When HT Damper Drives Instability?

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Main Equation

- In the air-bag single bunch approximation, beam equations of motion can be presented as in Ref [A. Chao, Eq. 6.183]:

$$\dot{X} = \hat{W} \cdot X - \hat{D} \cdot X$$

where X is a vector of the HT mode amplitudes,

$$\hat{W}_{lm} = -il\omega_s \delta_{lm} - i^{l-m}\kappa \int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau - \chi) J_m(\omega\tau - \chi)$$

$$\hat{D}_{lm} = i^{m-l} d J_l(\chi) J_m(\chi)$$

$\chi = \dfrac{Q'\delta p / p}{Q_s}$ is the HT phase, $\kappa = \dfrac{N_b r_0 c}{8\pi^2 \gamma Q_b}$ and d is the damper gain in units of the damping rate.

Eigen-System

- In 3-mode approximation, it leads to the Eigen-system problem:

$$\lambda X = \begin{pmatrix} i + r_1 - dJ_1^2(\chi) & -i\left[c - dJ_0(\chi)J_1(\chi)\right] & -dJ_1^2(\chi) \\ i\left[c - dJ_0(\chi)J_1(\chi)\right] & r_0 - dJ_0^2(\chi) & i\left[c - dJ_0(\chi)J_1(\chi)\right] \\ -dJ_1^2(\chi) & -i\left[c - dJ_0(\chi)J_1(\chi)\right] & -i + r_1 - dJ_1^2(\chi) \end{pmatrix}$$

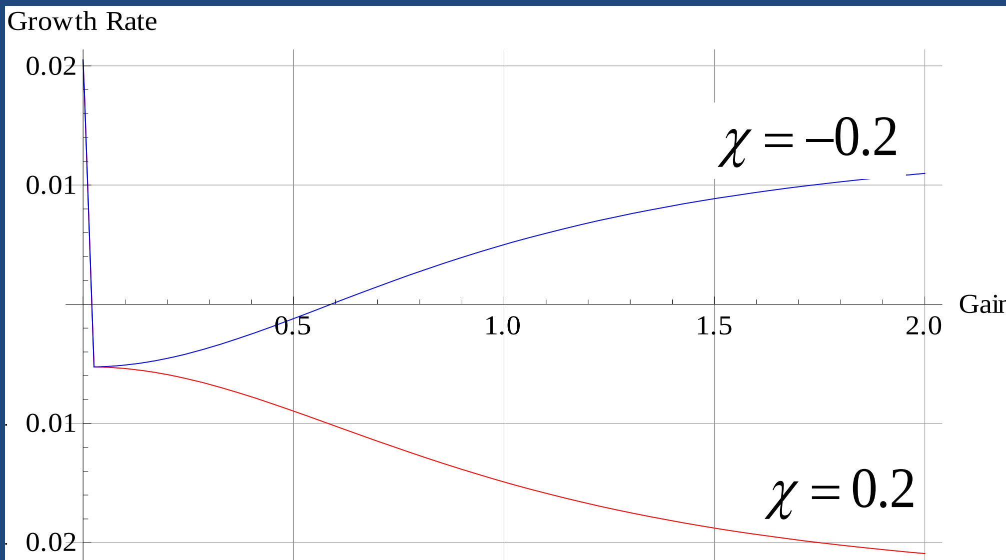
where

$$r_l = -\kappa \int_0^\infty d\omega \operatorname{Re} Z(\omega)\left[J_l^2(\omega\tau - \chi) - J_l^2(\omega\tau + \chi) \right]$$

$$c = 2\kappa \int_0^\infty d\omega \operatorname{Re} Z(\omega) J_0(\omega\tau - \chi) J_1(\omega\tau - \chi)$$

Examples

- Some typical examples

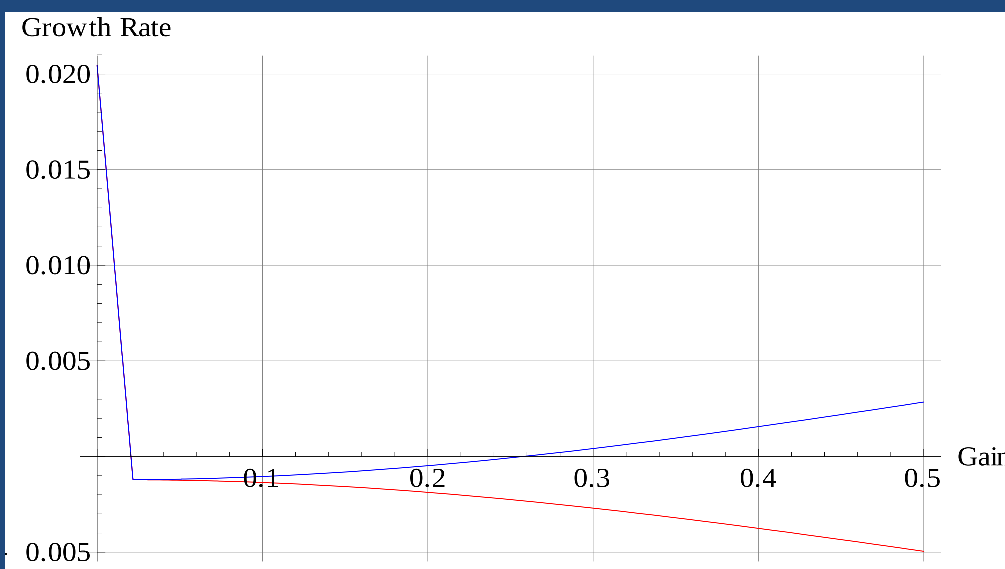


$r_0 = 0.02;$

$r_1 = -0.005;$

$c = 0.1;$

$\chi = \pm 0.2.$



$r_0 = 0.02;$

$r_1 = -0.001;$

$c = 0.1;$

$\chi = \pm 0.2.$

Conclusions

- For negative chromaticity, the damper may drive the beam unstable already at the gain several times lower than the synchrotron frequency.

- The exact value of the threshold gain depends also on the Landau damping of the first mode.

- MD proposal: at the negative chromaticity, to measure the threshold as the function of gain for different octupole settings.

Many thanks for everyone of you!