# Anomalous Diffusion for Phase Space Density Modification

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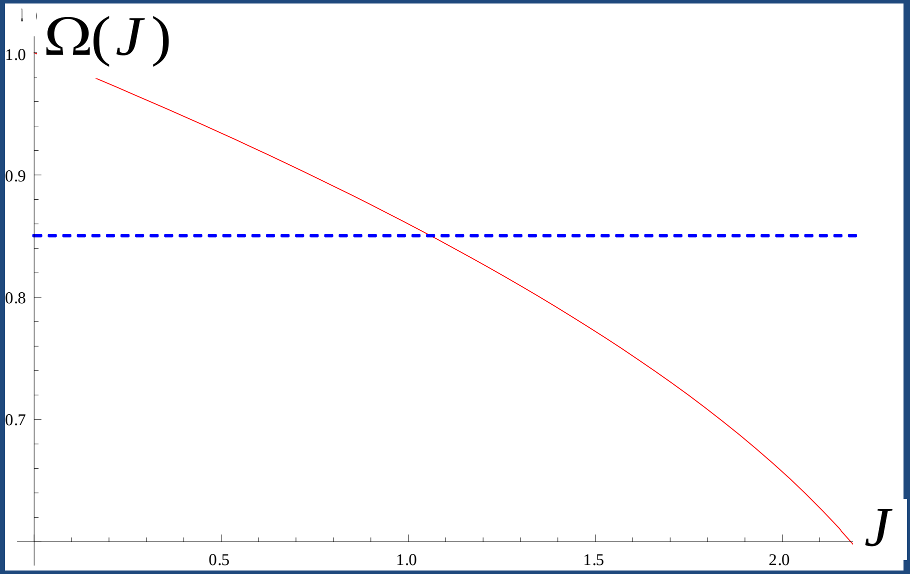
# Single-Harmonic RF

Synchrotron frequency

In a single-harmonic RF:

$$\Omega(J)/\Omega(0) \approx 1 - \frac{J}{\pi J_{\text{bucket}}}$$




Let the RF phase be modulated near the synchrotron frequency.
Then, equation of motion is:

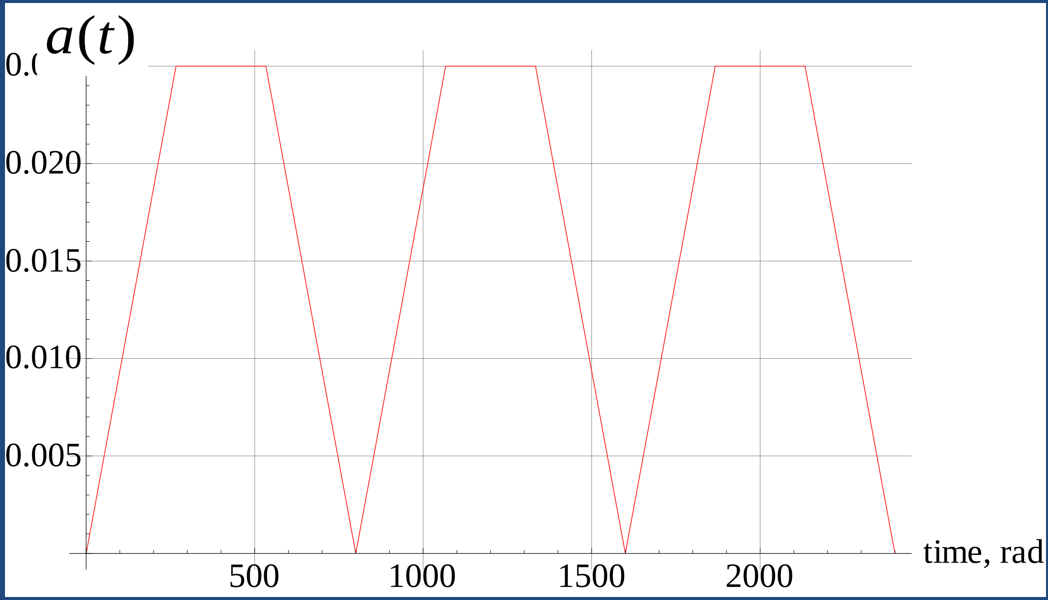
$$\ddot{z} + \sin\left[z - a(t)\sin\left((1-\varepsilon)t\right)\right] = 0$$

