Nested HT Method: Impedance, Damper, Radial Modes and Coupled Bunches

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special thanks - to
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CERN, LMC Aug 29, 2012

Contents

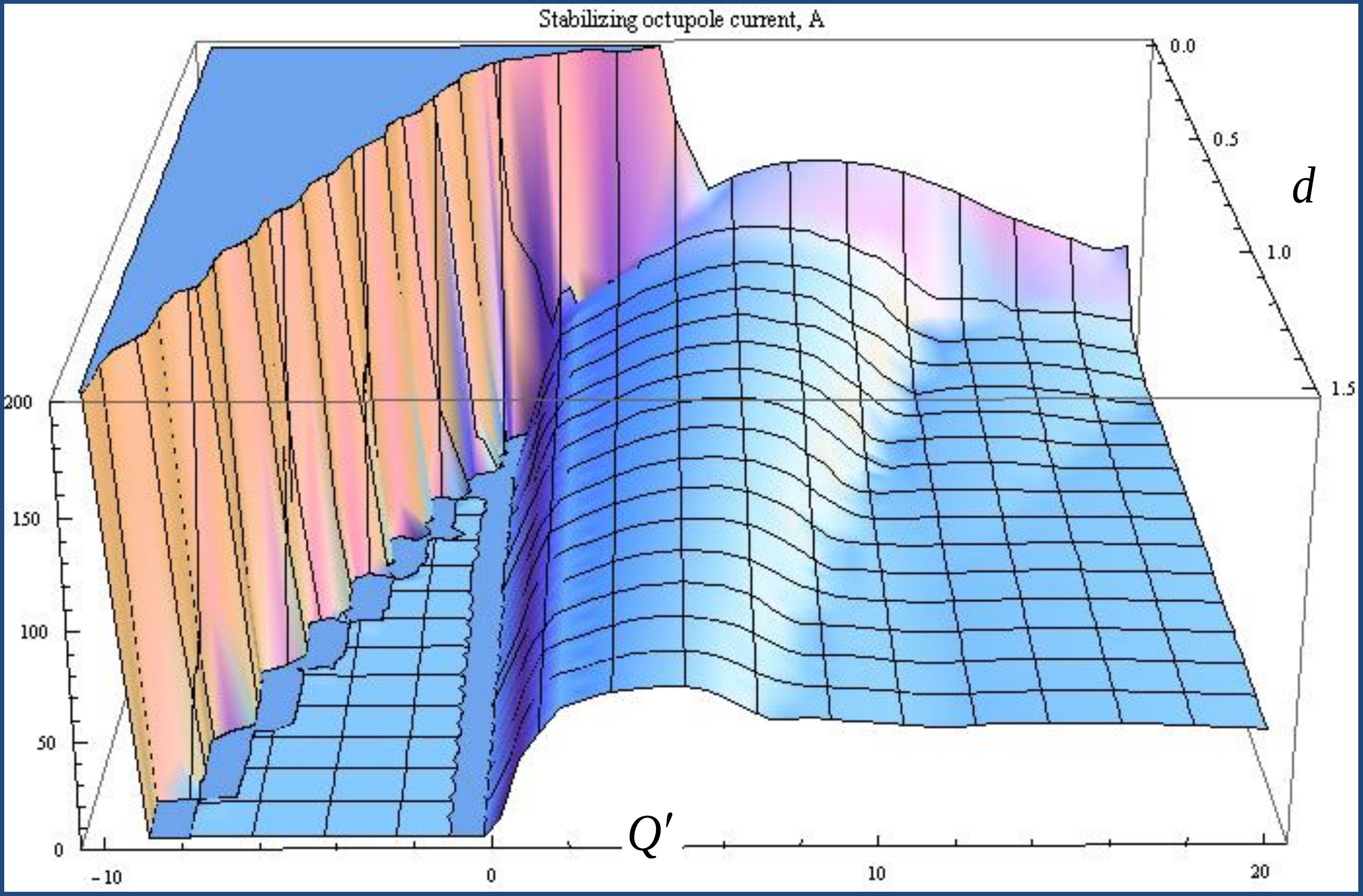
- The NHT method is developed to model multi-bunch transverse instabilities with all the azimuthal, radial and coupled-bunch modes.

- This presentation is a part of our common work with N. Mounet (developing a similar, but not identical approach) and S. White, developing a tracking code.

- Below is just a highlight of some recent results obtained by the NHT method. Its detailed presentation, together with results of N. Mounet and S. White, will be given in a special seminar.

- The main issues: dependence of the threshold octupole current on its polarity, the damper gain and its frequency profile, chromaticity, short- and long-range wakes, beam intensity and bunch separation.

