

# Past predictions for Longitudinal Loss of Landau Damping and Longitudinal Mode Coupling and comparison with the recent results from Alexey Burov

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## ◆ **LLD:**

- Reminder of AlexeyB's recent results (see ICE meeting, 10/08/11)
- Reminder on Sacherer's approach (for different distributions)
- Comparison with AlexeyB's results (LHC and Tevatron)
- Comparison with recent measurements in the LHC (ElenaS)

## ◆ **LMCI**

- Simple analytical models compared to HEADTAIL (SPS)
- Comparison with AlexeyB's results

## ◆ **Conclusions and outlook**

## REMINDER ON ALEXEYB'S RECENT RESULTS (1/2)

- ◆ **AlexeyB mentioned that without making the rigid-bunch approximation and using the Van Kampen modes, he found a huge (order of magnitude!) disagreement with previous works (following Sacherer's approach, i.e. rigid-bunch approximation)**
- ◆ **AlexeyB considered 3 distributions and the case of a constant inductive impedance above transition:**
  - **HP (Hofmann & Pedersen) distribution**
  - **Smooth (Sacherer's) distribution**
  - **The particular one of Tevatron with 7 coalesced bunches**

## REMINDER ON ALEXEYB'S RECENT RESULTS (2/2)

- ◆ **AlexeyB's new findings are the following:**
  - For HP: a threshold  $\sim 3$  times smaller than with the previous (Sacherer et al.) formalism
  - For the smooth Sacherer's distribution: a threshold  $\sim 1$  order of magnitude below (in the usual formalism it is only a factor  $\sim 2-3$  below, see next slides)
  - For the particular case of the Tevatron (using the "exact" phase space distribution): a threshold  $\sim 1$  order of magnitude below the smooth's one
  - Based on this analysis, AlexeyB's recommendation to fight this instability is to smoothen the core of the distribution as it is very effective (same qualitative result as with usual formalism, see next slides)

# REMINDER ON SACHERER'S APPROACH (1/10)

- See in particular "Methods for Computing Bunched-Beam Instabilities", CERN SI-BR/72-5, 1972 (<http://cdsweb.cern.ch/record/322545/files/CM-P00063598.pdf>)

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For the transverse case, the dispersion relation is

$$1 = - \frac{\tilde{K}(k)}{2Q_x} \int_{-\infty}^{\infty} \frac{g_0(\dot{z}) dz}{\omega - Q_x - k\dot{z}} \quad (20)$$

The Fourier transform of the kernel  $K(z)$  is related to either the longitudinal or transverse coupling impedance.

The more careful derivations of Landau<sup>16)</sup> (initial-value problem) or of Van Kampen and Case<sup>17)</sup> (normal-mode approach) give the usual prescription for treating the singularities in Eqs. (19) and (20).

*Discussion* The solution of the partial differential-integral Eq. (17) is relatively simple because (i) the variables can be separated, and (ii) the resulting integral equation reduces to an algebraic equation. In the bunched-beam case the variables also separate, but the integral equation analogous to (18) does not, in general, reduce to algebraic form.

#### 4. BUNCHED BEAMS

##### 4.1 Characteristics of solution

For linear external forces (no frequency spreads) and no self-forces, the orbits are circles in the  $z-\dot{z}/Q$  plane, and any initial distribution just rotates about the origin in this plane. Self-forces perturb the motion, but because they are usually weak in comparison with the external force, it is worth while to examine this seemingly trivial case in some detail.

The Vlasov equation reduces to

$$\frac{\partial g}{\partial t} - Q \frac{\partial g}{\partial \dot{z}} = 0, \quad (21)$$

which has the normal-mode type solutions

$$g_{mn} = R_{mn}(r) e^{-in\theta} e^{-i\omega_{mn}t}$$

$$\omega_{mn} = nQ$$

Because there are two degrees of freedom, there is a twofold infinity of solutions, as indicated by the indices  $m$  and  $n$ . Any distribution with circular symmetry but displaced from the origin (the rigid-dipole mode)

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$$-i\omega g_1 - Q_z \frac{\partial g_1}{\partial \theta} + F_1 \frac{\sin \theta}{Q_z} \frac{dg_0}{dr} = 0, \quad (9)$$

$$F_1(z) = \int K(z'-z) g_1(z', \dot{z}') dz' d\dot{z}'$$

The self-force  $F_1$  couples the various unperturbed modes

$$g_1 = R(r) e^{-in\theta} \quad (22)$$

together. However, the modes with different  $n$  oscillate with different frequencies, and as long as the self-force is small, we may ignore this coupling. This amounts to inserting Eq. (22) into Eq. (9) and keeping only the self-force term with the same harmonic  $n$ . We find an integral equation for  $R(r)$ ,

$$(\omega - nQ_z)R(r) + \frac{dg_0}{dr} \int_0^{\infty} G_n(r', r) R(r') r' dr' = 0 \quad (23)$$

where

$$G_n(r', r) = \frac{i}{2\pi} \int_0^{2\pi} K(r' \cos \theta' - r \cos \theta) e^{-in(\theta' - \theta)} \sin \theta d\theta d\theta'$$

*Discussion* This separation of angular and radial variables is valid as long as the frequency shifts produced by the self-force are small in comparison with the frequency separation  $Q_z$  between modes with different  $n$ . We end up with an integral equation analogous to the coasting-beam case (18), except that the integral is no longer a constant.

##### 4.3 Synthetic kernel<sup>18)</sup>

A common way to proceed is to simplify the self-force so that the integral in (23) is a constant. For example, this is accomplished if the self-force depends only on the location of the bunch centre-of-mass, and not on the details of the distribution,

$$F_1 = -2Q \Delta Q \bar{z} \\ = -2Q \Delta Q \int z g(z, \dot{z}) dz d\dot{z} \quad (24)$$

**Rigid-bunch approximation**

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Then

$$G_n(r', r) = \begin{cases} \pi Q \Delta Q r', & n = 1 \\ 0, & n \neq 1 \end{cases}$$

and when this is inserted into (23) we obtain the dispersion relation

$$1 = -\pi Q \Delta Q \int_0^{\infty} \frac{dg_0}{dr} r^2 dr, \quad (25)$$

where an amplitude dependence of  $Q_z$  is included. In the absence of Landau damping,  $Q_z = \text{constant}$  and we find

$$\omega = Q_z - \Delta Q, \quad (26)$$

which can serve to define  $\Delta Q$ .

*Discussion* Physically, we have replaced the actual interaction that generates a twofold infinity of modes and eigenfrequencies by a simplified interaction that excites only the rigid-dipole mode, as indicated in Fig. 5. We could also choose an interaction that excites only the breathing mode, or only one of the higher multipole modes. This approach

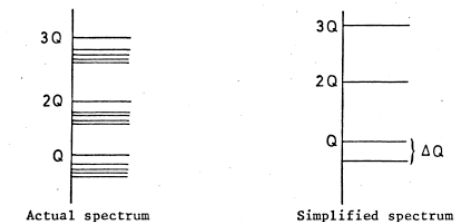


Fig. 5

is used in Refs. 10, 12, 13, and 14 to study transverse resistive-wall instabilities, and by many others. It seems to be the only practical method of including the effect of Landau damping in bunched beams. Dispersion relation (25) is evaluated in Ref. 4 for several distributions.