Slowly changed amplitude

Detuning

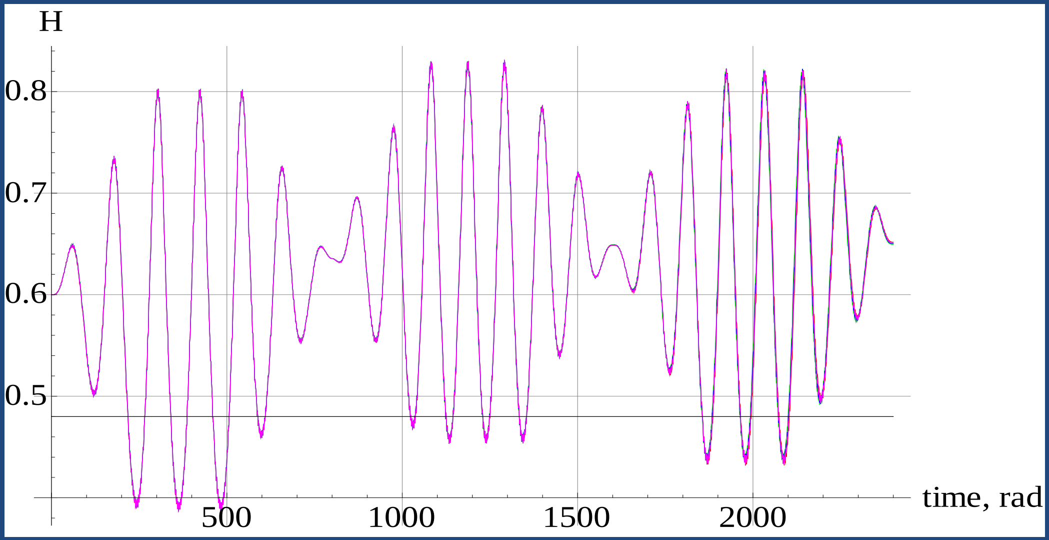
# Some solutions: nothing really special...