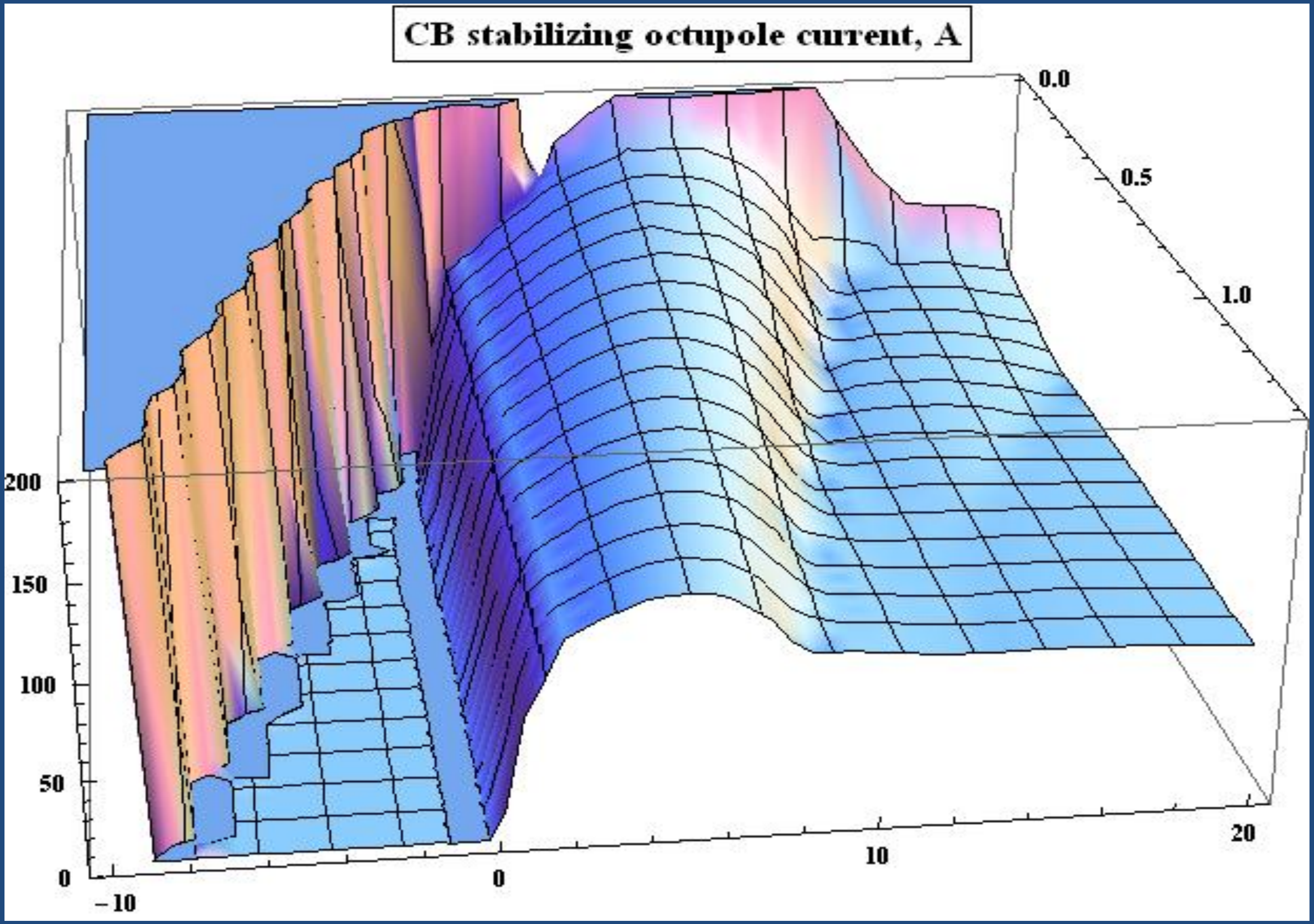
Threshold octupole current, negative polarity (old)



Threshold octupole current vs chromaticity and the damper gain.
The current ADT and 50 ns LHC beam are assumed.
Gain d=1.4 corresponds to 50 turns of the damping time at low CB frequency.