# REMINDER ON SACHERER'S APPROACH (2/10)

◆ See for instance:

■ [http://emetral.web.cern.ch/emetral/LongCohModes\\_26\\_02\\_03.ppt](http://emetral.web.cern.ch/emetral/LongCohModes_26_02_03.ppt)

■ <http://cdsweb.cern.ch/record/704810/files/ab-2004-002.pdf>

■ Dispersion relation

$$I_m^{-1}(\omega) = \Delta\omega_{cmm}^l$$

$$\Delta\omega_{cmm}^l = \frac{|m|}{|m|+1} \times \frac{j I_b \omega_s}{3 B^3 \hat{V}_T h \cos \phi_s} \times \left[ \frac{Z_l(p)}{p} \right]_{mm}^{eff}$$

$$B = f_0 \tau_b$$

$$I_m(\omega) = \frac{\int_0^\infty \frac{r^{2m}}{\omega - m\omega_s(r)} \frac{dg_0(r)}{dr} dr}{\int_0^\infty r^{2m} \frac{dg_0(r)}{dr} dr}$$

Sacherer formula

Dispersion integral

# REMINDER ON SACHERER'S APPROACH (3/10)

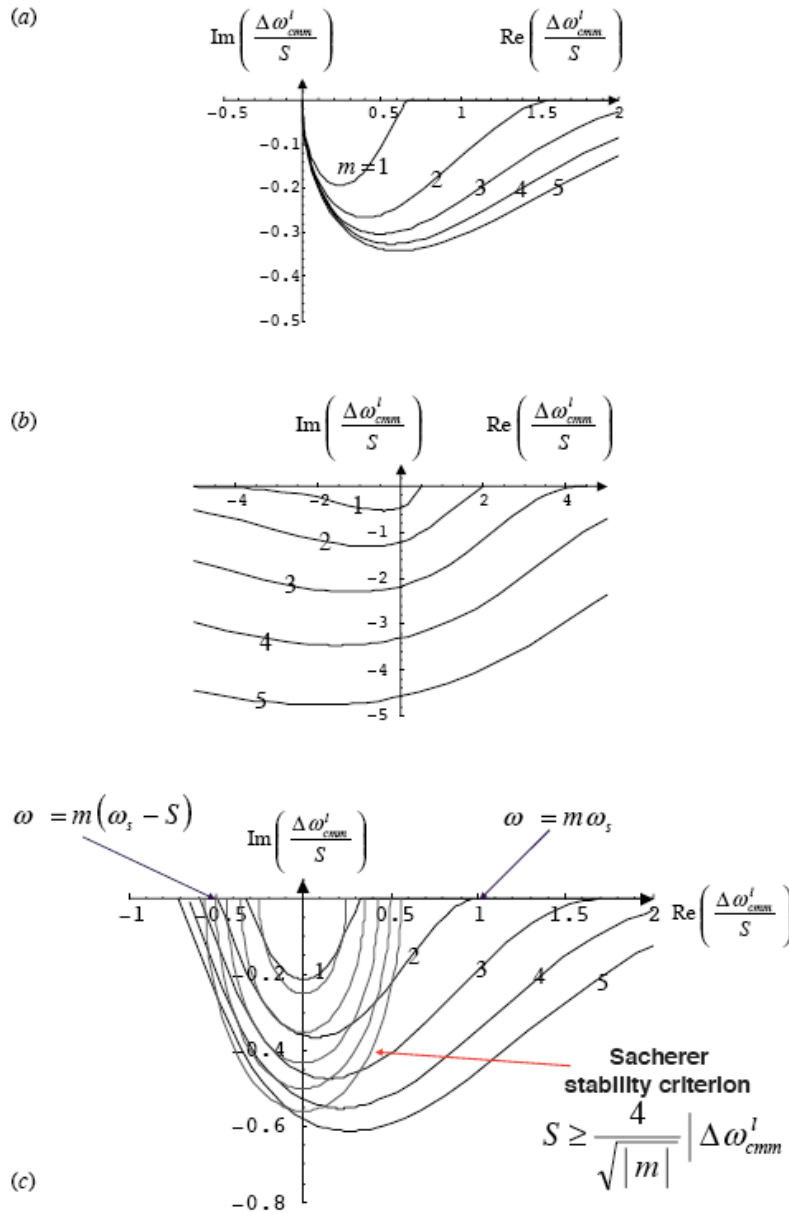


FIGURE 2. Stability diagrams for (a) a parabolic, (b) a Gaussian, and (c) the smooth distribution function  $g_0(\hat{\tau}) \propto (1-\hat{\tau}^2)^2$  used by Sacherer [3].

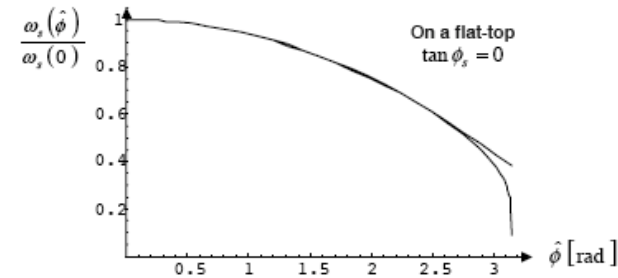


FIGURE 3. Exact and approximate (see Eq. (13)) full spreads between centre and edge of the bunch.

Several points of the stability diagram of Fig. 2(c) can be obtained analytically. The coherent synchrotron frequency shift is given by  $\Delta\omega'_{cmm}/S = m^2/(|m|+2)$  when  $\omega = m\omega_s$ , by  $\Delta\omega'_{cmm}/S = -|m|/(|m|+2)$  when  $\omega = m(\omega_s - S)$ , and by  $\text{Re}(\Delta\omega'_{cmm}/S) = 0$  when  $\omega \approx m(\omega_s - Sm/(m+1))$ .

The synchrotron amplitude distribution and line density of the three distribution functions are represented in Fig. 4.

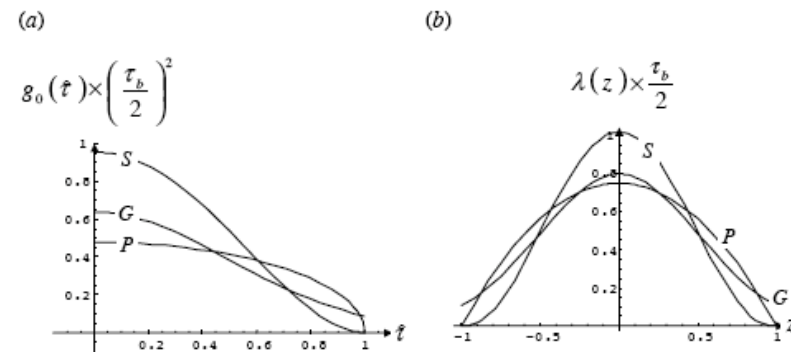


FIGURE 4. (a) Synchrotron amplitude distribution and (b) line density for a Parabolic, a Gaussian, and the smooth distribution used by Sacherer [3].

Here, the following definitions have been used:  $r = \hat{\tau}/(\tau_b/2)$ ,  $z = \tau/(\tau_b/2)$ ,  $\tau^2 + (\tau/\omega_{p0})^2 = \hat{\tau}^2$ ,  $\lambda(\tau) = \int g_0(\hat{\tau}) d(\tau/\omega_{p0})$ ,  $\int \lambda(\tau) d\tau = 1$ .

As can be seen in Eq. (2), the important function for the Landau damping mechanism is

$$\frac{\hat{\tau}^{2m} \frac{dg_0(\hat{\tau})}{d\hat{\tau}}}{\int_0^\infty \hat{\tau}^{2m} \frac{dg_0(\hat{\tau})}{d\hat{\tau}} d\hat{\tau}} \quad (14)$$

# REMINDER ON SACHERER'S APPROACH (4/10)

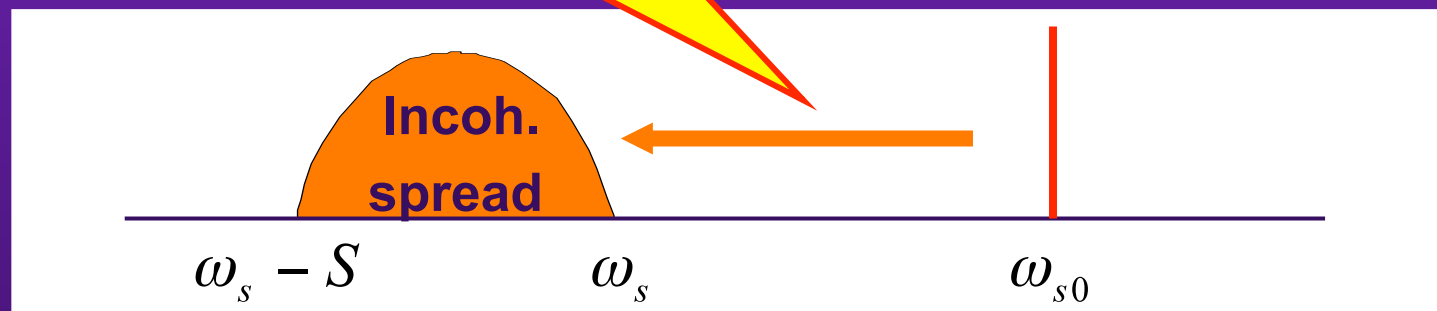
Reminder

Capacitive impedance Below Transition  
or  
Inductive impedance Above Transition

$$\Delta \omega_s^i < 0$$

$$\text{Re} \left( \frac{\Delta \omega_{cmm}^l}{S} \right) > 0$$

Incoherent  
synchrotron frequency  
shift  $\omega_s = \omega_{s0} + \Delta \omega_s^i$



$$S = \left( 1 + \frac{5}{3} \tan^2 \phi_s \right) \frac{\pi^2}{16} (h B)^2 \omega_s$$

