



Beta Beams,
EUROnu WP4



Collective Beta Beams



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Many thanks to: E. Benedetto, A. Chancé, E. Metral, N. Mounet, G. Rumolo, B. Salvant & E. Wildner

Outline

- Beta Beam Overview
- Collective Effects
 - ➔ Laslett's Tune Shifts
 - ➔ Wakefield Instabilities
 - ◆ HEADTAIL & MOSES
 - ◆ Intensity Thresholds
- Conclusion

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- **Collective Effects**
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 - ◆ **Intensity Thresholds**
- **Conclusion**

Beta Beams - Overview

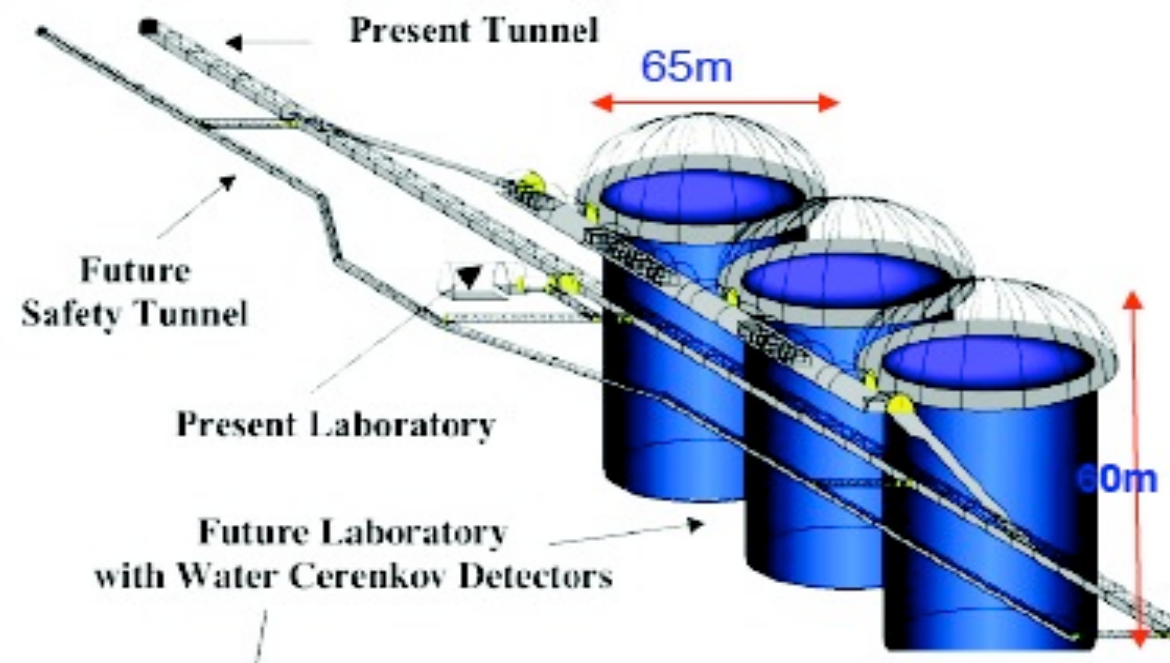
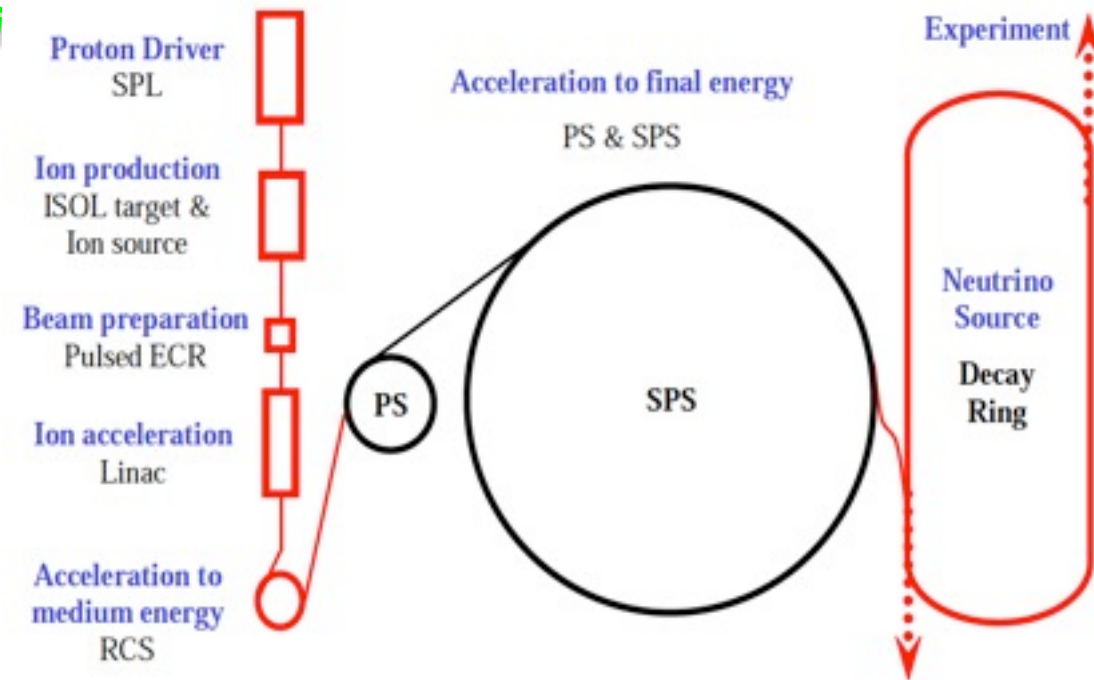
P. Zucchelli

- **Basic idea** (*Phys. Let. B, 532 (2002) 166-172*):
 - Accelerate radioactive ions to high γ
 - Let them β -decay in a Decay Ring (DR)
 - The DR has one straight section pointing in the direction of a neutrino detector \rightarrow
 - ν -beam with opening angle $1/\gamma$ and with known energy and ν -species

This gives only (anti) neutrinos from β^+ (β^-) decay:



- **Detector:**
 - Water Cerenkov detector enough to distinguish μ^+ and e^+ (μ^- and e^-), no need to distinguish μ^+ and μ^-



Choice of Ions

B B O V E R V I E W

- **Considerations**
 - **Pair of β^+ and β^- active ions**
for ν and anti- ν ...
 - **Production rates**
isol method or production ring
 - **Life time**
optimized for baseline $\sim 1s$
 - **Reactivity**
noble gases are good
 - **Q value**
defines ν -energy & baseline
 - **Low Z preferred**
*minimize accelerated mass per charge
reduce space charge problems*

Z →	0	1	2	3	4	5	6	7	8	9	10	11	12
n ↓	n	H	He	Li	Be	B	C	N	O	F	Ne	Na	Mg
0		¹ H	² He										
1	¹ n	² H	³ He	⁴ Li	⁵ Be	⁶ B	⁷ C	⁸ N	⁹ O	¹⁰ F	¹¹ Ne	¹² Na	¹³ Mg
2	² n	³ H	⁴ He	⁵ Li	⁶ Be	⁷ B	⁸ C	⁹ N	¹⁰ O	¹¹ F	¹² Ne	¹³ Na	¹⁴ Mg
3		⁴ H	⁵ He	⁶ Li	⁷ Be	⁸ B	⁹ C	¹⁰ N	¹¹ O	¹² F	¹³ Ne	¹⁴ Na	¹⁵ Mg
4	⁴ n	⁵ H	⁶ He	⁷ Li	⁸ Be	⁹ B	¹⁰ C	¹¹ N	¹² O	¹³ F	¹⁴ Ne	¹⁵ Na	¹⁶ Mg
5		⁶ H	⁷ He	⁸ Li	⁹ Be	¹⁰ B	¹¹ C	¹² N	¹³ O	¹⁴ F	¹⁵ Ne	¹⁶ Na	¹⁷ Mg
6		⁷ H	⁸ He	⁹ Li	¹⁰ Be	¹¹ B	¹² C	¹³ N	¹⁴ O	¹⁵ F	¹⁶ Ne	¹⁷ Na	¹⁸ Mg
7		⁸ H	⁹ He	¹⁰ Li	¹¹ Be	¹² B	¹³ C	¹⁴ N	¹⁵ O	¹⁶ F	¹⁷ Ne	¹⁸ Na	¹⁹ Mg
8		⁹ He	¹⁰ Li	¹¹ Be	¹² B	¹³ C	¹⁴ N	¹⁵ O	¹⁶ F	¹⁷ Ne	¹⁸ Na	¹⁹ Mg	²⁰ Al
9		¹⁰ He	¹¹ Li	¹² Be	¹³ B	¹⁴ C	¹⁵ N	¹⁶ O	¹⁷ F	¹⁸ Ne	¹⁹ Na	²⁰ Mg	²¹ Al
10		¹¹ He	¹² Li	¹³ Be	¹⁴ B	¹⁵ C	¹⁶ N	¹⁷ O	¹⁸ F	¹⁹ Ne	²⁰ Na	²¹ Mg	²² Al
11		¹² He	¹³ Li	¹⁴ Be	¹⁵ B	¹⁶ C	¹⁷ N	¹⁸ O	¹⁹ F	²⁰ Ne	²¹ Na	²² Mg	²³ Al
12		¹³ He	¹⁴ Li	¹⁵ Be	¹⁶ B	¹⁷ C	¹⁸ N	¹⁹ O	²⁰ F	²¹ Ne	²² Na	²³ Mg	²⁴ Al

FP6

¹⁸ Ne	n 8 z 10	⁶ He	n 4 z 2
J π	0+	J π	0+
Delta (MeV)	5.3172	Delta (MeV)	17.5951
T _{1/2}	1672.8 ms	T _{1/2}	806.715 ms
Decay	ec β^+ 100	Decay	β^- 100

Q = 3.30 MeV Q = 3.51 MeV

FP6: EURISOL
(CARE, contract number
RII3-CT-2003-506395)

EURISOL

FP7:

⁸ B	n 3 z 5	⁸ Li	n 5 z 3
J π	2+	J π	2+
Delta (MeV)	22.9215	Delta (MeV)	20.9468
T _{1/2}	770.3 ms	T _{1/2}	838.6 ms
Decay	ec β^+ 100 β^+ α 100	Decay	β^- 100 β^- α 100

Q = 13.9 MeV Q = 13.0 MeV

FP7: EUROnu
(Grant agreement no.:
212372)



“Q value” is the kinetic energy release of a particle at rest

E.g. for the neutron decay

$$Q = m_n - m_p - m_{\bar{\nu}} - m_e$$

Duty Factor

- To suppress atmospheric background detectors can only be open short time periods
 - Suppression Factor, $SF =$ opened time ratio of the detector
- The DR will be filled only with short bunches so that neutrinos are send only when the detector is opened
 - Duty Factor = filled ratio of the Decay Ring

Duty Factor = Suppression Factor

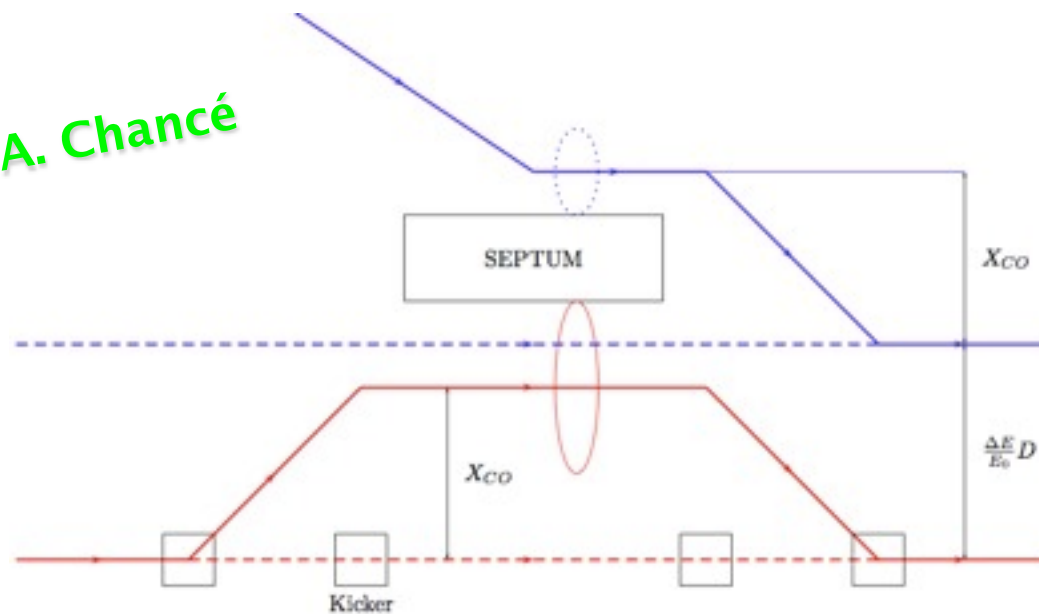


DR: Injection Scheme

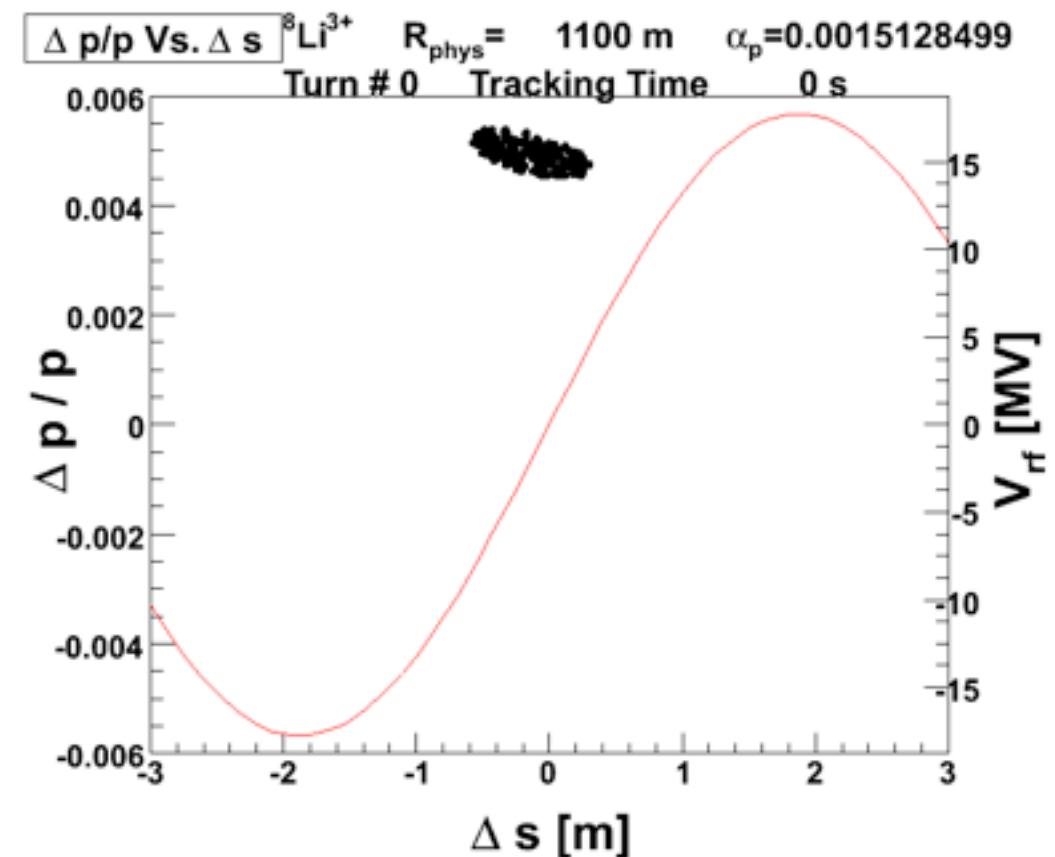
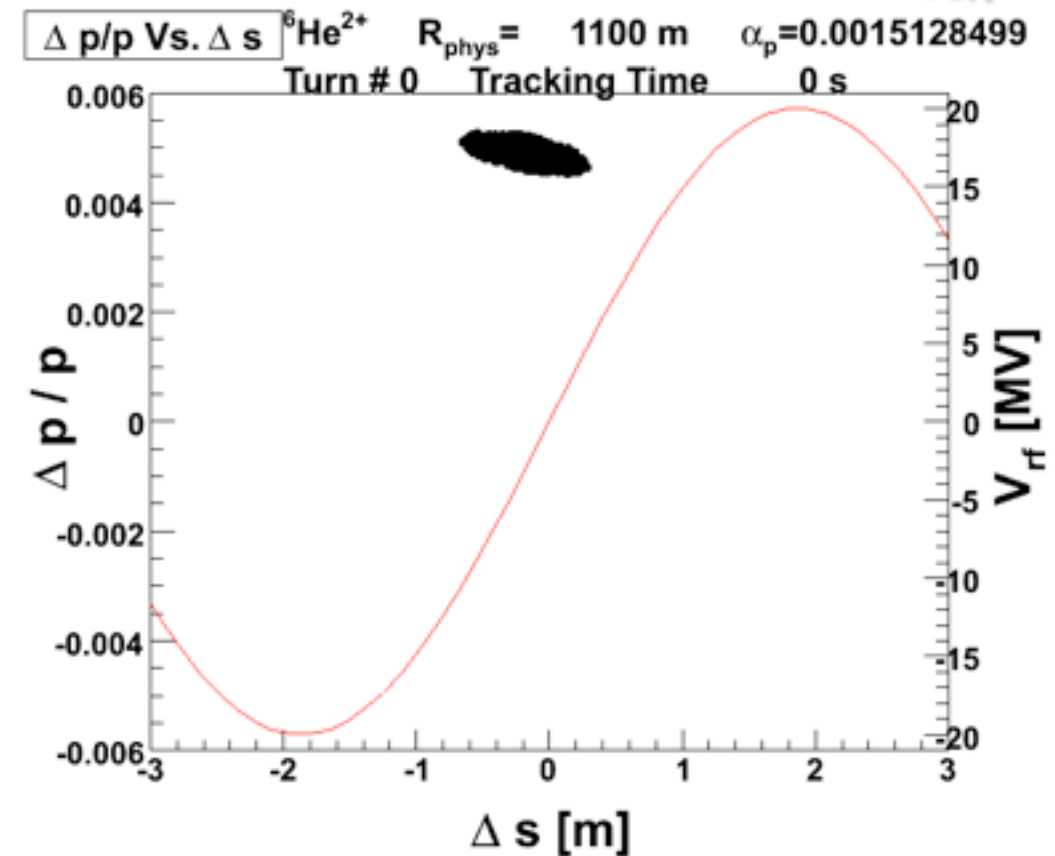
Daniel C. Heinrich

- **The new bunch is injected off momentum**
(separated by a septum magnet)

A. Chancé



- **After ¼ synchrotron turn it is “captured” by one RF system**
- **Then “merged” into the old bunch with the use of a 2nd RF system**
- **Collimation at $\Delta p/p = 2.5\%$**
 - ➔ **scrapes away ions not captured**
 - ➔ **limits the bunch size to protect the septum magnet**

