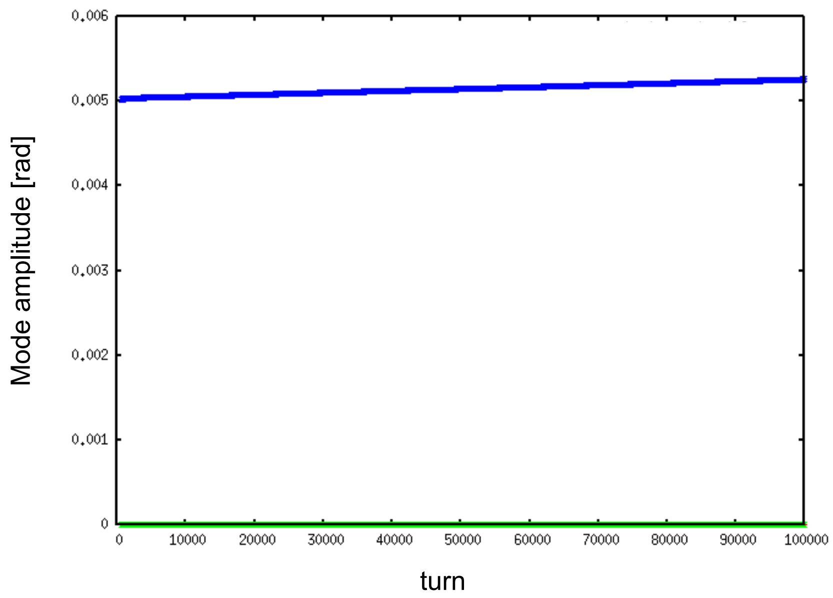
Damping time from exponential fit: 151 ms

Application to PS case: frequency domain FB

Benchmark:

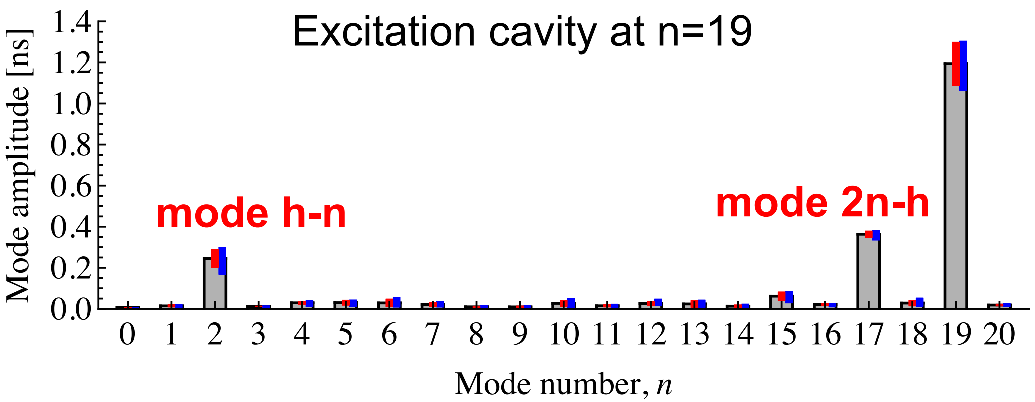
- **We finally use the FB in presence of the HOM with a damping rate equal to the growth rate.**





Application to PS case: external excitation

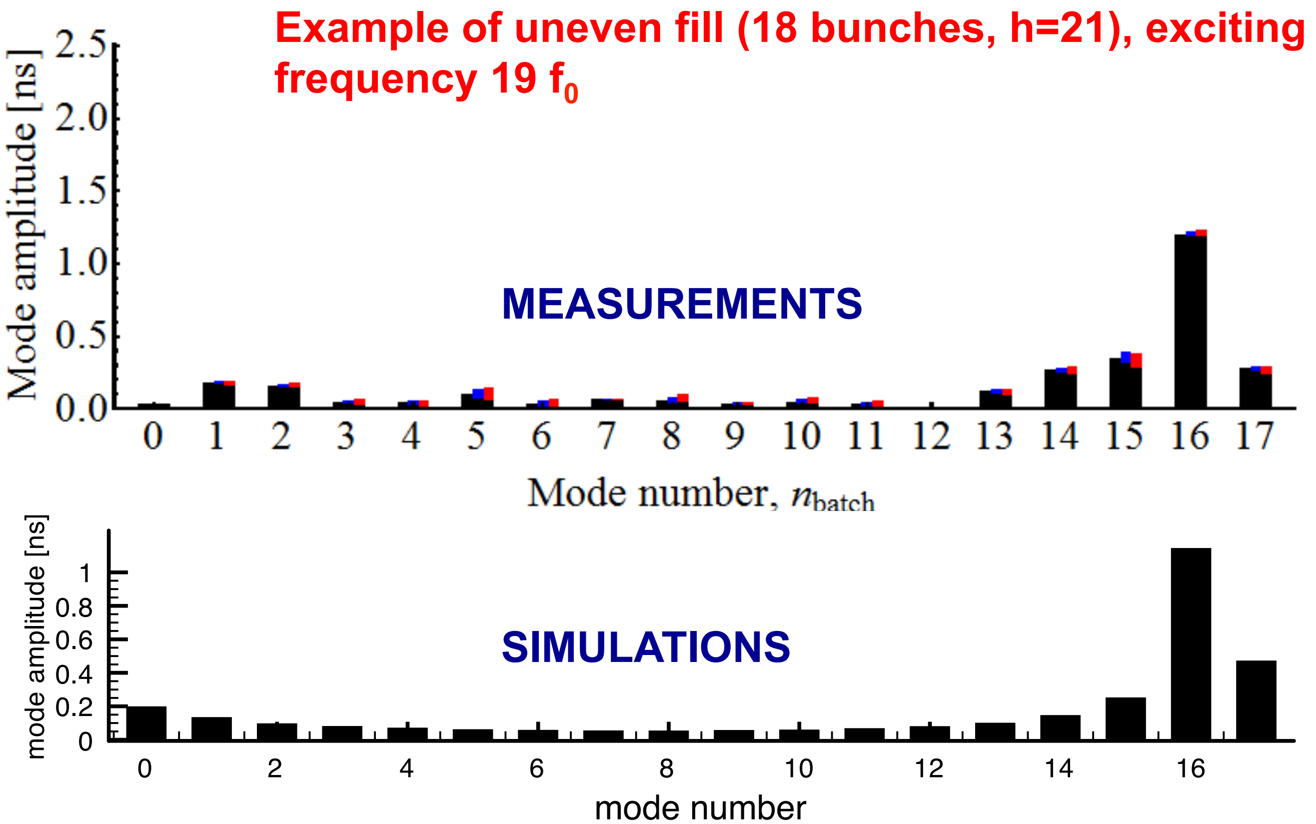
- A measurement program to study the behaviour of the PS coupled-bunch feedback started before LS1 (IPAC13 poster, H. Damerau et al.).
- To study CB oscillations, the low-level part of the existing FB has been connected to a spare 10 MHz accelerating cavity. As powerful longitudinal kicker (up to 20 kV), it is tunable from 2.8 MHz to 10 MHz, covering h = 6...21.
- To excite CB oscillation mode using the FB, a perturbation was injected to generate a sideband at $nf_0 \pm f_s$.



