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# **Consequences of an RF Power Trip in LHC**

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## **Abstract**

The consequences of an isolated RF power trip are analysed. We determine the beam current for which the induced cavity voltage or extracted beam power is sufficiently large as to require an immediate beam-dump. The possibility to re-inject, accelerate and coast a new beam with an unavailable RF power transmitter is also analysed.

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# 1 Introduction

We want to examine what happens if an LHC klystron trips in coast and, if it cannot be restarted, what are the conditions necessary for injecting a new and probably weaker beam.

## 2 Calculation of Cavity Voltage and Beam-Extracted Power

We base our calculations on the well-proven formula (for a derivation see Appendix) for the necessary generator incident wave  $I_g$  (complex) in equilibrium – neglecting the beam gap transients – for a given cavity voltage  $V_{acc}$ , the synchronous phase angle<sup>1</sup>  $\phi$ , the average DC beam current  $I_{b,DC}$  and the normalized bunch form factor<sup>2</sup>  $f_b$ , the main coupler  $Q_{ext}$ , cavity frequency  $\omega_0$  and detuning  $\Delta\omega$

$$(1) \quad \tilde{I}_g = \left( \frac{V_{acc}}{2 (R/Q)} \left( \frac{1}{Q_{ext}} + \frac{1}{Q_o} \right) + I_{b,DC} f_b \sin(\phi) \right) + i \left[ I_{b,DC} f_b \cos(\phi) - \frac{V_{acc} \Delta\omega}{(R/Q) \omega_o} \right]$$

and the reflected wave

$$(2) \quad \tilde{I}_r = \left( \frac{V_{acc}}{2 (R/Q)} \left( \frac{1}{Q_{ext}} - \frac{1}{Q_o} \right) - I_{b,DC} f_b \sin(\phi) \right) - i \left[ I_{b,DC} f_b \cos(\phi) - \frac{V_{acc} \Delta\omega}{(R/Q) \omega_o} \right]$$

The corresponding power can be calculated as

$$(3) \quad P_x = \frac{1}{2} Q_{ext} (R/Q) \left( \text{Re}^2[\tilde{I}_x] + \text{Im}^2[\tilde{I}_x] \right) \quad x \rightarrow g \text{ or } r$$

For superconducting<sup>3</sup> cavities  $Q_o$  is so large compared to  $Q_{ext}$  that we can neglect its inverse in the above formulas.

### 2.1 RF Trip in Coast

When a power generator trips,  $I_g$  becomes immediately zero and the system changes its equilibrium state with the characteristic time scale of  $\tau = Q_{ext}/\omega_0$  (see later). Since both real and imaginary parts of  $I_g$  become zero, we can add the squares of both equations getting the condition

$$(4) \quad \left( \frac{V_{acc}}{2 (R/Q) Q_{ext}} \right)^2 + \left( \frac{V_{acc} \Delta\omega}{(R/Q) \omega_o} \right)^2 = (I_{b,DC} f_b)^2 (\sin^2(\phi) + \cos^2(\phi)) = (I_{b,DC} f_b)^2$$

hence for the induced voltage

$$(5a) \quad V_{ind} = \frac{2 I_{b,DC} f_b (R/Q) Q_{ext}}{\sqrt{1 + (2\Delta\omega Q_{ext}/\omega_o)^2}}$$

The imaginary part of  $I_g$  and  $I_r$  are – apart from the sign – identical, hence the imaginary part of  $I_r$  is also zero. The real part of  $I_r$  has the same two components as the real part of  $I_g$  for which both components cancel, hence the real part of  $I_r$  is twice this component and we obtain straightforwardly

$$(6) \quad \tilde{I}_{r,ind} = \frac{2 I_{b,DC} f_b}{\sqrt{1 + (2\Delta\omega Q_{ext}/\omega_o)^2}}$$

and the ‘reflected’ power – in this case the power stripped off the beam – becomes

$$(7a) \quad P_r = \frac{2 (R/Q) Q_{ext} (I_{b,DC} f_b)^2}{1 + (2\Delta\omega Q_{ext}/\omega_o)^2}$$