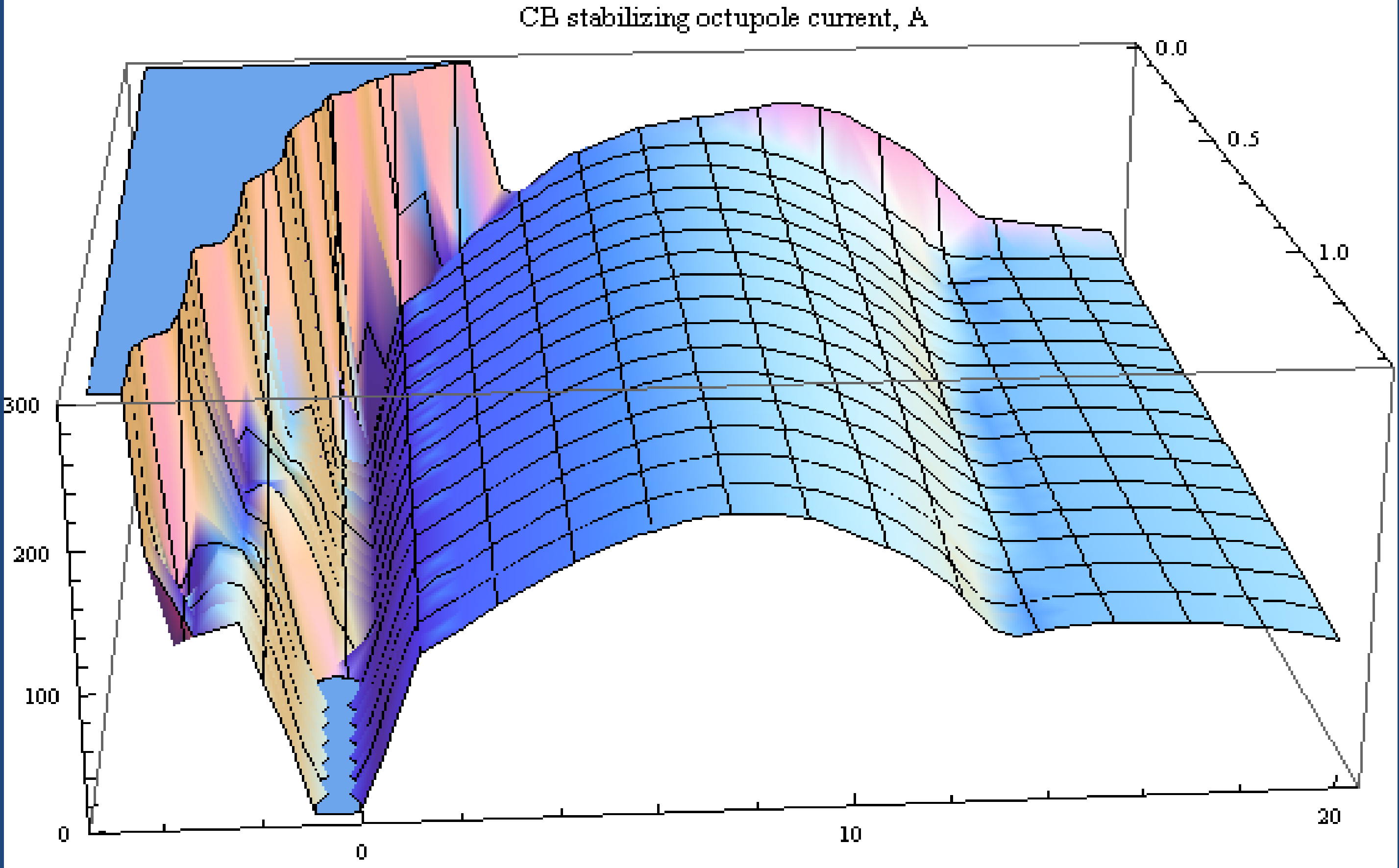
Note the area of zero-octupole stability.

Threshold octupole current, positive polarity (new)



The required current is ~ 1.7 times higher

2 times nominal impedance, Ioct<0 (old)