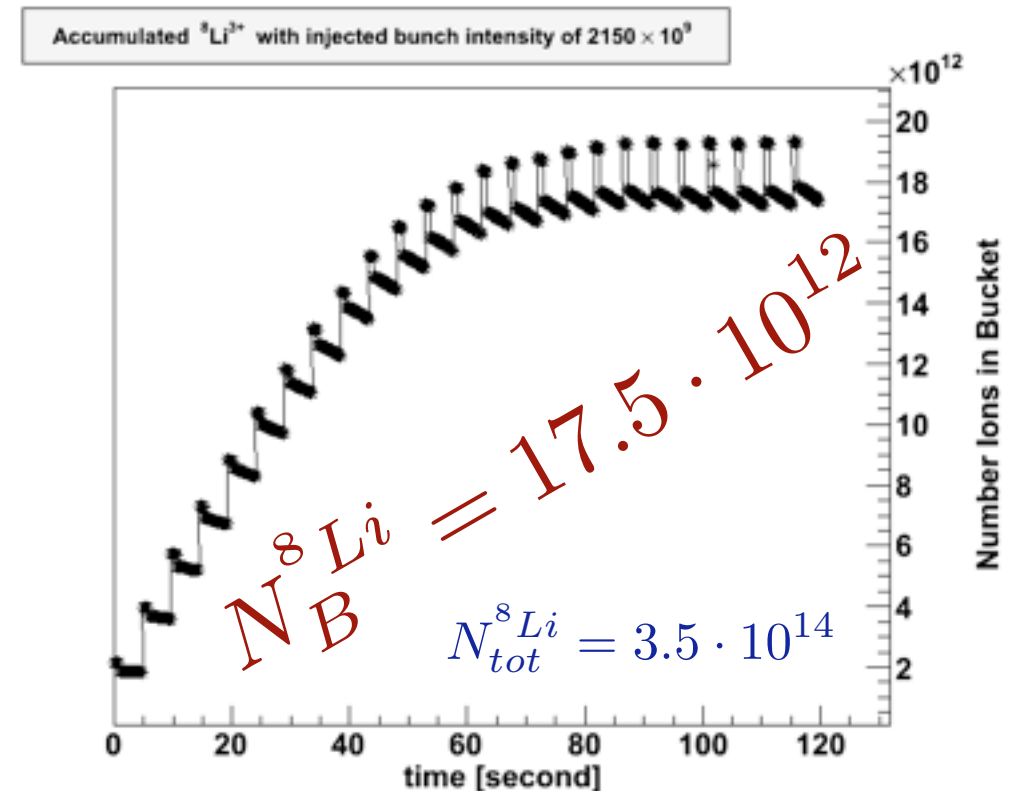
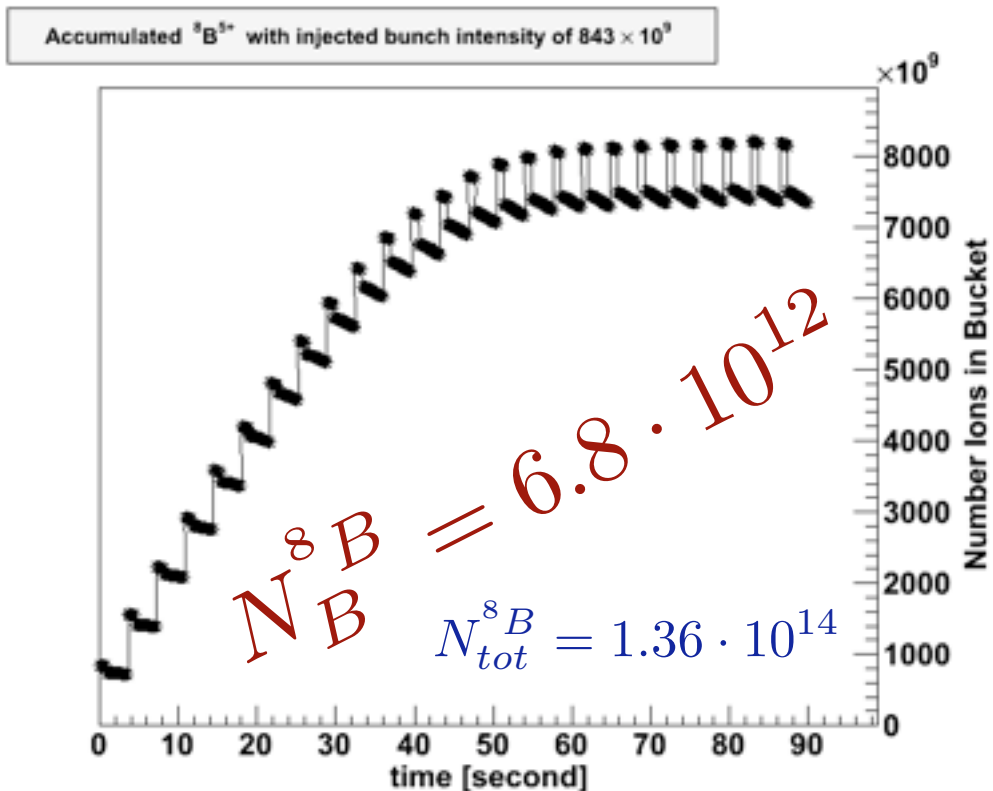
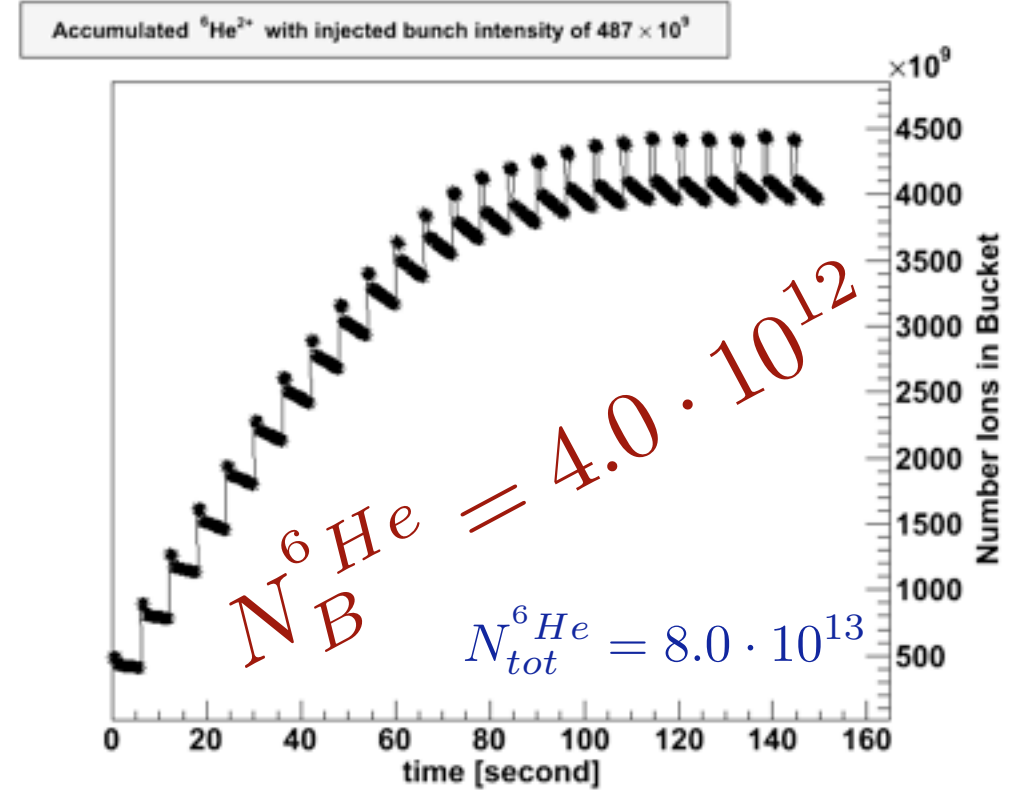
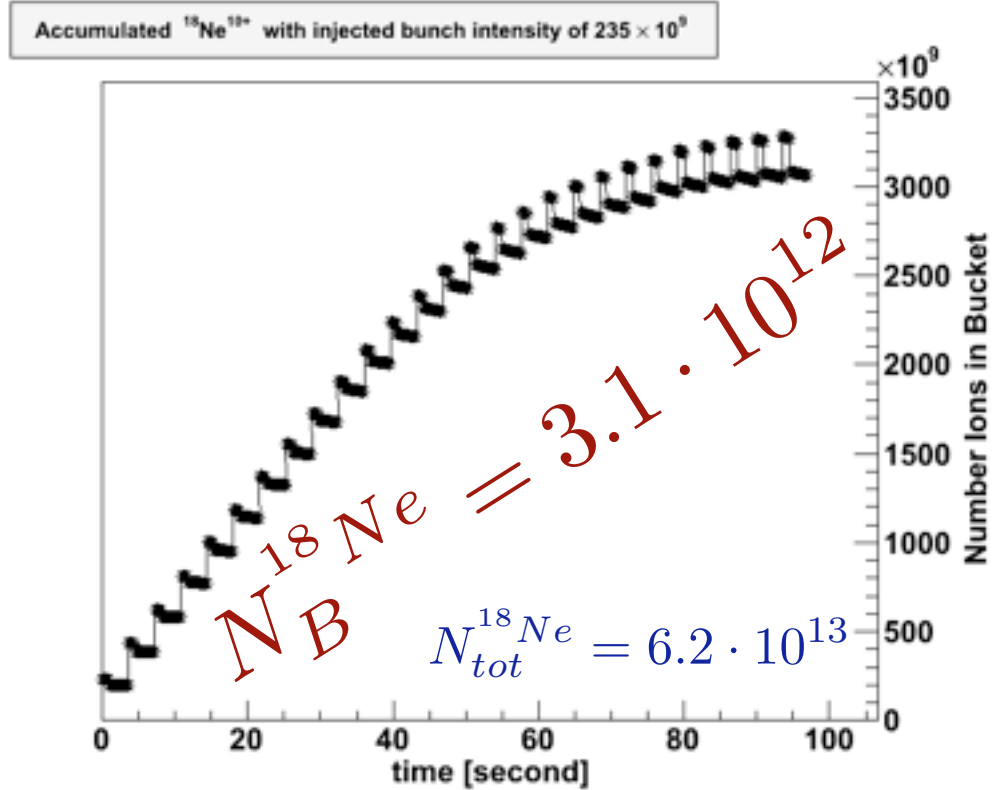


DR: Accumulation

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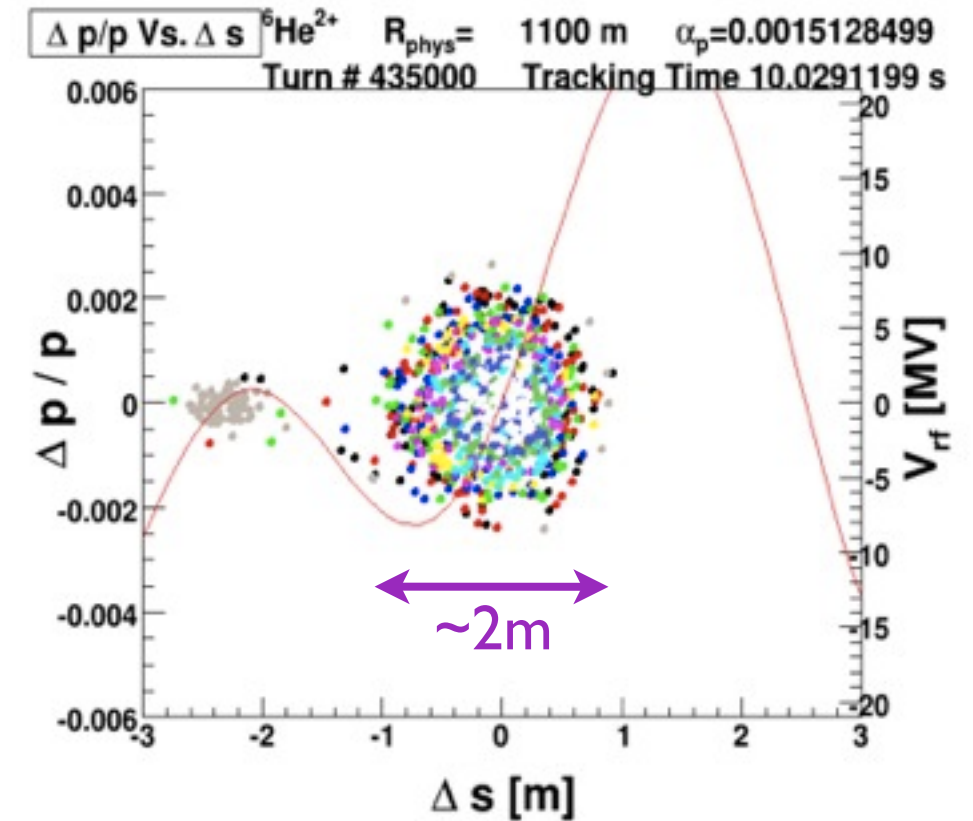
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- **Due to Collimation and Radioactive decay the number of ions per bunch saturates in the DR** (20 of these bunches gives N_{tot})

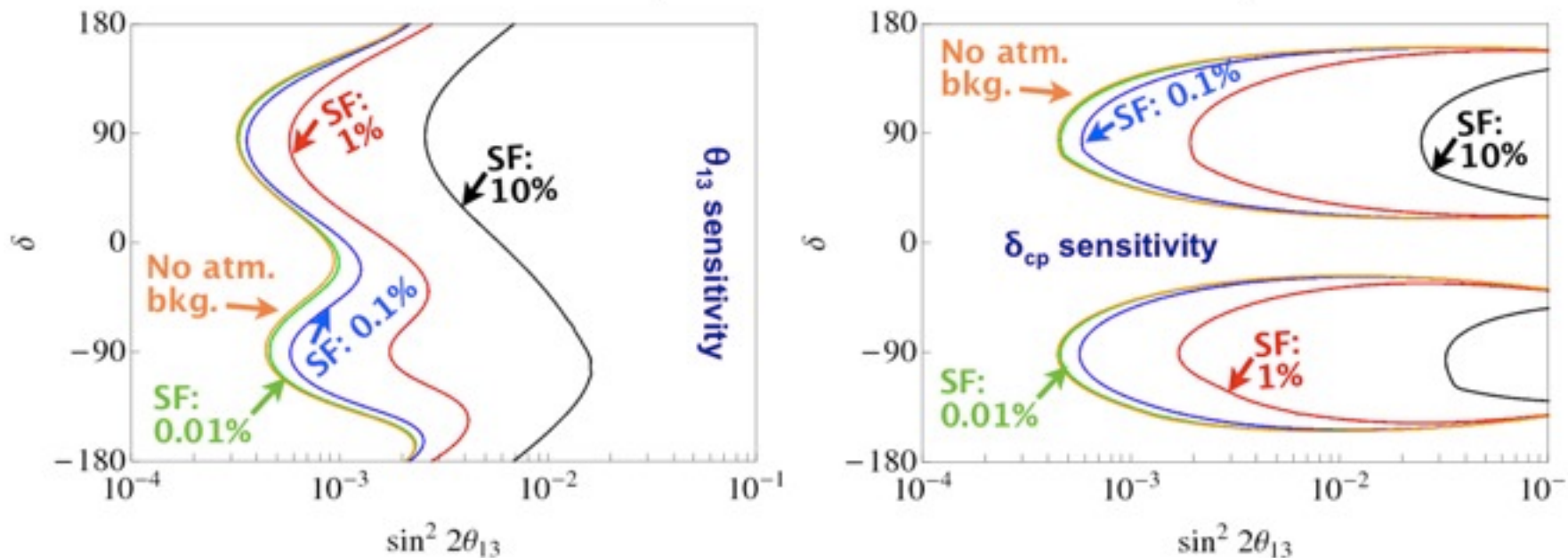


DR: Injection Scheme

- After “merging” the bunches are $\sim 2\text{m}$ long (all ions)
- 20 bunches from SPS to DR
 $\rightarrow \text{SF} = 20 \cdot 2\text{m} / 6911\text{m} = 0.58\%$
- I.e. between 0.1% and 1%
- With the intensities shown in previous slide we get these sensitivities:



Assumed Fluxes: 1.1×10^{18} ν /year from ^{18}Ne and 2.9×10^{18} anti- ν /year from ^6He



- **Good, BUT:**
 \rightarrow **What about collective effects?**

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BB Collective Effects

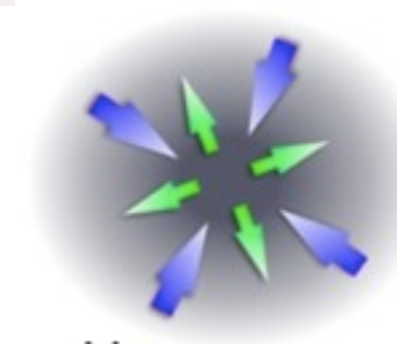
- High intensity ion beams are foreseen for the Beta Beam project
- High intensity bunches can have non-negligible amount of charges
 - ➔ Particles interact with each other and vacuum chamber
 - ◆ “Collective Effects”
- Collective Effects limit the final performance of accelerators
- The studies of instabilities of the ion beams are a crucial part of the Beta Beam project
- Studies will be done for all ions (FP6: ^{18}Ne & ^6He , FP7: ^8B & ^8Li) and for all machines (so far only DR)

Reasons for Instabilities

- Different reasons for Instabilities:

- ➔ Coulomb Forces

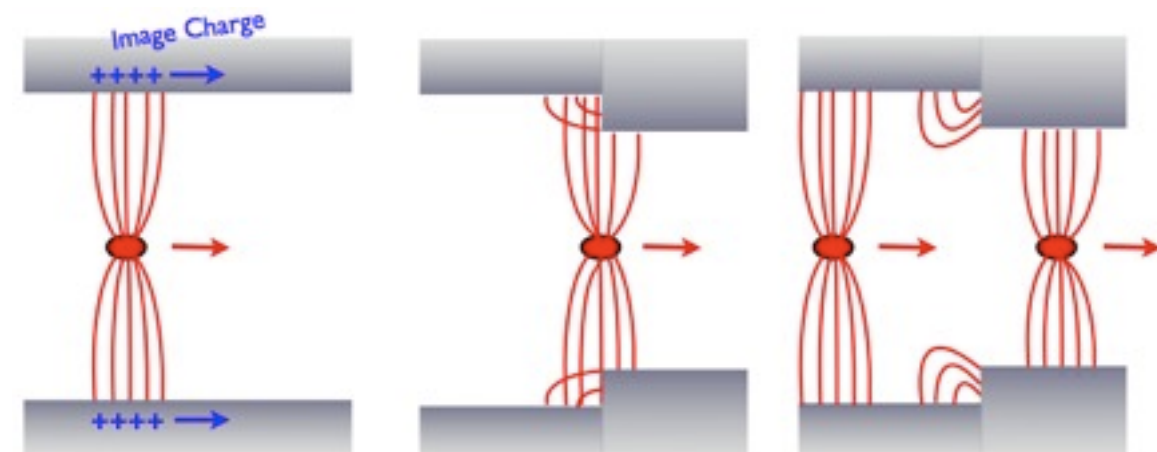
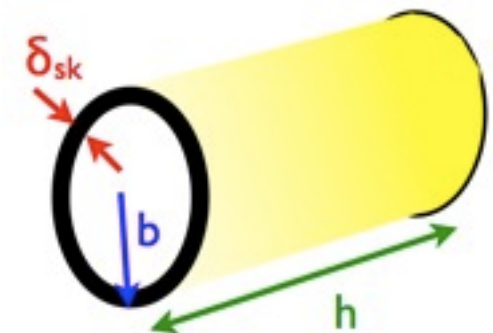
- within the bunch; “**Direct Space Charge**”
- between bunch and pipe; “**Image Field**”



“Laslett's
Tune Shifts”

- ➔ Wake Fields (= “Impedances” in frequency domain)

- due to resistive pipe; “**Resistive Wall Impedance**”
- due to pipe discontinuities; “**Resonance Impedance**”



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Laslett's Tune Shifts

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- **Direct Space Charge** forces are repulsive and proportional to distance from center → to be compared to quadrupoles → betatron tune shift

→ but $\Delta Q_{DSC_{x,y}} \propto \frac{1}{\gamma^2}$ since for relativistic

beams the repulsive **E forces** are cancelled by the contracting **B forces**



→ For DR ($\gamma=100$)
 $|\Delta Q_{DSC}| < 0.2$

$|\Delta Q_{DSC}| > 0.2$ could cause tune crossings over resonance lines → instabilities



- For PS with low γ ΔQ_{DSC} could be crucial (to be investigated)

SC	DR ¹⁸ Ne	DR ⁶ He
ΔQ_{dsc_x}	-0.0409	-0.0083
ΔQ_{dsc_y}	-0.0946	-0.0192
ΔQ_x^{incoh}	-0.0409	-0.0083
ΔQ_y^{incoh}	-0.0946	-0.0192
$\Delta Q_x^{coh p}$	-1.7470e-04	-3.5564e-05
$\Delta Q_y^{coh p}$	-3.1937e-04	-6.5016e-05
$\Delta Q_x^{coh np}$	-6.2768e-05	-1.2765e-05
$\Delta Q_y^{coh np}$	-1.1475e-04	-2.3337e-05

- **Image Fields** turned out to have even less effects in the DR

Outline

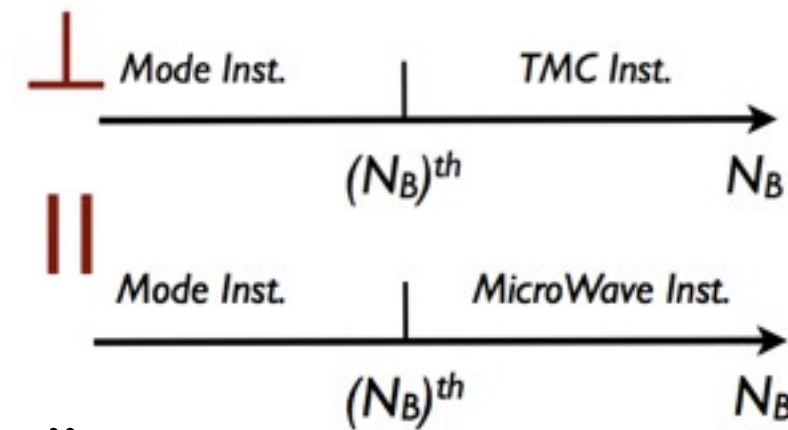
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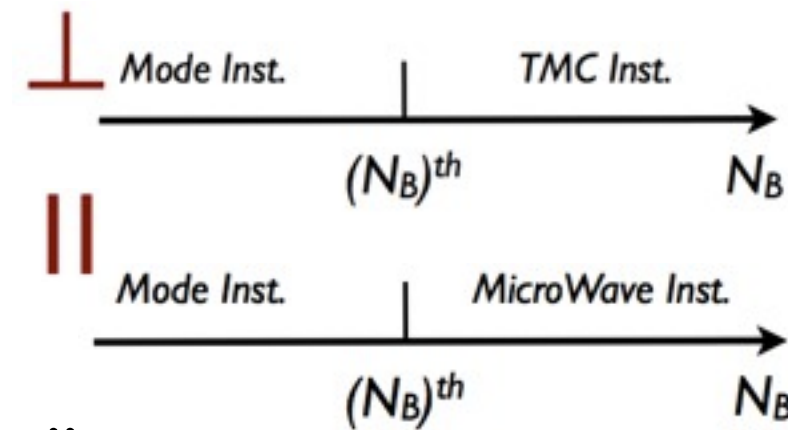
Wake Field Instabilities

- Wake Fields are described by
 - ➔ Wake Potential; $W(t)$, in time domain
 - ➔ Impedance; $Z(\omega) = \mathcal{F}[W(t)]$, in frequency domain
- Instabilities caused by $Z(\omega)$ are described by different theories depending on the intensity regime
 - If $N_b < N_b^{\text{th}}$ instabilities are “modest”
 - If $N_b > N_b^{\text{th}}$ instabilities will cause beam loss
- ➔ Important to find N_b^{th} since that is absolute maximum number ions we can have per bunch
- N_b^{th} will have to be found for each type of $Z(\omega)$;
 - Resistive Wall and Resonance Impedance
 - Longitudinal and Transversal Impedance