For a steady beam, cavity detuning is chosen such that the imaginary part of  $I_g$  becomes zero (see appendix (A19)), but for LHC with its large beam gaps we have to work with (about) half-detuning, i.e. we have in coast with  $\phi=180^\circ$  for the nominal cavity voltage  $V_{nom}$

$$(8) \quad \frac{\Delta\omega}{\omega_o} = - \frac{(R/Q) I_{b,DC} f_b}{2 V_{nom}}$$

<sup>1</sup> Convention for proton machines:  $\phi=0$  at the rising RF zero crossing

<sup>2</sup> Normalization such that for point bunches  $f_b=1$ , limiting condition  $-1 < f_b \leq 1$

<sup>3</sup> without quench, to be precise

Injecting this condition into (5a) and (7a) we obtain

$$(5b) \quad V_{ind} = \frac{2 I_{b,DC} f_b (R/Q) Q_{ext}}{\sqrt{1 + \left( \frac{(R/Q) Q_{ext} I_{b,DC} f_b}{V_{nom}} \right)^2}}$$

$$(7b) \quad P_r = \frac{2 (R/Q) Q_{ext} (I_{b,DC} f_b)^2}{1 + \left( \frac{(R/Q) Q_{ext} I_{b,DC} f_b}{V_{nom}} \right)^2}$$

Using the standard values,  $(R/Q)=45 \Omega$ ,  $Q_{ext}=100,000$  and  $V_{nom}=2$  MV, we get the following graphs

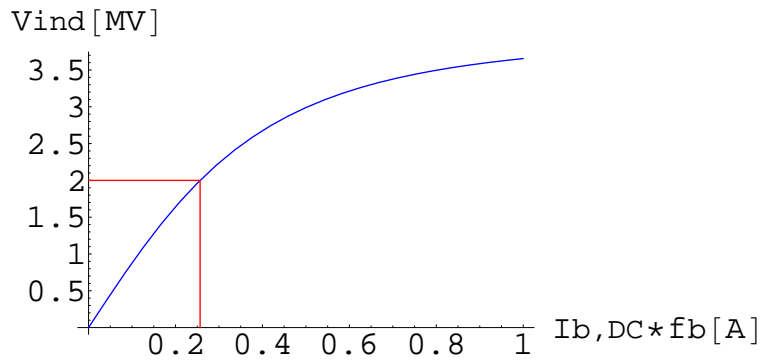


Fig. 1: Induced cavity voltage as function of the effective beam current, Asymptotic limit is 4 MV, 2 MV corresponding to  $I_{b,DC} f_b \approx 0.26$  A. (for LHC in coast at 400 MHz we have  $f_b \approx 0.9$ )

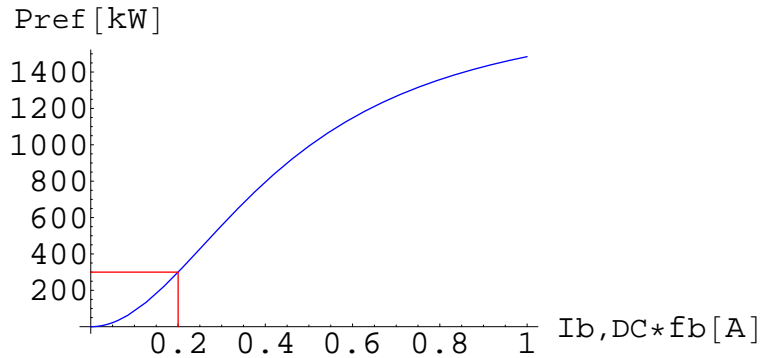


Fig. 2: Stripped off beam ('reflected') power as function of effective beam current. Asymptotic limit is 1780 kW, 300 kW corresponding to  $I_{b,DC} f_b \approx 0.20$  A. (for LHC in coast at 400 MHz we have  $f_b \approx 0.9$ )

We see that as the necessary half-detuning also increases with beam current, the induced voltage (and 'reflected' power) does not rise proportionally to (quadratic with) the beam current – as would otherwise be the case – but levels off. It can be seen easily that the asymptotic values will be

$$(8a) \quad V_{ind,\infty} = 2 V_{nom} = 4 \text{ MV and}$$

$$(8b) \quad P_{r,\infty} = 2 V_{nom}^2 / ((R/Q) Q_{ext}) \approx 1780 \text{ kW}$$

Normally the cavity voltage and beam current are in quadrature when  $\phi=180^\circ$  (as in coast in LHC) and there is no energy transfer. However, now the beam drives the cavity and the voltage builds up to produce maximum power transfer. To keep the ‘reflected’ power below 300 kW – to prevent overloading the circulator and load – the maximum effective current for the above conditions is about 0.20 A. To prevent the induced cavity voltage becoming larger than the nominal 2 MV, the maximum effective beam current is about 0.26 A, hence the power condition is more stringent. With  $f_b \approx 0.9$ , this means about 0.22 A, less than half the nominal current, can be accepted.