Approximated full spread  
between centre and edge of  
the bunch

# REMINDER ON SACHERER'S APPROACH (5/10)

Case of the dipole mode  $m = 1$

$$\Delta \omega_{c11}^l = U - jV$$

Motions  $\propto e^{j\omega t} \Rightarrow V > 0 \iff$  Instability

$$\Rightarrow \omega = \left( \omega_s - \frac{S}{2} \right) + U \frac{S^2 + 16(U^2 + V^2)}{16(U^2 + V^2)} + jV \frac{S^2 - 16(U^2 + V^2)}{16(U^2 + V^2)}$$

$\Rightarrow$  Stability criterion  $S \geq 4 \left| \Delta \omega_{c11}^l \right|$

Sacherer criterion recovered analytically

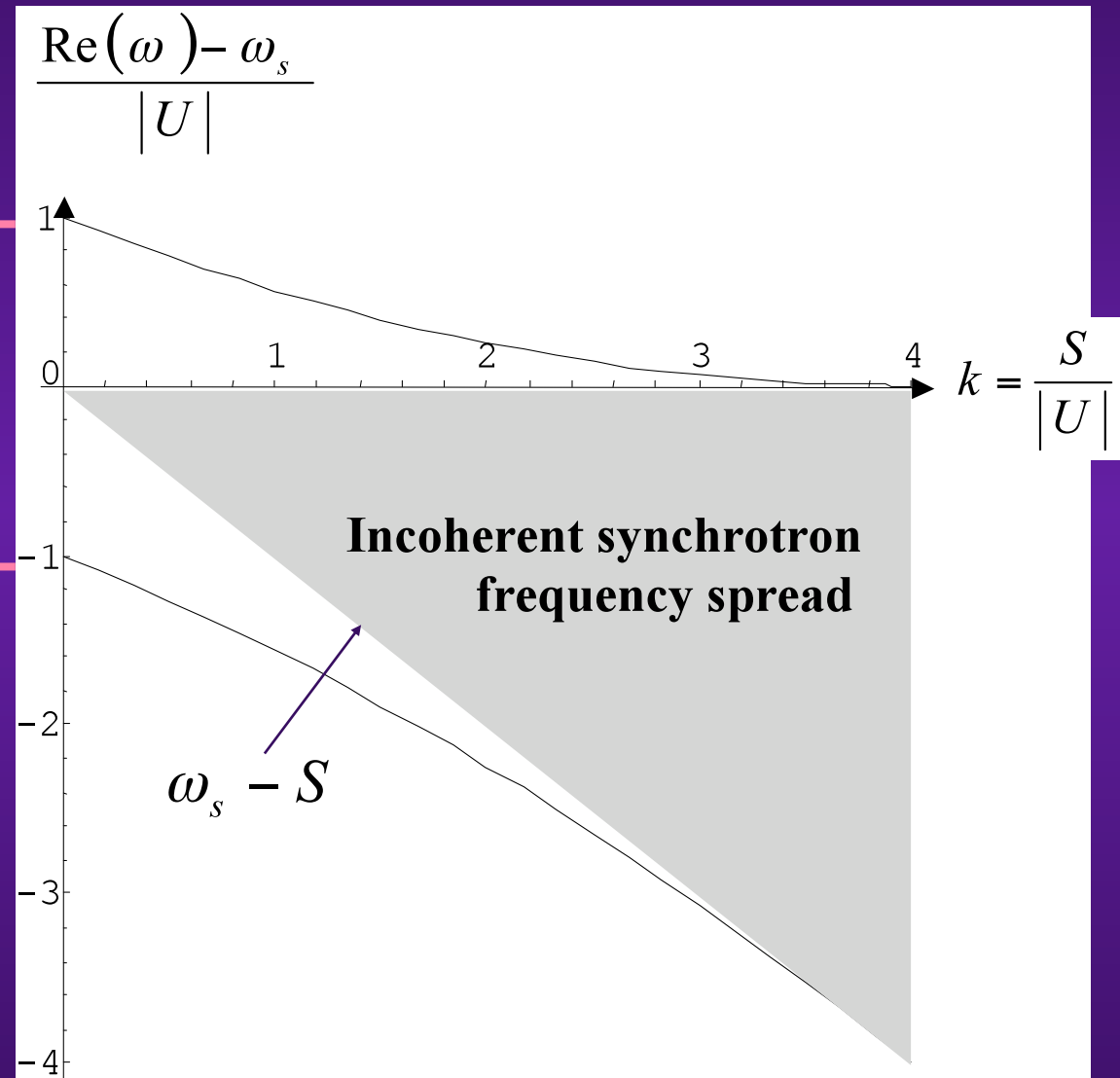
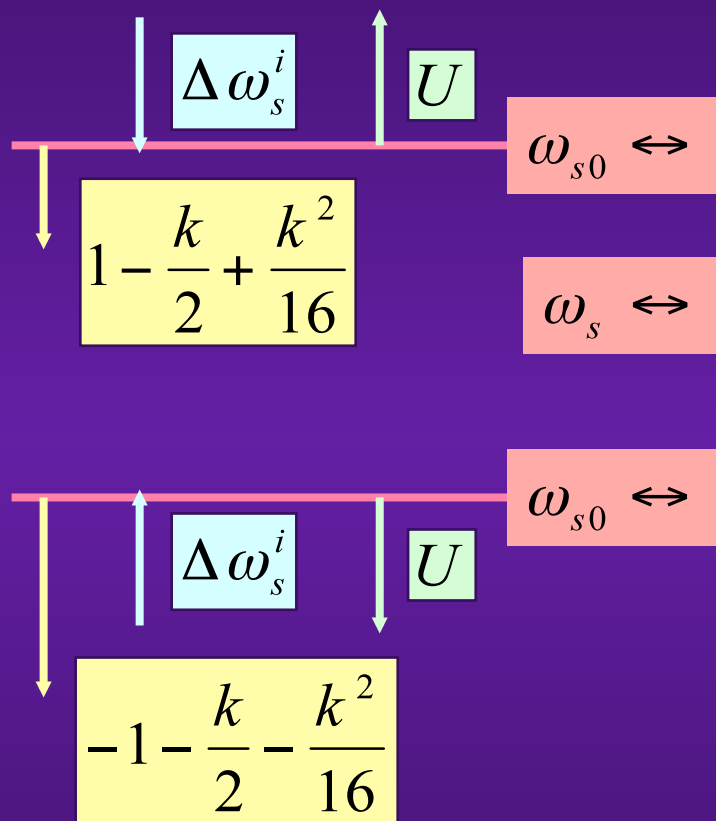
$$\text{Re}(\omega) = \omega_{s0} + \Delta \omega_s^i + U - \frac{S}{2} + \frac{S^2}{16U}$$

