(SOME) EFFECTS NEAR TRANSITION

E. Métral (CERN)

- Transition energy
- Longitudinal beam dynamics “far” below or above transition
- Transition crossing (with the example of the CERN PS machine)
- Transverse (slow) head-tail instability
- Fast (vertical) single-bunch instability
  - Crossing transition in the CERN PS
  - Injecting just above transition in the CERN SPS
- Conclusion
TRANSITION ENERGY (1/3)

- Momentum compaction factor

\[ \alpha_p = \frac{\text{d}C}{\text{d}p} \cdot \frac{C}{p} = \frac{1}{C} \int_{0}^{C} \frac{D_x(s)}{\rho(s)} \text{d}s \]

- In most circular machines, \( \alpha_p > 0 \)

- However, \( \alpha_p < 0 \) is also possible (e.g. CERN LEAR machine) => Called “Negative Momentum Compaction” (NMC) or “Imaginary \( \gamma_t \)” lattice

\[ \gamma_t = \frac{1}{\sqrt{\alpha_p}} \]
TRANSITION ENERGY (2/3)

◆ Assume $\alpha_p > 0$

- $dp > 0 \Rightarrow dC > 0$  
  $\Rightarrow$ What happens to the revolution frequency $f_{rev} = v/C$?
- $dp > 0 \Rightarrow dv > 0$

- At (very) high energy, $v \approx c_{\text{light}}$ and remains constant $\Rightarrow f_{rev}$
- At low energy, $v$ increases faster than $C$ $\Rightarrow f_{rev}$

There is an energy for which the velocity variation is compensated by the trajectory variation (i.e. $df_{rev} = 0$):

$\Rightarrow$ TRANSITION ENERGY
TRANSITION ENERGY (3/3)

- **Slip factor**

\[ \eta = - \frac{df_{rev}}{dp} \frac{f_{rev}}{p} = \alpha_p - \frac{1}{\gamma^2} = 1 - \frac{1}{\gamma_t^2 - \gamma^2} \]

- \( \eta < 0 \) below transition
- \( \eta = 0 \) at transition => Isochronous condition
- \( \eta > 0 \) above transition

Relativistic mass factor of the beam
LONGITUDINAL BEAM DYNAMICS
“FAR” BELOW OR ABOVE TRANSITION (1/2)

- (Bucket) separatrices: Below transition
- Above transition

\[
\phi_s = \begin{cases} 
0^\circ & \phi_s = 30^\circ \\
60^\circ & \phi_s = 85^\circ \\
180^\circ & \phi_s = 150^\circ \\
120^\circ & \phi_s = 95^\circ 
\end{cases}
\]

\[\Omega_s = \Omega_{rev} \left( -\frac{e\hat{V}_{RF} h \eta \cos \phi_s}{2\pi \beta^2 E_{total}} \right)^{1/2}\]

\[\phi_s \Rightarrow \pi - \phi_s\]
**LONGITUDINAL BEAM DYNAMICS**

“FAR” BELOW OR ABOVE TRANSITION (2/2)

- **Particle trajectories**: Below transition

\[ \phi_s = 0^\circ \]

\[ \phi_s = 30^\circ \]
TRANSITION CROSSING (1/9)

◆ “Far” below or above transition \(\Leftrightarrow\) Adiabaticity condition \(\frac{1}{\Omega_s^2} \left| \frac{d\Omega_s}{dt} \right| \ll 1\)

◆ “Close” to transition, the adiabaticity condition is not satisfied \(\Rightarrow\) Non-adiabatic synchrotron motion

- When the time is close enough to transition, the particle will not be able to catch up with the rapid modification of the bucket shape

- Nonadiabatic time

\[
T_c = \left( \frac{\beta^2 E_{\text{rest}} \gamma_t^4}{4 \pi f_{\text{rev}} T h \hat{V}_{RF} |\cos \phi_s|} \right)^{1/3}
\]