When the trip arises, the voltage builds up so quickly that the tuner has no time to move sufficiently to prevent this build-up, see illustration in Figs. 3 and 4. Thus the high power RF equipment would have to withstand this excess power during a significant fraction of a second. It is certainly risky to rely on the RF power hardware withstanding this without damage.

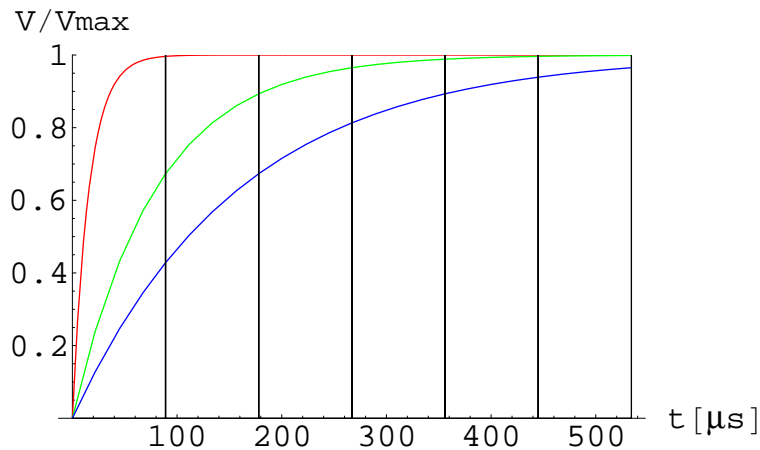


Fig. 3: Rise of the cavity field (0 - 500  $\mu$ s) for  $Q_{ext}=20,000$  (minimum, fastest, in red),  $Q_{ext}=100,000$  (for coast, in green) and  $Q_{ext}=200,000$  (maximum, slowest, in blue). (The vertical ‘ticks’ indicate one turn of about 89  $\mu$ s)

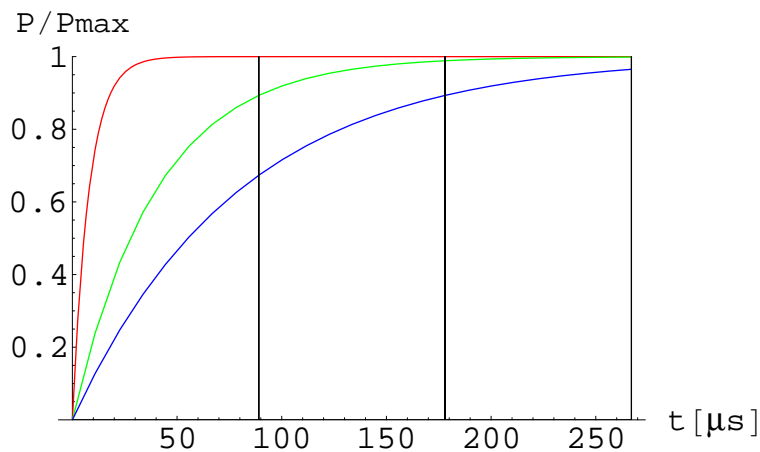


Fig. 4: Rise of the extracted beam power (0 - 250  $\mu$ s) for  $Q_{ext}=20,000$  (minimum, fastest, in red),  $Q_{ext}=100,000$  (for coast, in green) and  $Q_{ext}=200,000$  (maximum, slowest, in blue). (The vertical ‘ticks’ indicate one turn of about 89  $\mu$ s)

## 2.2 Re-injection of a New Beam

To reduce the induced voltage and stripped off power we can lower  $Q_{ext}$  of the coupler to its minimum (20,000) and detune the cavity to a maximum. We assume that within the tuning range 50 kHz freedom is still guaranteed at least in one direction, i.e. from (5a) and (7a) we conclude

$$(9a) \quad V_{ind} = 0.35 \text{ MV/A} \cdot (I_b f_b)$$

$$(9b) \quad P_r = 69 \text{ kW/A}^2 (I_b f_b)^2$$

For the *nominal* (average) DC current of 0.55 A this means with  $f_b=0.9$  that we expect 0.17 MV induced voltage and 17 kW stripped off power, which should be tolerable without problems for the hardware. Also the remaining 7 cavities should be able to supply 2.3 MV (6.3 MV/m) instead of 2 MV (5.5 MV/m) to replace the missing single cavity for the total voltage.