Generalization in the presence of frequency spread

$$|U| \gg V$$

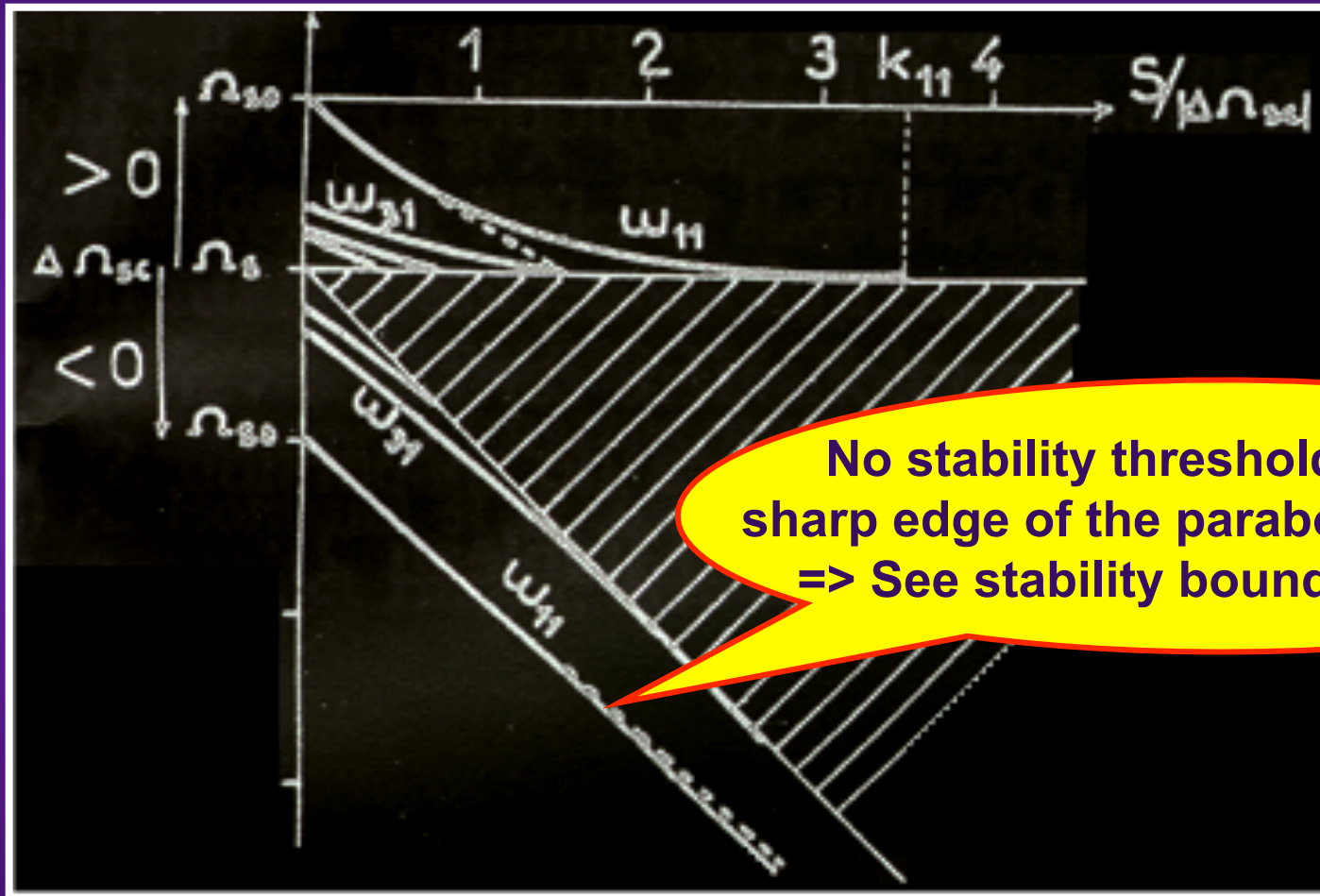


# REMINDER ON SACHERER'S APPROACH (6/10)



# REMINDER ON SACHERER'S APPROACH (7/10)

Reminder : Besnier's picture (in 1979 for a parabolic bunch)



## REMINDER ON SACHERER'S APPROACH (8/10)

- ◆ Some quantitative results (for the rigid-bunch approximation) for an inductive impedance above transition (i.e. case of AlexeyB):

- HP =>

$$\Delta \omega_{c11}^{l,th} \approx 0.7 S$$

- Gaussian =>

$$\Delta \omega_{c11}^{l,th} \approx 0.5 S$$

- Smooth Sacherer =>

$\Delta \omega_{c11}^{l,th} \approx 0.3 S$  => Slightly more than a factor 2 lower than HP => I agree with AlexeyB

- Approx. Sacherer => criterion I usually use

$$\Delta \omega_{c11}^{l,th} \approx 0.25 S$$

## REMINDER ON SACHERER'S APPROACH (9/10)

- ◆ => HP distribution is the best (as found by AlexeyB) because we discuss the particles in the centre of the distribution (as we consider the case of an inductive impedance above transition or space charge below) => It is indeed very efficient to flatten the distribution in the centre to increase Landau damping
- ◆ However, for the case of an inductive impedance below transition or space charge above, it would be the opposite as in this case, due to the sharp edge of the HP distribution, no Landau damping is provided and smooth (long) tails are preferred in this case

## REMINDER ON SACHERER'S APPROACH (10/10)

- ◆ Consider the case of an inductive impedance above transition (or space charge below), the HP distribution and the dipole mode only:
  - Computing the incoherent tune shift at small amplitudes at the intensity threshold gives (to use the same parameter as AlexeyB)

$$\frac{\Delta \omega_{s,th}^i}{\omega_{s0}} \approx -0.4 \times (h B)^2$$

- The intensity threshold can be expressed as

$$N_b^{th} = \frac{\Delta \omega_{s,th}^i}{\omega_{s0}} \times \frac{2 \pi^2 B^3 \hat{V}_T h \cos \phi_s}{3 e f_0 j \frac{Z_l}{n}}$$

# COMPARISON WITH ALEXEYB'S RESULTS (1/3)

## ◆ Numerical application for the LHC at top energy:

- $Z_1 / n = 0.1 \text{ j } \Omega$
- $f_0 = 11245 \text{ Hz}$
- $h = 35640$
- $V_{RF} = 16 \text{ MV}$
- Total ( $4 \sigma$ ) bunch length = 1 ns  $\Rightarrow B = 1.1245E-5$
- $f_{s0} = 23 \text{ Hz}$ ,  $\omega_{s0} = 144.5 \text{ rad/s}$
- $S = 14.3 \text{ rad/s}$

All consistent with what AlexeyB reported

AlexeyB found  $\sim 5.7E11 \text{ p/b}$ ,  
i.e. factor  $\sim 3$  less

AlexeyB found  $\sim 2.2\%$ ,  
i.e. also a factor  $\sim 3$  less

■  $\Rightarrow$  HP:

$$\frac{\Delta \omega_{s,th}^i}{\omega_{s0}} \approx -6.3 \%$$

$$N_b^{th} \approx 1.9 \cdot 10^{12} \text{ p/b}$$

■  $\Rightarrow$  Sacherer's approximation (which I usually use)

$$N_b^{th} \approx 6.7 \cdot 10^{11} \text{ p/b}$$

## COMPARISON WITH ALEXEYB'S RESULTS (2/3)

### ◆ Numerical application for the Tevatron at both injection and top energy:

- $Z_1 / n = 1.5 \text{ j } \Omega$
- $f_0 = 47619 \text{ Hz}$  (53 MHz RF frequency )
- $h = 1113$
- $V_{RF} = 1 \text{ MV}$  (both at injection - 150 GeV - and top - 980 GeV -)
- Total ( $4 \sigma$ ) bunch length = 15 / 9 ns at injection / top energy
- $f_{s0} = 90 \text{ Hz}$ ,  $w_{s0} = 565.5 \text{ rad/s}$
- $S = 14.3 \text{ rad/s}$

Measured unstable bunches  
at  $\sim 2E11$  p/b at injection

All consistent with what  
AlexeyB reported

#### ■ => HP:

- Injection energy
- Top energy

$$\frac{\Delta \omega_{s,th}^i}{\omega_{s0}} \approx -25 \%$$

$$N_b^{th} \approx 5.8 \cdot 10^{13} \text{ p/b}$$

$$\frac{\Delta \omega_{s,th}^i}{\omega_{s0}} \approx -9 \%$$

$$N_b^{th} \approx 4.5 \cdot 10^{12} \text{ p/b}$$

## COMPARISON WITH ALEXEY'S RESULTS (3/3)

### ◆ Comparison below and above transition (for inductive impedance):

#### ■ HP:

- **Me (and Besnier etc.): Always unstable**
- **AlexeyB: Same result (at least very very small as it is a numerical result and depends on grid point etc.)**

**=> Similar result between the 2 approaches**

#### ■ Smooth:

- **Me (and Sacherer etc.): Same AT and BT**
- **AlexeyB: Almost the same (~ 30% higher BT)**

**=> Similar result between the 2 approaches**

I would say then that the important point is the rigid-approximation and maybe not so much the potential well (to explain the difference with "usual" theories)...