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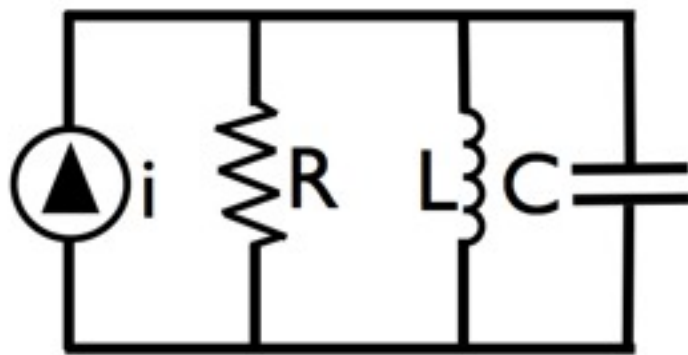
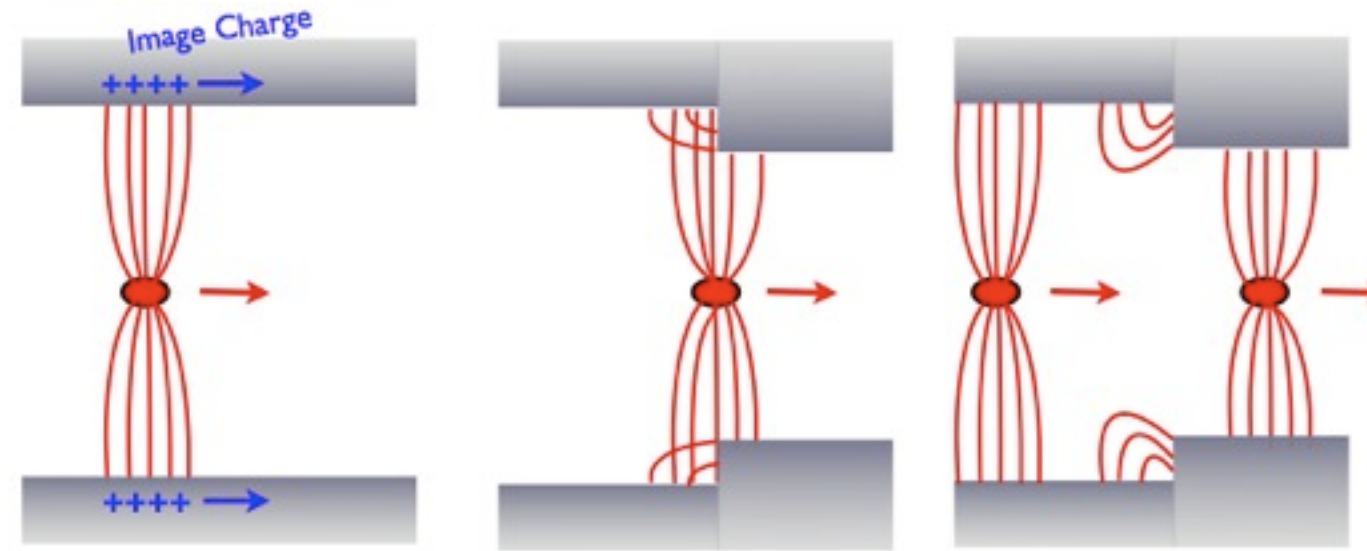


DR

^{18}Ne & ^6He

Resonance Impedance

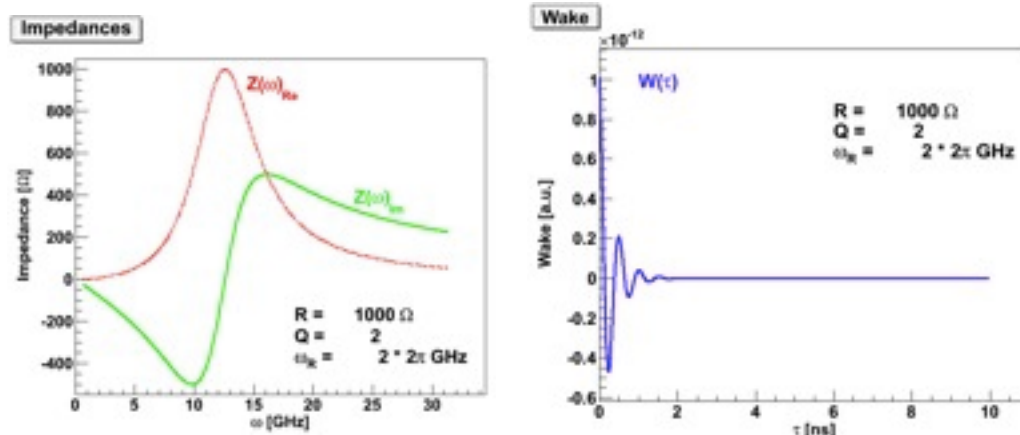
- Wake fields can be trapped in e.g. cavities in the beam pipe → Resonance Impedance → Can be modeled with an RLC circuit:



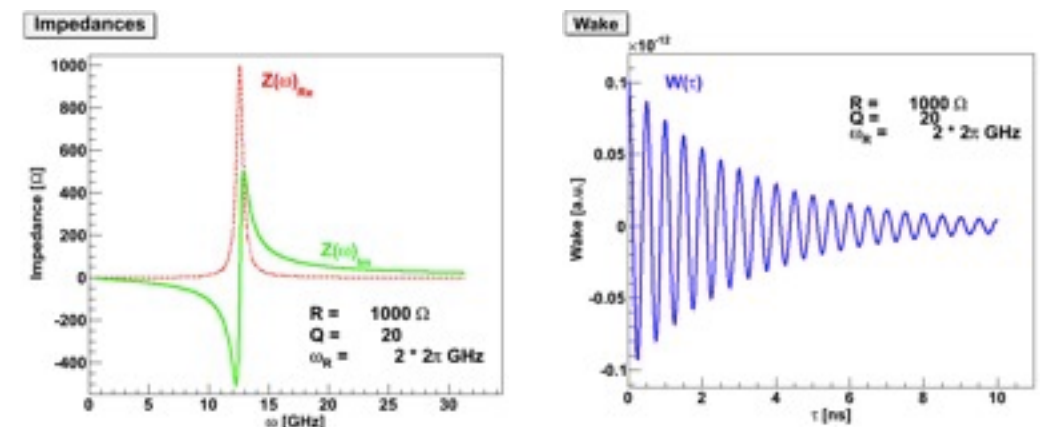
$$Z_{\perp}(\omega) = \frac{R_{\perp} \frac{\omega_r}{\omega}}{1 + iQ \left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r} \right)}$$

Q = Quality Factor, R_{\perp} = Shunt Impedance, ω_r = Resonance Angular Frequency

Broad Band (low Q)

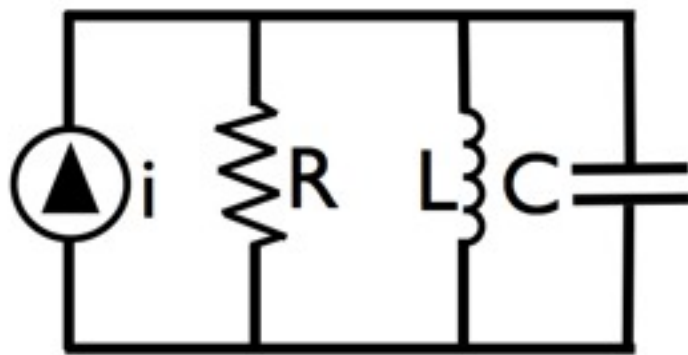
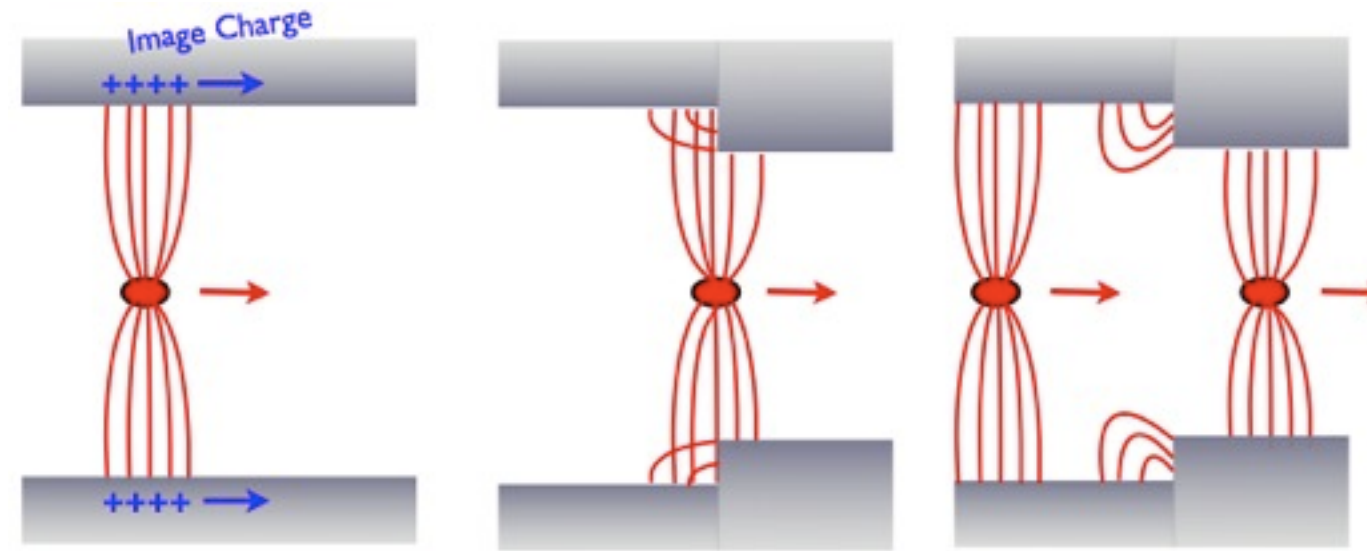


Narrow Band (high Q)



Resonance Impedance

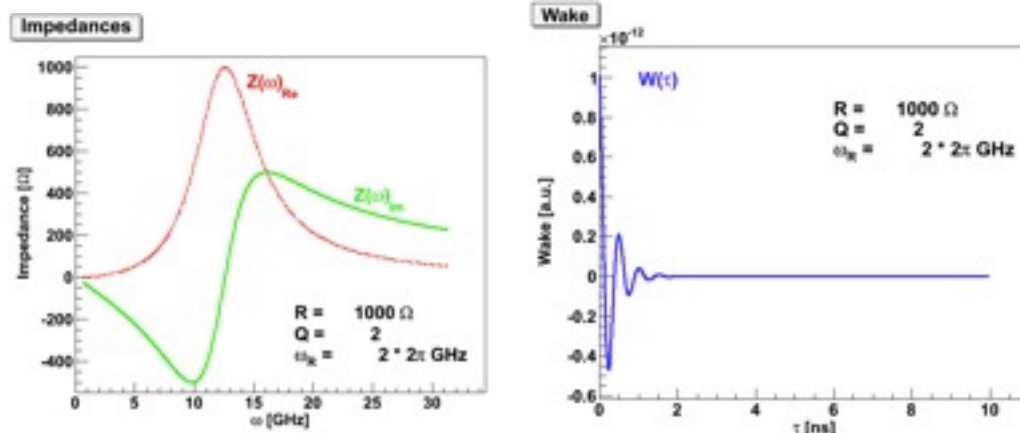
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Q = Quality Factor, R_{\perp} = Shunt Impedance, ω_r = Resonance Angular Frequency

Broad Band (low Q)



Will only show results for

- ^{18}Ne & ^6He in Decay Ring
- Transversal
- Resonance
- Broad Band (Q=1)

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DR Intensity Threshold

- Three different ways to find N_b^{th} :

➔ A theoretical equation, “Coasting Beam Eq.”:

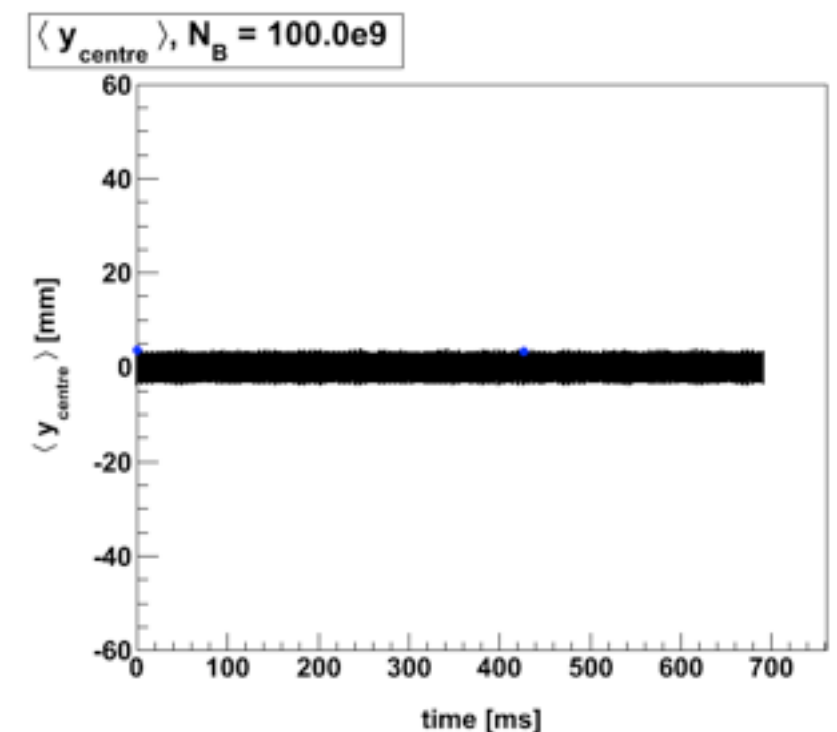
$$N_{b_{x,y}}^{\text{th}} = \frac{32}{3\sqrt{2}\pi} \frac{Q_{x,y} |\eta| \varepsilon_l^{2\sigma} \omega_r}{Z^2 \beta^2 c} \left(\Re \left[Z_{\perp_{x,y}}^{BB} \right]_{\text{max}} \right)^{-1} \left(1 + \frac{\omega_{\xi_{x,y}}}{\omega_r} \right)$$

➔ A program calculating the theoretical instability rise time depending on the intensity, “MOSES”

➔ A multi-particle tracking simulation, “HEADTAIL”

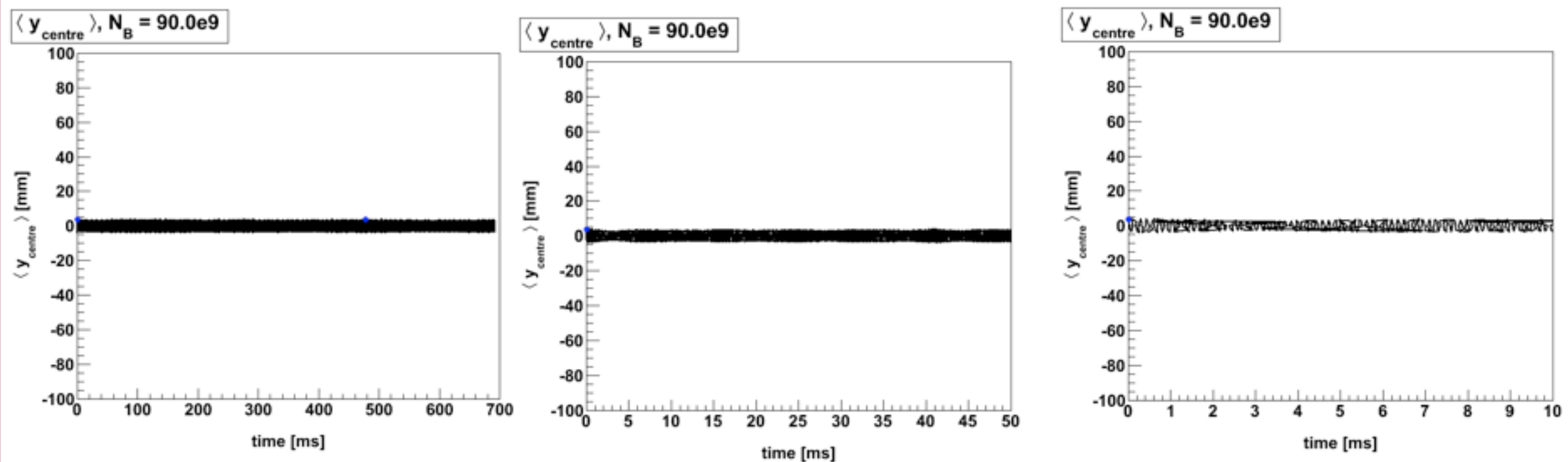
◆ One of HEADTAIL’s output, the vertical mean beam center, is shown here for different bunch intensities

◆ Exponential least square fit is used to get the Growth Rate, $1/\tau$

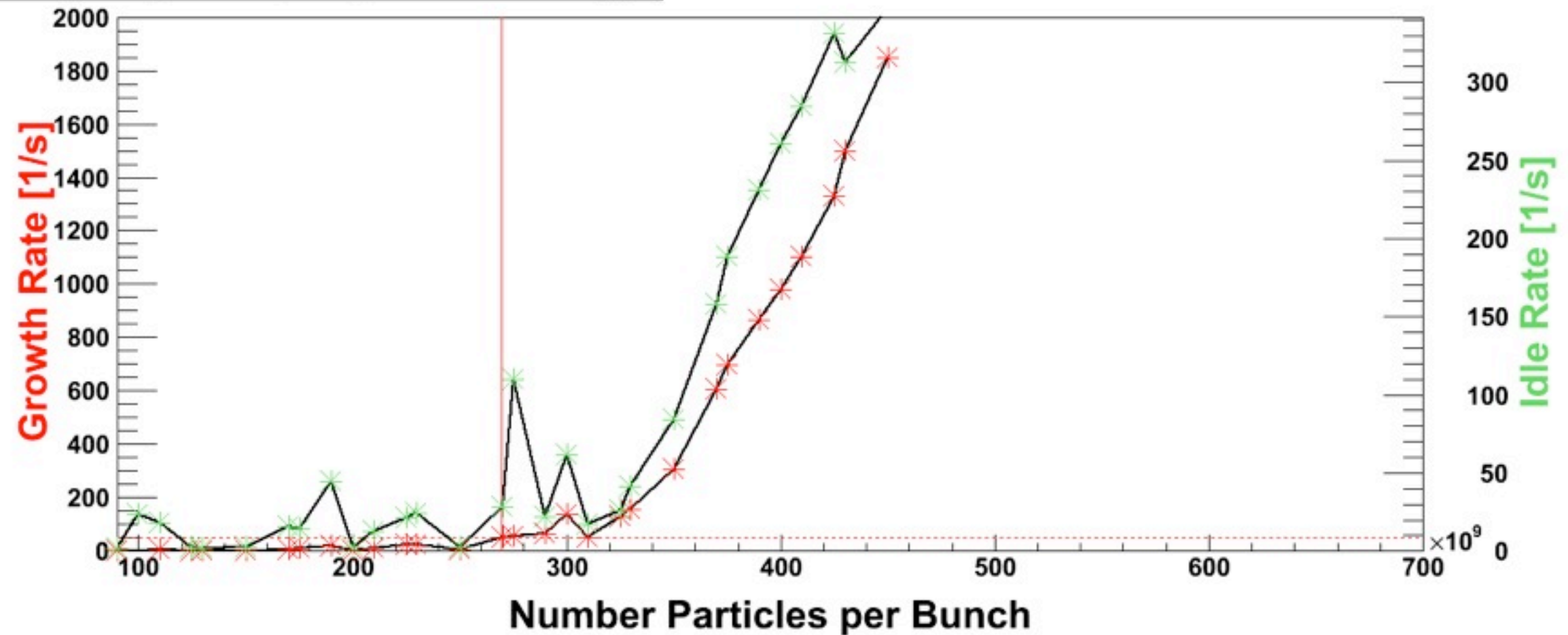


Growth Rate from HEADTAIL

HEADTAIL / MODES



$1/\tau$ vs. N_B for $\epsilon_1 = 22d0$ [eVs] (DR ${}^6\text{He}$ with BB_\perp)