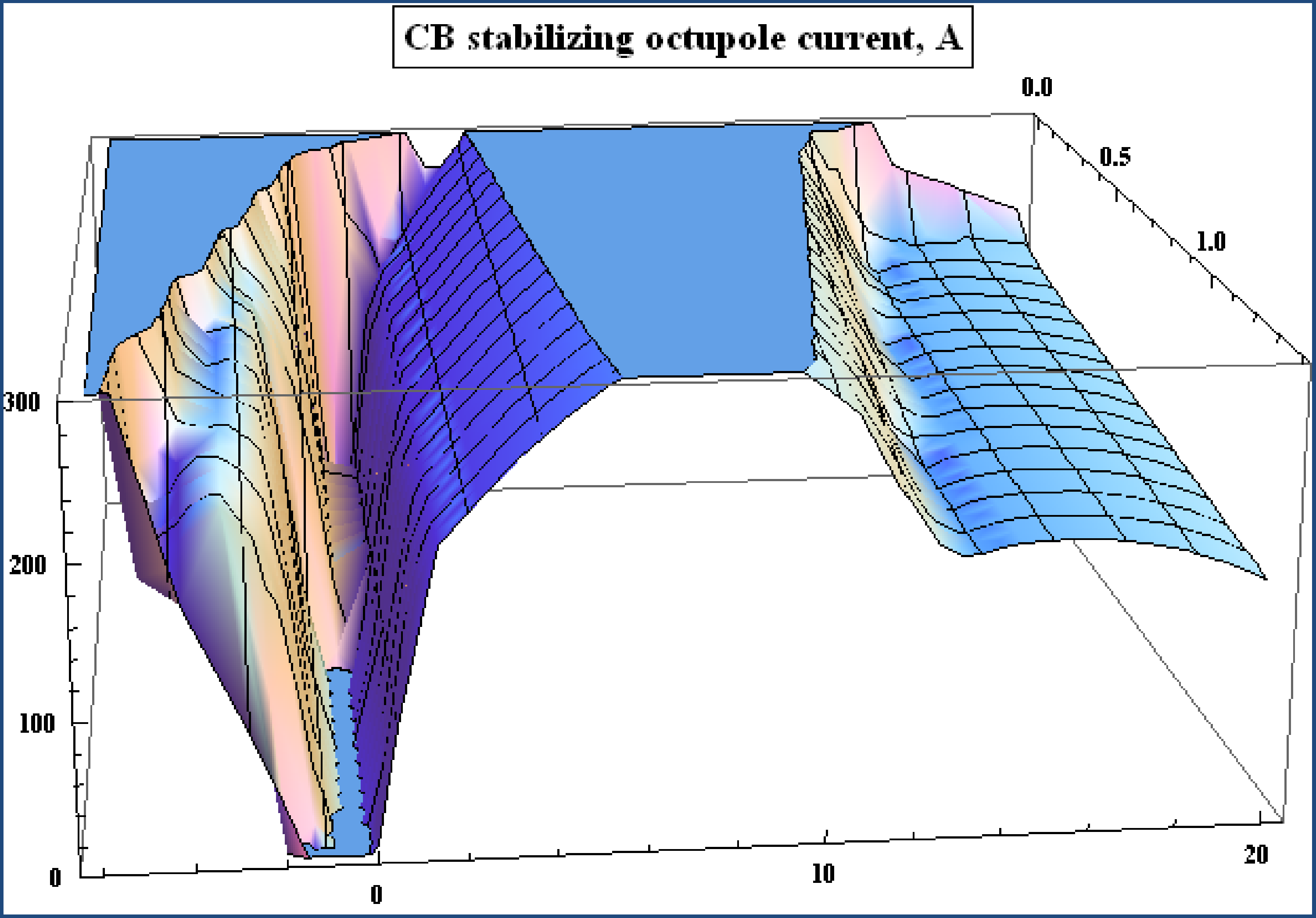


Threshold octupole current

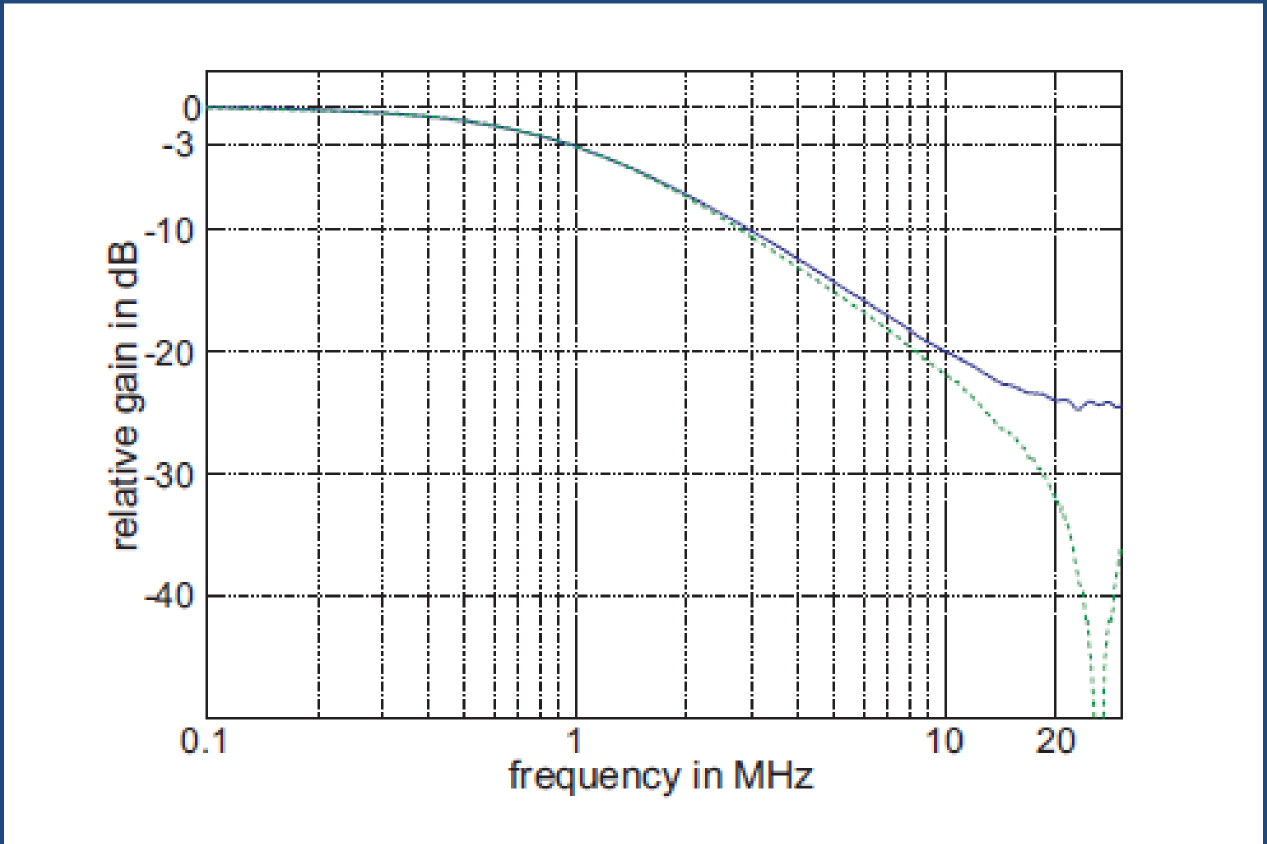
The zero-octupole stability area almost disappeared at that gain!

If the real impedance is doubled, then we are staying in the right area of chromaticity>12.