# COMPARISON WITH LHC MEASUREMENTS BY ELENAS

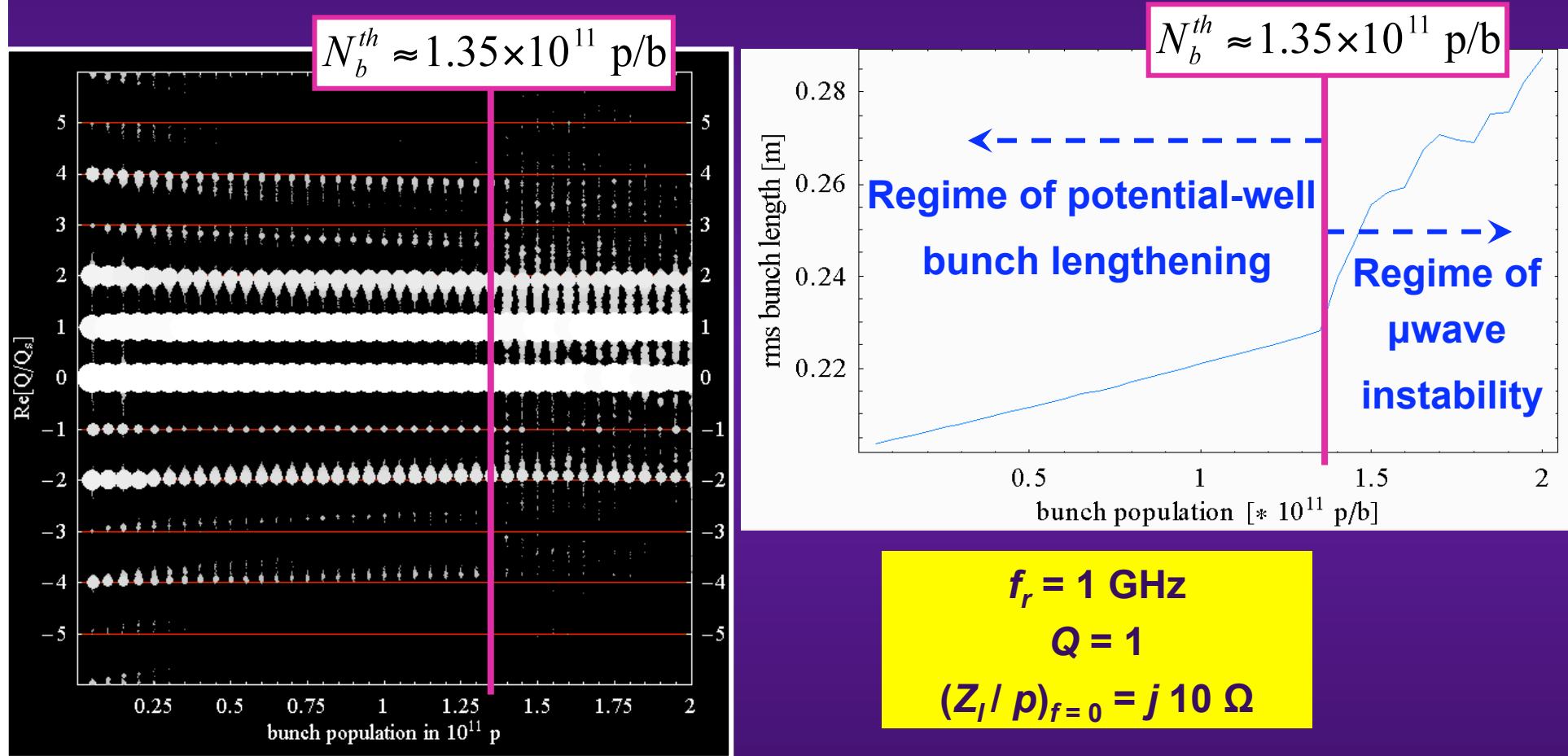
- ◆ **See ICE meeting (01/09/10) + IPAC11 paper**

=> Seems close to “usual” theories, leading to a  $Z_1 / n$  close to the theoretical value ( $\sim 0.1 \text{ j } \Omega$ , including the resistive part of the collimators)

=> To be followed up in more detail (exact shape of the distribution etc. as it is very sensitive)

# SIMPLE LMCI MODEL COMPARED TO HEADTAIL (1/8)

## ◆ Case of the SPS studied in the past



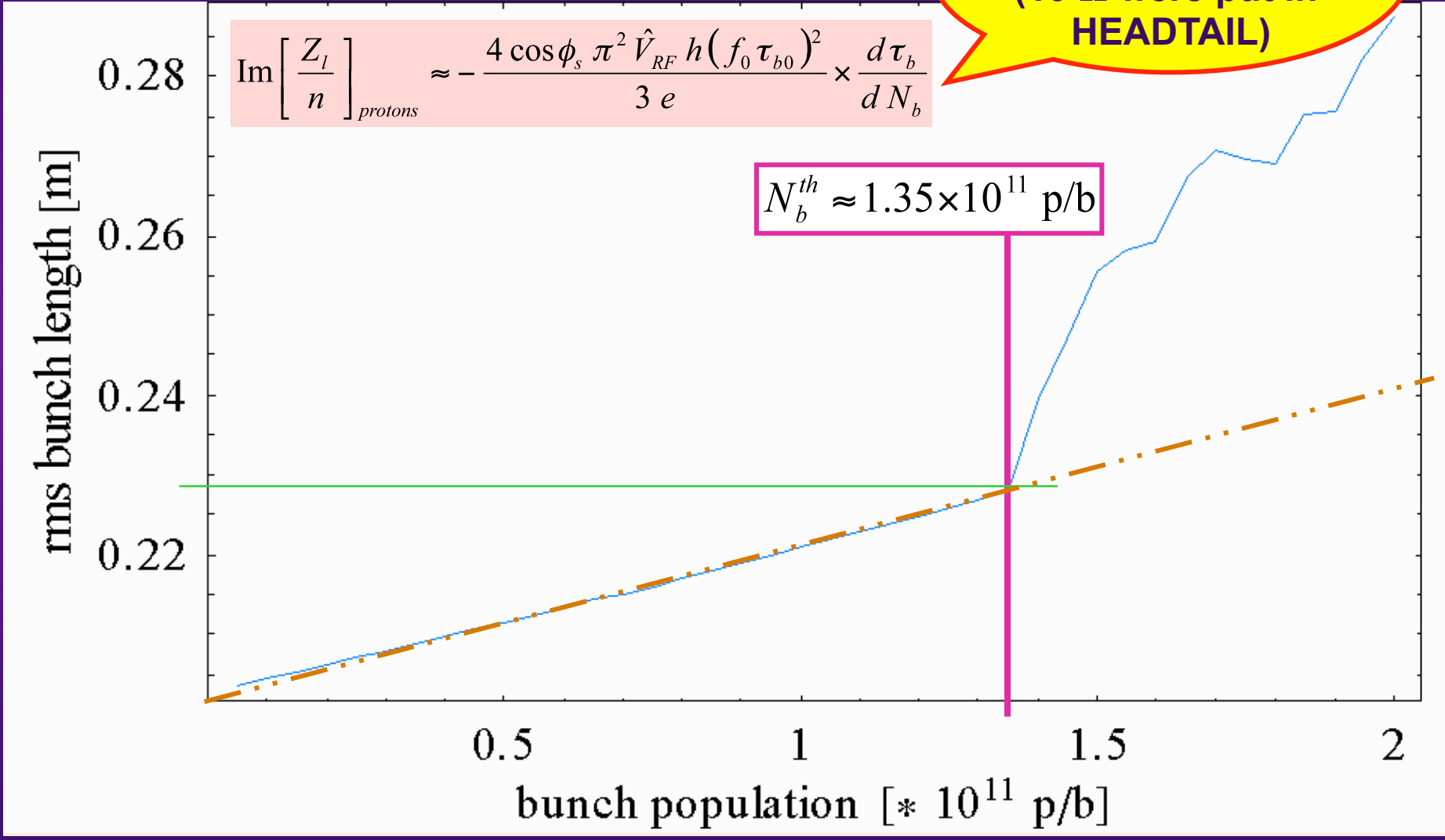
*Benoit Salvant*

# SIMPLE LMCI MODEL COMPARED TO HEADTAIL (2/8)

$\text{Im}[Z_l/n]_{\text{deduced}} \sim 8.4 \Omega$   
 (10  $\Omega$  were put in HEADTAIL)

$$\text{Im} \left[ \frac{Z_l}{n} \right]_{\text{protons}} \approx - \frac{4 \cos \phi_s \pi^2 \hat{V}_{RF} h (f_0 \tau_{b0})^2}{3 e} \times \frac{d\tau_b}{dN_b}$$