$\varepsilon = 0.03$



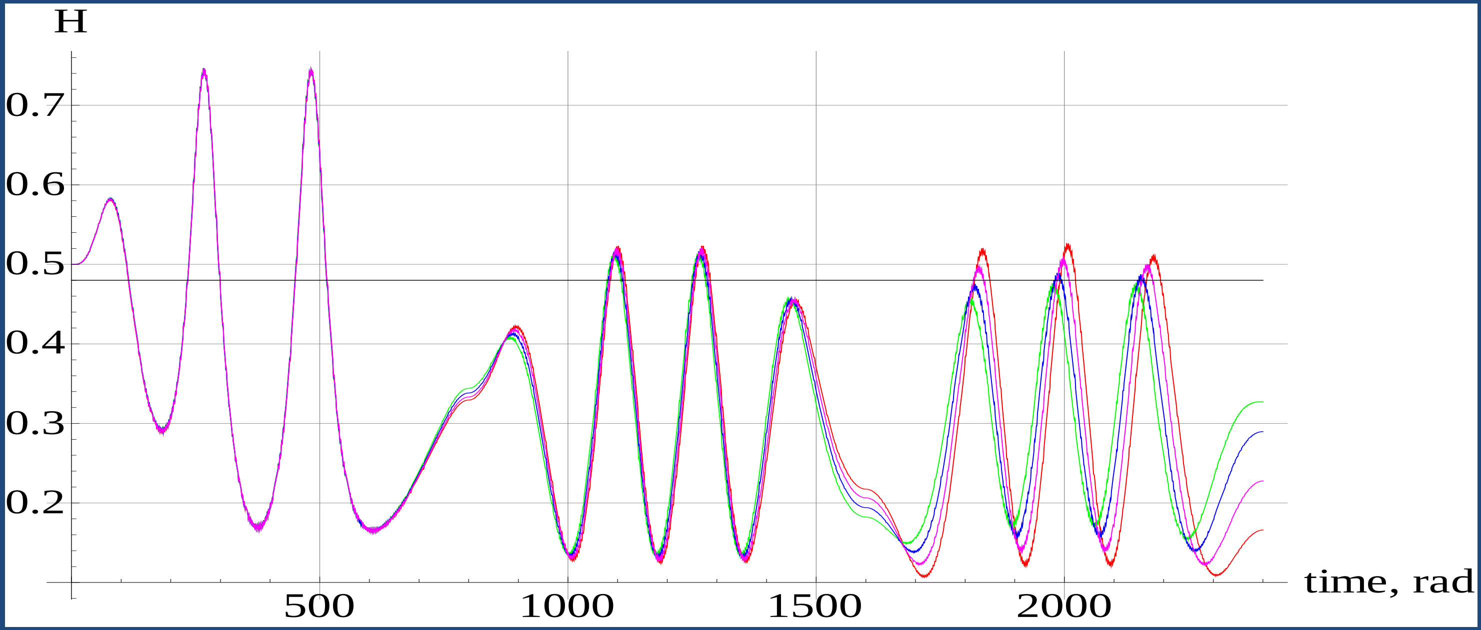

$$H_{1-4}(0) = 0.6$$
$$\phi_1(0) = 0.85\pi / 2$$
$$\phi_2(0) = 0.86\pi / 2$$
$$\phi_3(0) = 0.87\pi / 2$$
$$\phi_4(0) = 0.88\pi / 2$$