\(\sim 2\) ms for the nTOF bunch in the CERN PS
TRANSITION CROSSING (2/9)

- nTOF bunch in the CERN PS

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Average machine radius: $R$ [m]</td>
<td>100</td>
</tr>
<tr>
<td>Bending dipole radius: $\rho$ [m]</td>
<td>70</td>
</tr>
<tr>
<td>$\dot{B}$ [T/s]</td>
<td>2.2</td>
</tr>
<tr>
<td>$\hat{V}_{RF}$ [kV]</td>
<td>200</td>
</tr>
<tr>
<td>$h$</td>
<td>8</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>0.027</td>
</tr>
<tr>
<td>Longitudinal (total) emittance: $\varepsilon_L$ [eVs]</td>
<td>2</td>
</tr>
<tr>
<td>Number of protons/bunch: $N_b$ [1E10 p/b]</td>
<td>800</td>
</tr>
<tr>
<td>Norm. rms. transverse emittance: $\varepsilon_{x,y}^*$ [$\mu m$]</td>
<td>5</td>
</tr>
<tr>
<td>Trans. average betatron function: $\beta_{x,y}$ [m]</td>
<td>16</td>
</tr>
<tr>
<td>Beam pipe [cm $\times$ cm]</td>
<td>3.5 $\times$ 7</td>
</tr>
<tr>
<td>Trans. tunes: $Q_{x,y}$</td>
<td>6.25</td>
</tr>
</tbody>
</table>

=> $\gamma_t \approx 6.1$
TRANSITION CROSSING (4/9)

- **Longitudinal mismatch** (due to the longitudinal Space Charge): IN STATIC

![Diagram showing the transition crossing with different conditions: Adiabatic without SC, Nonadiabatic without SC, Nonadiabatic with SC below transition, and Nonadiabatic with SC above transition.]
TRANSITION CROSSING (5/9)

- **Longitudinal mismatch** (due to the longitudinal Space Charge): IN DYNAMIC (i.e. crossing transition)
TRANSITION CROSSING (6/9)

- Longitudinal mismatch (due to the inductive part of the longitudinal Broad-Band impedance): IN DYNAMIC (i.e. crossing transition)

\[ Z_{lBB} / n = j 20 \ \Omega \]
TRANSITION CROSSING (7/9)

- Remedies
  - Avoid crossing transition in the design phase $\Rightarrow \alpha_p < 0$
  - If transition crossing cannot be avoided, the "$\gamma_t$ jump" is the only (known) method to overcome all the intensity limitations $\Rightarrow$ Artificial increase of the transition crossing speed by means of fast pulsed quadrupoles (at non zero dispersion locations)

\[
\dot{\gamma} = 49.9 \text{ s}^{-1}
\]

Effective crossing speed ~ 50 times faster with the $\gamma_t$ jump
TRANSITION CROSSING (8/9)

- Asymmetric or symmetric $\gamma_t$ jump?

![Graph showing the transition crossing with nonadiabatic and nonadiabatic with SC cases]
TRANSITION CROSSING (9/9)

With $\gamma_j$ jump

Without $\gamma_j$ jump

-0.02
-0.01
0
0.01

Time [ms]

-0.0075
-0.005
-0.0025
0
0.0025
0.005
0.0075
0.01

Time [ms]
TRANSVERSE (SLOW) HEAD-TAIL INSTABILITY (1/2)

- If the **sign of the chromaticity** (which is equal to ~ -1 for an uncorrected machine like the PS) is not changed (in both transverse planes) above transition, a (single-bunch) head-tail instability may develop.

  - This instability can be damped through Landau damping using octupoles, which introduce some amplitude detunings. This method was first used in the past to stabilize the PS beams.

  - However, the better method of changing the sign of the chromaticities (and keeping them to small positive values) by acting on the optics with sextupoles was then adopted, and it has become a routine operation at the CERN PS for many years.