With the lowest  $Q_{ext}$ , the dead cavity produces an impedance of  $\approx 0.9 \text{ M}\Omega$  in the ring, limiting the stable beam current at injection to about nominal for a 0.7 eVs bunch [1].

### 3. Conclusion

If the coasting beam is above about half the nominal beam current, stripped off power risks destroying the RF power hardware; hence the beam should be dumped immediately. Even if the current is below this value, the beam will encounter a significant synchronous phase angle jump, e.g. 2 MV plus the induced decelerating voltage have suddenly to be supplied by the 7 remaining cavities, corresponding to a phase angle of more than  $8^\circ$ . The consequences of the corresponding beam perturbations are difficult to estimate.

For a new beam one might envisage detuning the cavity and lowering  $Q_{ext}$  of the coupler for the cavity with the dead klystron so that the induced voltage and reflected power become small. However, even with the minimum  $Q_{ext}$  of  $\approx 20,000$ , this cavity presents an impedance of  $0.9 \text{ M}\Omega$  in the ring.

A feedback system summing up all cavity voltages and acting with the available ones might improve this situation; however the technical difficulty is considerable, having in mind that for reliability reasons the present system is made of (twice) eight clearly separated individual cavity-klystron systems.

### References

[1] E. Shaposhnikova: "Longitudinal Beam Parameters during Acceleration in the LHC", CERN, LHC project note 242.

## Appendix: The Fundamental Beam-Cavity-Generator Relations

We assume steady state currents and voltage, hence transients are not considered. We base our calculations on the following lumped circuit model. The cavity is presented by the LCR-block, the coupler by the transmission line with impedance  $Z$  on which we have the incident (generator) wave current  $I_g$  and the reflected wave current  $I_r$ . The beam is presented as a current source. The cavity is excited to voltage  $V$  under these conditions.

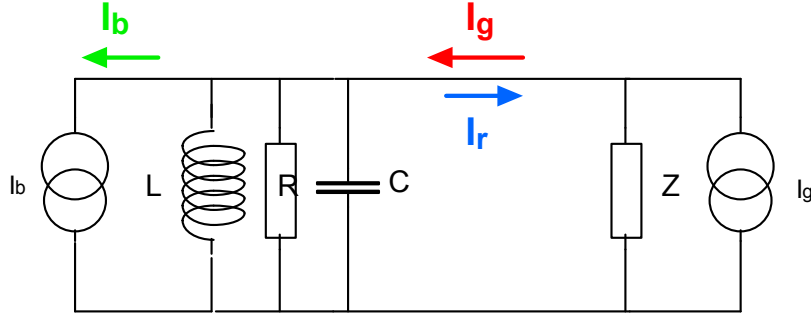


Fig. A1: the lumped circuit model, cavity modelled by LCR, coupler by transmission line of impedance  $Z$

The generator emits a wave with frequency  $\omega$  and implicitly we assume that *all* dynamic variables are proportional to  $\exp(i\omega t)$ . The cavity is tuned to  $\omega_0$ , not necessarily identical to  $\omega$ .