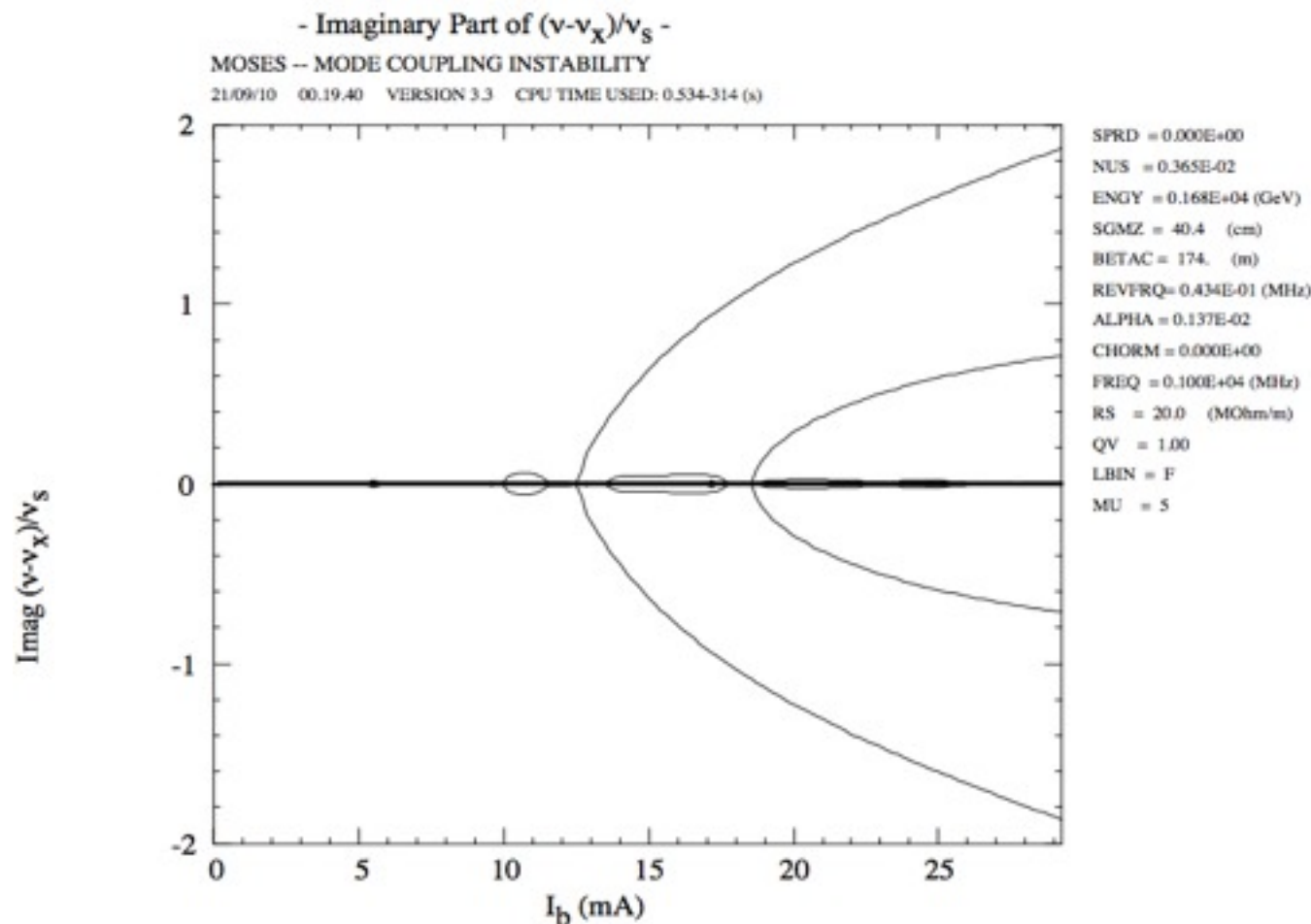
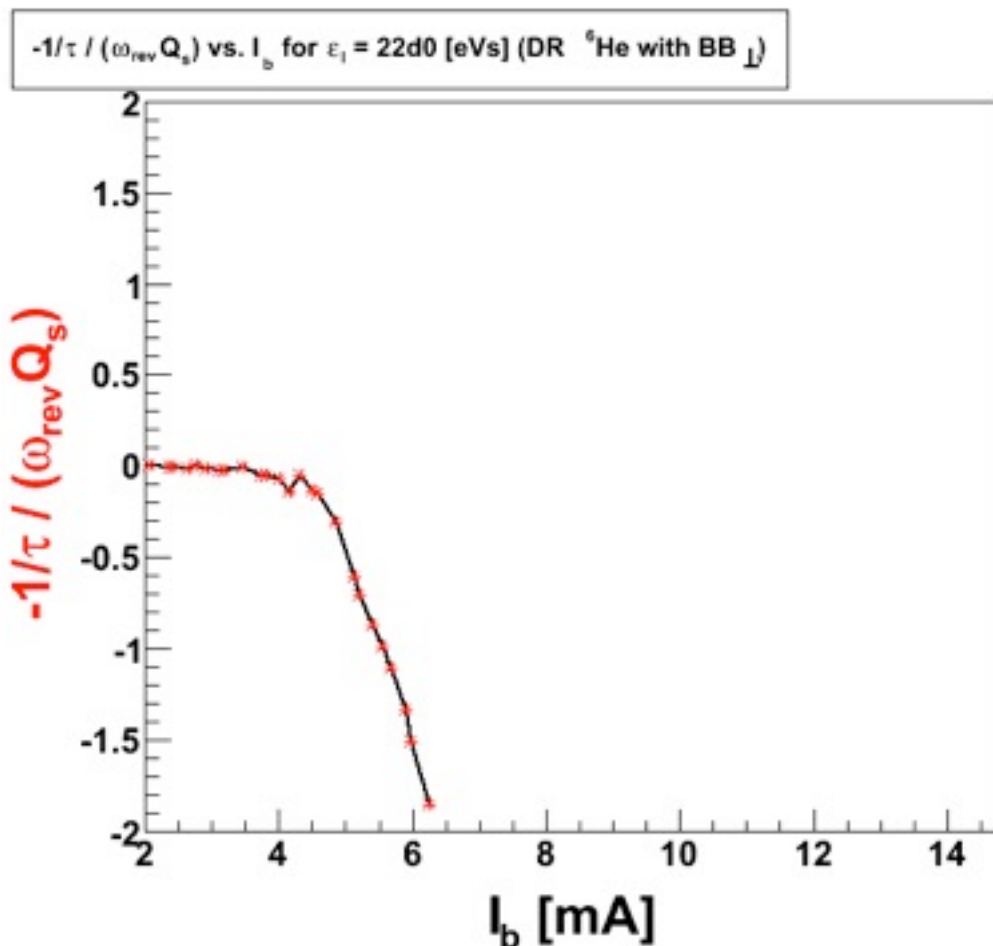
MOSES' Scales

HEADTAIL / MOSES

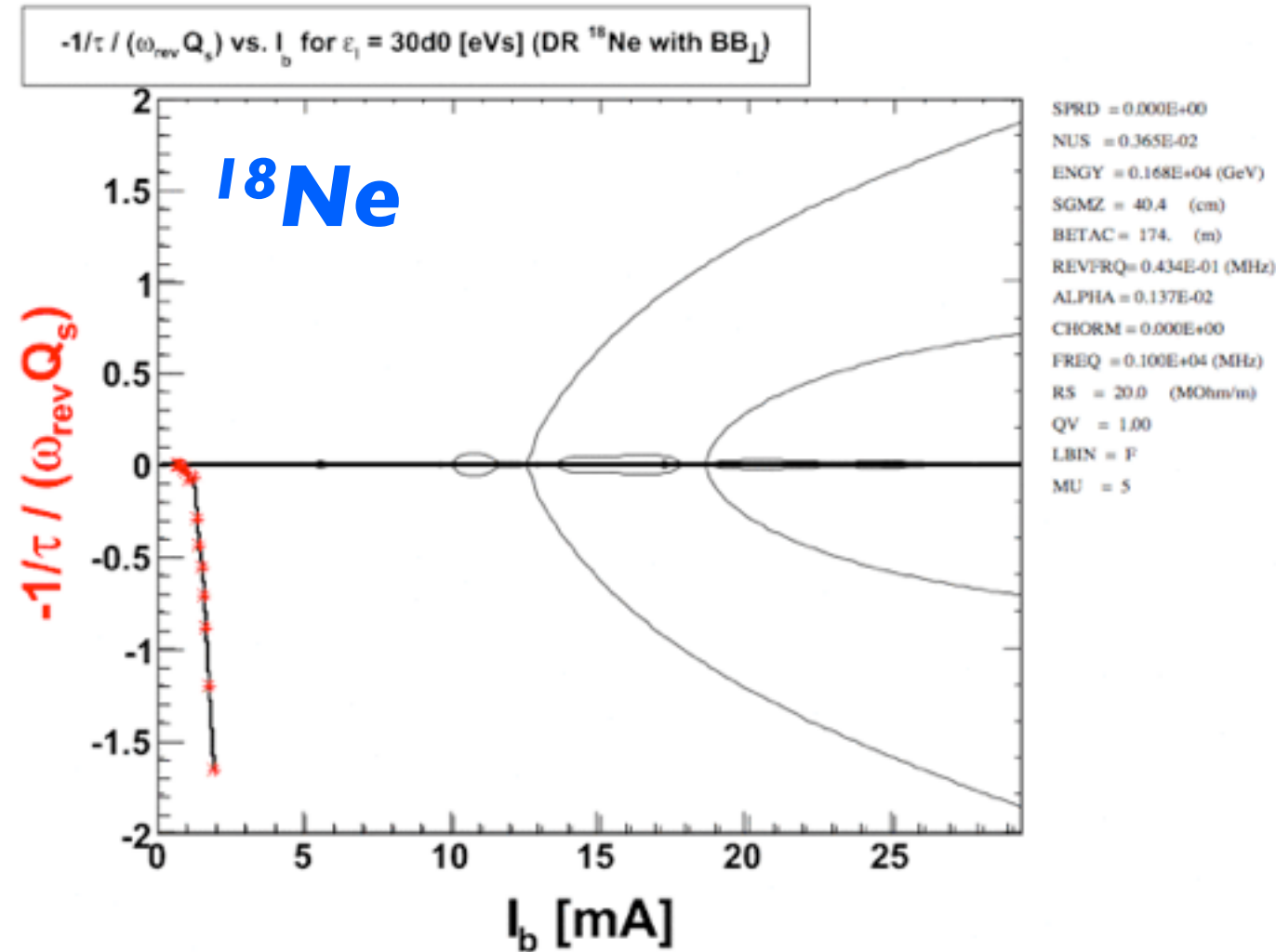
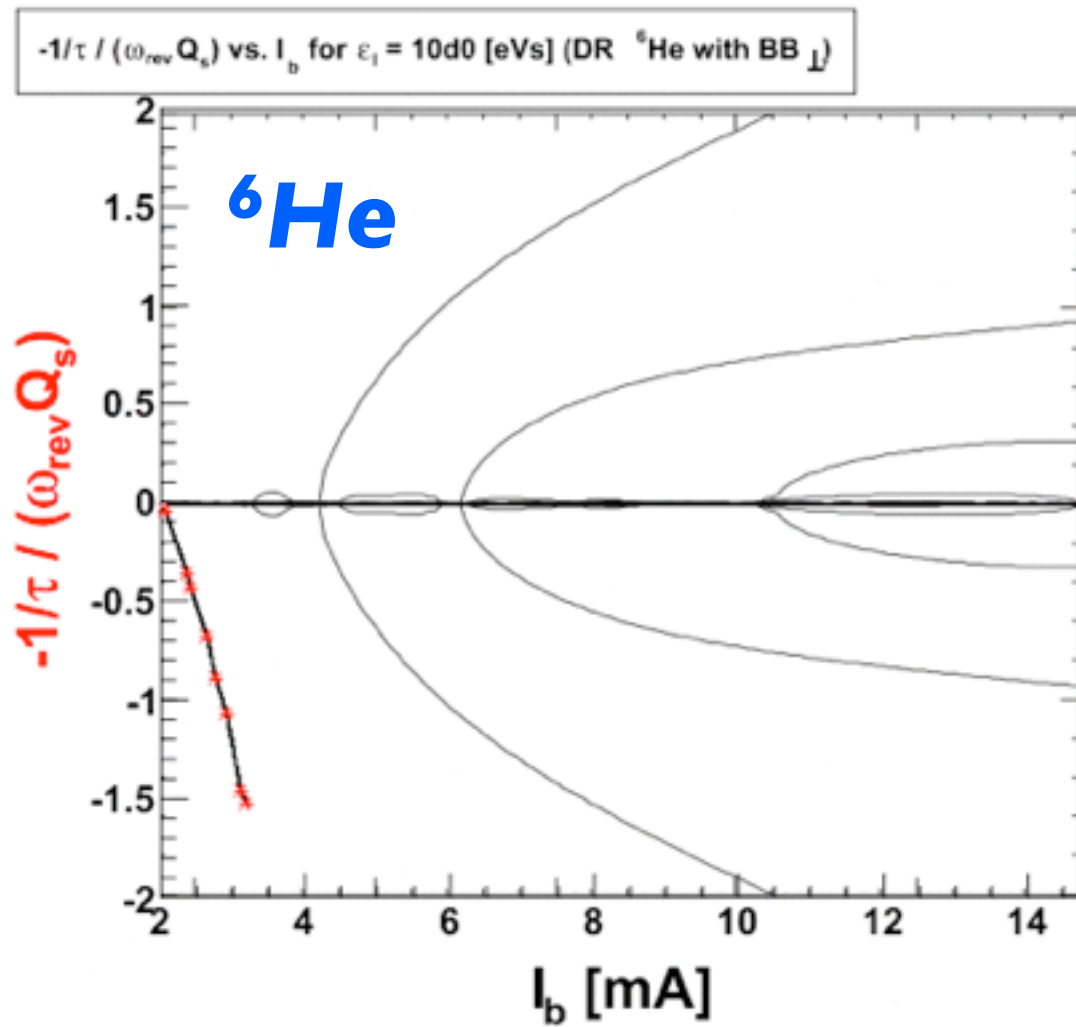
- **To get same scale as MOSES' plots:**

- **X-axis:** $\frac{1}{\tau} \rightarrow \frac{\Im[\Delta Q]}{Q_s} = -\frac{1/\tau}{\omega_{rev} Q_s}$ **since** $1/\tau = -\Im[\Delta Q] \omega_{rev}$

- **Y-axis:** $N_B \rightarrow \frac{ZeN_B}{T_{rev}} = I_b$



Emittance Scan - No Z Correction



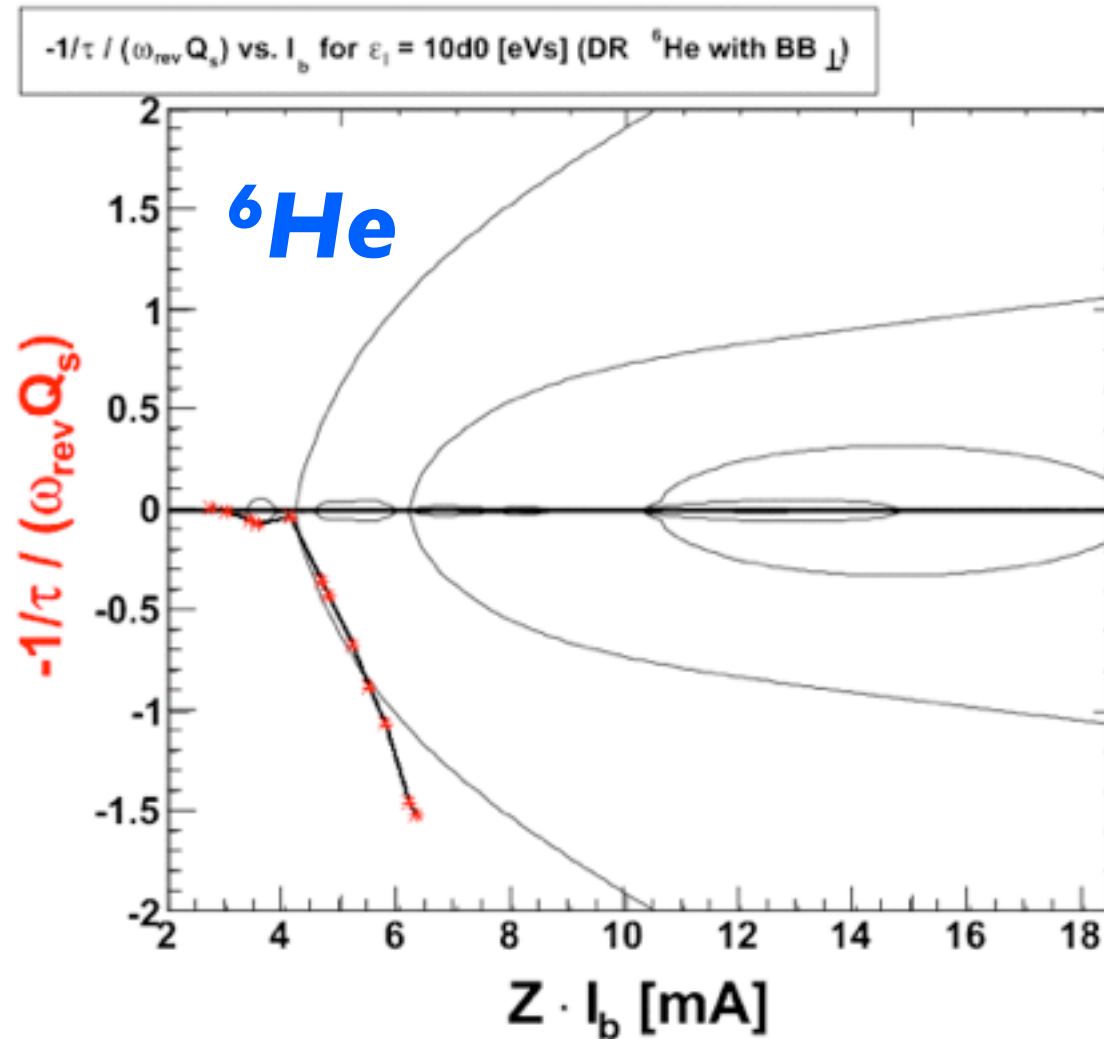
Z Correction

- To get the ion equivalent results from MOSES
 - ➔ Let's assume for simplicity MOSES solves Sacherer's equation for protons, then we see

$$\left(\frac{1}{\tau}\right)_{\perp x,y}^{m,n} = \frac{-1}{|n|+1} \frac{ZeI_b C \langle \beta_{x,y} \rangle \omega_{rev}}{4\pi E_{tot} L_b} \frac{\sum_{p=-\infty}^{\infty} \Re [Z_{\perp}(\omega_p)] h_{|n|}(\omega_p - \omega_{\xi})}{\sum_{p=-\infty}^{\infty} h_{|n|}(\omega_p - \omega_{\xi})}$$

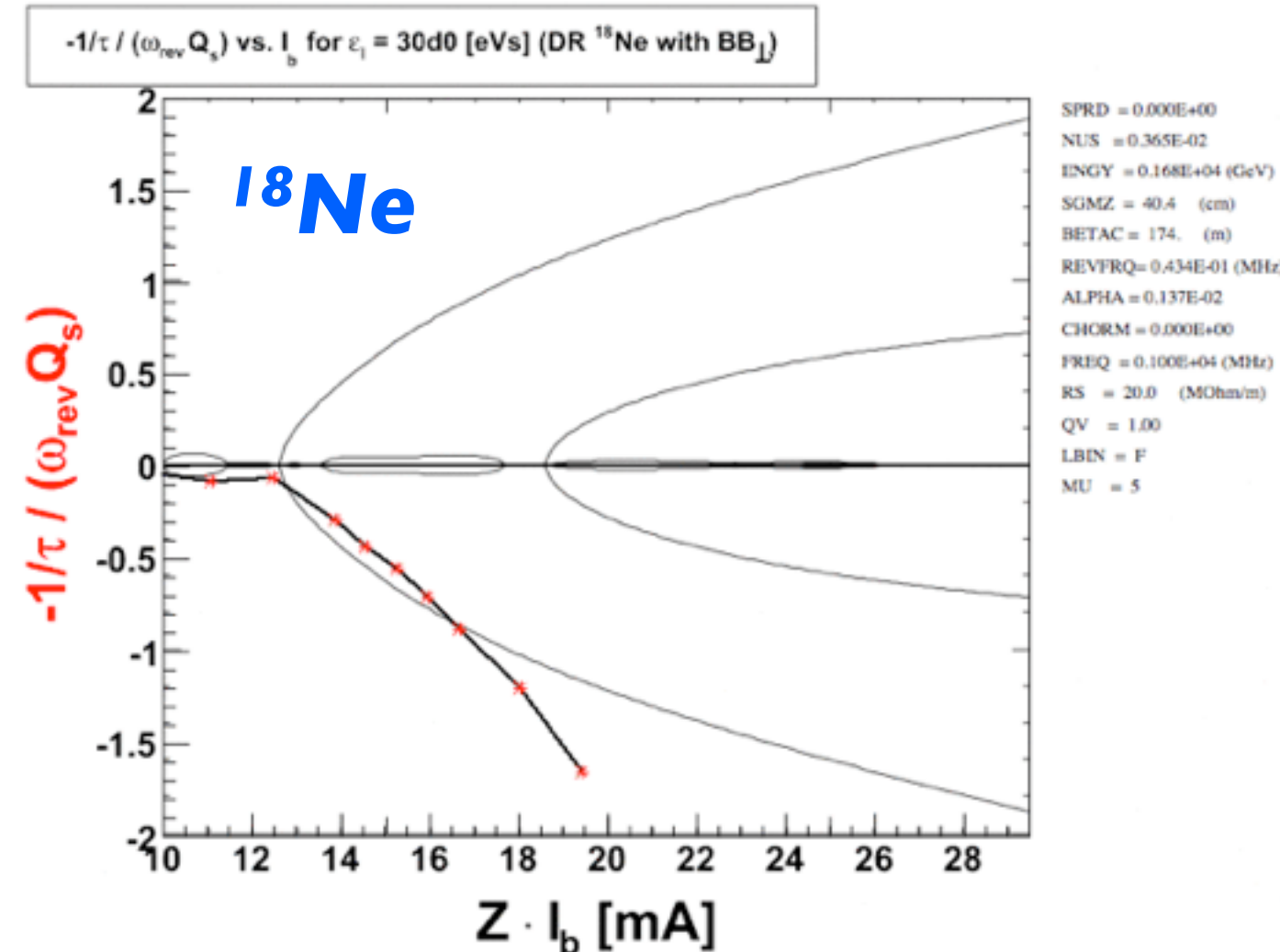
- ➔ that the current, I_b , should be multiplied by the charge number, Z , to get the correct growth rate
- ➔ We also see that the mass number, A , is included in the total energy, E_{tot}
- ➔ So the correction is: Multiply x-axis with Z