  => **The chromatic frequency should be (slightly) positive to avoid the head-tail mode 0 (most critical) from developing**

\[
f_{\xi_y} = Q_y f_{rev} \xi_y \eta
\]
TRANSVERSE (SLOW) HEAD-TAIL INSTABILITY (2/2)

- Example of transverse (slow) head-tail instability observed in the CERN PS at injection (below transition)
  - Consecutive traces at a pick-up superimposed
  - Standing-wave patterns with 4 nodes (called “mode 4”)

**Measurements**

**Analytical prediction**

(for a bunch going through the centre of the pick-up)
A simple formula is obtained from 5 seemingly diverse formalisms (in the absence of space charge and transverse feedback), assuming i) a Broad-Band impedance and ii) the long-bunch regime:

- Coasting-Beam approach with peak values
- Fast Blow-Up
- Beam Break-Up
- Post Head-Tail
- Transverse Mode-Coupling Instability (TMCI)
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (2/14)

- Example of TMCI

![Diagram](Image)

Bunch coherent motion $\propto e^{\frac{t}{\tau_{instab}}}$

- Synchrotron period
- Instability rise-time

*Courtesy of Benoit Salvant*

$Q_s \rightarrow 0$ approaching transition
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (3/14)

- i) 1\textsuperscript{st} assumption: Broad-Band impedance
ii) 2\textsuperscript{nd} assumption: long-bunch regime

\[ N_{b,th}^y \]

- Short-bunch regime
- Long-bunch regime

From TMCI theory

\[ 2 f_r \tau_b \]
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (5/14)

- Simple formula (with the 2 assumptions):

\[
N_{b,th}^y \propto \frac{f_r}{|Z_y|} |\eta| Q_y \varepsilon_L \left(1 + \frac{f_{\xi_y}}{f_r}\right)
\]

- Increase the chromatic frequency
- Chromaticity jump in case transition has to be crossed

- Try to decrease the impedance and/or increase the resonance frequency => Impedance reduction campaign

- Increase the beam longitudinal emittance (when possible)

- Change the optics to increase the betatron tune (decrease the beta function at critical impedances) and/or go further away from transition => New optics needed

* No dependence on $Q_s$!
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (6/14)

- In the PS: even in the presence of the $\gamma_t$ jump, together with the change of the sign of both chromaticities when transition is crossed, a fast vertical single-bunch instability is observed (with the nTOF bunch) when no longitudinal emittance blow-up is applied before transition.

\[ \Sigma, \Delta R, \Delta V \text{ signals} \]

- Head stable
- Tail unstable

$\sim 700$ MHz

Time (10 ns/div)

$\Rightarrow$ Instability suppressed by increasing the longitudinal emittance
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (7/14)

=> Similar observation on other beams (e.g. below with the beam for the Antiproton Decelerator) when the longitudinal emittance is too small
In the SPS

Synchrotron period ≈ 7 ms

Instability (initially) suppressed by increasing the chromaticity

\[ \xi_y = 0 \]

\[ \xi_y = 0.8 \]
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (9/14)

⇒ Travelling-wave pattern along the bunch

$\xi_y = 0.14$

$\langle y \rangle$ [a.u.]

Head

Tail

$1^{st}$ trace (in red) = turn 2

Last trace = turn 150

Every turn shown
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (10/14)

\[ \xi_y = 2.04 \]

First trace (in red) = turn 2
Last trace = turn 150
Every turn shown
◆ $\gamma_t$ was recently modified in the SPS to increase the TMCI intensity threshold above the foreseen intensities for the future upgrade.