For any transmission line we have the relations

$$(A1) \quad V = Z (\vec{I} + \vec{I}) \Rightarrow V = Z (I_g + I_r)$$

which is equivalent to

$$(A2) \quad I_r = \frac{V}{Z} - I_g$$

With the RF beam current  $I_{b,RF}$  the current  $I_{LCR}$  through the LCR-block is

$$(A3a) \quad I_{LCR} = I_g - I_r - I_{b,RF}$$

or, injecting (A2)

$$(A3b) \quad I_{LCR} = 2I_g - I_{b,RF} - \frac{V}{Z}$$

On the other hand we can express the same current  $I_{LCR}$  as the sum of the currents through its parallel elements, all proportional to the excitation voltage  $V$

$$(A4) \quad I_{LCR} = I_L + I_C + I_R = V \left( \frac{1}{i\omega L} + i\omega C + \frac{1}{R} \right)$$

Combining (A4) and (A3b) we get

$$(A5) \quad V \left( i\omega C \left( 1 - \frac{1}{\omega^2 LC} \right) + \left( \frac{1}{R} + \frac{1}{Z} \right) \right) = 2I_g - I_{b,RF}$$

We have the general relation  $LC = 1/\omega_0^2$  for the resonance frequency<sup>4</sup> hence

$$(A6) \quad V \left( i C \frac{\omega^2 - \omega_0^2}{\omega} + \left( \frac{1}{R} + \frac{1}{Z} \right) \right) = 2I_g - I_{b,RF}$$

We can assume that the detuning of the cavity,  $\Delta\omega = \omega_0 - \omega$ , is very small compared to  $\omega$ . Then we can approximate

$$(A7) \quad \omega^2 - \omega_0^2 = \omega^2 - (\omega + \Delta\omega)^2 \approx -2 \omega \Delta\omega$$

and (A6) becomes

<sup>4</sup> R only changes this relation significantly for extremely small  $Q_0$ , not the case in our context

$$(A8) \quad V \left( -2i \Delta\omega C + \left( \frac{1}{R} + \frac{1}{Z} \right) \right) = 2I_g - I_{b,RF}$$

Now we have to express the lumped circuit quantities C, R and Z by cavity quantities, L being already expressed by C with  $LC = 1/\omega_0^2$ .

Carrying a charge q from one plate of a capacitor to the other one, results in the voltage change

$$(A9) \quad \Delta V = \frac{q}{C}$$

while a charge q travelling through a cavity with (R/Q) leaves a voltage

$$(A10) \quad \Delta V = q \omega (R/Q)$$

This induces the equivalence

$$(A11) \quad C = \frac{1}{\omega (R/Q)} \Rightarrow L = \frac{(R/Q)}{\omega}$$

For any resonator the relation  $Q_0 = \omega RC$  holds, hence injecting (A11) yields

$$(A12) \quad R = Q_0 (R/Q)$$

The transmission line, modelling the power coupler, presents an energy leak corresponding to a resistor Z in parallel to the resistor R. In fact the coupler's  $Q_{ext}$  is just defined as the Q-value of a cavity of infinite  $Q_0$  fed by this coupler, hence we have

$$(A13) \quad Z = Q_{ext} (R/Q)$$

Filling in these cavity quantities in the lumped circuit equation (A8) we obtain

$$(A14) \quad V \left( -i \frac{\Delta\omega}{\omega (R/Q)} + \frac{1}{2(R/Q)} \left( \frac{1}{Q_0} + \frac{1}{Q_{ext}} \right) \right) = I_g - \frac{1}{2} I_{b,RF}$$

$I_{b,RF}$  is complex and can be expressed as sum of real and imaginary part. We agree to define the complex phase of all waves such that  $V$  is *purely real*. The synchronous phase angle is the angle of the RF voltage when the *beam* arrives. In electron machines – where one works generally close to maximum energy transfer – the phase angle  $\phi_{elec}$  is called zero if beam and voltage are in phase. Therefore for  $V$  proportional to  $\exp(i\omega t)$ , the beam RF current has to be proportional to  $\exp(i\omega t - i\phi_{elec})$ .

For proton machines the phase angle  $\phi$  is called zero at the rising zero crossing of the RF voltage, i.e. we have  $\phi = \phi_{elec} - 90^\circ$ . Using the latter convention for the further calculations we get

$$(A15a) \quad I_{b,RF} = |I_{b,RF}| \left( \cos(\phi_{elec}) - i \cdot \sin(\phi_{elec}) \right) = |I_{b,RF}| \left( \sin(\phi) + i \cdot \cos(\phi) \right)$$