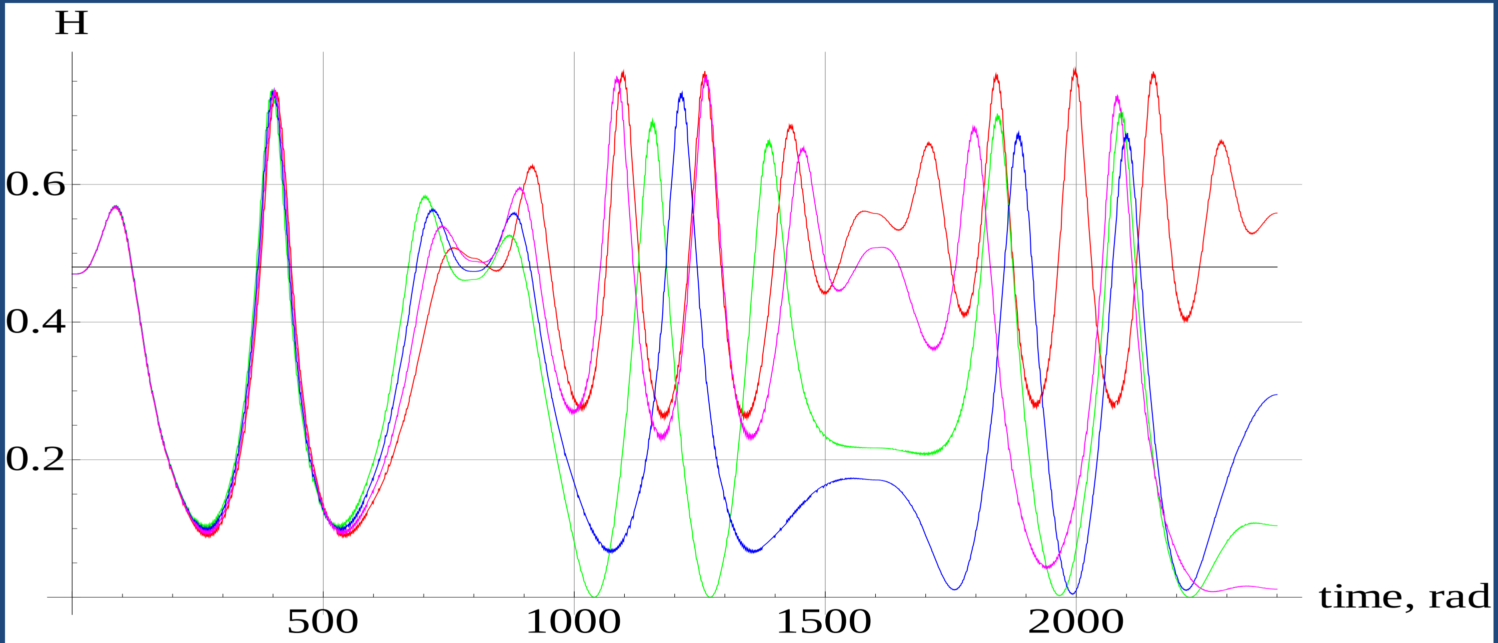
$$H = p^2 / 2 + 1 - \cos z$$



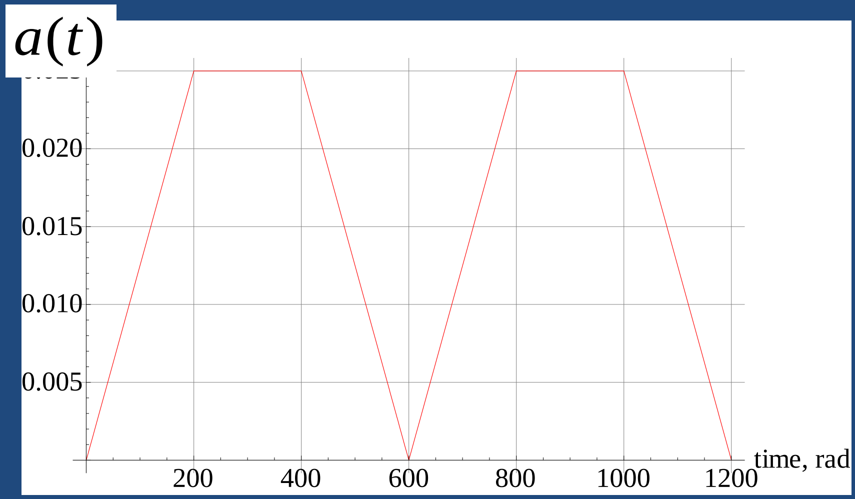

The close trajectories remain close.

# At the same time, for other particles...



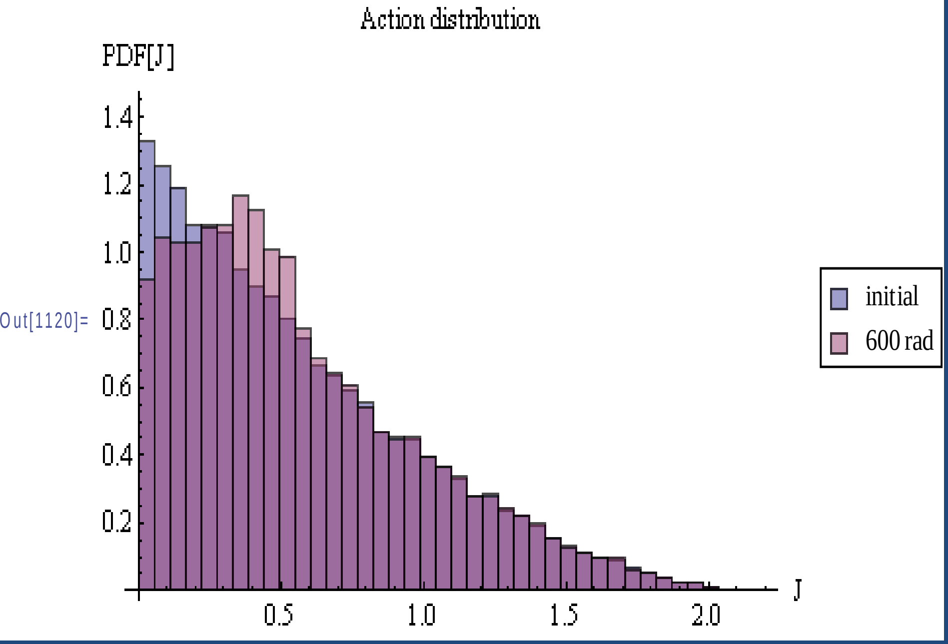
# Distribution function is changed (2 cycles):