Example: mode spectrum of 21 bunches in h = 21, excited at the upper SB of $19f_0$

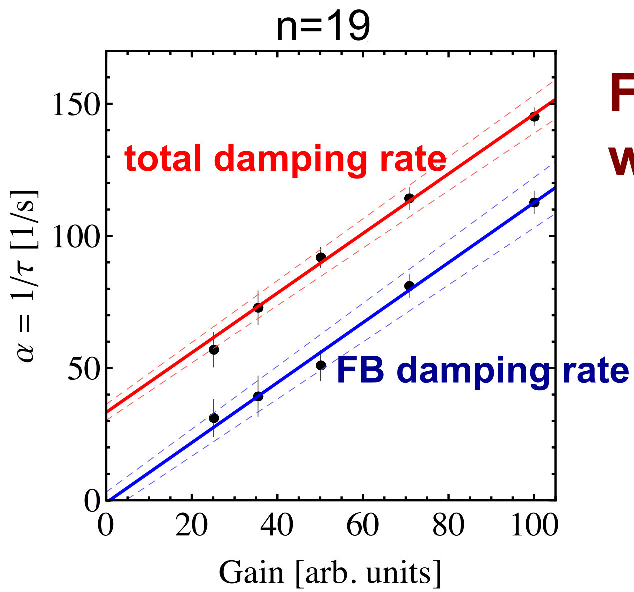
Application to PS case: external excitation

- An external excitation was implemented in the simulation code to be used as alternative to a HOM to excite the beam.



Application to PS case: external excitation

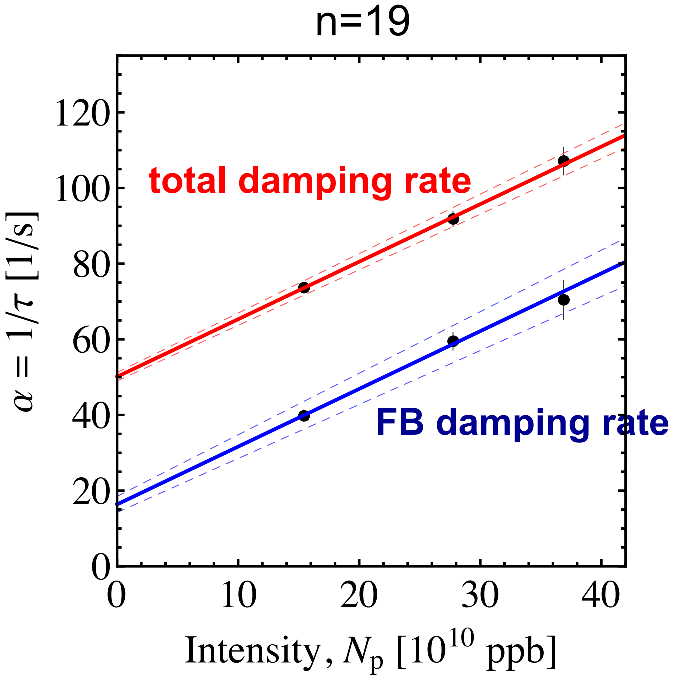
- By using LHC-type beams, where only 18 bunches are accelerated in h=21, leaving a gap of three empty buckets for extraction purposes, the damping rates have been measured versus intensity, longitudinal emittance and FB gain.
- Additionally, damping rates without FB were measured to disentangle the contribution of the FB from natural damping.



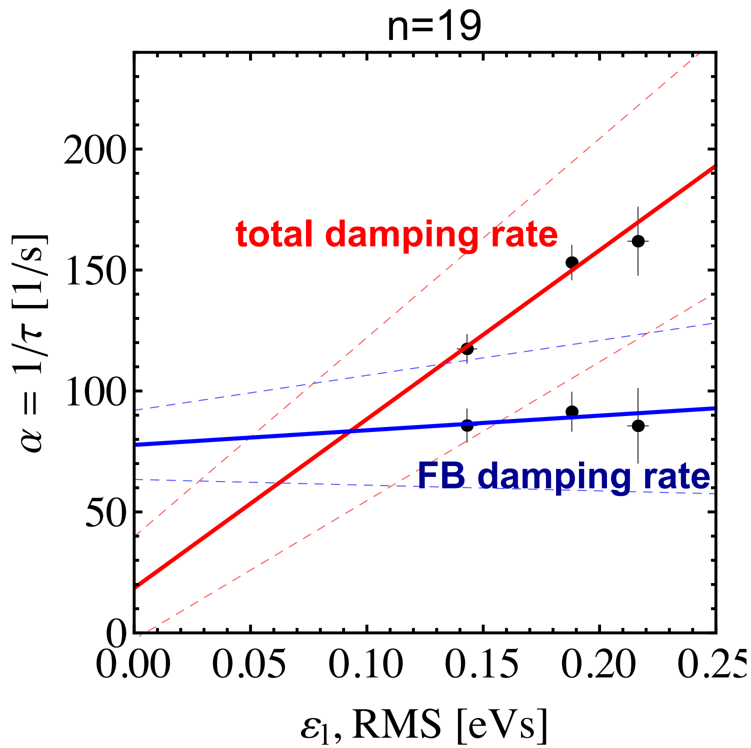
FB damping rate linear with FB gain

Application to PS case: external excitation

FB damping rate vs bunch intensity (∝ beam spectrum)