$N_b^{th} \approx 1.35 \times 10^{11} \text{ p/b}$



# SIMPLE LMCI MODEL COMPARED TO HEADTAIL (3/8)

$$\frac{|Z_i^{BB} / p|}{1.2} \times \left[ 1 - \text{Sgn}(\eta) \times \frac{3}{4} \left( \frac{|Z_i^{SC} / p|}{|Z_i^{BB} / p|} - 1 \right) \right]^{1/4}$$
$$\leq \frac{(E/e)\beta^2 |\eta|}{I_{p0}} \times \left( \frac{\Delta p}{p_0} \right)_{\text{FWHH},0}^2$$

Exactly the same as  
KSB in our case

=>

$$\left( N_b^{th} \right)_{theory} \approx 0.7 \times 10^{11} \text{ p/b}$$

i.e. theoretical  
prediction is ~ 2 times  
lower compared to  
HEADTAIL

# SIMPLE LMCI MODEL COMPARED TO HEADTAIL (4/8)

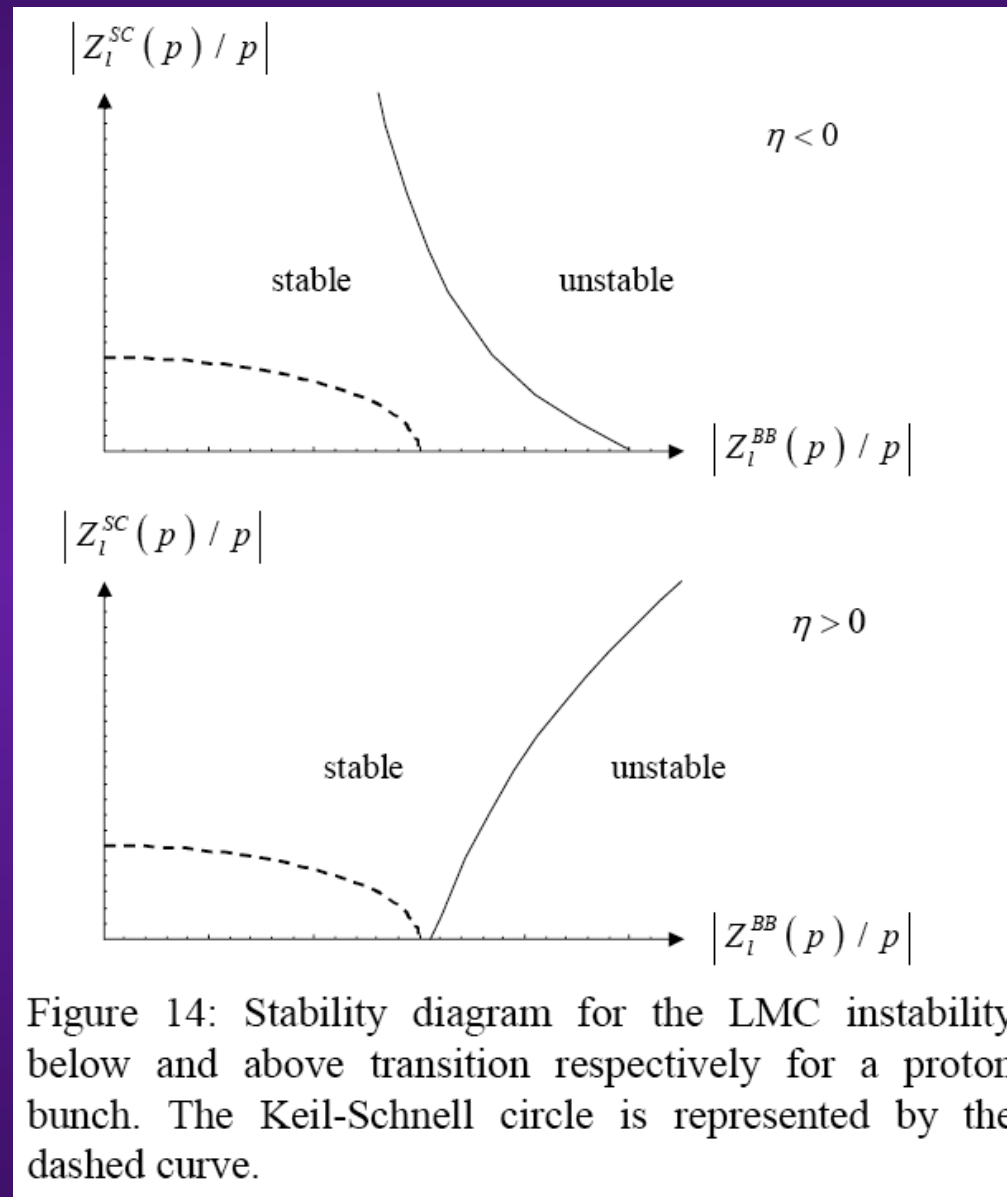


Figure 14: Stability diagram for the LMC instability below and above transition respectively for a proton bunch. The Keil-Schnell circle is represented by the dashed curve.

# SIMPLE LMCI MODEL COMPARED TO HEADTAIL (5/8)

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Flag_for_bunch_particles(1->protons_2->positrons_3&4->ions): 1
Average_electron_cloud_density_along_the_ring(1/m^3): 1.e+12
Number_of_particles_per_bunch: 1e+9
Horizontal_beta_function_at_the_kick_sections_[m]: 40.
Vertical_beta_function_at_the_kick_sections_[m]: 40.
Bunch_length_(rms_value)_[m]: 0.21
Normalized_horizontal_emittance_(rms_value)_[um]: 3.0
Normalized_vertical_emittance_(rms_value)_[um]: 3.0
Longitudinal_momentum_spread: 0.00093
Synchrotron_tune: 0.00324
Momentum_compaction_factor: 0.00192
Ring_circumference_length_[m]: 6911.
Relativistic_gamma: 27.7286
Number_of_kick_sections: 1
Number_of_laps: 30000
Multiplication_factor_for_pipe_axes: 10
Multiplication_factor_for_pipe_axes: 10
Longitud_extension_of_the_bunch_(+/-N*sigma_z): 2.
Horizontal_tune: 26.13
Vertical_tune: 26.18
Horizontal_chromaticity_[Q'x]: 0.
Vertical_chromaticity_[Q'y]: 0.
Flag_for_synchrotron_motion: 1
Scale_factor_for_electrons_size: 4
Switch_for_wake_fields: 1
Switch_for_pipe_geometry(0->round_1->flat): 0
Number_of_turns_for_the_wake: 1
Res_frequency_of_broad_band_resonator_[GHz]: 1.
Transverse_quality_factor: 1.
Transverse_shunt_impedance_[MOhm/m]: 0.
Res_frequency_of_longitudinal_resonator_[MHz]: 1000.
Longitudinal_quality_factor: 1.
Longitudinal_shunt_impedance_[MOhm]: 0.23
Flag_for_the_tune_spread(0->no_1->space_charge_2->random): 0
Flag_for_the_e-field_calc_method(0->no_1->soft_Gauss_2->PIC): 0
Magnetic_field(0->no_1->dipole_2->solenoid_3->combined): 0
Switch_for_initial_kick: 0
x-kick_amplitude_at_t=0_[sigmas]: 0.0
y-kick_amplitude_at_t=0_[sigmas]: 0.0
Flag_for_the_proton_space_charge: 0
Flag_for_the_sc-rotation(0->local_centroid_1->bunch_centroid): 0
Solenoid_field_[T]: 0.