2 times nominal impedance, Ioct>0 (new)



Gain frequency profile

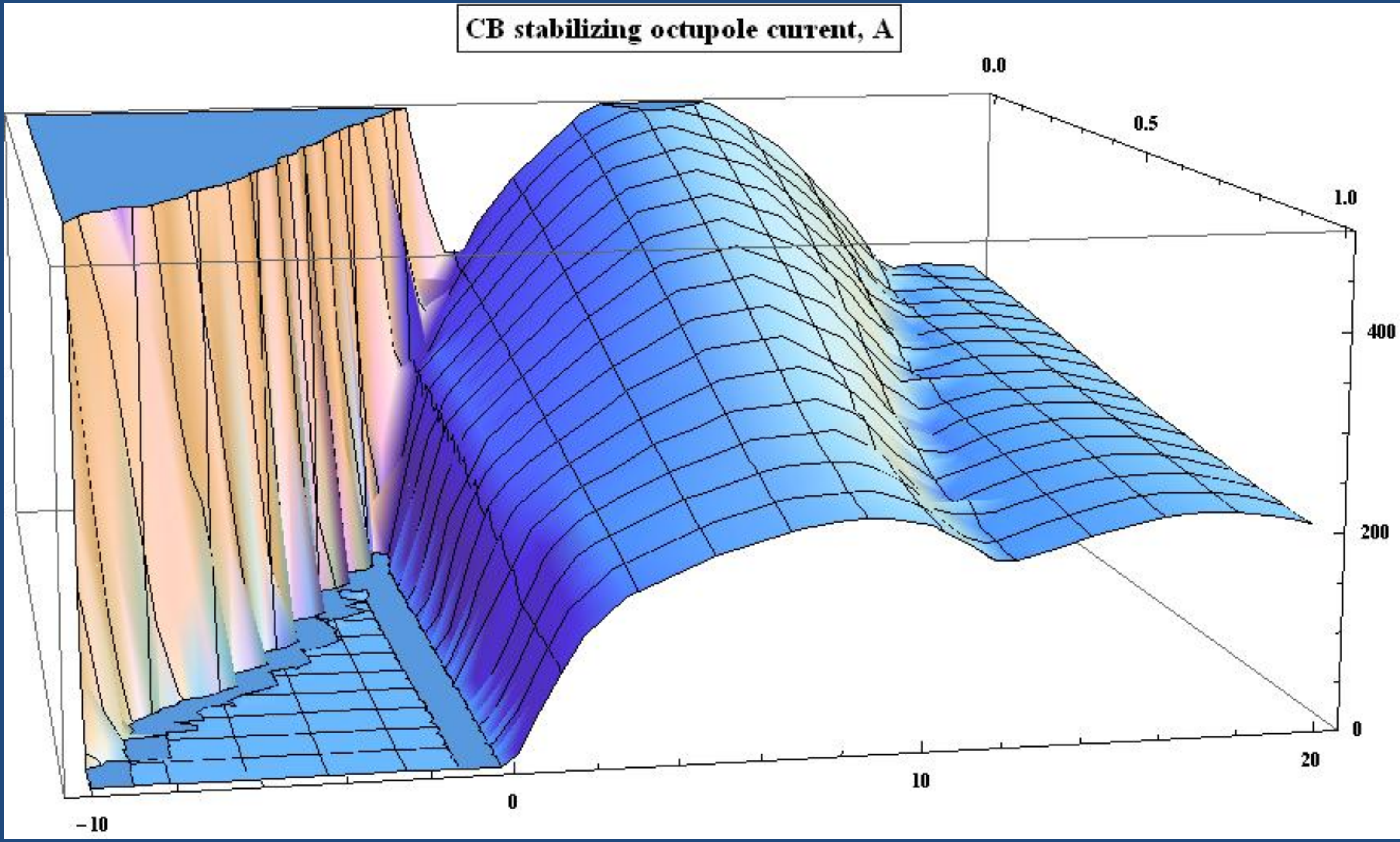


Current ADT frequency profile (W. Hofle, D. Valuch) .

At the maximal CB frequency 10 MHz it drops 10 times.