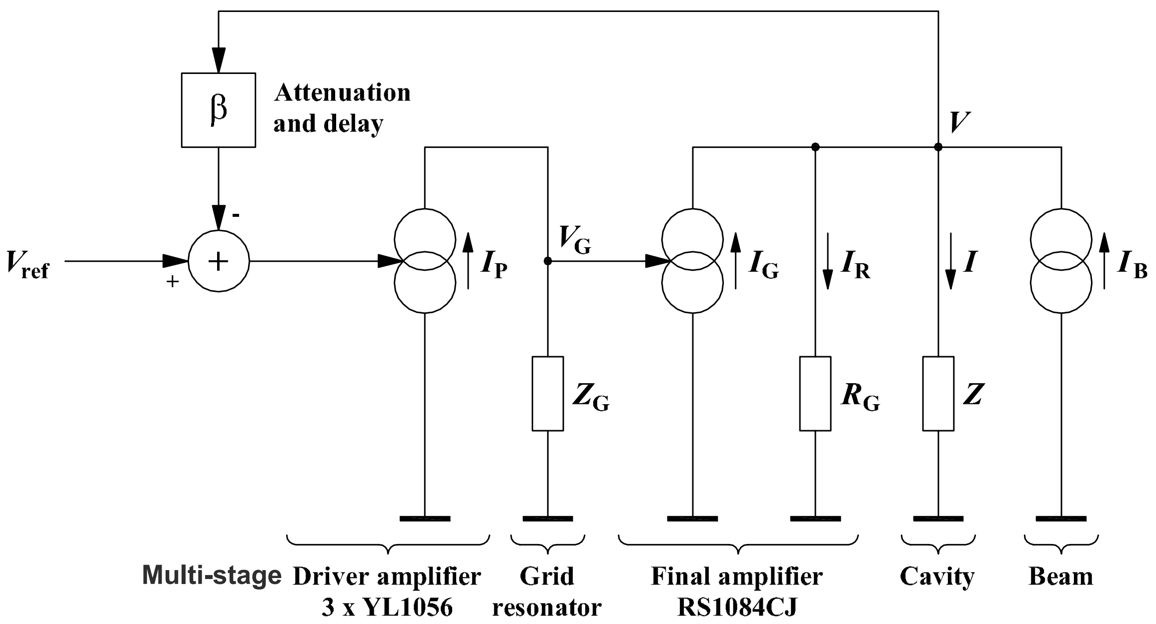
FB damping rate constant with longitudinal emittance



The FB damping rate does not cross the origin, this could be due to saturation effects in the analogue front-end

Impedance model and comparison with measurements

- The main sources of longitudinal coupled bunch instability in the PS are thought to be the 10 MHz RF cavities.
- The coupling impedance of the cavities is not a simple resonator due to the feedback loop and the power amplifier.



Multi-stage

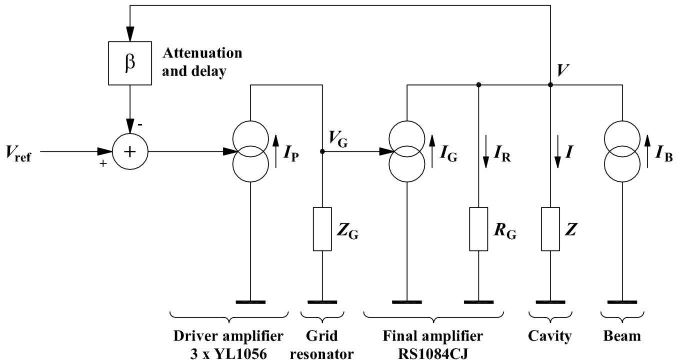
Proceedings of PAC07, Albuquerque, New Mexico, USA