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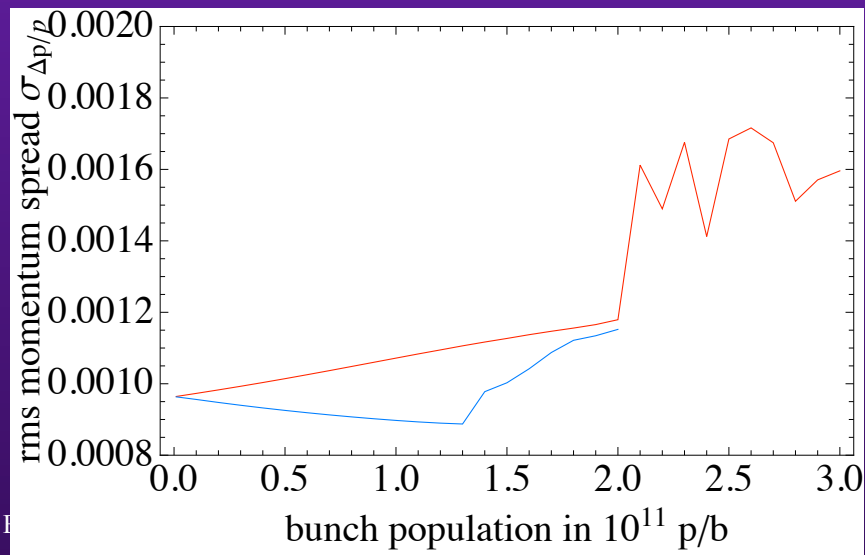
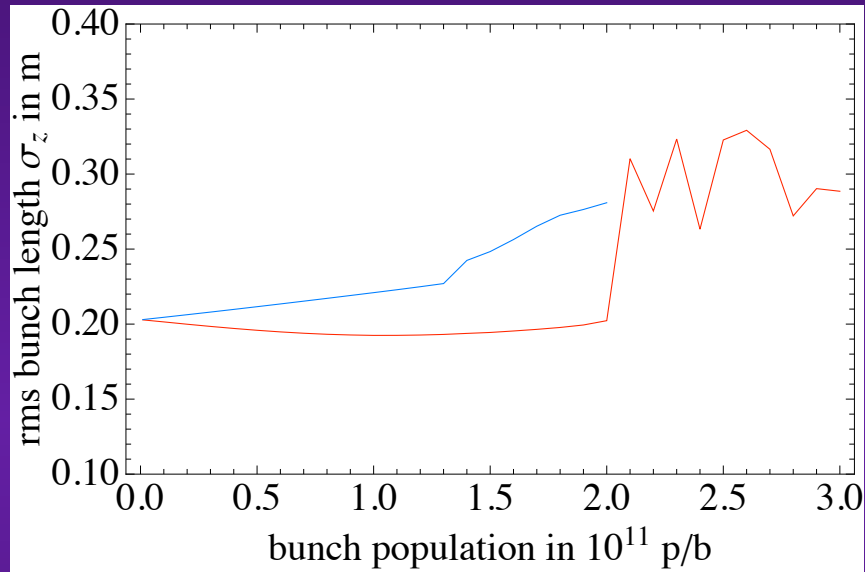
Above Transition

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Flag_for_the_sc-rotation(0->local_centroid_1->bunch_centroid): 0
Solenoid_field_[T]: 0.
Switch_for_amplitude_detuning: 0
Coherent_centroid_motion(0->off_1->on): 1
El_distrib(1->Rect_2->Ellip_3->[1_strp]_4->[2_strp]_5->Parab): 1
Linear_coupling_switch(1->on_0->off): 0
Linear_coupling_coefficient_[1/m]: 0.0015
Average_dispersion_function_in_the_ring_[m]: 0.0
Position_of_the_stripes_[units_of_sigmax]: 3.0
Width_of_the_stripes_[units_of_sigmax]: 0.5
Kick_in_the_longitudinal_direction_[m]: 0.001
Number_of_turns_between_two_bunch_shape_acquisitions: 10000
Main_rf_voltage_[V]: 0.6e+6
Main_rf_harmonic_number: 4620
Initial_2nd_rf_voltage_[V]: 0.
Final_2nd_rf_cavity_voltage_[V]: 0.7e+6
Harmonic_number_of_2nd_rf: 18480
Relative_phase_between_cavities: 0.
Start_turn_for_2nd_rf_ramp: 30000
End_turn_for_2nd_rf_ramp: 40000
Sextupolar_kick_switch(1->on_0->off): 0
Sextupole_strength_[1/m^2]: -0.254564
Dispersion_at_the_sextupoles_[m]: 2.24
Switch_for_losses(0->no_losses_1->losses): 0
Second_order_horizontal_chromaticity_(Qx''): 0.
Second_order_vertical_chromaticity_(Qy''): 0.
Switch_for_boundary_conditions(0->open_space_1->rect_box): 1
Switch_for_random_phase_advance(0->no_1->yes): 0
Switch_for_e-cooler(0->no_e-cooler_1->tuned_e-cooler): 0
Length_of_the_e-cooler_[m]: 3.
Switch_for_the_damper: 0
Damper_x_gain: 0.1
Damper_x_noise_amplitude: 1e-5
Damper_y_gain: 0.1
Damper_y_noise_amplitude: 1e-5
Conductivity_of_the_resistive_wall_[1/Ohm/m]: 1.e6
Length_of_the_resistive_wall_[m]: 0.
Switch_for_beta: 0
Switch_for_wake_table: 0
Linear_Rate_of_Change_of_Momentum_[GeV/c/sec]: 0.
Second_Order_Momentum_Compaction_Factor: 0.

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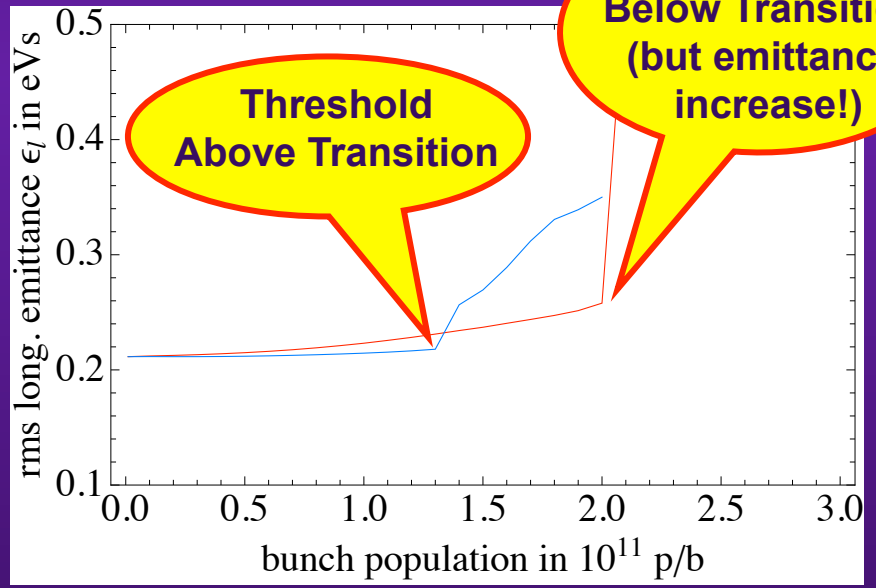
# SIMPLE LMCI MODEL COMPARED TO HEADTAIL (6/8)