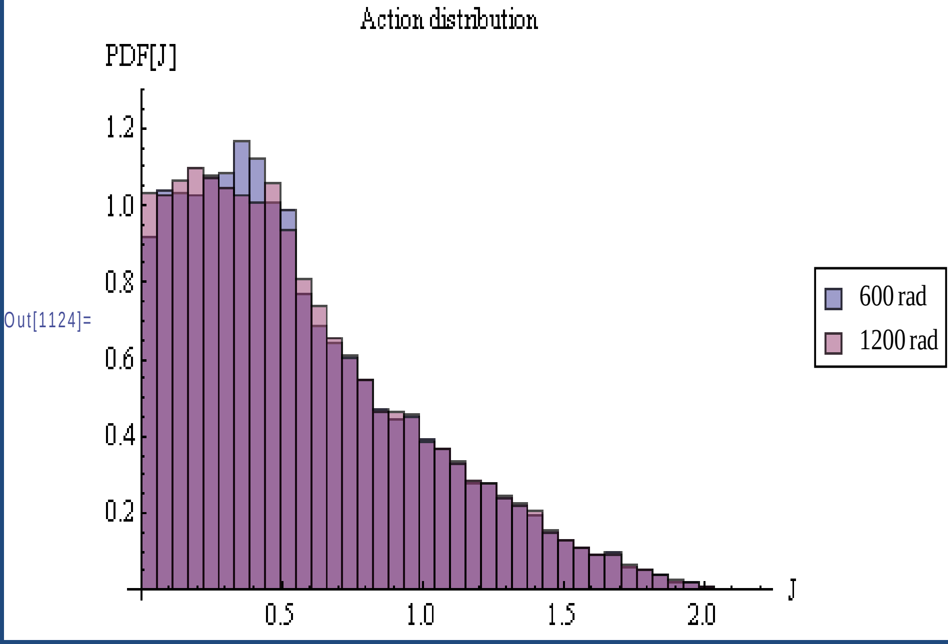



The affected area

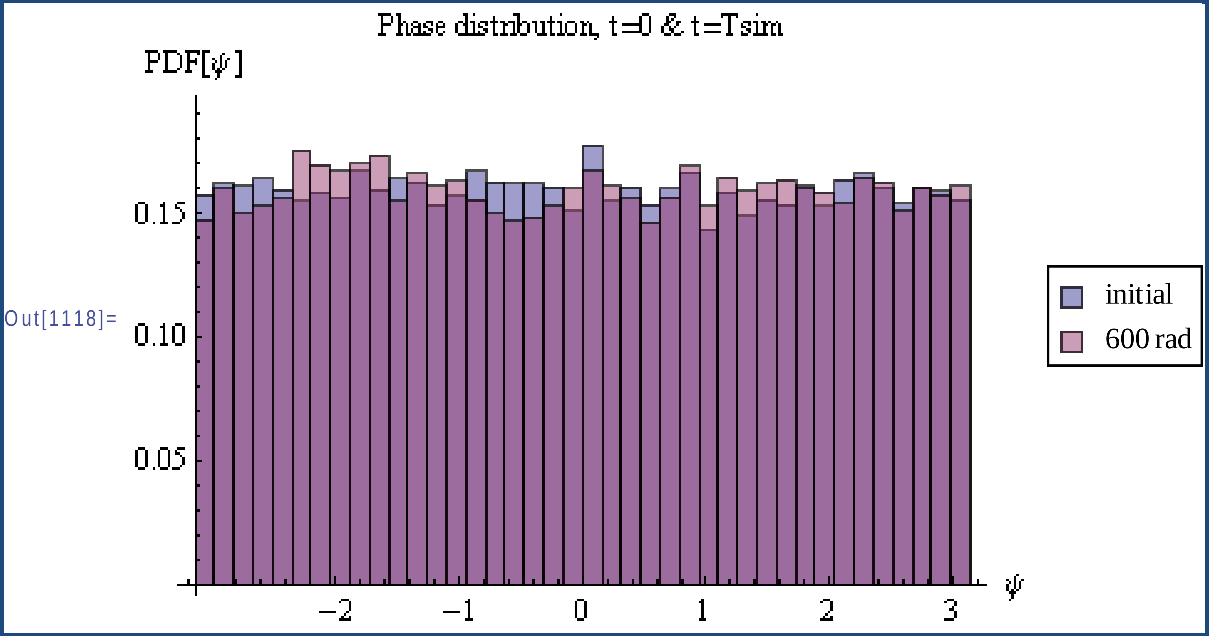
$$J_{\text{diff}} / J_{\text{bucket}} \approx 6\varepsilon$$