LONGITUDINAL COUPLED-BUNCH INSTABILITIES IN THE CERN PS

H. Damerau, et al.

Impedance model and comparison with measurements

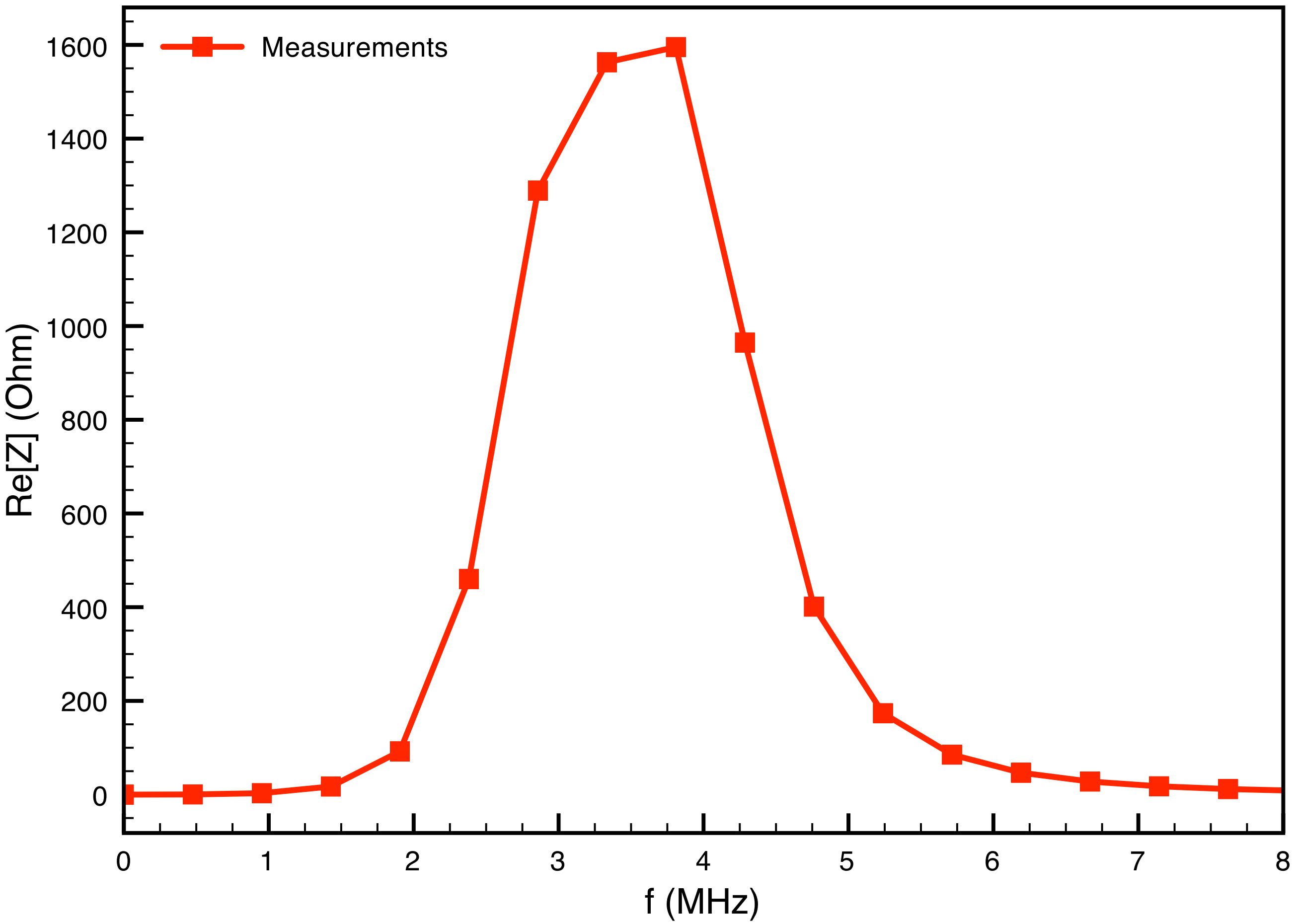
- According to the model, the impedance seen by the beam can be written as



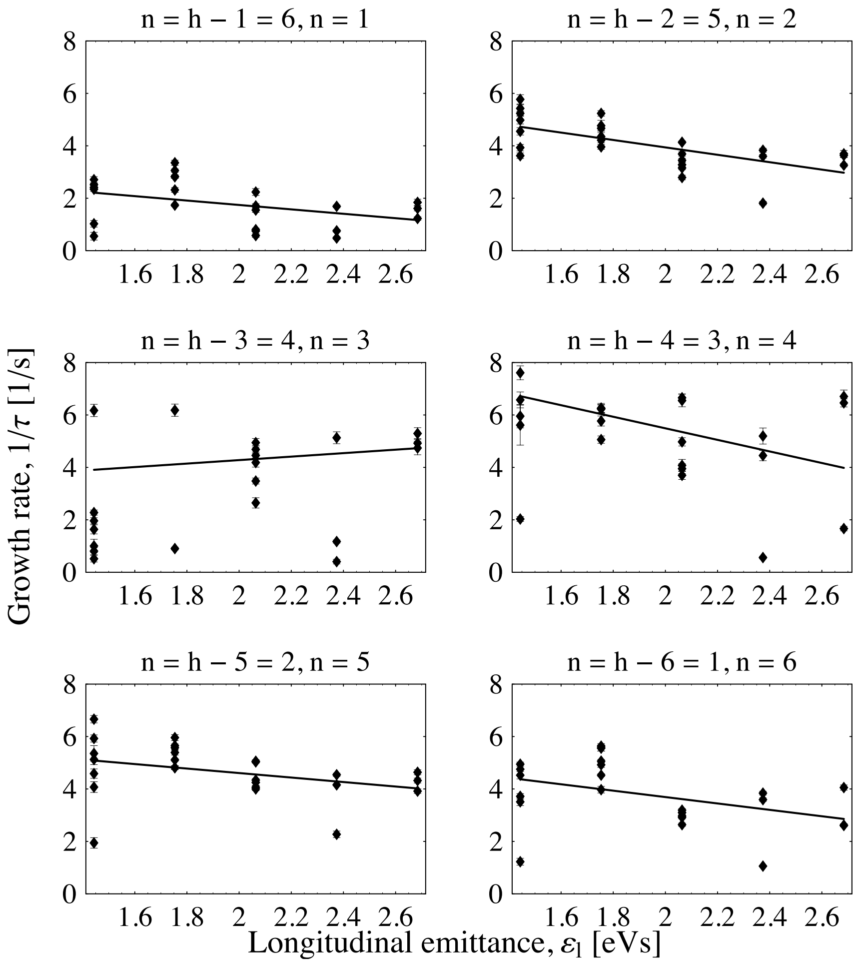
$$Z_c = \frac{dV}{dI_B} = \frac{1}{Z_G g_P g_G \beta + (R_G + Z)/(R_G Z)}$$

- $Z_G(\omega)$ = impedance of the grid resonator, R_g = resistor modelling the loading of the cavity by the output impedance of the amplifier, g_P, g_G = effective trans-conductances of driver and final amplifier, β = attenuation and delay of the feedback loop.
- The parameters of the model have been matched to reproduce the measured open and closed loop transfer functions of six over the ten cavities.