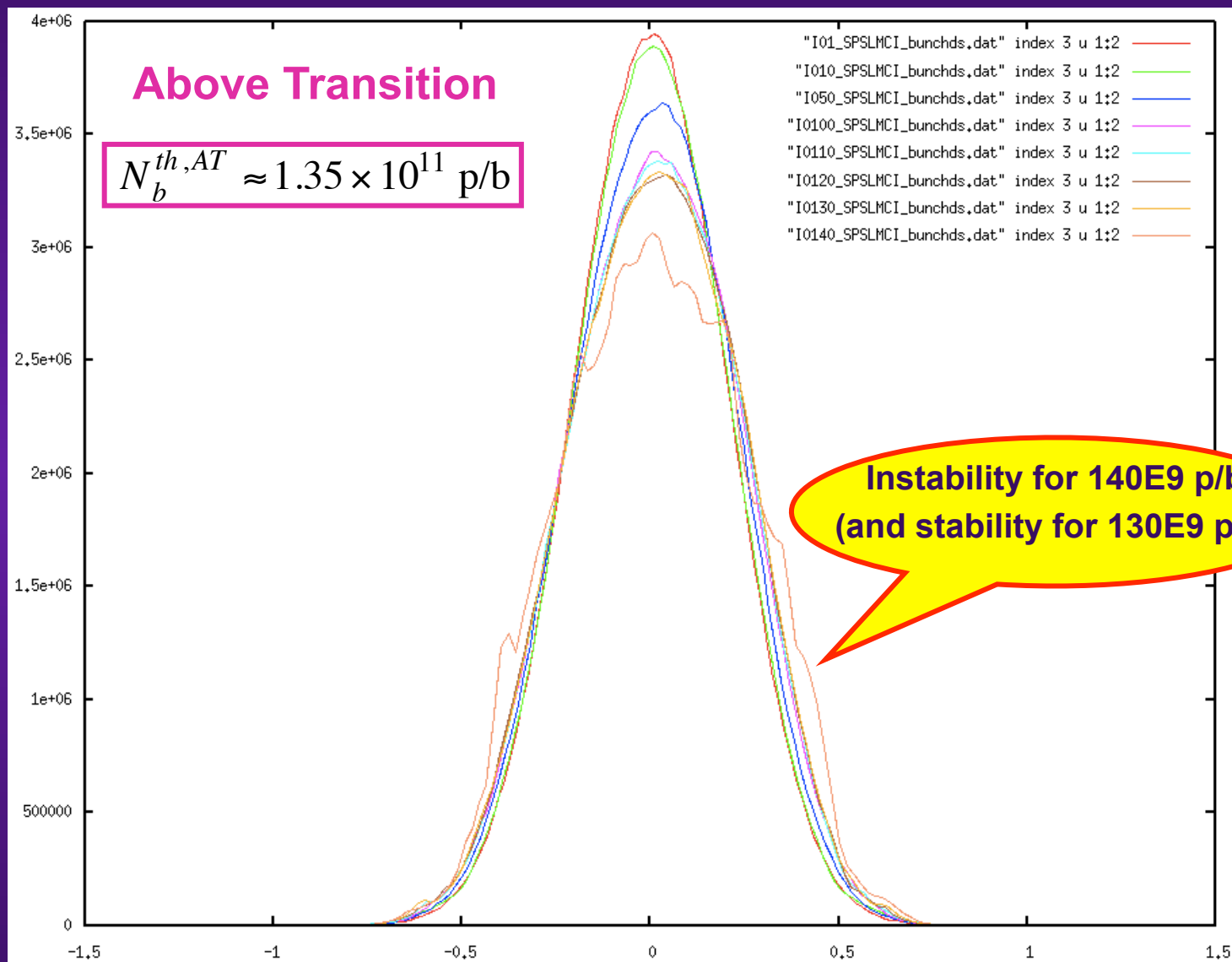
$$|\eta|_{BT} = |\eta|_{AT} = 6.2 \times 10^{-4}$$

$$\alpha_{p,AT} = 0.00192$$

$$\alpha_{p,BT} = 0.00068$$

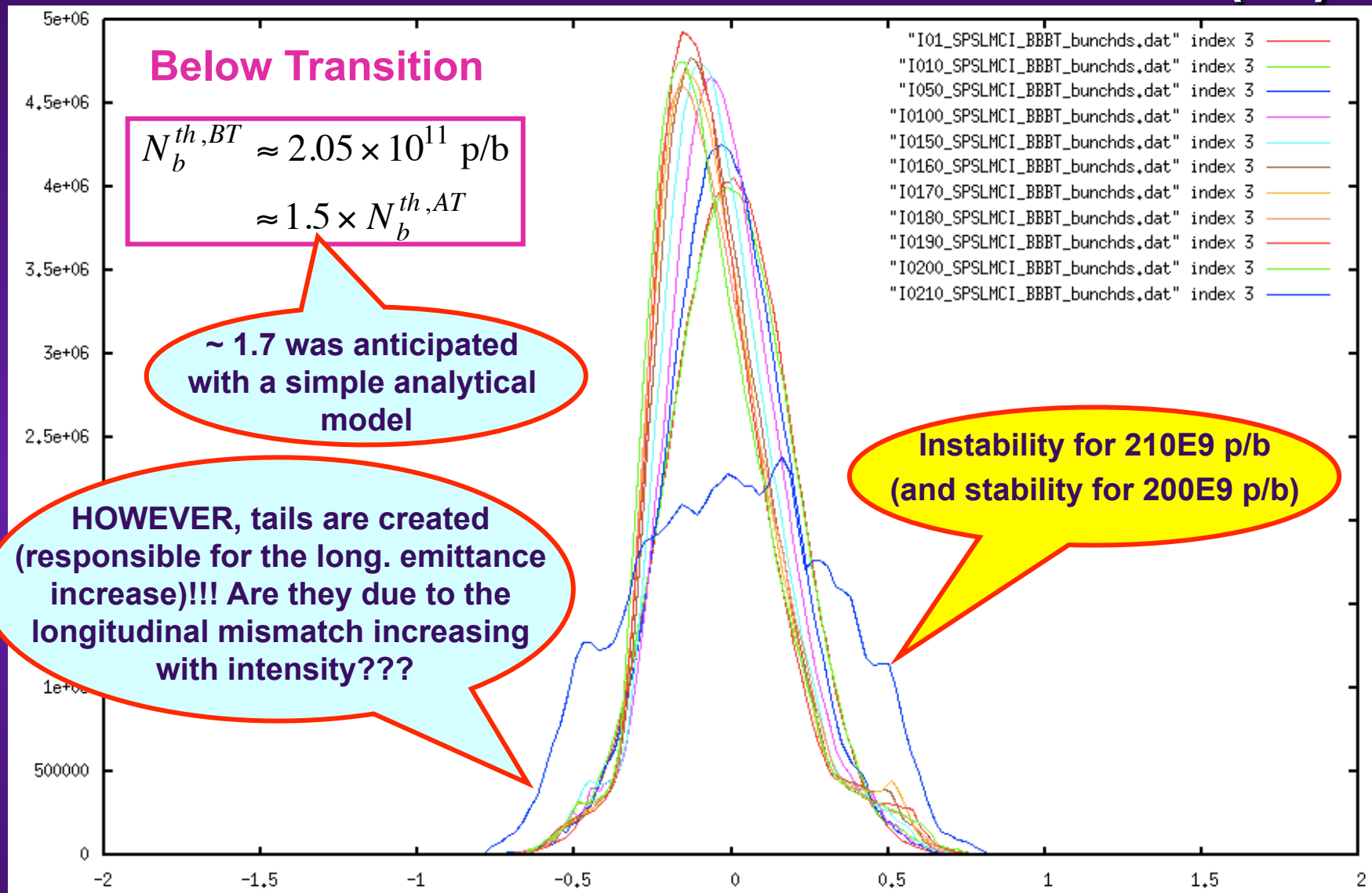


# SIMPLE LMCI MODEL COMPARED TO HEADTAIL (7/8)





# SIMPLE LMCI MODEL COMPARED TO HEADTAIL (8/8)



**In fact, looking at the evolution of the longitudinal distribution, it seems as if the core of the bunch is shortening but not the tails**

# COMPARISON WITH ALEXEYB'S RESULTS

## ◆ Results from AlexeyB:

- $Z_1 / n$  (from the slope) = 5.6 j  $\Omega$  (to be compared to the 10 j  $\Omega$  put in HEADTAIL and the 8.4 j  $\Omega$  found with simple formula)

- LLD threshold (above transition):

$$N_b^{th} \approx 1.1 \cdot 10^{10} \text{ p/b}$$

i.e. ~ 10 times lower  
than LMCI

$$\frac{\Delta \omega_{s,th}^i}{\omega_{s0}} \approx -3.7 \%$$

- LLD threshold (below transition): ~ 8 times higher, i.e. close to LMCI => In this case the mode-coupling analysis should be included in AlexeyB's calculation as it is not negligible anymore!

# CONCLUSIONS AND OUTLOOK

- ◆ **Very interesting new results by AlexeyB, which successfully explained the observations of dancing bunches in the Tevatron**
- ◆ **As mentioned by AlexeyB, they qualitatively agree with previous analyses etc. but the numerical factor can be very large!**
- ◆ **Let's try and understand what happens in the LHC and why the intensity threshold for LLD seems close to "usual" theories, whereas it should be much smaller according to AlexeyB (for the smooth approximation we used) => Detail analysis of the (centre of the) distribution as it is very very sensitive**
- ◆ **Concerning the LMCI studies above and below transition, started in the past for the SPS with a broad-band impedance:**
  - **Check with HEADTAIL the effect of space charge in addition to the broad-band impedance => Hugo Day**
  - **Detailed comparison with AlexeyB**

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