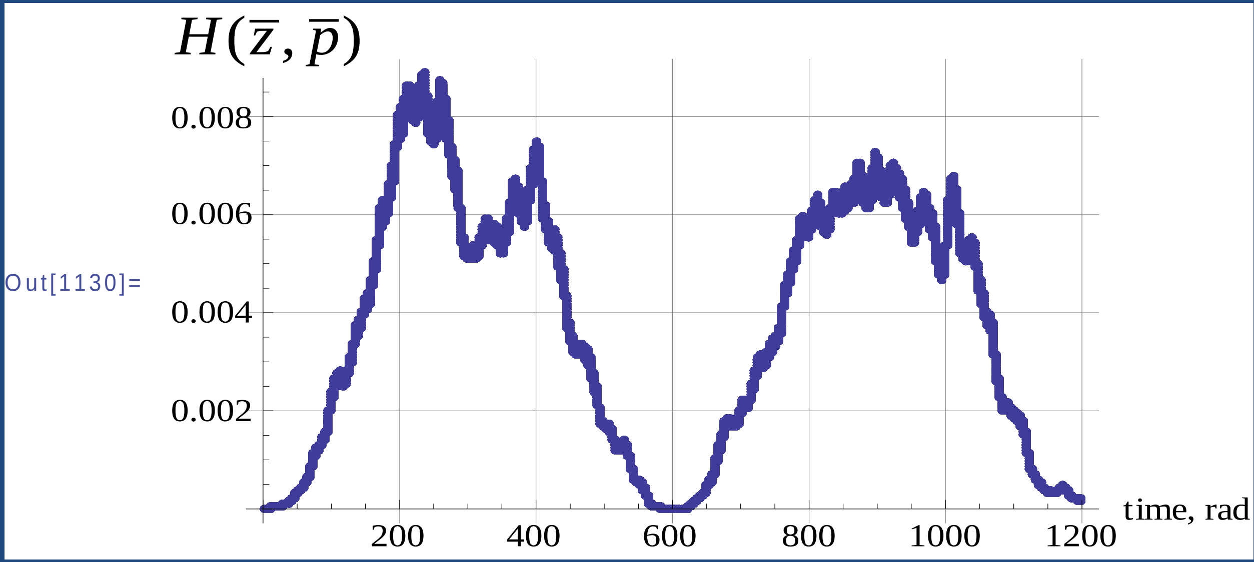
$$J_{\text{bucket}} = 8/\pi$$



# No coherent motion is excited:



# H. Huang et al, PRE 48, 4678 (1993)

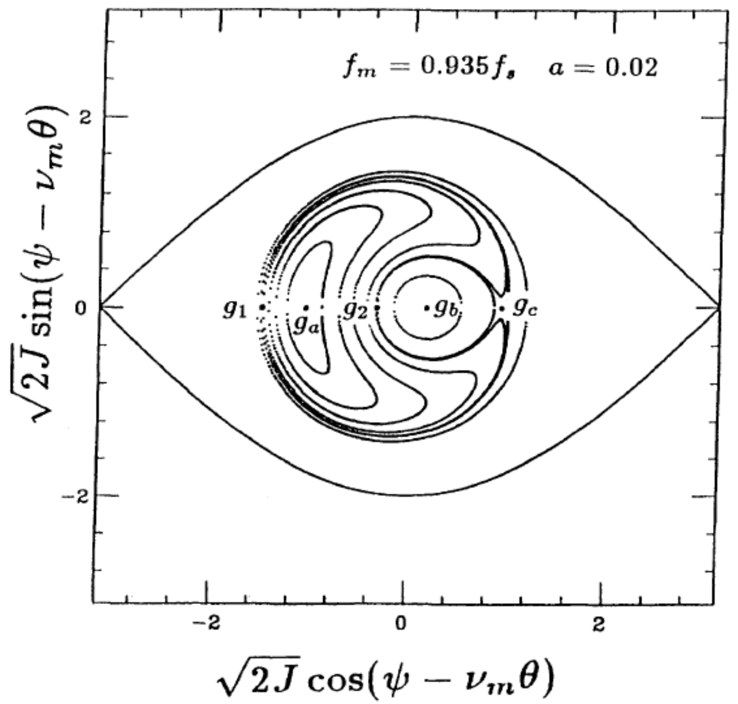



FIG. 2. The tori of the time averaged Hamiltonian at $\nu_m = 0.935\nu_s$ and $a = 0.02$. The separatrix for two resonant islands is the crescent shaped curve with cusps reaching the unstable fixed point.

Chaotic behavior follows from the separatrix crossing.
During the shaking cycle, the inner island shrinks to zero, and then grows again.
Particles outside the outer border do not cross the separatrix, and their distribution is not changed.

# Conclusions

- Anomalous diffusion gives an instrument for precise local flattening of the phase space density.

- Using that, the core may be flattened while the tails stay the same.

- Good precision for detuning is important.