Impedance model and comparison with measurements



Impedance model and comparison with measurements

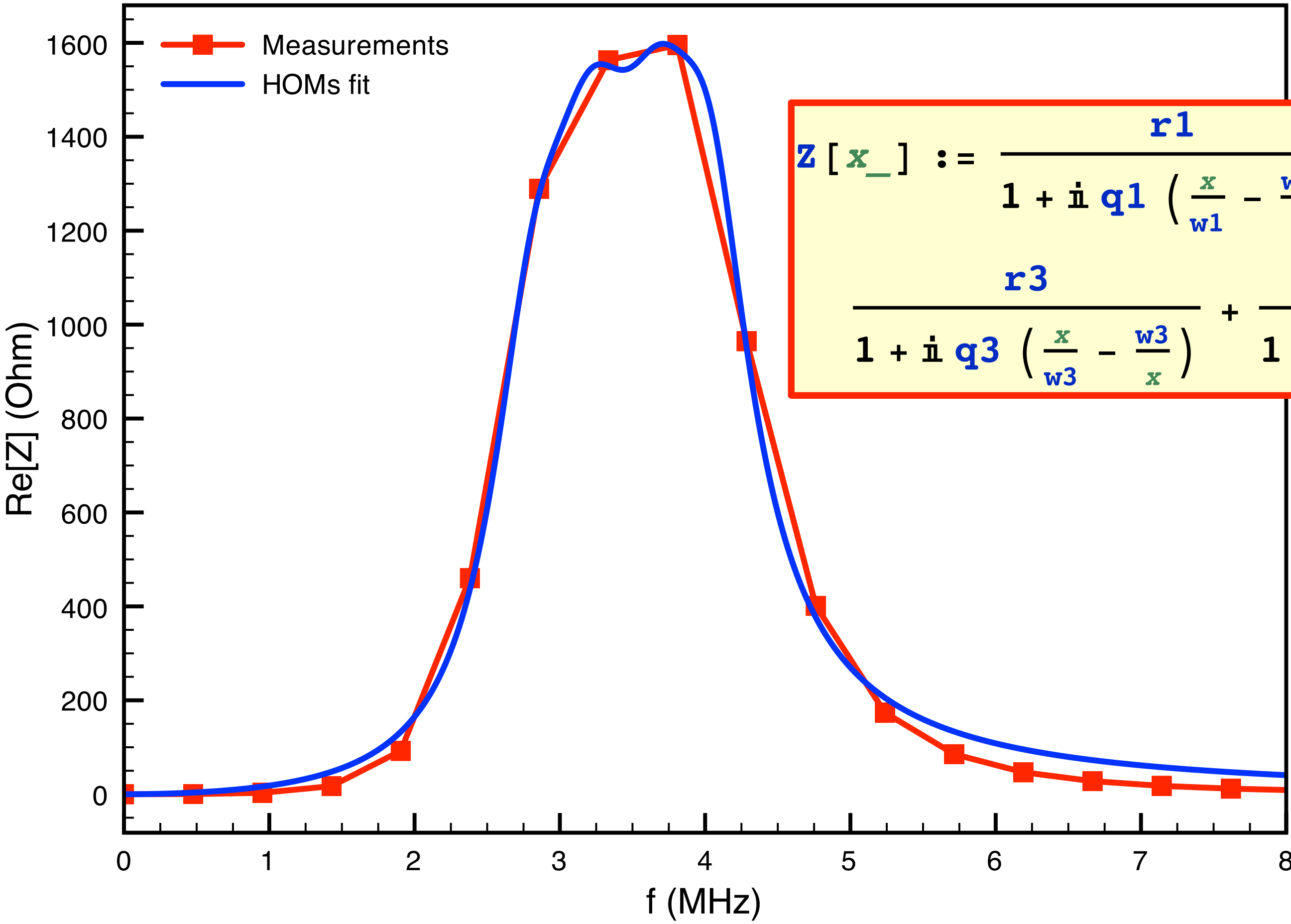


- According to the PAC07 paper, the growth rates range from about 1 s^{-1} to 5 s^{-1}, with a weak dependence on the longitudinal emittance.

- The unstable modes are n=1, 2, 3

Mode number	$n = 1$	$n = 2$	$n = 3$
Growth rate, $1/\tau$	$2.5\,\mathrm{s}^{-1}$	$3.0\,\mathrm{s}^{-1}$	$1.0\,\mathrm{s}^{-1}$

Impedance model and comparison with measurements



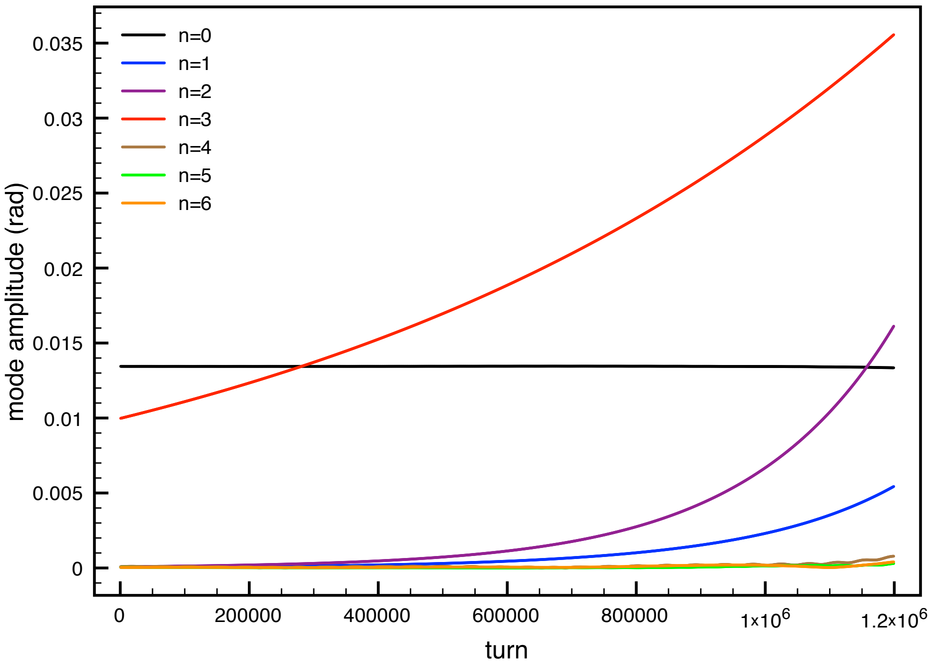
$$Z[x\_] := \frac{r1}{1 + i\, q1 \left(\frac{x}{w1} - \frac{w1}{x}\right)} + \frac{r2}{1 + i\, q2 \left(\frac{x}{w2} - \frac{w2}{x}\right)} + \frac{r3}{1 + i\, q3 \left(\frac{x}{w3} - \frac{w3}{x}\right)} + \frac{r4}{1 + i\, q4 \left(\frac{x}{w4} - \frac{w4}{x}\right)}$$