◆ Simple rough estimate of $\gamma_t$ for machines made of simple FODO cells:

  - Approximating the machine radius by the bending radius, yields

    $$D_x \approx \frac{\rho}{Q_x^2}$$

  - Inserting this in the definition of $\alpha_p$ (and then expressing $\gamma_t$) yields

    $$\gamma_t \approx Q_x$$

=> If one wants to modify $\gamma_t$ (increase or decrease its value), one should modify the horizontal tune.
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (12/14)

- TMCI intensity threshold with the old (Q26) optics at injection: $\approx 1.7 \times 10^{11}$ p/b

- Predictions going from Q26 to the new (Q20) optics:
  
  - **Q26:** $|\eta| Q_y = 0.62 \times 10^{-3} \times 26.13 \approx 0.0162 \quad \gamma_t = 22.8$
  
  - **Q20:** $|\eta| Q_y = 1.80 \times 10^{-3} \times 20.13 \approx 0.0362 \quad \gamma_t = 18$

=> A gain of a factor $0.0362 / 0.0162 \approx 2.2$ in the intensity threshold was expected.
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (13/14)

- Measurements

=> Good agreement with simple formula

Gain of a factor $4.5 / 1.7 \approx 2.6$

Q26 optics: $\gamma_t = 22.8$

Q20 optics: $\gamma_t = 18$

*Courtesy of Benoit Salvant et al.*

*Courtesy of Hannes Bartosik et al.*
FAST (VERTICAL) SINGLE-BUNCH INSTABILITY (14/14)

- Very good agreement between measurements and simulations

![Graph showing measurements and simulations]

**N.B.**

<table>
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<tr>
<th>Measurements</th>
<th>Headtail simulations</th>
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<tbody>
<tr>
<td>4.5x10^{11} p/b @ 0.35 eVs</td>
<td>Island of slow instability</td>
</tr>
</tbody>
</table>

**Courtesy of Hannes Bartosik et al.**

=> Intensity threshold with the new (Q20) optics: \( \sim 4.5 \times 10^{11} \) p/b
CONCLUSION (1/2)

◆ Stability of synchrotron motion (when crossing transition): $\phi_s \Rightarrow \pi - \phi_s$

◆ Head-Tail instability (when crossing transition): change the sign of the chromaticity (both planes) => Positive chromatic frequency

$$\frac{\xi_y}{\eta} f_{\xi_y} = Q_y f_{rev} \frac{\xi_y}{\eta} > 0$$

◆ Transverse Mode-Coupling Instability => With the 2 assumptions: i) Broad-Band impedance and ii) long-bunch regime (effects of space charge and transverse feedback still under discussion…)

- Impedance reduction
- Increase longitudinal emittance (as in the PS)
- Increase the |slip factor| (as in the SPS) and/or the tune
- Increase the chromatic frequency (below or above transition)
- “Chromaticity jump” when transition needs to be crossed => Not only the sign needs to be changed (for Head-Tail reason) but the shape could be optimised (for TMCI reason)
CONCLUSION (2/2)

◆ Increasing the |slip factor| also helps for i) the Longitudinal Mode-Coupling Instability and ii) the fast single-bunch electron cloud instability

◆ Attractive operation of synchrotrons under an isochronous or quasi-isochronous condition to (naturally) achieve very short bunches

=> Requires

- An accurate control of the first high-order component of the momentum compaction factor to provide the necessary momentum acceptance

\[
C(\delta) = C_0 \left[ 1 + \alpha_0 \delta \left( 1 + \alpha_1 \delta + \alpha_2 \delta^2 + \ldots \right) \right] \quad \delta = \Delta p / p
\]

- Effective ways to damp all the collective instabilities
REFERENCES

◆ E. Métral and D. Möhl, Transition Crossing, Volume I of "Fifty years of the CERN Proton Synchrotron", CERN–2011–004, June 2011, p. 59
  and all the references therein

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  Supervisor: Simone Gilardoni

◆ Detailed studies for the CERN SPS => Hannes Bartosik, Beam Dynamics and Optics Studies for the LHC Injectors Upgrade, CERN-THESIS-2013-25
  Supervisor: Yannis Papaphilippou