Emittance Scan - Z Correction



- **Z Correction seems to work perfect only for big growth rate**

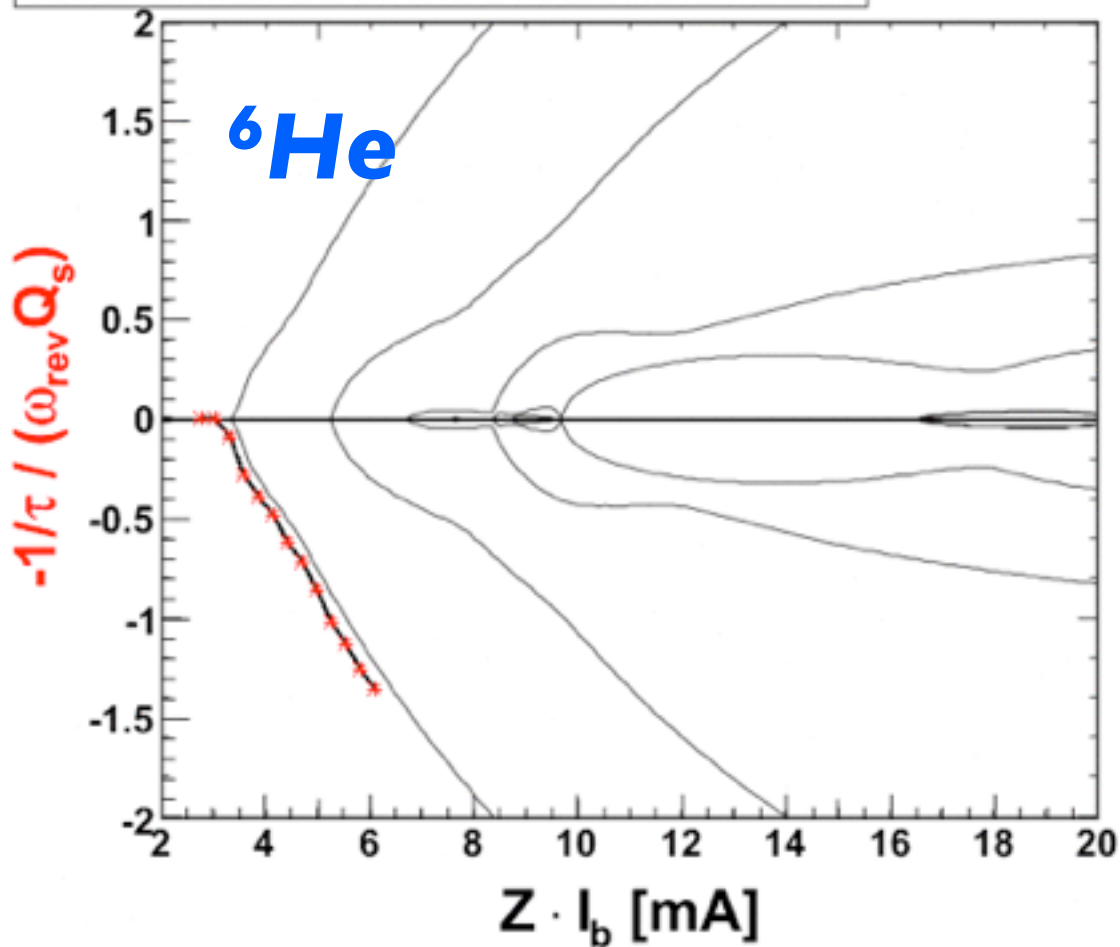
➔ **Any other idea... ?**



Res. Freq. Scan - Z Correction

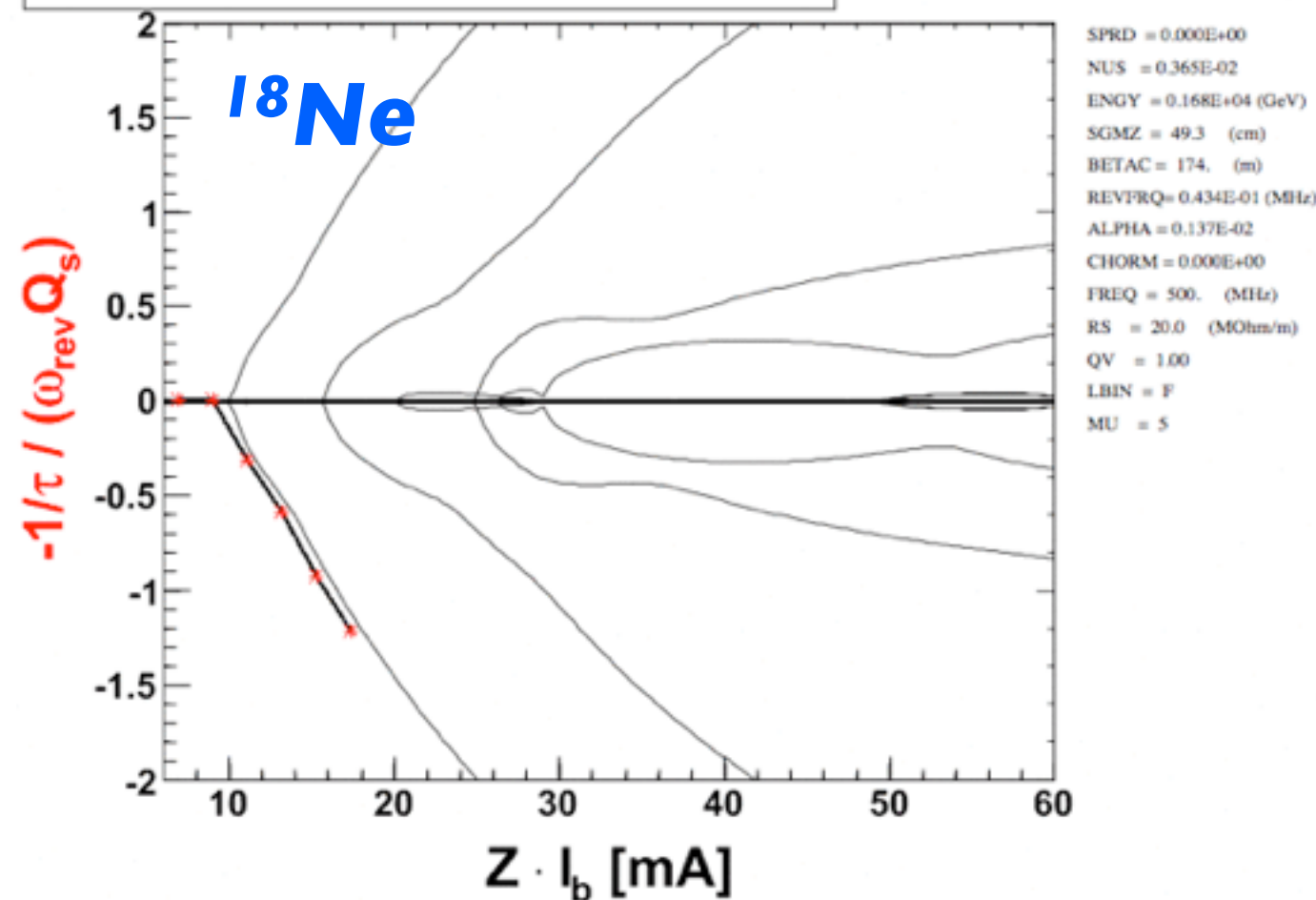
HEADTAIL / MODES

$-1/\tau / (\omega_{rev} Q_s)$ vs. I_b for $f_{r1} = 0d5$ [GHz] (DR ${}^6\text{He}$ with BB_{\perp})



- **Z Correction seems to work perfect only for big growth rate**
- ➔ **Any other idea... ?**

$-1/\tau / (\omega_{rev} Q_s)$ vs. I_b for $f_{r1} = 0d5$ [GHz] (DR ${}^{18}\text{Ne}$ with BB_{\perp})



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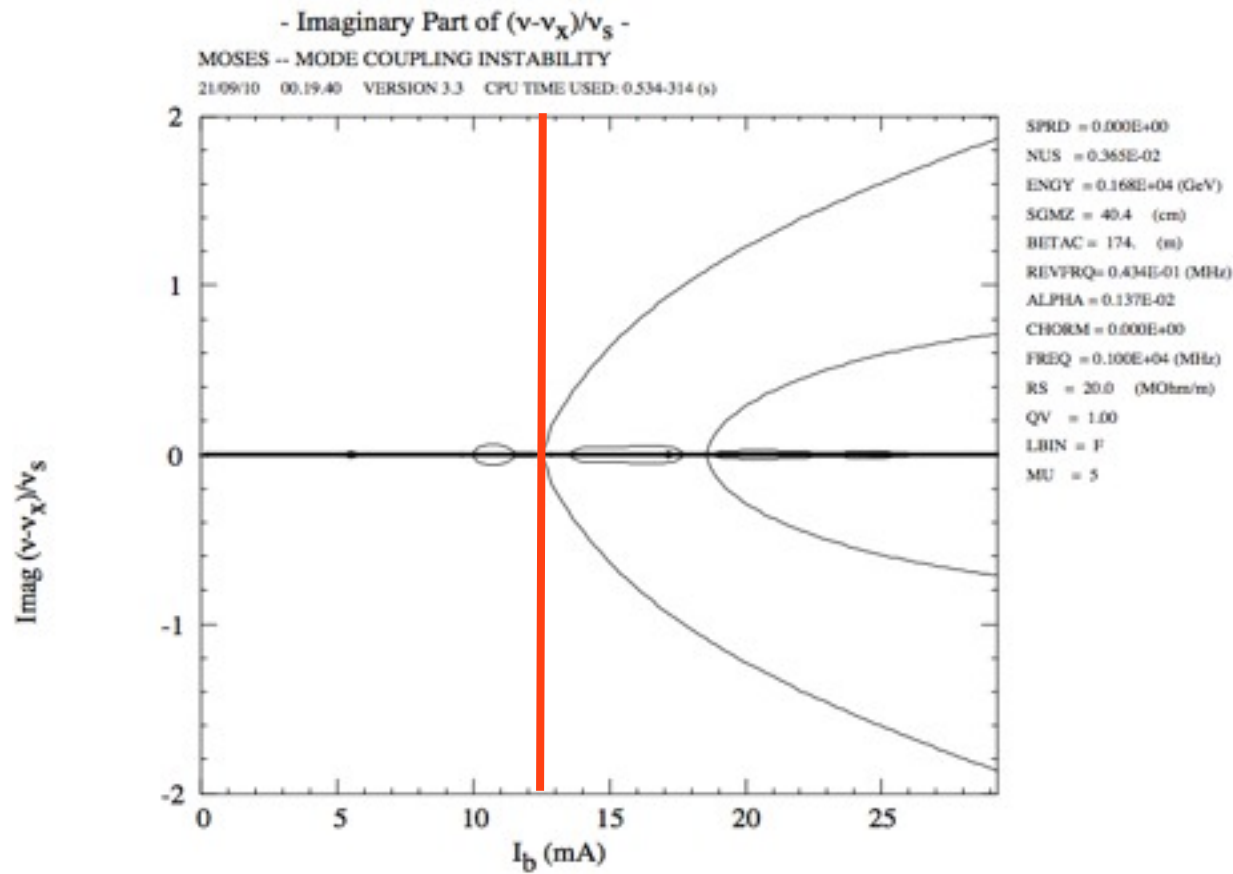
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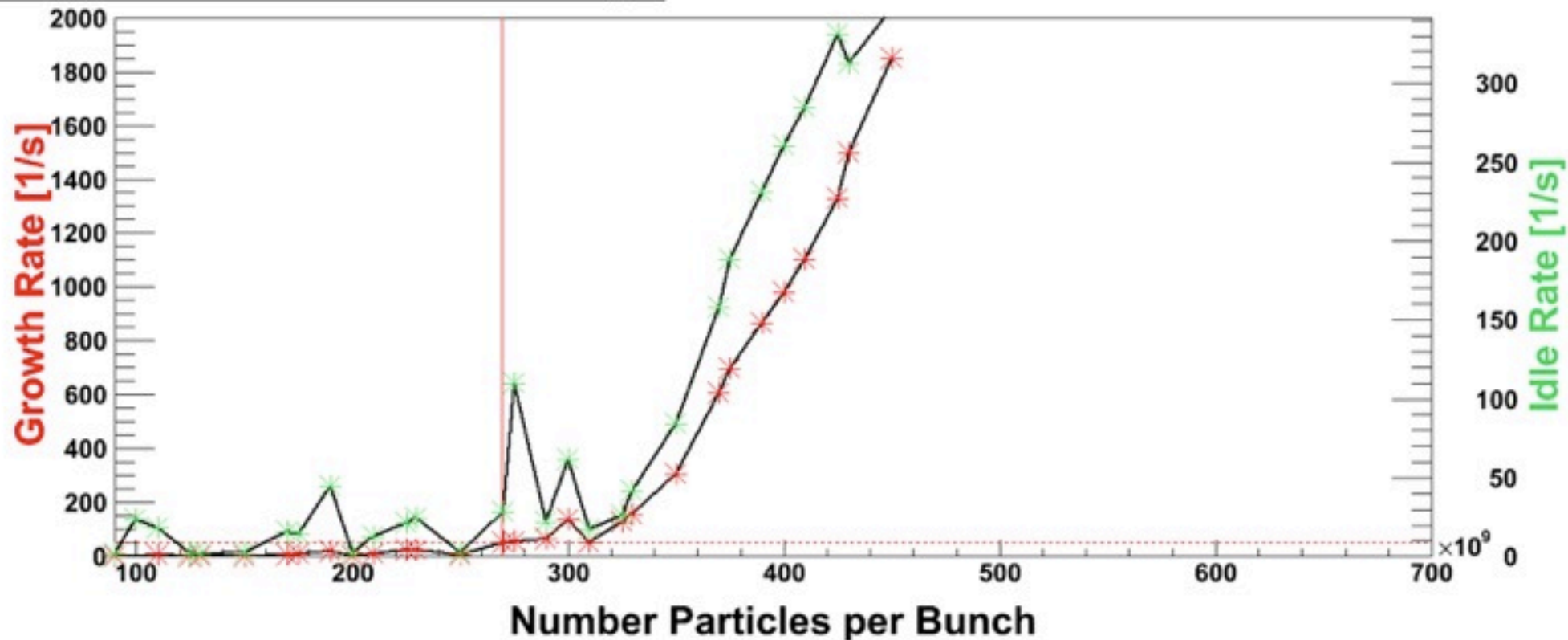
Intensity Threshold

T H R E S H O L D



- **MOSES**: Get the ion equivalent threshold $I_b^{th} = I_b^{th} / Z$ and then $N_b^{th} = T_{rev} I_b^{th} / Ze$
- **HEADTAIL**: Define threshold growth rate $(1/\tau)^{th} = 50 \text{ Hz}$

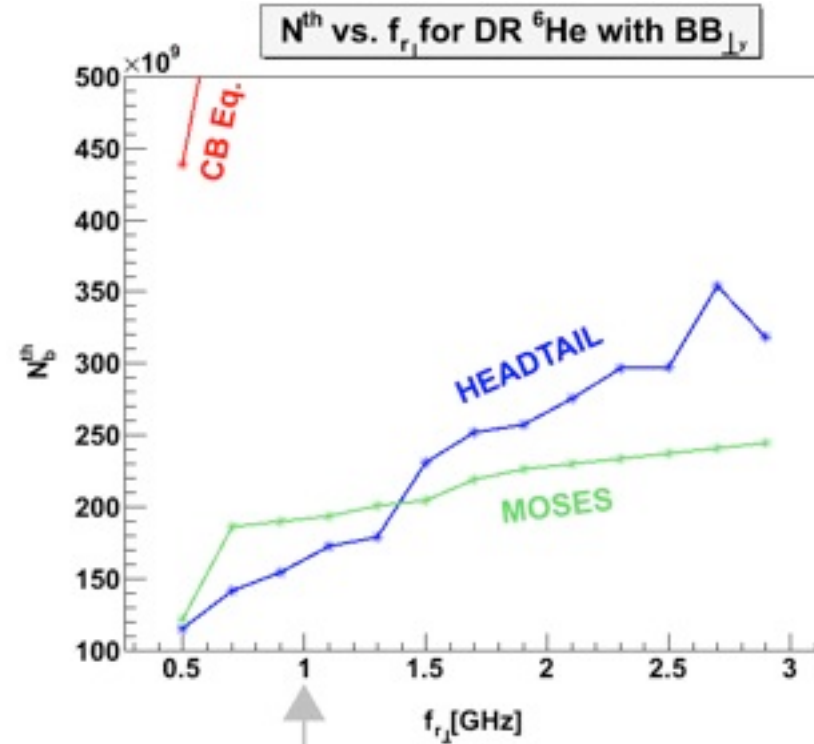
1/τ vs. N_B for ε₁ = 22d0 [eVs] (DR ⁶He with BB₁)



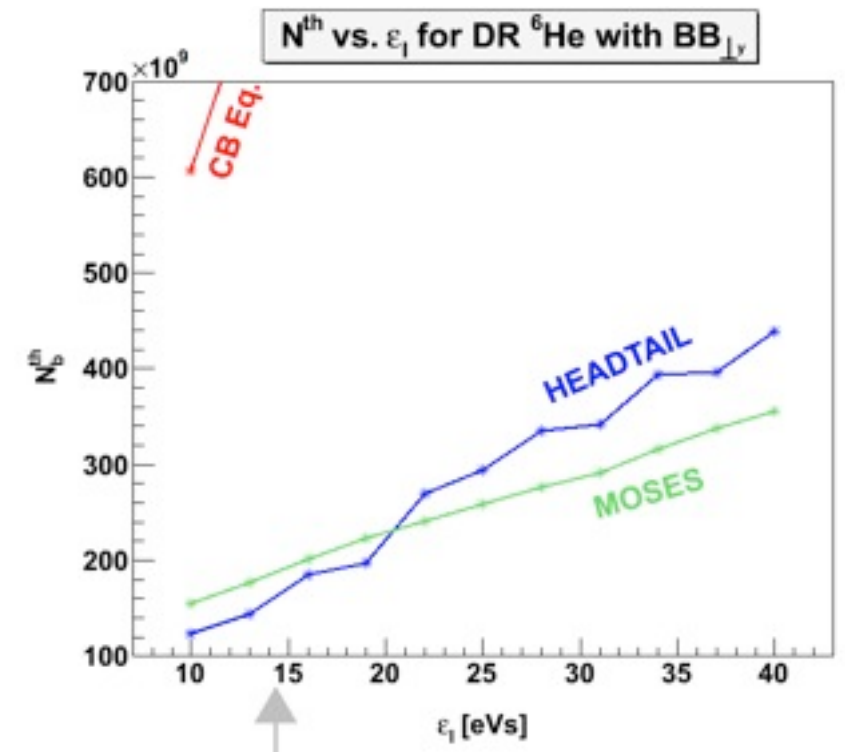
Scans for ${}^6\text{He}$ in DR

T H R E S H O L D

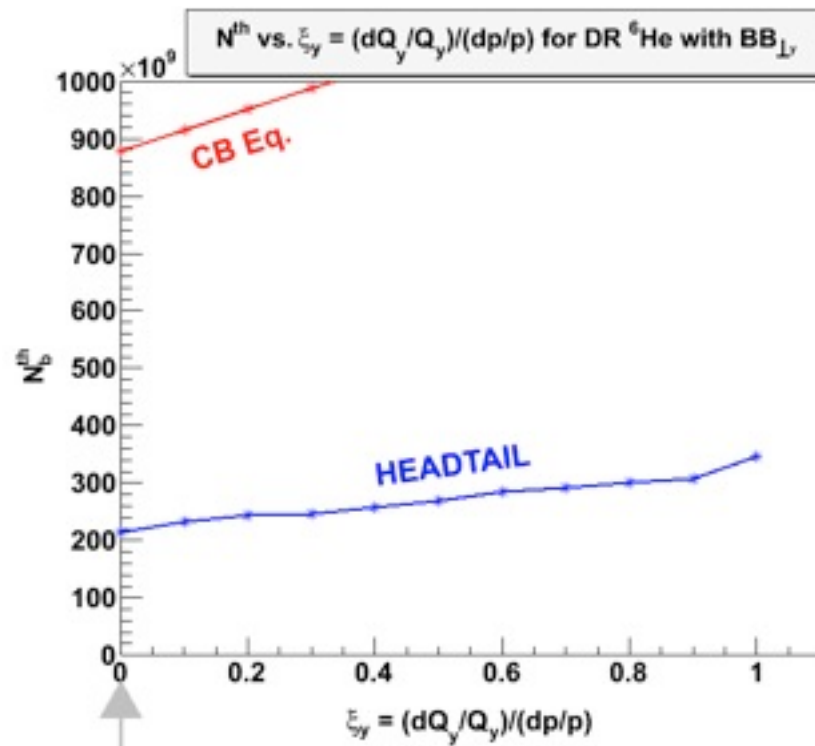
- **Scan over Resonance Frequency**



- **Scan over Longitudinal Emittance**



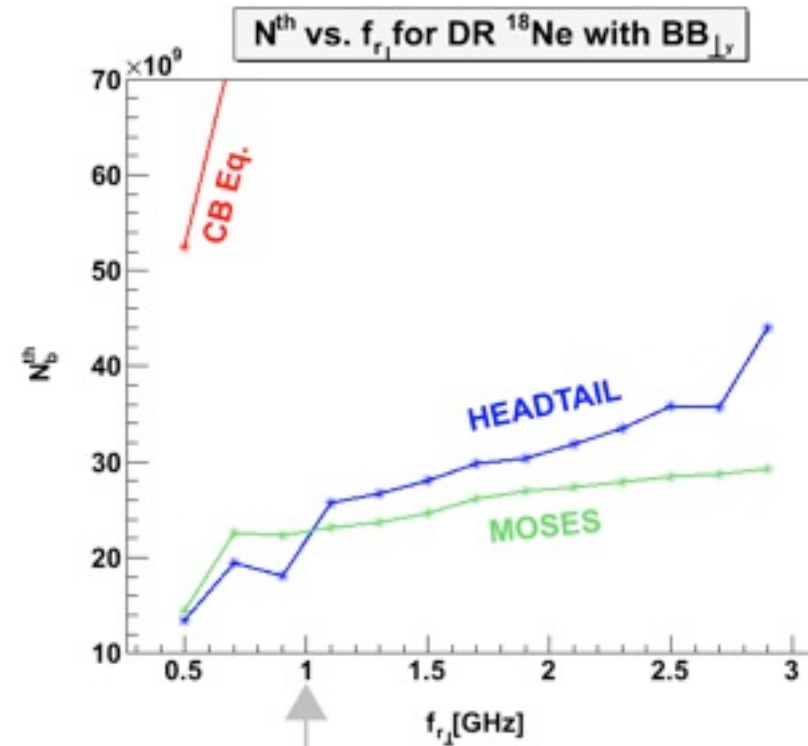
- **Scan over Chromaticity with $(I/\tau)^{\text{th}} = 100 \text{ Hz}$**