Impedance model and comparison with measurements

- Simulation results with:

h=7 (7 equally spaced bunches), average energy = 13 GeV, ppp = 9×10^{12}

```
5.9696D0   747.343D0       2.548643616294442D7 ---------- Q  --  RS  --  WR
4.0167D0   821.522D0       2.038646876515147D7 ---------- Q  --  RS  --  WR
4.76129D0   728.578D0      2.328470555619313D7 ---------- Q  --  RS  --  WR
3.77121D0   733.168D0      1.781578444557525D7 ---------- Q  --  RS  --  WR
```



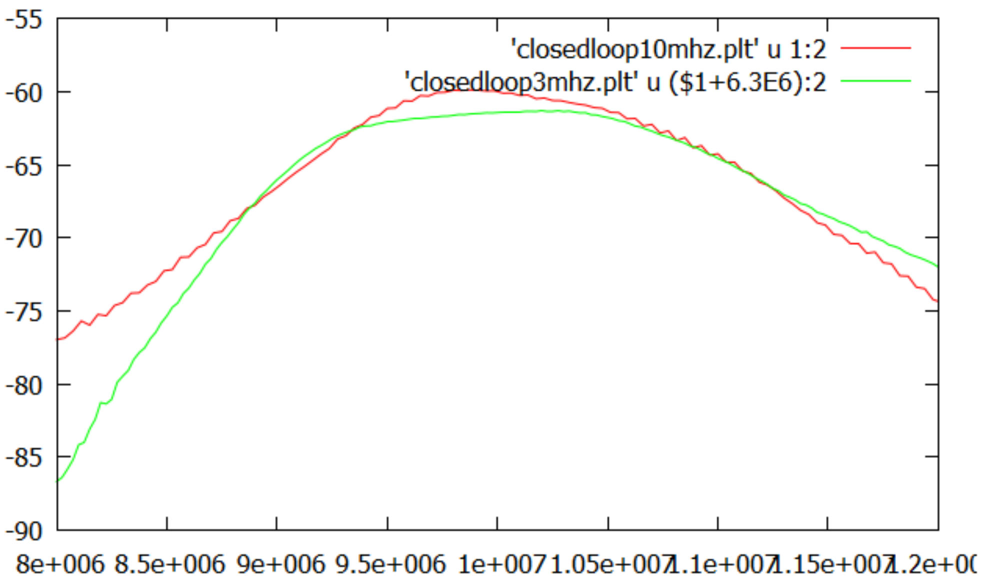
Simulation's results			
Mode Number	1	2	3
Growth Rate	1.84 s^{-1}	2.14 s^{-1}	0.52 s^{-1}

Impedance model and comparison with measurements

- The same work has been done with h=21 and 18 bunches: the closed loop transfer function @ 10 MHz has been measured and compared with that @ 3.3 MHz.

- The result confirmed that we could use the same impedance model, just by shifting the frequencies.