The complex Fourier spectrum of a repetitive beam passage for infinitely short bunches has equal line height (the DC current) for all frequencies from  $-\infty$  to  $+\infty$ . For the corresponding real spectrum (having no negative lines) equivalent positive and negative lines of the complex spectrum exactly add up; hence they are twice the DC current. But there is one unique line, the zero-frequency line, with the unchanged magnitude. Hence for point bunches any such line of  $I_{b,RF}$  corresponds to  $2 I_{b,DC}$ . For bunches of finite length this factor 2 will become lower for higher frequency-lines<sup>5</sup> and to take this effect into account, we add a *relative* bunch form factor  $f_b$  that is *normalized to 1 for infinitely short bunches*. Then we obtain for the general case

$$(A15b) \quad I_{b,RF} = 2 I_{b,DC} f_b \left( \sin(\phi) + i \cos(\phi) \right)$$

Combining (A14) and (A15b) yields the real and imaginary part of the incident wave  $I_g$  creating together with the beam current the voltage  $V$

$$(A16) \quad I_g = \left( \frac{V}{2(R/Q)} \left( \frac{1}{Q_0} + \frac{1}{Q_{ext}} \right) + I_{b,DC} f_b \sin(\phi) \right) + i \cdot \left( I_{b,DC} f_b \cos(\phi) - \frac{V \Delta\omega}{\omega (R/Q)} \right)$$

Using (A2) and (A13) we also get the reflected current

$$(A17) \quad I_r = \left( \frac{V}{2(R/Q)} \left( \frac{1}{Q_{ext}} - \frac{1}{Q_0} \right) - I_{b,DC} f_b \sin(\phi) \right) - i \cdot \left( I_{b,DC} f_b \cos(\phi) - \frac{V \Delta\omega}{\omega (R/Q)} \right)$$

Evidently, we have the usual power-current relation for alternating current ( $I$  and  $V$  are *peak* values)

$$(A18) \quad P_x = \frac{1}{2} R_x |I_x|^2 \Rightarrow P_{g,r} = \frac{1}{2} Z |I_{g,r}|^2 = \frac{1}{2} (R/Q) Q_{ext} |I_{g,r}|^2$$

<sup>5</sup> and may even invert its sign but stays always absolutely below 2.

We also see that the steady state optimum detuning, which minimizes the (absolute) reflected current, is obtained for imaginary zero component of  $I_r$ , hence

$$(A19) \quad \frac{\Delta\omega_{opt}}{\omega} = \frac{I_{b,DC} f_b \cos(\phi) (R/Q)}{V}$$

Above transition energy coast takes place at  $\phi = 180^\circ$ , hence  $\Delta\omega$  will be negative in LHC.

Furthermore we can (if technically foreseen) adapt  $Q_{ext}$  so that for a given beam current the real part of the reflected current also disappears with the condition

$$(A20a) \quad \frac{1}{Q_{ext,opt}} = 2 \frac{(R/Q) I_{b,DC} f_b \sin(\phi)}{V} - \frac{1}{Q_0}$$

and for a *superconducting* cavity with  $Q_0 \gg Q_{ext}$  we simply obtain

$$(A20b) \quad Q_{ext,opt} = \frac{V}{2 (R/Q) I_{b,DC} f_b \sin(\phi)}$$

For completeness we also write the basic formula for electron machine convention of  $\phi_{elec}$

**Electron machine convention:**  $\phi_{elec} = 0$  when beam arrives at the maximum of V

$$(A21a) \quad I_g = \left( \frac{V}{2(R/Q)} \left( \frac{1}{Q_0} + \frac{1}{Q_{ext}} \right) + I_{b,DC} f_b \cos(\phi_{elec}) \right) - i \cdot \left( I_{b,DC} f_b \sin(\phi_{elec}) + \frac{V \Delta\omega}{\omega (R/Q)} \right)$$

$$(A21b) \quad I_r = \left( \frac{V}{2(R/Q)} \left( \frac{1}{Q_{ext}} - \frac{1}{Q_0} \right) - I_{b,DC} f_b \cos(\phi_{elec}) \right) + i \cdot \left( I_{b,DC} f_b \sin(\phi_{elec}) + \frac{V \Delta\omega}{\omega (R/Q)} \right)$$

$$(A22a) \quad \frac{\Delta\omega_{opt}}{\omega} = \frac{I_{b,DC} f_b \sin(\phi_{elec}) (R/Q)}{V}$$

$$(A22b) \quad Q_{ext,opt} = \frac{V}{2(R/Q) I_{b,DC} f_b \cos(\phi_{elec})}$$