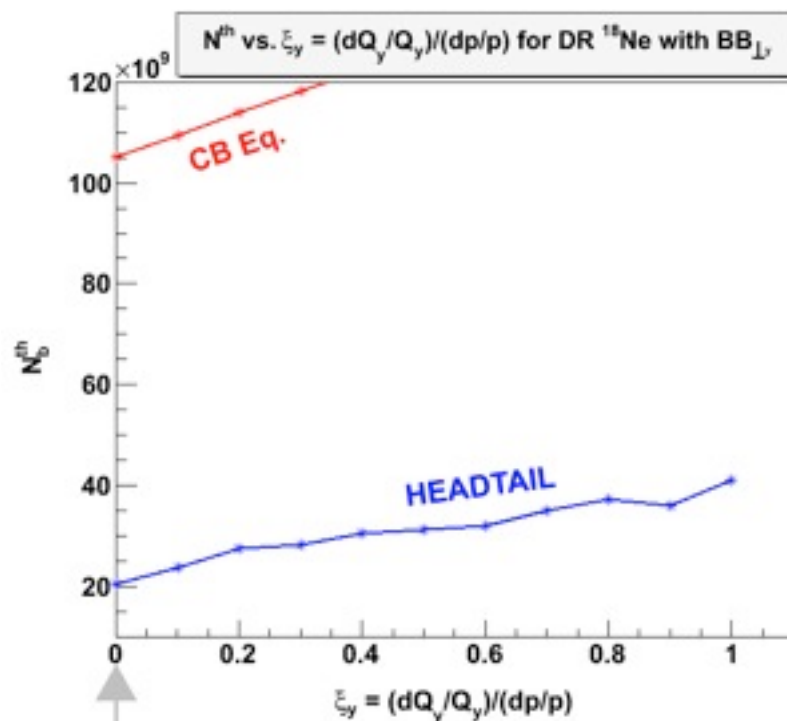
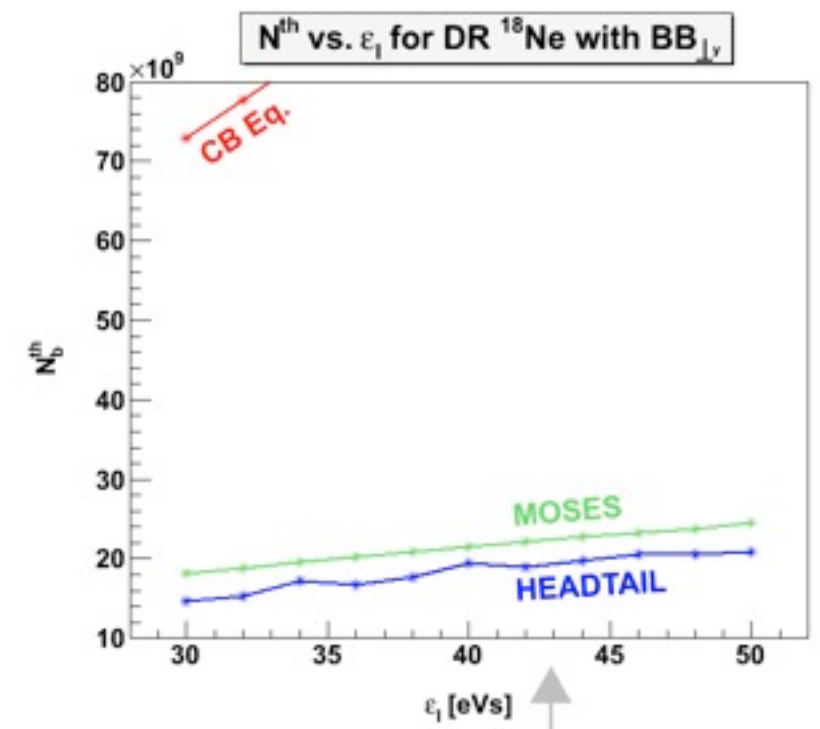
Scans for ^{18}Ne in DR

T H R E S H O L D

- **Scan over Resonance Frequency**



- **Scan over Longitudinal Emittance**



- **Scan over Chromaticity with $(I/\tau)^{\text{th}} = 100 \text{ Hz}$**



N_b^{th} vs. R_{\perp} in DR

- **None of the parameters we scanned over so far, ϵ_l , f_r and ξ , seem to manage to improve N_B^{th} up to the level we want:**

$$N_B^{6\text{He}} = 4.0 \cdot 10^{12}$$

$$N_B^{18\text{Ne}} = 3.1 \cdot 10^{12}$$

- **Let's see how much smaller R_{\perp} have to be compared to $R_{\perp}^{\text{sps}} = 20 \text{ M}\Omega/\text{m}$ to allow N_B^{th}**

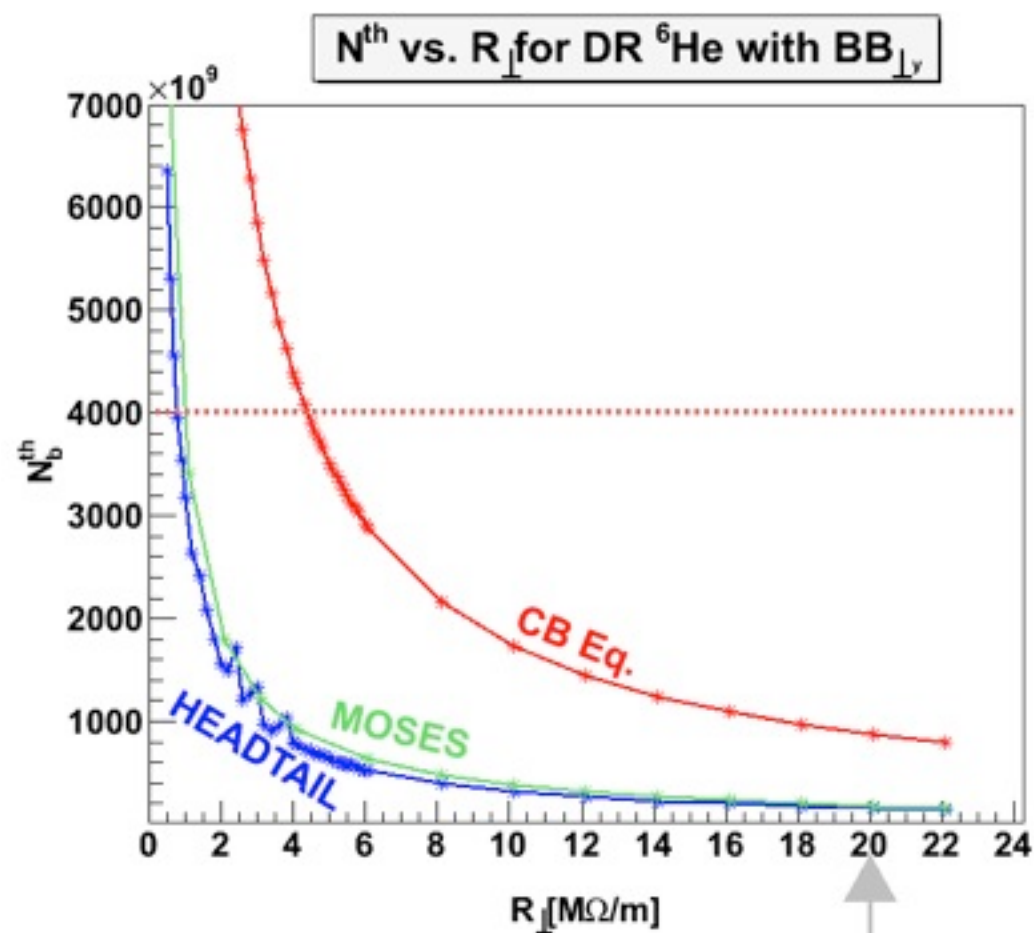
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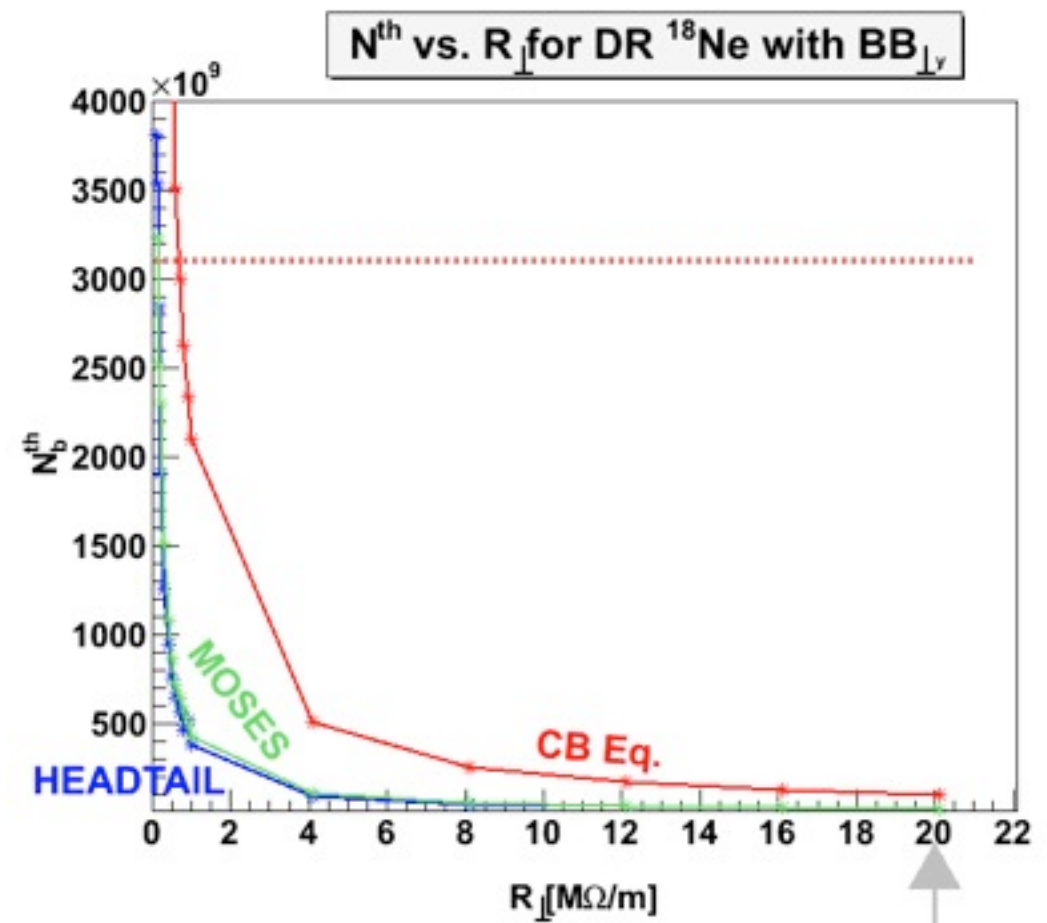
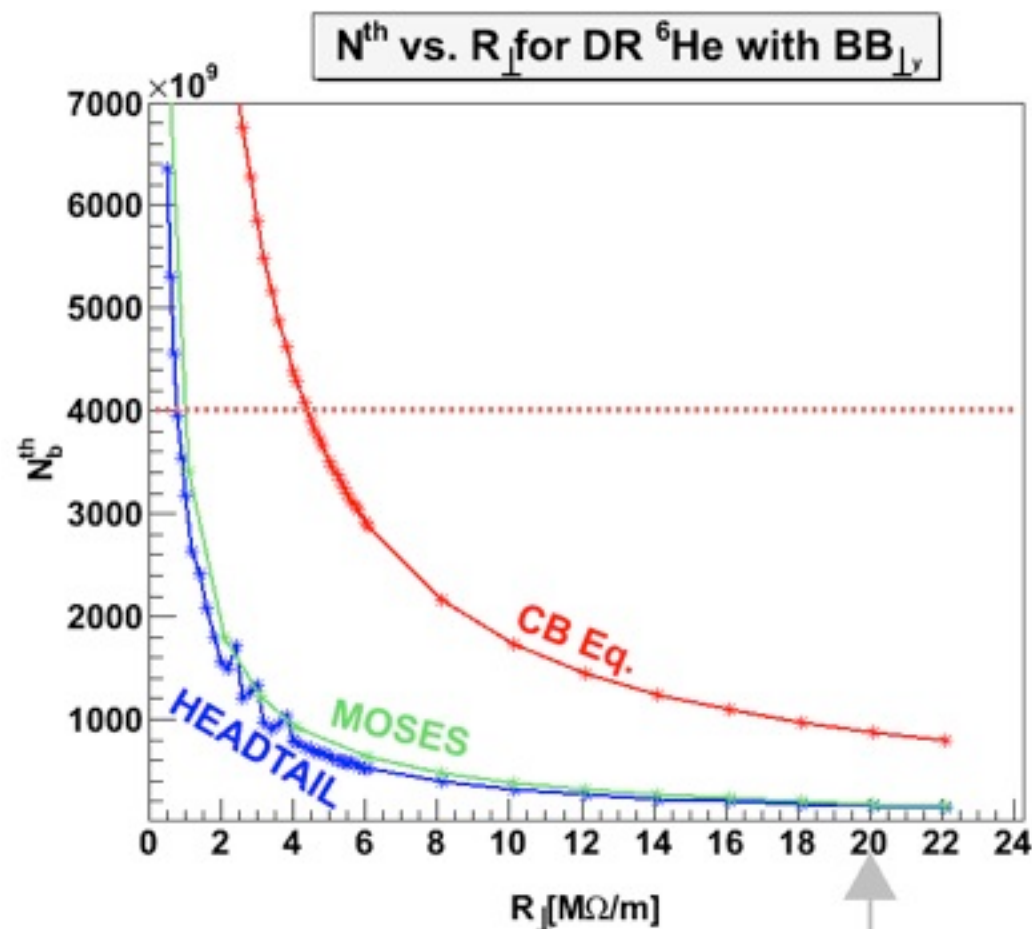
N_b^{th} vs. R_{\perp} in DR

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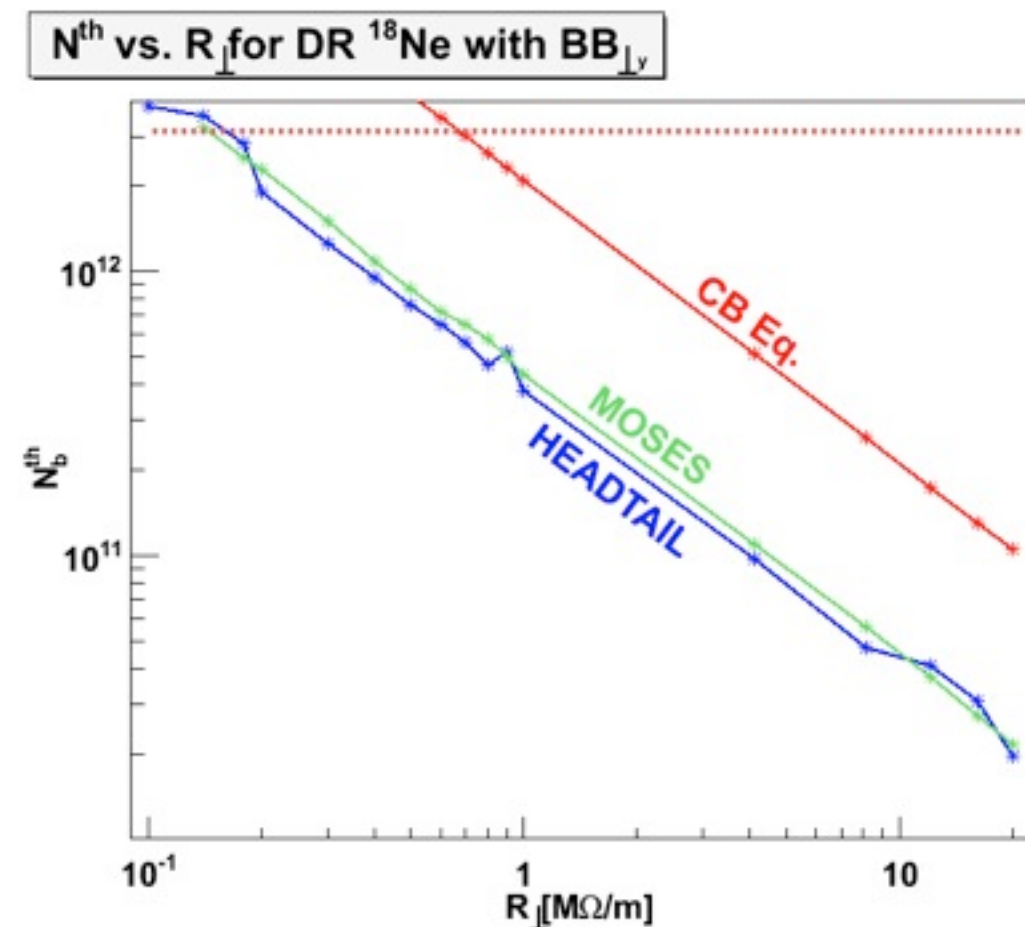
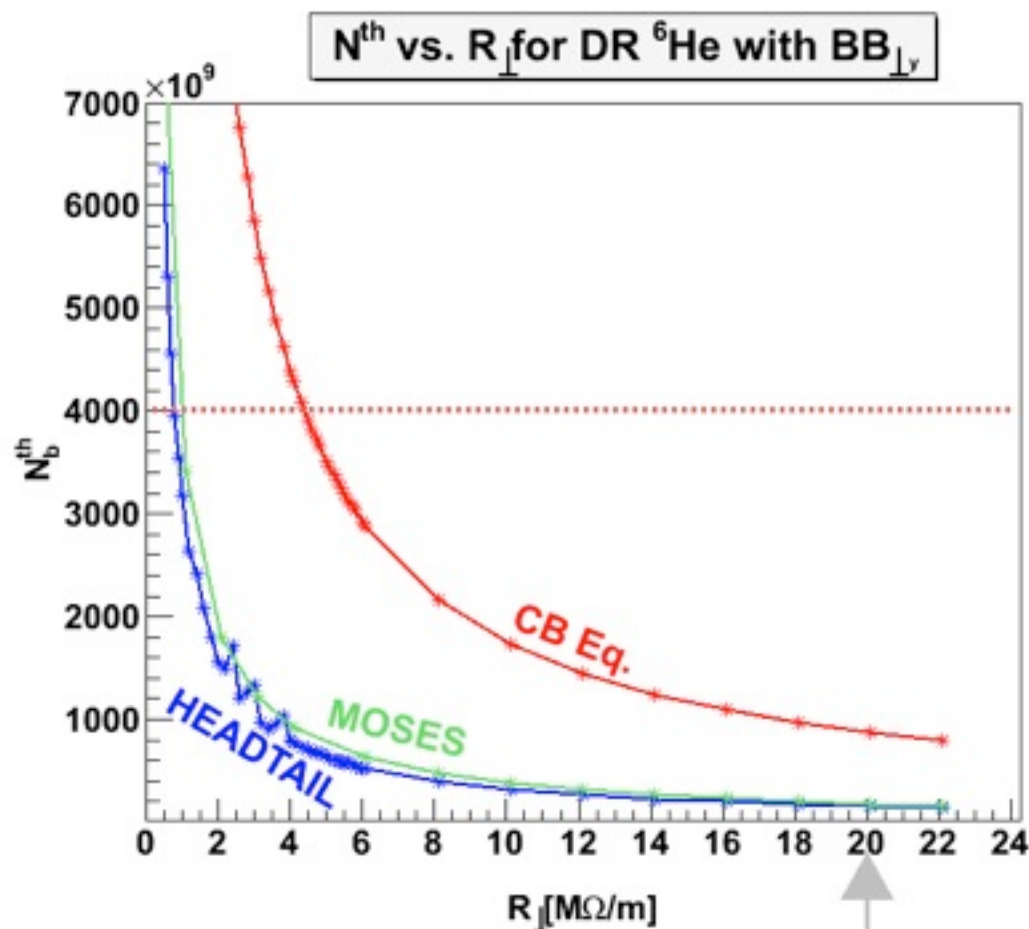
N_b^{th} vs. R_{\perp} in DR

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Outline

- Beta Beam Overview
- Collective Effects
 - ➔ Laslett's Tune Shifts
 - ➔ Wakefield Instabilities
 - ◆ HEADTAIL & MOSES
 - ◆ Intensity Thresholds
- Conclusion

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- Beta Beam Overview
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- Conclusion

Conclusion

- Direct Space Charge effect will not limit the performance of the Decay Ring (Laslett's Equations)
- We have a very challenging upper limit of the DR's Transversal Shunt Impedance, R_{\perp} :
 - ➔ 10 (100) times smaller than SPS for ${}^6\text{He}$ (${}^{18}\text{Ne}$)
... based on HEADTAIL and MOSES studies
- This study, that was completely based on parameters from "FP6", suggests a re-optimization of the Beta Beam design

Note under preparation:

<http://chansen.web.cern.ch/chansen/PUBLICATIONS/bbCollective.pdf>

SVN: <http://svnweb.cern.ch/world/wsvn/bbcollective>

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Backup Slides

Input Parameters

TABLE 2. Input parameters from previous Beta Beam Decay Ring design report [10].

Parameters	Description	DR ¹⁸ Ne	DR ⁶ He
Z	Charge Number	10	2
A	Mass Number	18	6
h	Harmonic Number	924	924
C [m]	Circumference	6911.6	6911.6
ρ [m]	Magnetic Radius	155.6	155.6
γ_{tr}	Gamma at Transition	27.00	27.00
V_{RF} [MV]	Voltage	1.196e+01	2.000e+01
dB/dt [T/s]	Magnetic Ramp	0.00	0.00
γ	Relativistic Gamma	100.0	100.0
δ_{max}	Maximum Momentum Spread	2.50e-03	2.50e-03
E_{rest} [MeV]	Rest Energy	16767.10	5605.54
M	Number Bunches per Batch	20	20
L_b [m]	Full Bunch Length	1.970	1.970
N_b	Number Ions per Injected Bunch	2.35e+11	4.87e+11
N_B	Average Number Ions per Bunch	3.10e+12	4.00e+12
m_r	Merges Ratio	20	15
$t_{1/2}$ [s]	Half Life at Rest	1.67	0.81
T_c [s]	Revolution Time	3.60	6.00
Q_x	Horizontal Tune	22.23	22.23
Q_y	Vertical Tune	12.16	12.16
$\langle\beta\rangle_x$ [m]	Average Horizontal Betatron Function	148.25	148.25
$\langle\beta\rangle_y$ [m]	Average Vertical Betatron Function	173.64	173.64
$\langle D\rangle_x$ [m]	Average Dispersion	-0.60	-0.60
ξ_x	Horizontal Chromaticity	0.0	0.0
ξ_y	Vertical Chromaticity	0.0	0.0
$\epsilon_{N_x}(1\sigma)$ [π m-rad]	Normalized Horizontal Emittance	1.48e-05	1.48e-05
$\epsilon_{N_y}(1\sigma)$ [π m-rad]	Normalized Vertical Emittance	7.90e-06	7.90e-06
ϵ_l (full) [eVs]	Full Longitudinal Emittance	42.89	14.36
b_x [cm]	Horizontal Beam Pipe Size	16.0	16.0
b_y [cm]	Vertical Beam Pipe Size	16.0	16.0
ρ_{res} [Ω m]	Resistivity	1.0e-07	1.0e-07

TABLE 3. Assumed impedance input parameters.