$$Z_c = \frac{dV}{dI_B} = \frac{1}{Z_G g_P g_G \beta + (R_G + Z)/(R_G Z)}$$

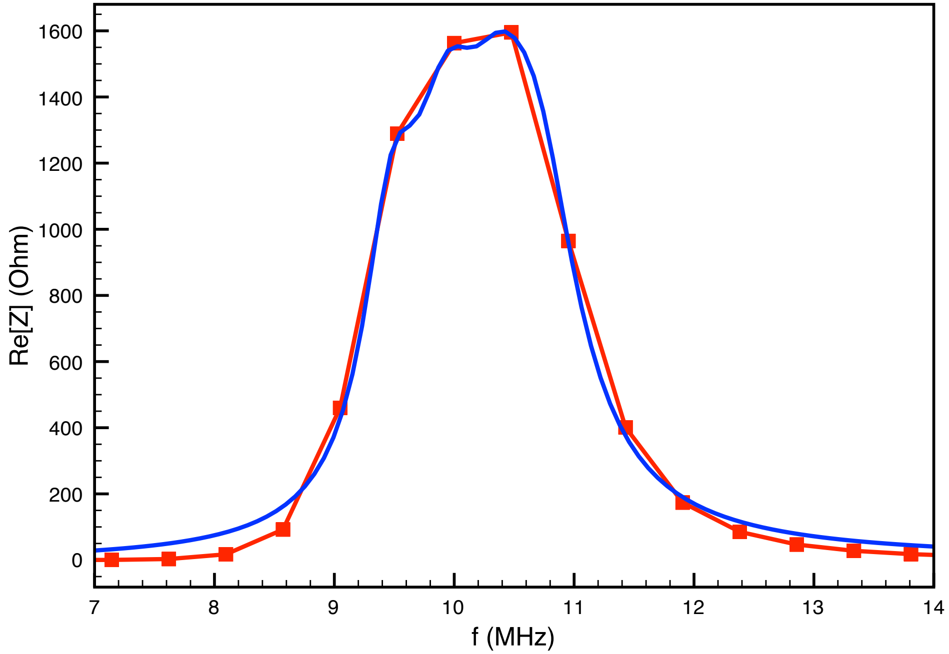


Impedance model and comparison with measurements

- We obtained the new fit

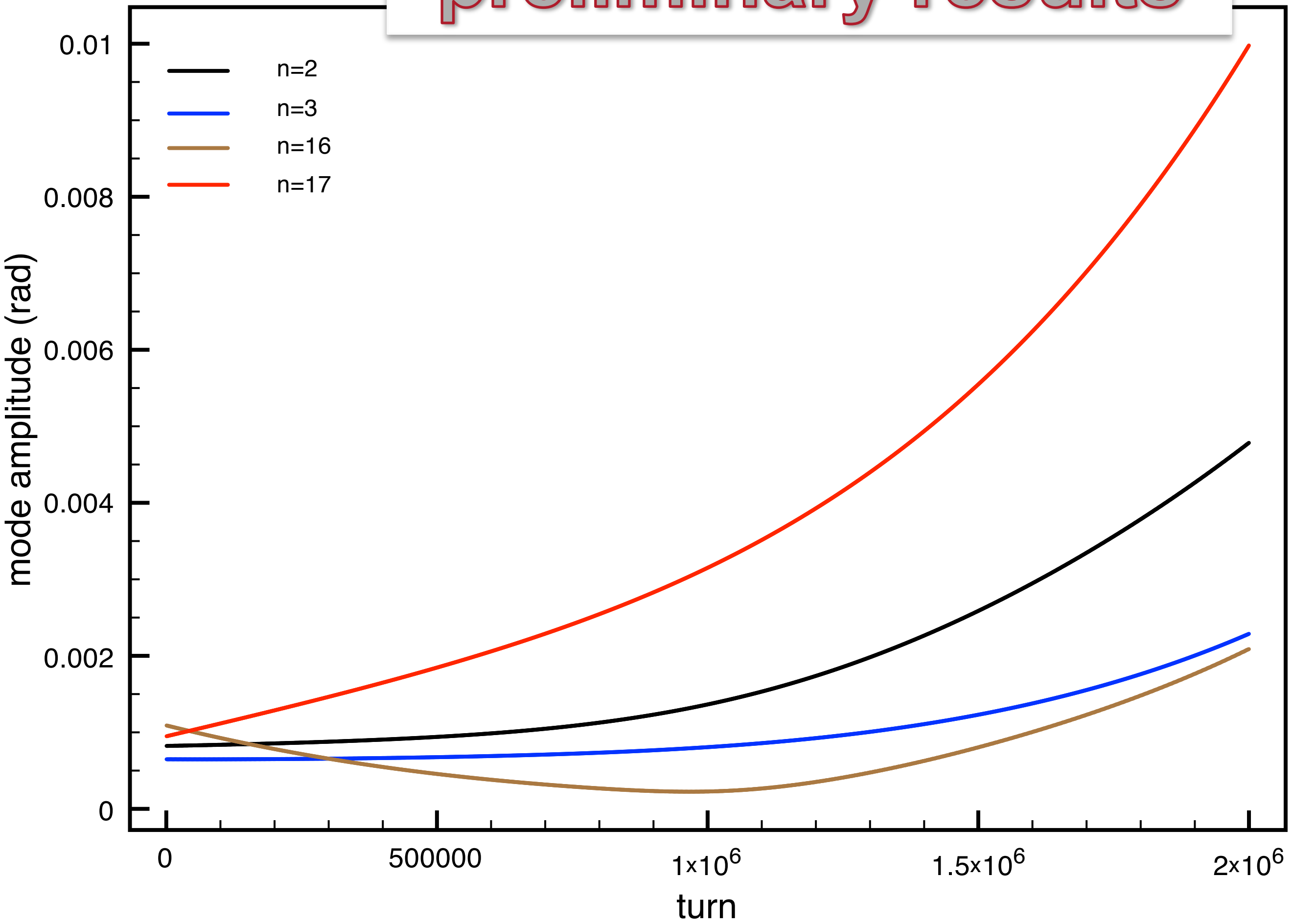
```
12.4459D0  750.35D0      6.535949310329977D7 ---------- Q  --  RS  --  WR
13.5905D0  824.954D0     6.252406268492149D7 ---------- Q  --  RS  --  WR
16.3653D0  732.464D0     5.968143726442426D7 ---------- Q  --  RS  --  WR
13.0183D0  735.866D0     6.252646101896113D7 ---------- Q  --  RS  --  WR
```

- we used $9x10^{12}$ ppp in 18, non equally spaced bunches (h=21)