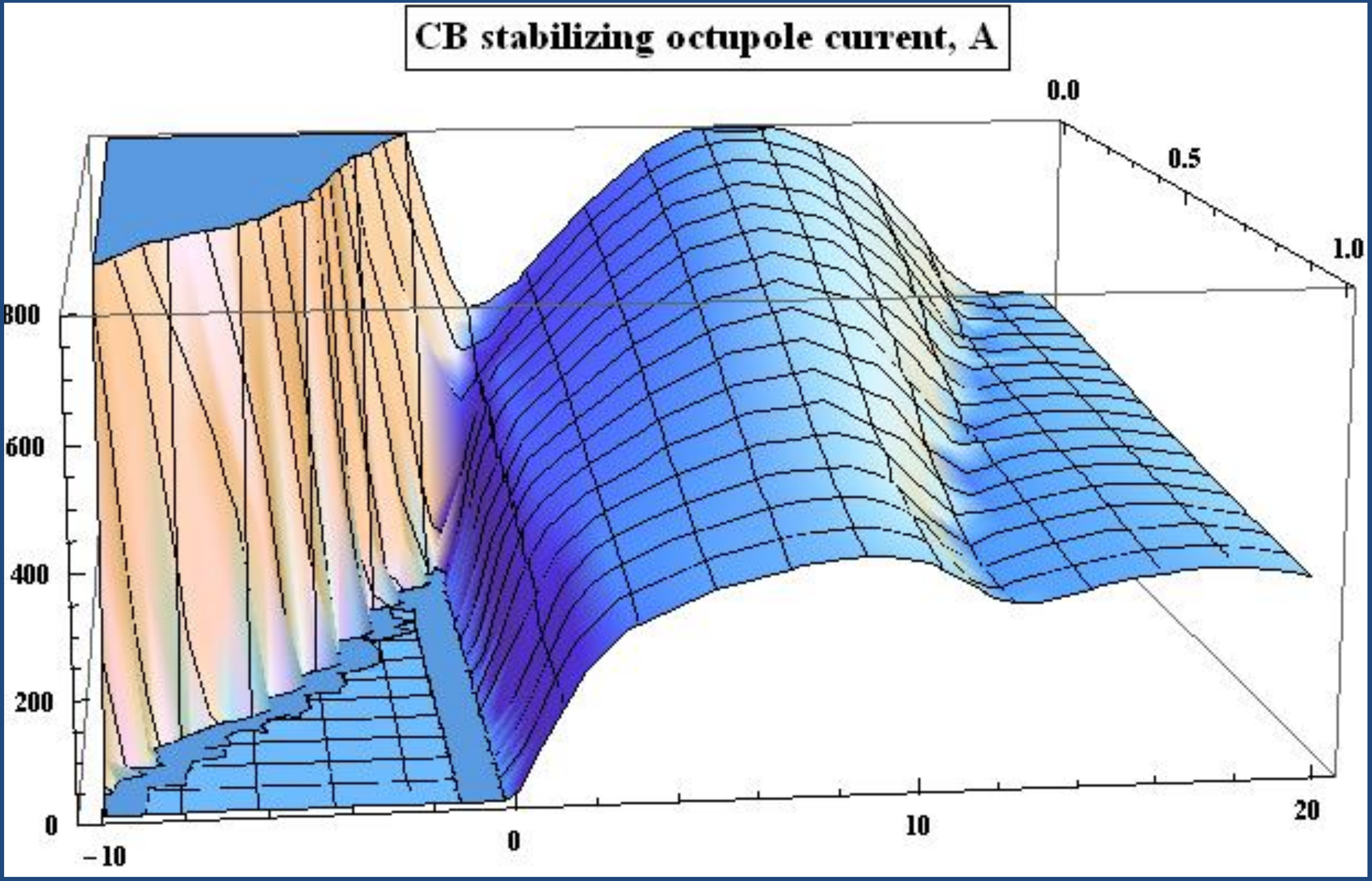
It can be shown that for the LHC beam the optimal gain profile is flat.
This profile is almost equivalent to a flat one with 10 times smaller gain.

Flat gain and 4 times nominal impedance, Ioct<0



The zero-octupole stability area is regained!

Flat gain and 4 times nominal impedance, Ioct>0



Summary: power of the model

- Method of nested head-tail modes (NHT) is implemented on a base of Mathematica. It allows to find coherent tunes for all the modes, solving the eigenproblem at its 3D set (azimuthal $\otimes$ radial $\otimes$ coupled-bunch).

- The external tables: impedance/wake, ADT frequency profile, stability diagram.

- Based on that, the threshold octupole current as a function of the gain amplitude and chromaticity is calculated.

- To test any new beam, impedance or gain profile, it takes only 25 min on my 3 years old laptop.

Summary: first results

- A good agreement (20% or better) with a single-bunch tracking code of Simon is seen.

- Cross-check with previously started Nicolas-Alexey (NA) computation (multiple bunches, no radial modes so far) is ongoing, showing an importance of parallel approaches.

- A recommendation of NA to increase chromaticity and keep the max gain, already showed its practical value, is confirmed by the NHT computations.

- Assuming 2 times higher impedance as nominal model of Nicolas, the reliable area of zero-octupole stability is excluded at current ADT.

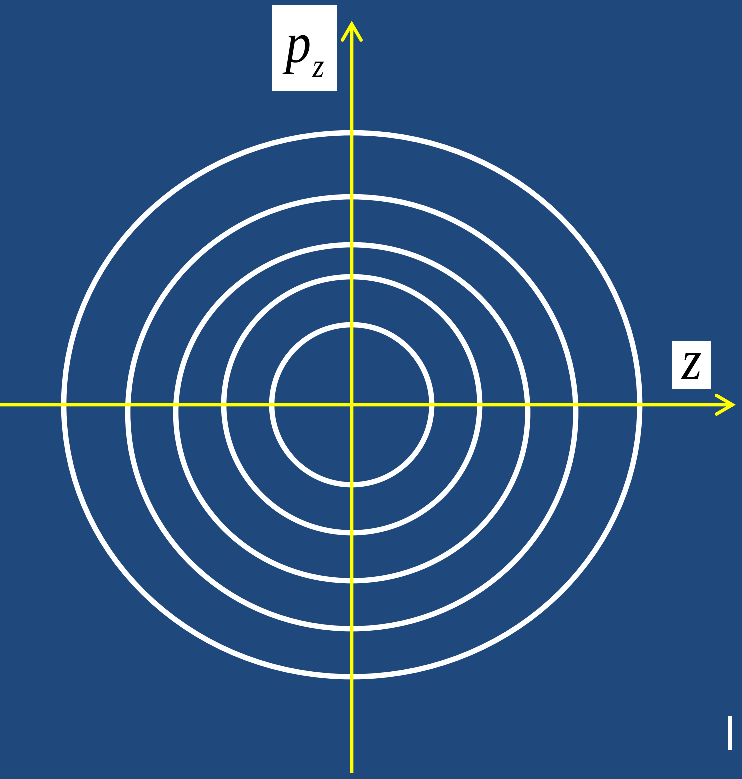
- For optimal flat ADT frequency profile this highly desirable area can be regained even for 4 times higher impedance!