Parameters	Description	DR ¹⁸ Ne	DR ⁶ He
$Q_{ }$	Longitudinal Quality Factor	1.00	1.00
$\omega_{r, }$ [GHz]	Longitudinal Angular Resonance Frequency	6.28	6.28
$ Z_{ }/n $ [Ω] = $\lim_{\omega \rightarrow 0} \frac{ Z(\omega) }{\omega/\omega_{rv}}$		10.00	10.00
$R_{s, }$ [M Ω] = $\frac{ Z/n Q\omega_r}{\omega_{rv}}$	Longitudinal Shunt Impedance	0.231	0.231
Q_{\perp}	Transverse Quality Factor	1.00	1.00
$\omega_{r,\perp}$ [GHz]	Transverse Angular Resonance Frequency	6.28	6.28
$R_{s,\perp}$ [M Ω /m]	Transverse Shunt Impedance	20.00	20.00

Calculated Values

TABLE 4. Calculated values.

		DR ¹⁸ Ne	DR ⁶ He
r_0 [m] = $r_p Z^2 / A$	<i>Ion Radius</i>	8.53e-18	1.02e-18
E_{tot} [GeV] = $\gamma \cdot E_{rest}$	<i>Total Energy</i>	1676.71	560.55
$\beta = \sqrt{1 - 1/\gamma^2}$	<i>Relativistic Beta</i>	1.00	1.00
$\eta = (1/\gamma_{tr})^2 - (1/\gamma)^2$	<i>Phase Slip Factor</i>	1.27e-03	1.27e-03
T_{rev} [μ s] = $C/(\beta c)$	<i>Revolution Time</i>	23.0558	23.0558
R [m] = $C/2\pi$	<i>Machine Radius</i>	1100.02	1100.02
ω_{rev} [MHz] = $2\pi/T_{rev}$	<i>Angular Revolution Frequency</i>	0.27	0.27
$\sigma_\delta = \delta_{max}/2$	<i>1 Sigma Momentum Spread</i>	1.25e-03	1.25e-03
τ_b [ns] = $L_b/(\beta c)$	<i>Full Bunch Length</i>	6.57	6.57
\hat{I} [A] = ZeN_B/τ_b	<i>Peak Current</i>	755.80	195.04
I_b [A] = ZeN_B/T_{rev}	<i>Beam Current</i>	0.22	0.06
$\epsilon_l^{2\sigma}$ [eVs] = $\frac{\pi}{2}\beta^2 E_{tot} \tau_b \delta_{max}$	<i>2 Sigma Longitudinal Emittance</i>	43.27	14.46
$Q_s = \sqrt{\frac{hZeV \eta \cos \phi_s }{2\pi\beta^2 E_{tot}}}$	<i>Synchrotron Tune</i>	0.00	0.00
ω_s [kHz] = $Q_s \cdot \omega_{rev}$	<i>Synchrotron Angular Frequency</i>	1.00	1.00
ω_x [MHz] = $Q_x \cdot \omega_{rev}$	<i>Horizontal Betatron Angular Frequency</i>	6.06	6.06
ω_y [MHz] = $Q_y \cdot \omega_{rev}$	<i>Vertical Betatron Angular Frequency</i>	6.06	6.06
ω_c [GHz] = $\beta c/b_{min(x,y)}$	<i>Cut-Off Angular Frequency</i>	1.87	1.87
$\Delta Q_{\xi_x} = \xi_x \delta_{max} Q_x$	<i>Horizontal Tune Shift due to Chromaticity</i>	0.0	0.0
$\Delta Q_{\xi_y} = \xi_y \delta_{max} Q_y$	<i>Vertical Tune Shift due to Chromaticity</i>	0.0	0.0
ω_{ξ_x} [MHz] = $\xi_x Q_x \omega_{rev}/\eta$	<i>Horizontal Chromatic Angular Frequency</i>	2.38e+02	2.38e+02
ω_{ξ_y} [MHz] = $\xi_y Q_y \omega_{rev}/\eta$	<i>Vertical Chromatic Angular Frequency</i>	1.30e+02	1.30e+02

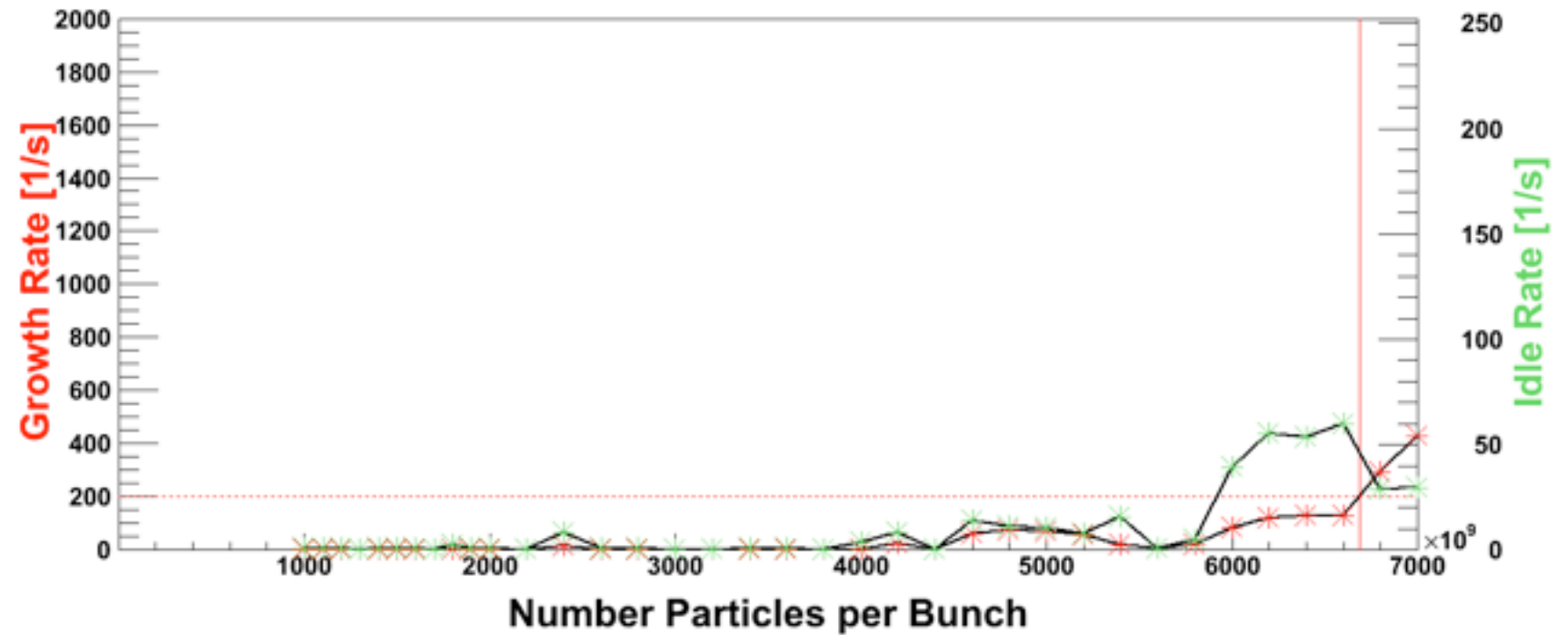
N_b^{th} vs. R_{\perp} for ${}^6\text{He}$ in DR

COLLECTIVE

- **$N_b > N_b^{\text{th}}$ when rise times < 5 ms** (growth rate > 200 Hz)

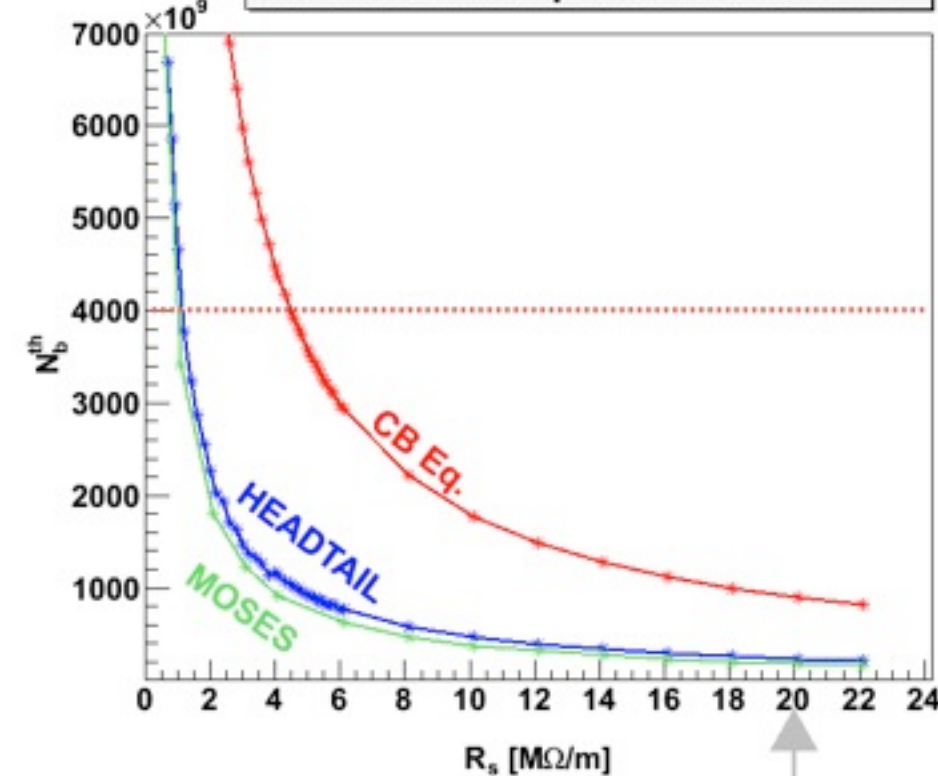
- **G.R. vs. N_b^{th} for different R_{\perp}** →

1/τ vs. N_b for $R_{\perp} = 0.7 \text{ M}\Omega/\text{m}$



- **R_{\perp} could be lowered to increase the intensity limit**
- **But a factor 10 smaller than $R_{\perp}^{\text{sps}} = 20 \text{ M}\Omega/\text{m}$ is very challenging**

N_b^{th} vs. Shunt Impedance for DR ${}^6\text{He}$



MOSES is developed for protons, so to get the ion equivalent the intensity limits, $N_b^{\text{th}}_{\text{moses}}$, was divided by a factor Z

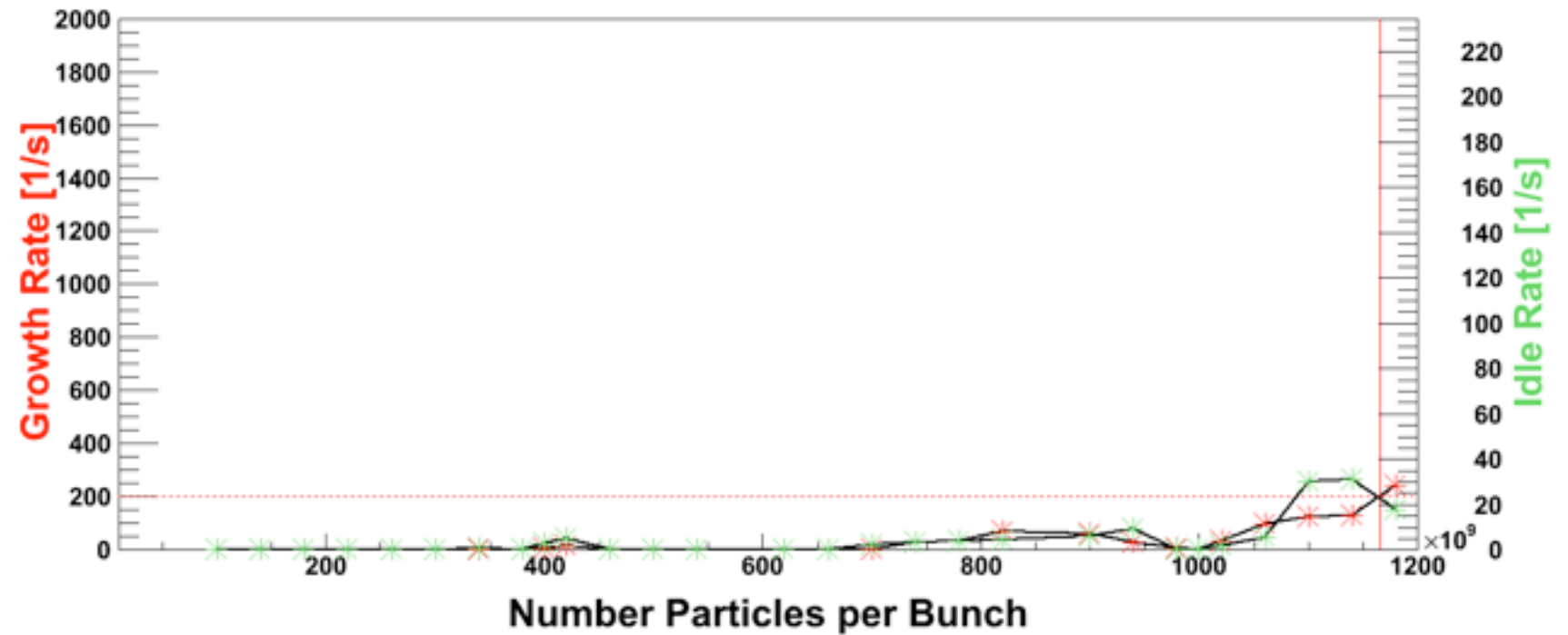
N_b^{th} vs. R_{\perp} for ${}^6\text{He}$ in DR

COLLECTIVE

- **$N_b > N_b^{\text{th}}$ when rise times < 5 ms** (growth rate > 200 Hz)

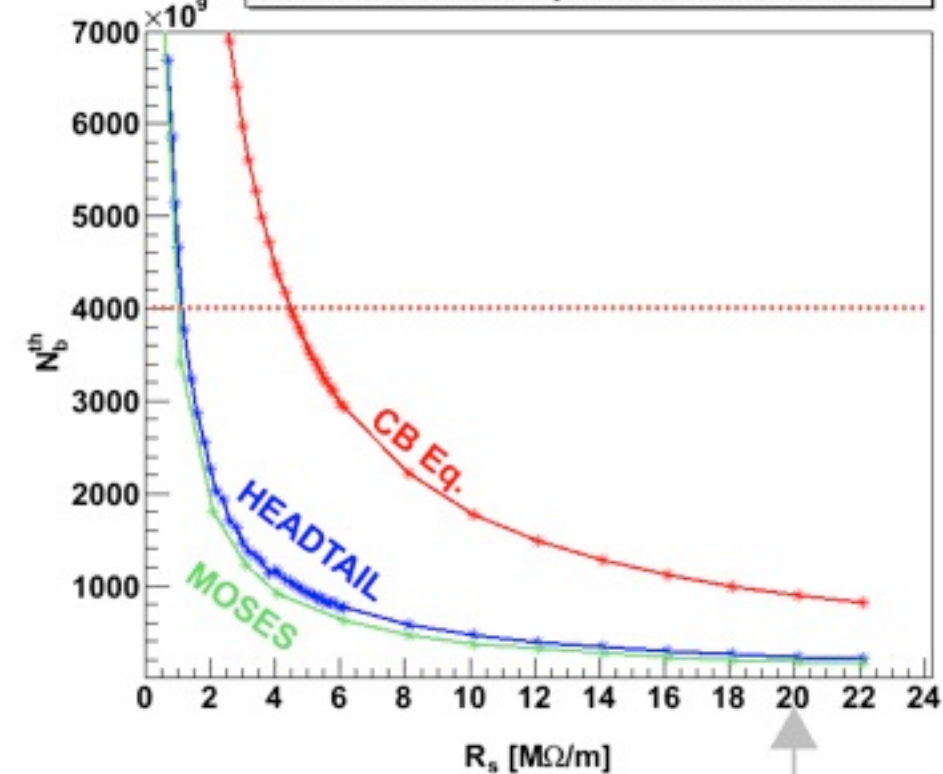
- **G.R. vs. N_b^{th} for different $R_{\perp} \rightarrow$**

1/τ vs. N_b for $R_s^{\perp} = 4d0$ MΩ/m



- **R_{\perp} could be lowered to increase the intensity limit**
- **But a factor 10 smaller than $R_{\perp}^{\text{sps}} = 20$ MΩ/m is very challenging**

N_b^{th} vs. Shunt Impedance for DR ${}^6\text{He}$



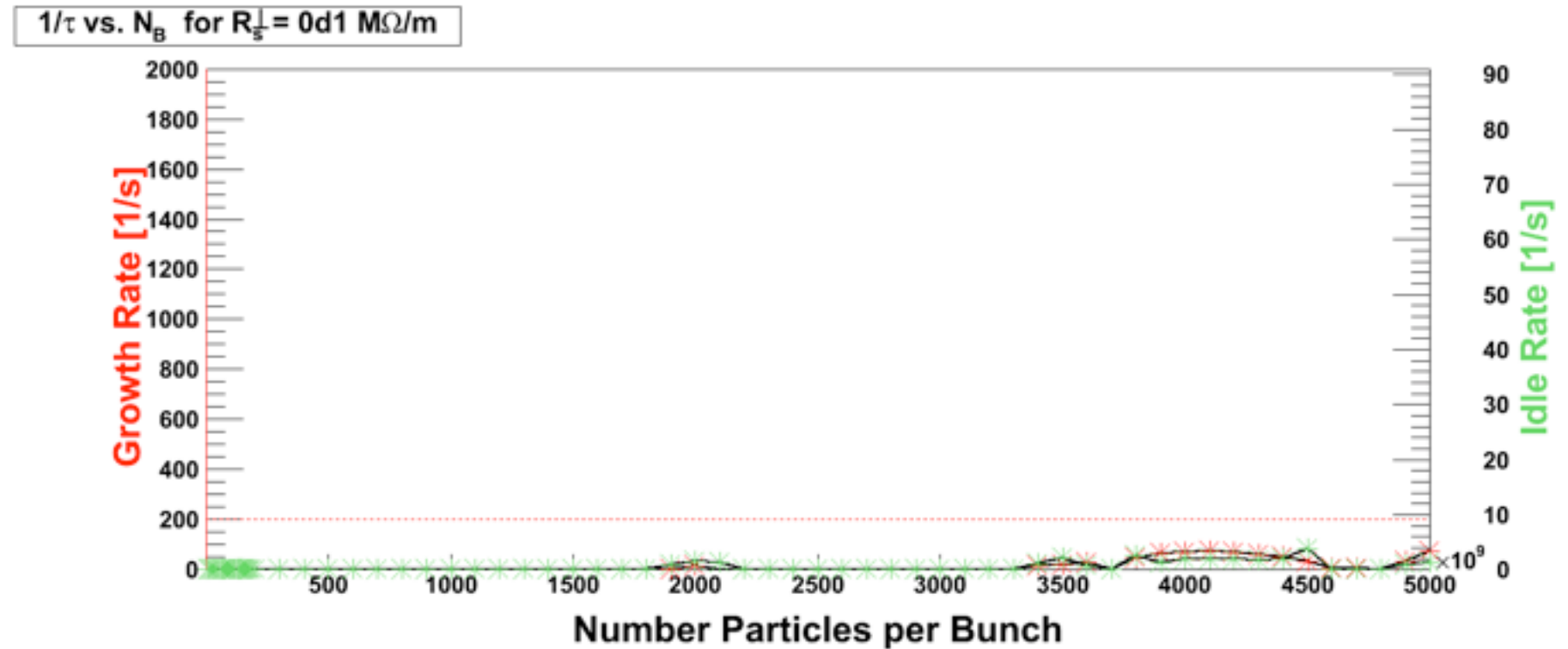
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N_b^{th} vs. R_{\perp} for ^{18}Ne in DR

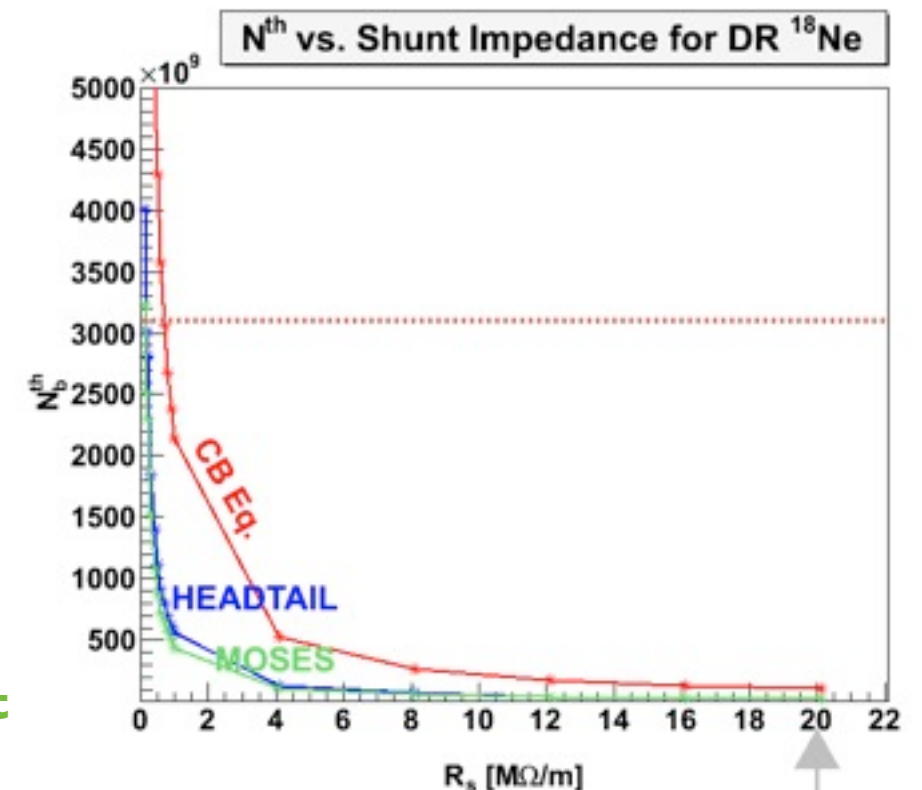
COLLECTIVE

- **$N_b > N_b^{\text{th}}$ when rise times < 5 ms** (growth rate > 200 Hz)