Impedance model and comparison with measurements

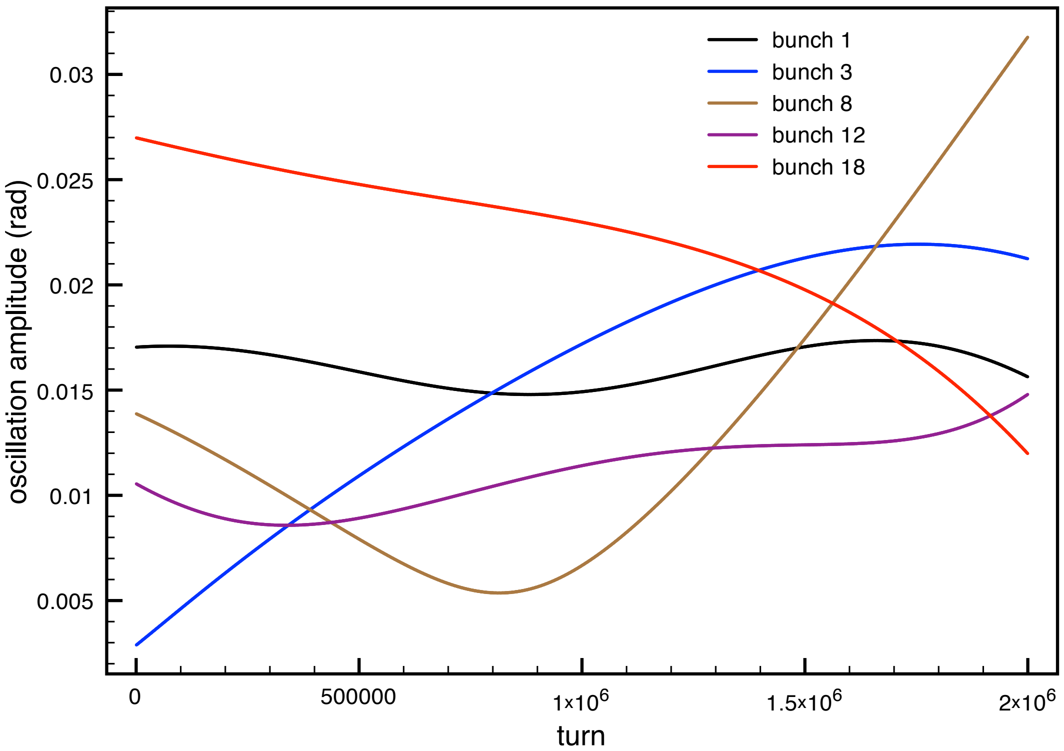
preliminary results

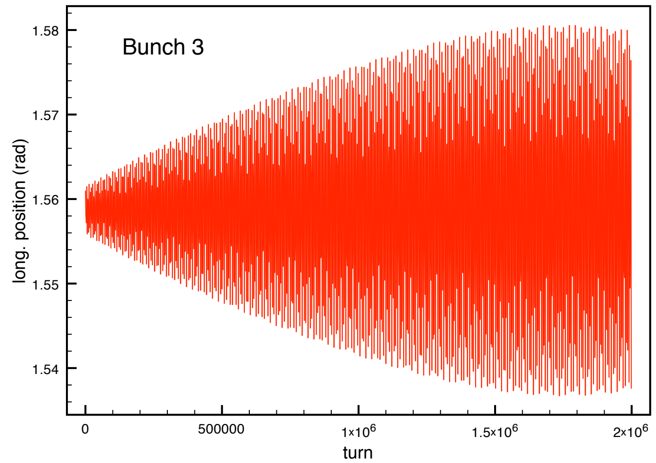


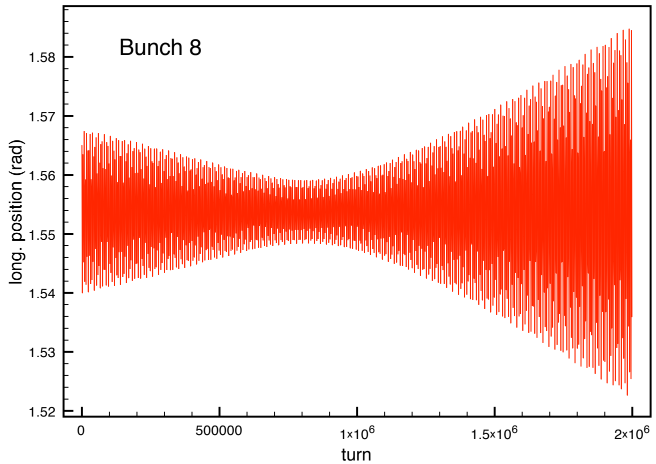
- The 'mode pattern' depends on the initial conditions
- Oscillation amplitude depends on the bunch number in the train

Impedance model and comparison with measurements

preliminary results







Impedance model and comparison with measurements

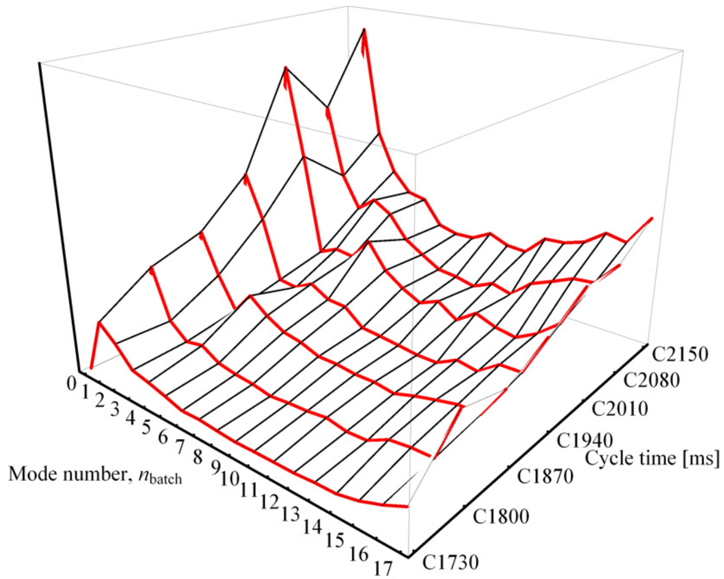
preliminary results

Proceedings of HB2010, Morschach, Switzerland MOPD52

LONGITUDINAL PERFORMANCE WITH HIGH-DENSITY BEAMS FOR THE LHC IN THE CERN PS

H. Damerau*, S. Hancock, M. Schokker, CERN, Geneva, Switzerland

- **The work continues …**



Conclusions …

- **The Longitudinal Coupled Bunch simulation Code (LCBC) has been benchmarked with the PS parameters.**
- **The code has been validated with the PS data obtained with external excitation.**
- **A frequency domain longitudinal FB system has been implemented and tested.**
- **The 10 MHz cavities impedance model has been implemented.**
- **Comparisons with measurements at h=7 show good agreement.**

… and future work

- **Simulations with PS nominal beam conditions.**
- **Simulations using PS-LIU beam parameters with longitudinal FB.**
- **Find the maximum required FB voltage with the PS-LIU beam parameters.**
- **Benchmark with HeadTail (only for longitudinal dipolar coupled bunch instability without FB).**


THANK YOU
FOR YOUR ATTENTION!