Next steps

- To include train structure;
- To include quadrupolar wakes/impedances;
- To include coupled bunches in tracking code of Simon White using the same idea as for this code; to include real batches and trains there.
- At every stage to reach agreement with the parallel NA computations and Simon W. + Head-Tail tracking.
- To apply this ‘predicting machine’ for SPS, PS, PSB…

Many thanks for your attention!

Nested Head-Tail Basis



$$\psi_{l\alpha} \propto \exp(il\phi + i\chi_\alpha \cos\phi - i\Omega_l t)\,;$$

$$\chi_\alpha = \frac{Q'\omega_0 r_\alpha}{c\eta}\;;$$

$$\Omega_l = \omega_b + l\omega_s\;.$$

I am using n_r equally populated rings which radii r_α are chosen to reflect the phase space density.

Main Equation, single bunch

- In the water-bag single bunch approximation, beam equations of motion can be presented as in Ref [A. Chao, Eq. 6.183]:

$$\dot{X} = \hat{M}_0 \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X$$

where X is a vector of the HT mode amplitudes,

$$(\hat{M}_0 + \hat{Z})_{lm\alpha\beta} = -il\delta_{lm}\delta_{\alpha\beta} - i^{l-m}\frac{\kappa}{n_r}\int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau_\alpha - \chi_\alpha) J_m(\omega\tau_\beta - \chi_\beta)$$

$$\hat{D}_{lm\alpha\beta} = -i^{l-m}\frac{d}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta)$$

d is the damper gain in units of the damping rate, $\kappa = \frac{N_b r_0 R_0}{8\pi^2 \gamma Q_b Q_s}$

time is in units of the angular synchrotron frequency.

Coupled Equidistant Bunches

Main idea:

For LHC, the bunches are well-separated, so the wake function of the neighbor bunch can be taken as flat within the bunch length.

The only difference between the bunches is CB mode phase advance, otherwise they are all identical.

Thus, the CB kick felt by any bunch is proportional to its own offset, so the CB matrix $\hat{C}$ has the same structure as the damper matrix $\hat{D}$:

$$\dot{X} = \hat{M}_0 \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X + \hat{C} \cdot X;$$

$$\hat{D}_{lm\alpha\beta} = -i^{l-m} \frac{d_\mu}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta); \qquad \hat{C} = 2\pi i \kappa W(\varphi_\mu) \hat{D} / d_\mu;$$

$$W(\varphi_\mu) = \sum_{k=1}^{\infty} W(-ks_0) \exp(ik\varphi_\mu).$$

Wake and impedance are determined according to A. Chao book.