- **G.R. vs. N_b^{th} for different $R_{\perp} \rightarrow$**



- **R_{\perp} could be lowered to increase the intensity limit**
- **But a factor 100 smaller than $R_{\perp}^{\text{sps}} = 20 \text{ M}\Omega/\text{m}$ is very challenging**



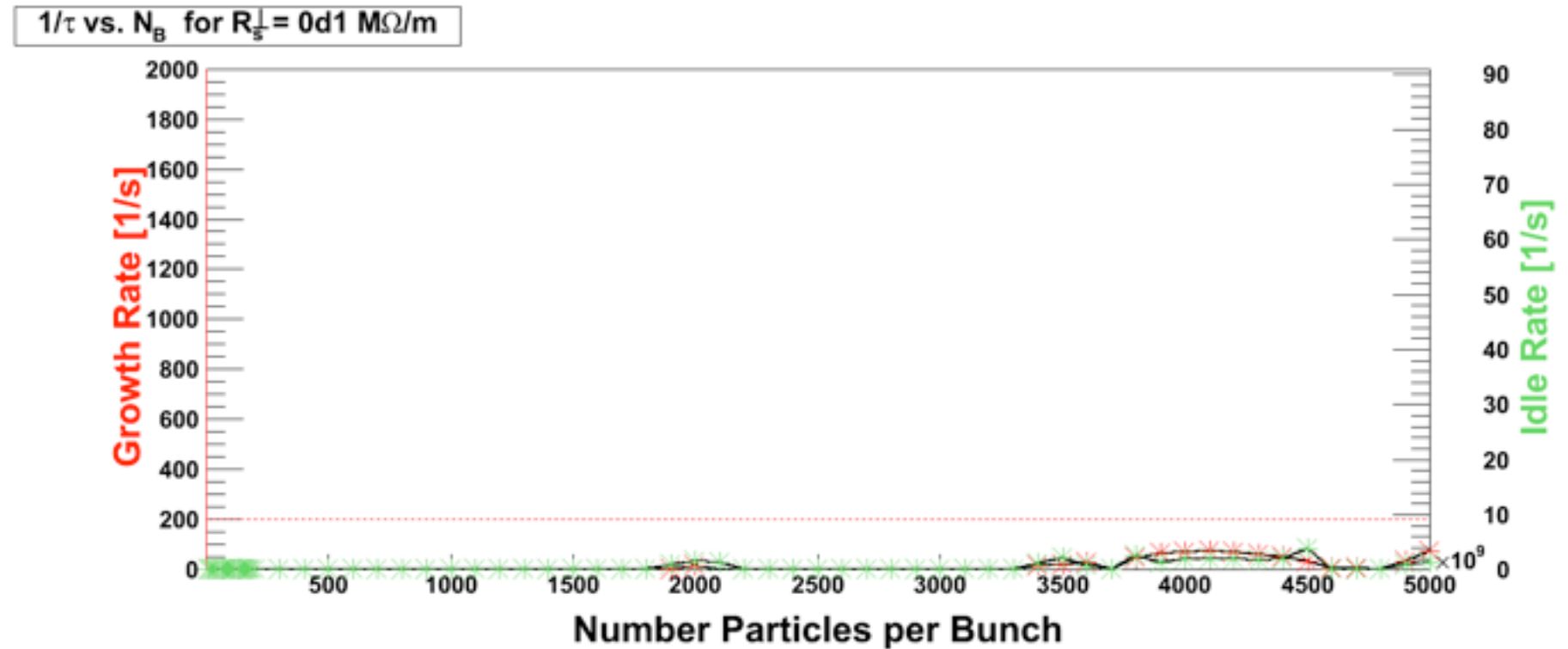
MOSES is developed for protons, so to get the ion equivalent the intensity limits, $N^{\text{th}}_{\text{moses}}$, was divided by a factor Z

N_b^{th} vs. R_{\perp} for ^{18}Ne in DR

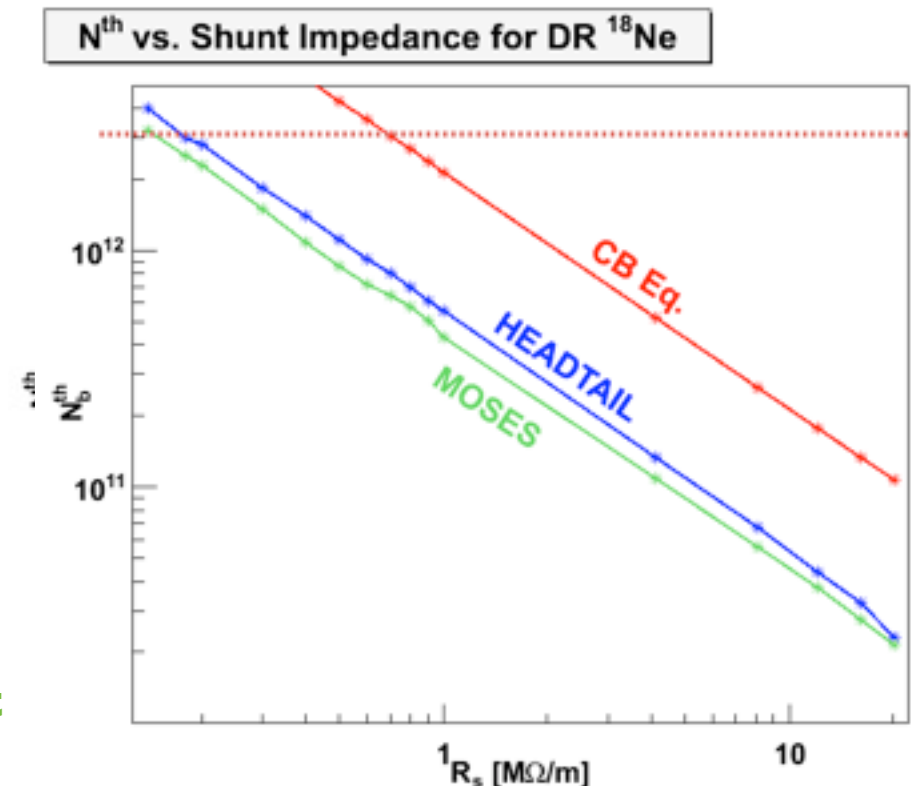
COLLECTIVE

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- **But a factor 100 smaller than $R_{\perp}^{\text{sps}} = 20 \text{ M}\Omega/\text{m}$ is very challenging**



MOSES is developed for protons, so to get the ion equivalent the intensity limits, $N_b^{\text{th}}_{\text{moses}}$, was divided by a factor Z

Growth Rate From Eq. of Motion

$$\ddot{y} + (Q_y \omega_{rev})^2 y = 0$$

$$\ddot{y} + (Q_y \omega_{rev})^2 y = Ky$$

$$y(t) = A_1 e^{i(Q_y + \Delta Q_y) \omega_{rev} t} + A_2 e^{-i(Q_y + \Delta Q_y) \omega_{rev} t}$$

$$\Delta Q_y = -K / (2Q_y \omega_{rev}^2)$$

$$\bar{y}(t) = A e^{i(Q_y + \Delta Q_y) \omega_{rev} t}$$

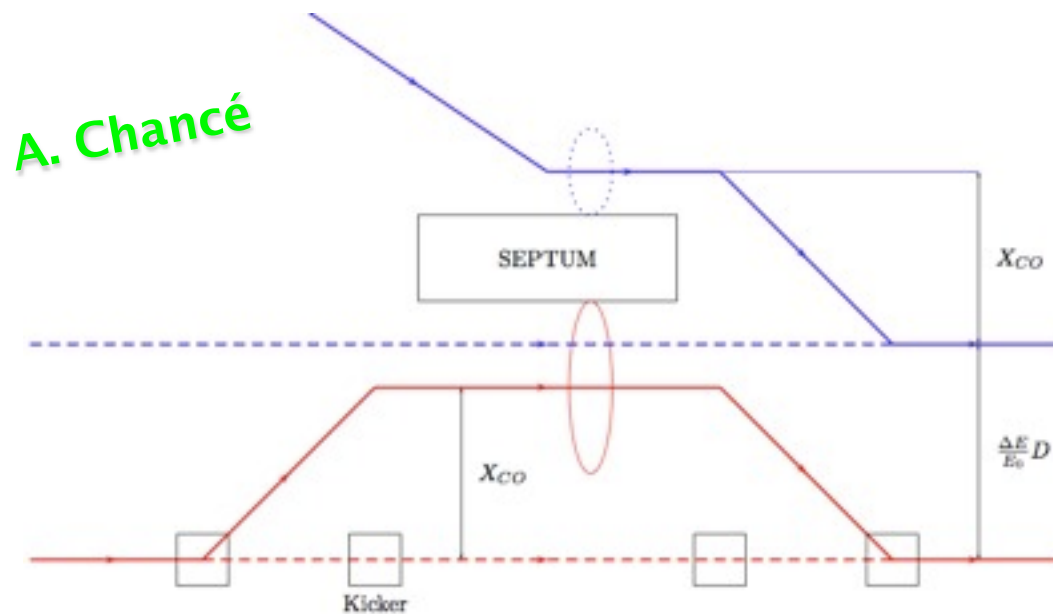
$$= A e^{i(Q_y + \Re[\Delta Q_y]) \omega_{rev} t} e^{-\Im[\Delta Q_y] \omega_{rev} t}$$

$$1/\tau \equiv -\Im[\Delta Q_y] \omega_{rev}$$

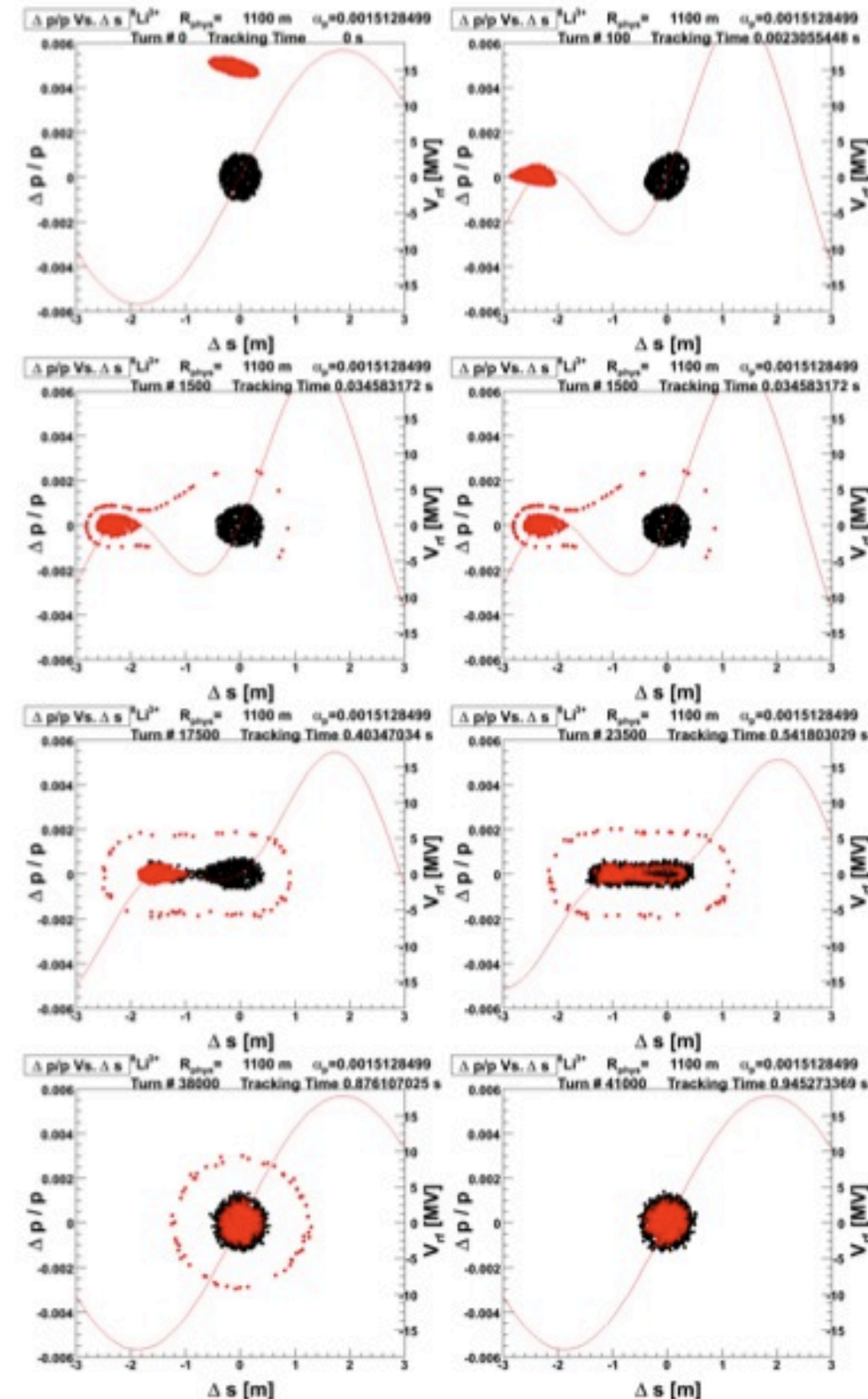
DR: Injection Scheme

Daniel C. Heinrich

- **The new bunch is injected off momentum**
(separated by a septum magnet)



- **After 1/4 synchrotron turn it is “captured” by one RF system**
- **Then “merged” into the old bunch with the use of a 2nd RF system**
- **Collimation at $\Delta p/p = 2.5\%$**
 - ➔ **scrapes away ions not captured**
 - ➔ **limits the bunch size to protect the septum magnet**



D R I N J E C T I O N

DR: RF Program

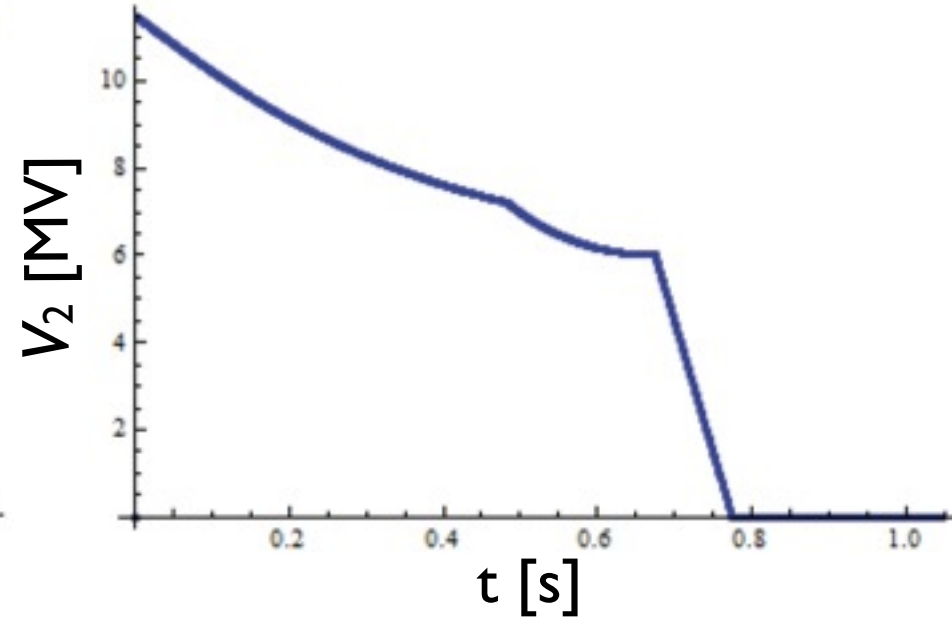
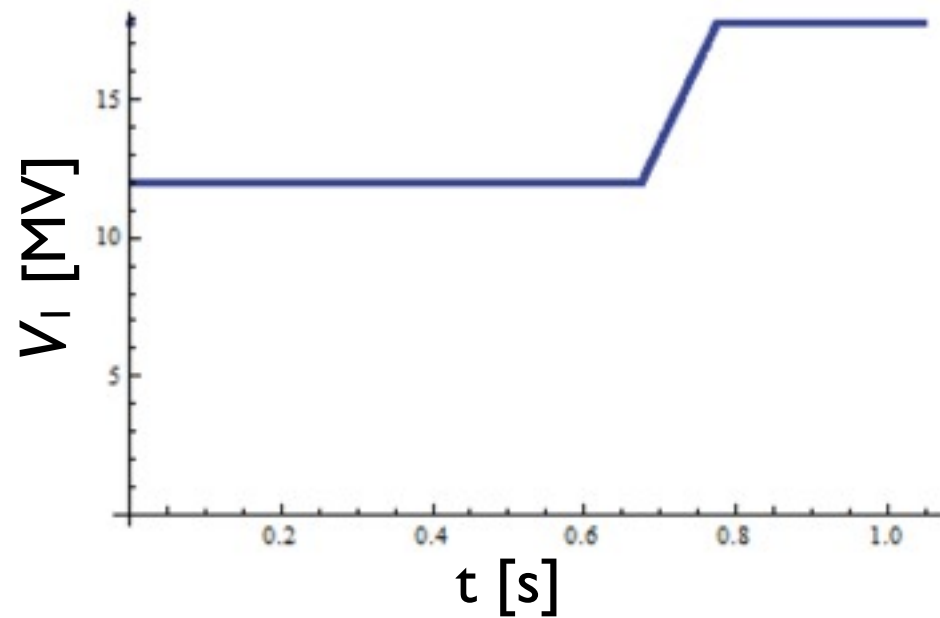
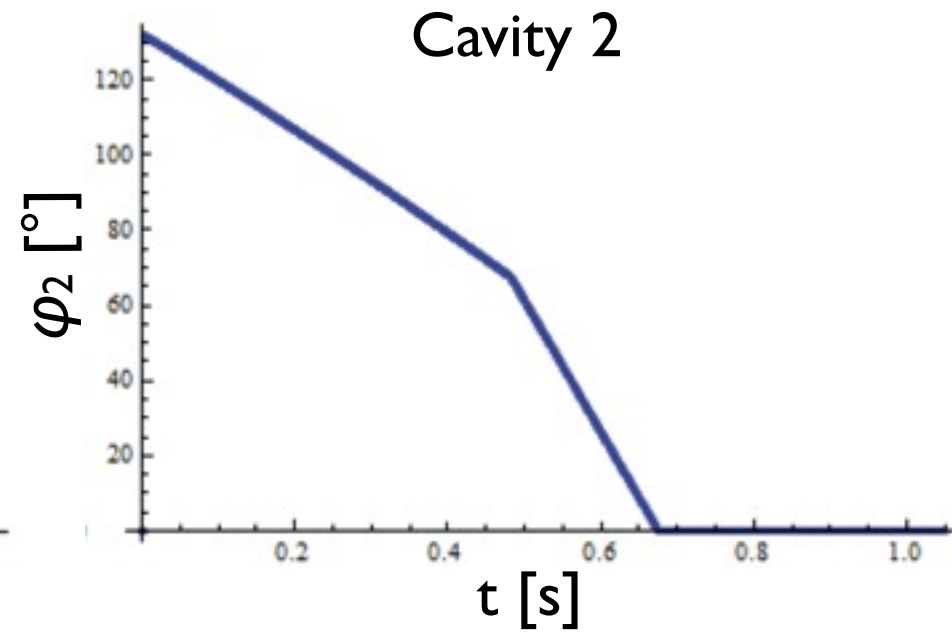
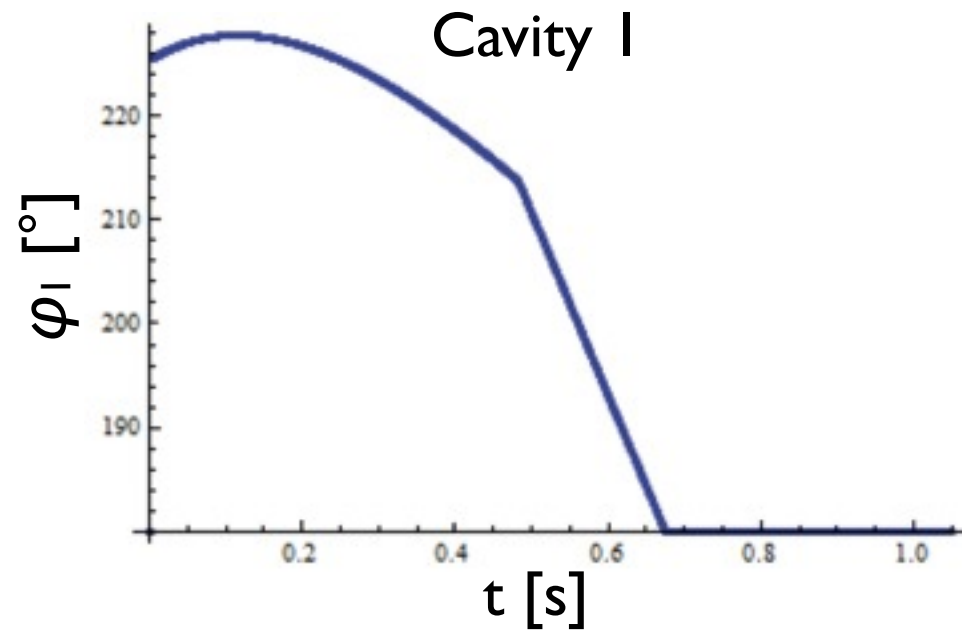
Daniel C.
Heinrich

A. Chancé

- **The**

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RF Program for ^8Li



DR: RF Program

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$1/\tau$ vs. N_B for $R_s = 6d1 \text{ M}\Omega/\text{m}$