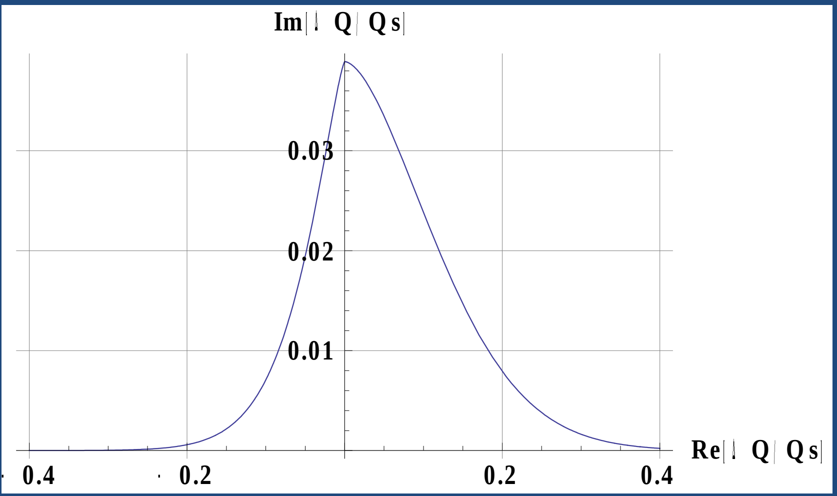
Analysis of solutions

1. For every given gain and chromaticity, the eigensystem is found for the LHC impedance table (Nicolas M.).

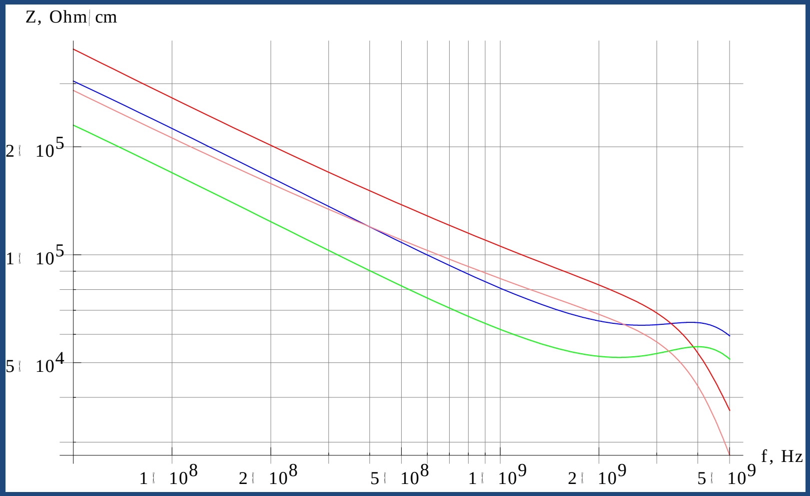
2. The complex tune shifts are found from the eigenvalues $\Delta\Omega_{l\alpha} = \Omega_{l\alpha} - l$

3. The stabilizing octupole current is found from the stability diagram (Xavier B.) for every mode, then max is taken.

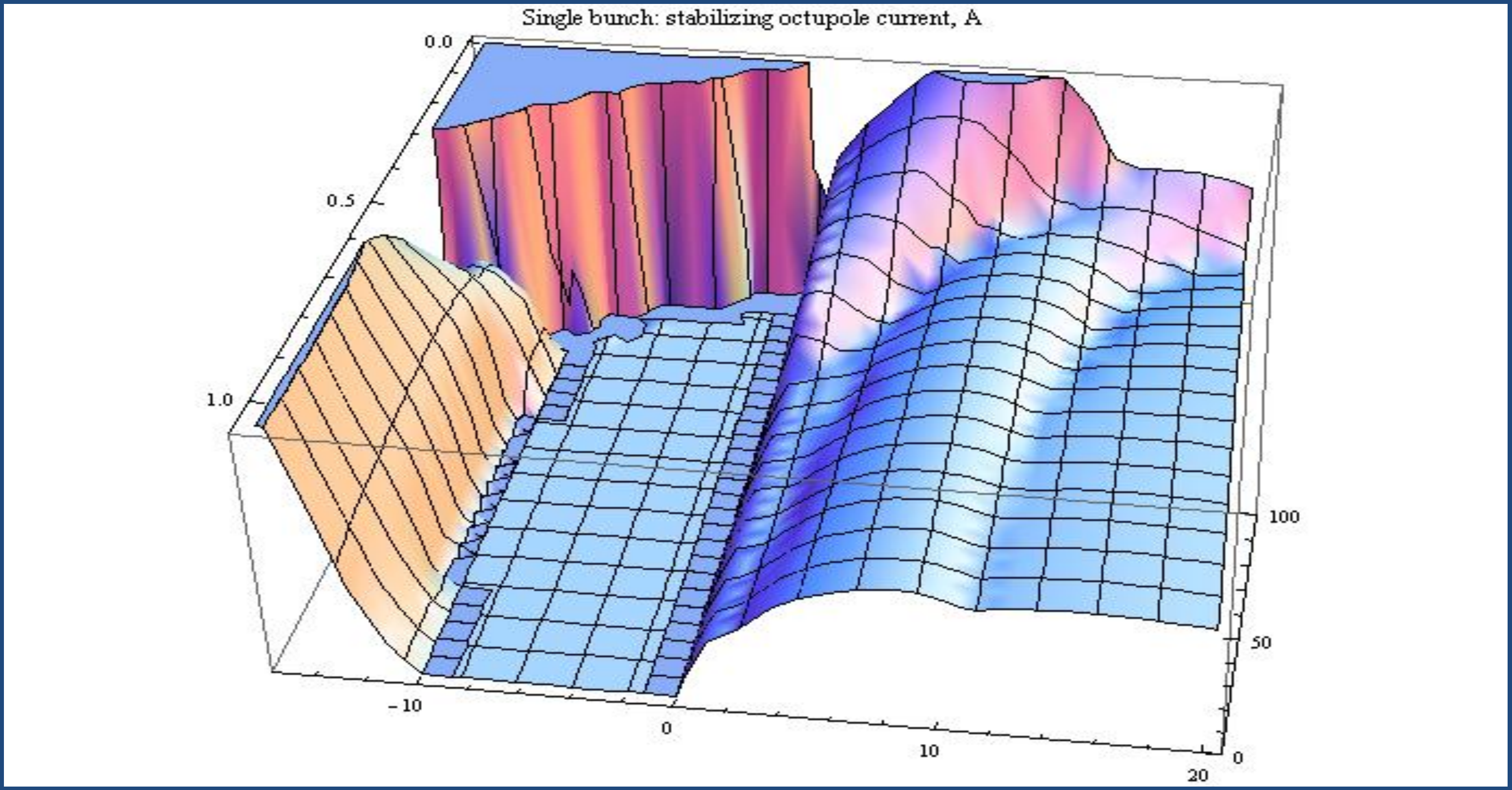
Stability diagram at -200 A of octupoles



Impedances (Nicolas M.)



Single Bunch, Ioct<0



Octupole current vs gain and chromaticity.

Note: the beam is stable for **gain>0.2** and $-10 \le Q' \le 0.$

Gain is measured in omega_s units, currently its maximum value is 1.4 at low CB frequency (=> 50 turns damping time), and 10 times smaller at 10 MHz.